



Banff International Research Station

for Mathematical Innovation and Discovery

Applications of Iwasawa Algebras

March 3 - 8, 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

The first talk each day has a kind of survey character. The five evening talks on Tuesday and Thursday are 20 minute presentations. All other lectures are 60 minutes.

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30 Buffet Dinner, Sally Borden Building
20:00 Informal gathering in 2nd floor lounge, Corbett Hall
Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45 Breakfast
8:45–9:00 Introduction and Welcome by BIRS Station Manager
9:00–10:00 Pottharst: *Review of (ϕ, Γ) -modules*
10:00–10:30 Coffee Break
10:30–11:30 Loeffler: *ϵ -isomorphisms for crystalline p -adic Galois representations*
11:30–13:00 Lunch
13:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
13:50 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:00–15:00 Nakamura: *A generalization of Kato's local epsilon conjecture for (ϕ, Γ) -modules over the Robba ring*
15:00–15:30 Coffee Break
15:30–16:30 Berger: *Multivariable Lubin-Tate (ϕ, Γ) -modules*
17:30–19:30 Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Greenberg: <i>Some Questions Concerning Iwasawa Algebras and Their Modules</i>
10:00–10:30	Coffee Break
10:30–11:30	Zerbes: <i>Euler systems for Rankin-Selberg convolutions of modular forms</i>
11:30–13:30	Lunch
14:00–15:00	Popescu: <i>Explicit ℓ-adic models of Tate sequences and applications</i>
15:00–15:30	Coffee Break
15:30–16:30	Nickel: <i>Equivariant Iwasawa theory and non-abelian Stark-type conjectures</i>
17:30–19:30	Dinner
20:00	Riedel: <i>On Perrin-Riou's exponential maps and reciprocity laws</i>
20:30	Schmitt: <i>On the interpolation part of a local main conjecture with a view towards a global twist statement</i>
21:00	Shekhar: <i>Congruence formula for certain dihedral twists</i>

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Kakde: <i>K-theory for Iwasawa algebras and related rings</i>
10:00–10:15	Coffee Break
10:15–11:15	Witte: <i>Iwasawa algebras and localisation sequences in higher algebraic K-theory</i>
11:30–12:30	Sharifi: <i>Iwasawa algebras and Duality</i>
12:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Zabradi: <i>Applications of Iwasawa algebras to representation theory</i>
10:00–10:30	Coffee Break
10:30–11:30	Kohlhaase: <i>Modular Admissibility</i>
11:30–13:30	Lunch
14:00–15:00	Ardakov: <i>Duflo's theorem and two-sided ideals in Iwasawa algebras</i>
15:00–15:30	Coffee Break
15:30–16:30	Schmidt: <i>Localisation for p-adic analytic representations</i>
17:30–19:30	Dinner
20:00	Jha: <i>Fine Selmer group of Hida deformations</i>
20:30	Sprung: <i>Modesty in the size of the Tate-Shafarevich group</i>

Friday

7:00–9:00	Breakfast
9:00–10:00	Chinburg: <i>Equivariant coherent Euler characteristics and Iwasawa Theory</i>
10:00–10:30	Coffee Break
10:30–11:30	Ochiai: <i>Coleman map for families of Galois representations</i>
11:30–13:30	Lunch

**Checkout by
12 noon.**

** Participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

ABSTRACTS

- Ardakov: *Duflo's theorem and two-sided ideals in Iwasawa algebras*

The classical theorem of Duflo states that every primitive ideal in the enveloping algebra of a complex semisimple Lie algebra annihilates a highest weight module. I will describe work in progress (joint with Simon Wadsley) towards an analogue of this result for affinoid enveloping algebras, and explain how to prove directly (in certain cases) that the corresponding highest weight affinoid modules are faithful over the Iwasawa algebra KG . Using this line of attack, we eventually expect to show that every non-zero two-sided ideal in KG must have finite codimension over the ground field K whenever the group G is almost simple.

- Berger: *Multivariable Lubin-Tate (ϕ, Γ) -modules*

I will explain the construction of some (ϕ, Γ) -modules over a ring of power series in several variables that is naturally attached to a Lubin-Tate formal module. I will also discuss the possible connection with the p -adic Langlands program for $GL(2, F)$, with F a finite unramified extension of \mathbb{Q}_p , using the p -adic Fourier theory of Schneider and Teitelbaum.

- Chinburg: *Equivariant coherent Euler characteristics and Iwasawa Theory*

This talk will be a survey of some of the ideas and techniques common to Iwasawa theory and the study of equivariant coherent Euler characteristics. I will begin with a dictionary between the noncommutative Iwasawa Main Conjecture on the one hand and Thomason's approach to coherent Lefschetz-Riemann-Roch Theorems on the other. I will then discuss some results about the K_1 and SK_1 groups of group rings which are useful in both contexts. I will end with a description of how such results enter into work with G. Pappas and M.J. Taylor on non-commutative Riemann-Roch Theorems.

- Greenberg: *Some Questions Concerning Iwasawa Algebras and Their Modules*

We will describe a number of rather basic questions concerning module theory for Iwasawa algebras which are motivated by arithmetic applications. In particular, we are interested in the behavior of the Mordell-Weil group for an elliptic curve in a p -adic Lie extension.

- Jha: *Fine Selmer group of Hida deformations*

Fine Selmer group is an arithmetic module which is studied, in different manes, in cyclotomic Iwasawa theory and in Iwasawa theory of elliptic curve. In this talk, we will study fine Selmer group for elliptic curve and modular forms and Λ -adic forms. Inspired by some deep classical conjectures of Iwasawa and Greenberg in cyclotomic Iwasawa theory, Coates and Sujatha have proposed certain conjectures regarding the structures of the fine Selmer group of an elliptic curve. We will formulate the analogues of these conjectures for modular forms and for Λ -adic forms. We will also compare the usual Selmer group in a Hida family of congruent modular forms.

- Kakde: *K -theory for Iwasawa algebras and related rings*

This talk is a survey of known results and important open problems (according to my taste and knowledge) about lower K -groups (K_0 and K_1 , both absolute and relative) of Iwasawa algebras and certain localisations. Not much is known about higher K -groups of Iwasawa algebras even in the commutative case. If time permits I will say something about higher K -theory of Iwasawa algebras and how these groups may play a role in Iwasawa theory.

- Kohlhaase: *Modular Admissibility*

The mod- p local Langlands correspondence requires the study of smooth representations of p -adic reductive groups on vector spaces over fields of characteristic p . For such representations, many (non-)finiteness phenomena still lack a satisfying explanation. We report on ongoing work concerning the

admissibility of supersingular representations. In part, our study relies on the structure theory of Iwasawa algebras of p -adic Lie groups, as developed by Lazard.

- Loeffler: *ϵ -isomorphisms for crystalline p -adic Galois representations*

Fukaya and Kato have formulated a very general local Tamagawa number conjecture for families of Galois representations of p -adic fields, asserting that a certain isomorphism of determinants (involving the Bloch-Kato exponential maps and local ϵ -factors) can be interpolated p -adically. I will report on joint work with Otmar Venjakob and Sarah Zerbes in which we prove this conjecture for families of twists of a fixed crystalline representation, using the “two-variable” generalization of Perrin-Riou’s theory developed in my earlier work with Zerbes.

- Nakamura: *A generalization of Kato’s local epsilon conjecture for (ϕ, Γ) -modules over the Robba ring*

In my previous work, I developed the theory of Bloch-Kato’s and Perrin-Riou’s exponential maps in the framework of (ϕ, Γ) -modules over the Robba ring. As an application of this work (and of the recent work of Kedlaya-Pottharst-Xiao on the finiteness of cohomologies of families of (ϕ, Γ) -modules), I will talk about a generalization of Kato’s epsilon conjecture for families of (ϕ, Γ) -modules over the Robba ring. In particular, I will prove our conjecture for families of trianguline (ϕ, Γ) -modules. I also compare our results with the previous known results in the case of p -adic representations (e.g. rank one case proved by Kato, crystalline case proved by Benois-Berger).

- Nickel: *Equivariant Iwasawa theory and non-abelian Stark-type conjectures*

We discuss different formulations of the equivariant Iwasawa main conjecture attached to an extension of totally real fields F/k with Galois group G , where k is a number field and G is a p -adic Lie group of dimension 1 for an odd prime p . In particular, we consider a non-abelian variant of the conjecture as formulated (in the abelian case) by Greither and Popescu. We show that these formulations are equivalent and hold if Iwasawa’s μ -invariant vanishes using results of Kakde and Ritter-Weiss. Under mild hypotheses, we use this to prove non-abelian generalisations of Brumer’s conjecture and a strong version of the Coates-Sinnott conjecture provided that $\mu = 0$.

- Ochiai: *Coleman map for families of Galois representations*

When we have a family of nearly ordinary p -adic Galois representations of the absolute Galois group of a p -adic field, we expect to have an interpolation of Kato’s dual exponential map called Coleman map. Coleman is expected to be a machinery which transforms the (conjectural) Euler systems to p -adic L -functions and it gives an important step to prove the Iwasawa Main Conjecture. After stating general conjectural pictures on the existence of Euler system and Coleman map, I will state some results related to the context for some deformations constructed by Hida theory.

- Popescu: *Explicit ℓ -adic models of Tate sequences and applications*

We will explain how our recent joint work with Greither in equivariant Iwasawa theory leads to a natural construction of explicit ℓ -adic models of Tate sequences for Galois extensions of arbitrary global fields. The applications of this construction include: (new and rather explicit) proofs of the ETNC for Artin motives, under strong hypotheses for number fields and unconditionally for function fields; the proof of a vast generalization of a conjecture of Darmon on special values of global L -functions, whose particular case pertaining to order of vanishing one L -functions was recently proved by Mazur and Rubin via the theory of (rank one) Kolyvagin systems. The new class of Iwasawa modules constructed and studied in our recent work with Greither plays a crucial role in all these considerations.

- Pottharst: *Review of (ϕ, Γ) -modules*

We review the theory of (ϕ, Γ) -modules, and the way they make the link between p -adic Hodge theory and Galois cohomology explicit. We focus on the case of base field \mathbb{Q}_p , both for simplicity and to

keep the role of Iwasawa algebras clear; even with this restriction, however, there are still a few useful variants to sort out.

- Riedel: *On Perrin-Riou's exponential maps and reciprocity laws*

We give a construction of a "big exponential map" in the sense of Perrin-Riou in the setting of cyclotomic Iwasawa theory for de Rham (φ, Γ) -modules. Additionally, we construct two pairings (one "coming from cohomology", and one "coming from convolution of measures") following Perrin-Riou/Colmez/... and show a reciprocity law for semi-stable (φ, Γ) -modules over an unramified extension K/\mathbb{Q}_p . We apply these results in the example of a semi-stable elliptic curve over \mathbb{Q}_p .

- Schmidt: *Localisation for p -adic analytic representations*

We survey joint work with D. Patel/M. Strauch. Let G be a p -adic reductive group. A localization theory for locally analytic G -representations should encompass the Schneider-Stuhler theory for smooth representations and should have links to the localization theory of Beilinson-Bernstein and Brylinski-Kashiwara for Lie algebra representations. We will discuss approaches to such a theory via Berkovich spaces and formal semistable models of flag varieties.

- Schmitt: *On the interpolation part of a local main conjecture with a view towards a global twist statement*

In this talk an interpolation property is given for the element \mathcal{E}_u satisfying the local main conjecture that was constructed in Venjakob's work "On The Non-Commutative Local Main Conjecture For Elliptic Curves With Complex Multiplication". This element is part of a conjectured global result suggested by Kato, which relates the p -adic L -function of a CM elliptic curve to twists of the p -adic L -function associated to the Tate Motive and of the element \mathcal{E}_u .

- Sharifi: *Iwasawa algebras and Duality*

Tate and Poitou-Tate duality for cohomology groups of Galois groups of local and global fields, respectively, take place between the cohomology group of a finite module and another cohomology group for the Tate twist of the Pontryagin dual of the module. Nekovar obtained related duality theorems, replacing the Pontryagin dual by a Grothendieck dual, in a derived category of Galois modules that are also finitely generated modules over a complete, commutative local Noetherian ring with finite residue field. I will explain an extension of this to p -adic Lie extensions of local and global fields, which is to say, to a case in which the commutative ring is replaced by a possibly noncommutative Iwasawa algebra over it. This is joint work with Meng Fai Lim.

- Shekhar: *Congruence formula for certain dihedral twists*

In this talk we shall discuss about a congruence formula for the special values of certain dihedral twists of two primitive modular forms of weight two with isomorphic residual Galois representation at a prime p . This is a joint work with R. Sujatha.

- Sprung: *Modesty in the size of the Tate-Shafarevich group*

A version of BSD for number fields relates the growth of Sha to special values of p -adic L -functions which in general do not live in Iwasawa algebras. We rewrite these p -adic L -functions in terms of new elements of Iwasawa algebras, and as an application exhibit growth formulas for Sha and discover some mysterious behavior when scrutinizing modular forms of weight two.

- Witte: *Iwasawa algebras and localisation sequences in higher algebraic K -theory*

Let G be the semi-direct product of a p -adic Lie group H and $\Gamma = \mathbb{Z}_p$ acting on H . In this talk, we will show that the localisation sequence of higher K -Theory for the Iwasawa algebra of G with respect to the localisation at Venjakob's canonical Ore set S splits into short split exact sequences. We will also discuss the localisation at S^* and some implications concerning non-commutative Iwasawa theory.

- Zabradi: *Applications of Iwasawa algebras to representation theory*

Motivated by the hypothetical p -adic Langlands correspondence we briefly introduce both p -adic Banach-space representations and locally analytic representations of p -adic groups. We explain why Iwasawa algebras (and the fact that they are noetherian) play an important role defining admissibility in the Banach case. On the other hand, the definition of locally analytic admissibility relies on the Frechet-Stein structure on the locally analytic distribution algebra. We also introduce certain topological localisations of these rings (ie. the Iwasawa algebra and the locally analytic distribution algebra) motivated by their one-variable commutative analogues such as Fontaine's ring $\mathcal{O}_{\mathcal{E}}$ and its variants: overconvergent and Robba rings. We outline how these localisations are useful for relating the objects on the automorphic side to (φ, Γ) -modules.

- Zerbes: *Euler systems for Rankin-Selberg convolutions of modular forms*

I will describe the construction of an Euler system over the cyclotomic tower for the product of the Galois representations attached to two weight 2 modular forms, generalizing work of Beilinson, Flach, and Bertolini-Darmon-Rotger. I will further describe some applications of the Euler system for bounding the fine Selmer group, and I will explain the conjectural relation of the Euler system to so-called 'higher rank Euler systems', a concept that was introduced by Rubin and Perrin-Riou. This is joint work with Antonio Lei and David Loeffler.