Banff International Research Station
for Mathematical Innovation and Discovery

Geometric Variational Problems
December 15-20, 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00-9:45	Yu Yuan
9:50-10:35	Robert Haslhofer
10:35-11:00	Coffee Break, TCPL
11:00-11:45	Jeff Streets
11:45–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00-14:45	Albert Chau
14:45-15:10	Coffee Break, TCPL
15:10-15:55	Robert Gulliver
16:00-16:45	Jacob Bernstein
16:50-17:35	Jaigyoung Choe
17:35–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00-9:45	Spyros Alexakis
9:50-10:35	Huy Nguyen
10:35-11:00	Coffee Break, TCPL
11:00-11:45	Andrea Mondino
11:45	Group Photo; meet in foyer of TCPL
11:50–13:30	Lunch
13:30-14:15	Miles Simon
14:20-15:05	Ben Sharp
15:05-15:30	Coffee Break, TCPL
15:30-16:15	Eric Woolgar
16:20-17:05	Philippe Castillon
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00-9:45	Brian Krummel
9:50-10:35	Costante Bellettini
10:35-11:00	Coffee Break, TCPL
11:00-11:45	Chikako Mese
11:45–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00-9:45	Natasa Sesum
9:50-10:35	Frédéric Robert
10:35-11:00	Coffee Break, TCPL
11:00-11:45	Christine Breiner
11:45–13:30	Lunch
13:30-14:15	Felix Schulze
14:20-15:05	Brett Kotschwar
15:05-15:30	Coffee Break, TCPL
15:30-16:15	Glen Wheeler
16:20-17:05	Tom Ilmanen
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00-10:30	Informal Discussions
10:30-11:00	Coffee Break, TCPL
11:00-11:45	Informal Discussions
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Geometric variational problems
June 23-28, 2013

ABSTRACTS

Speaker: **Spyros Alexakis** (University of Toronto)
Title: *Boundary regularity and bubbling for Willmore surfaces with bounded weighted energy*
Abstract: We consider the space of Willmore surfaces in H^3 and in the unit ball in E^3, with boundaries lying on the sphere. We obtain boundary regularity and bubbling criteria for such surfaces in terms of a certain weighting of the traceless total curvature. The weight function here blows up logarithmicaly at the boundary. Analogous questions could also be posed in the context of asymptotically hyperbolic Einstein metrics and for harmonic maps.

Speaker: **Costante Bellettini** (IAS/ Princeton University)
Title: *Calibrations of degree 2 and blow up analysis*
Abstract: Calibrated currents naturally appear when dealing with several geometric questions, some aspects of which require a deep understanding of regularity properties of calibrated currents. We will review some of these issues, then focusing on the two-dimensional case where we will show a surprising connection with pseudo-holomorphic curves and perform a blow up analysis, where we will be interested in the uniqueness of tangent cones and in the rate of decay for the mass ratio.

Speaker: **Jacob Bernstein** (Johns Hopkins University)
Title: *The topology of the limits of a sequence of embedded minimal disks*
Abstract: The work of Colding-Minicozzi says that a sequence of embedded minimal disks in a three-manifold subconverges to a minimal lamination away from a closed set of singular points. Motivated by this, a great deal of recent work has been done constructing examples of possible limits. Notably, Hoffman-White have produced examples where some of the leaves are annuli – leading them to ask whether leaves of more complicated topology were possible. We will show that under natural geometric conditions on the ambient three-manifold the answer is no. This is joint work with G. Tinaglia.

Speaker: **Christine Breiner** (Fordham University)
Title: *Compactness theory for biharmonic maps into spheres*
Abstract: Critical points for the functional $E(u) = \int |\Delta u|^2$ are called biharmonic maps and are natural fourth order analogues of harmonic maps. Compactness theory for harmonic maps in two dimensions is well understood. In this talk we will discuss recent work with T. Lamm in which we determine energy quantization and the C^0 limit picture for sequence of approximate biharmonic maps from four dimensional manifolds into spheres. The analogous results for harmonic maps is true even for general targets and relies on results of Sacks-Uhlenbeck, Jost, and Parker. We will review the main ideas behind the proof of energy quantization and the C^0 limit picture for harmonic maps and explain some of the pitfalls that arise when trying to extend this proof to the biharmonic setting. We will then explain what we gain by mapping into spheres and sketch a few aspects of the proof.

Speaker: **Philippe Castillon** (Université de Montpellier II/ UBC)
Title: *Submanifolds, isoperimetric inequalities and optimal transport*
Abstract: In this talk we shall see how to use optimal transport method to prove isoperimetric inequalities on submanifolds of the Euclidean space. In particular, I will give a new proof of the Michael-simon inequality with a constant better than those previously known. This work relies on the description of a solution to the Monge problem when the initial measure is supported in a submanifold and the target one in a linear subspace.

Speaker: **Albert Chau** (University of British Columbia)
Title: *The Kähler Ricci flow on complete Kähler manifolds with unbounded curvature*
Abstract: Consider the Kähler Ricci flow on a complete non-compact Kähler manifold (M, g). A classical result of W.-X. Shi says that the flow has a short time solution if g has bounded curvature. This talk addresses the problem of finding a solution when g has unbounded curvature. I will begin by discussing some a priori estimates and general existence results for the flow. I will then discuss applications in the case g is a $U(n)$ invariant Kähler metric on $\mathbb{C}^n$. The talk is based on joint work with L.F. Tam and K.F Li.

Speaker: **Jaigyoung Choe** (Korea Institute for Advanced Study)
Title: *Stable capillary hypersurfaces in a wedge*
Abstract: Let Σ be an immersed stable CMC hypersurface in a wedge bounded by two hyperplanes in $\mathbb{R}^n$. Suppose that Σ meets those two hyperplanes in constant contact angles and is disjoint from the edge of the wedge. We will show that if $\partial\Sigma$ is embedded for $n = 3$, or if $\partial\Sigma$ is convex for $n = 4$, then Σ is part of the sphere.

Speaker: **Robert Gulliver** (University of Minnesota)
Title: *Branch points of minimizing nonorientable surfaces*
Abstract: The construction of minimal surfaces in a Riemannian manifold is accomplished by minimizing energy under Plateau boundary conditions, which allow reparameterization of the given boundary curves, if any. The minimizer is a conformally parameterized minimal surface with possible isolated singularities, called *branch points*. True branch points are those which are locally not a branched covering of an immersed surface. They are only possible on an area-minimizing minimal surface when the codimension is at least 2. The absence of false branch points, or more generally of ramified branch points, requires a global topological hypothesis, the Douglas hypothesis; but that has so far only been shown to suffice for orientable surfaces. A branch point is *ramified* if in every neighborhood of the point there are two open sets which define the same piece of surface. In this talk, we will outline these arguments for oriented surfaces, extend a basic theorem to non-orientable surfaces, and then discuss ramified branch points of non-orientable minimal surfaces, focusing on surfaces of small Euler characteristic, using the Riemann-Hurwitz formula. For example, we will show that in codimension one, a mapping from the projective plane, which minimizes area among homotopically non-trivial mappings, is an immersion.

Speaker: **Robert Haslhofer** (Courant Institute)
Title: *Mean curvature flow of mean convex hypersurfaces*
Abstract: In the last 15 years, White and Huisken-Sinestrari developed a far-reaching structure theory for the mean curvature flow of mean convex hypersurfaces. Their papers provide a package of estimates and structural results that yield a precise description of singularities and of high curvature regions in a mean convex flow. In the present talk, we explain a new treatment of the theory of mean convex (and k-convex) flows. This includes: (1) an estimate for derivatives of curvatures, (2) a convexity estimate, (3) a cylindrical estimate, (4) a global convergence theorem, (5) a structure theorem for ancient solutions, and (6) a partial regularity theorem. Our new proofs are both more elementary and substantially shorter than the original arguments. Our estimates are local and universal. A key ingredient in our new approach is the new non-collapsing result of Andrews. Some parts are also inspired by the work of Perelman. This is joint work with Bruce Kleiner.

Speaker: **Tom Ilmanen** (ETH Zürich)
Title:
Abstract:

Speaker: **Brett Kotschwar** (Arizona State University)
Title: *A frequency approach to unique continuation for the Ricci flow*

Abstract: I will discuss an alternative approach, based on the consideration of an appropriate frequency-type quantity, to certain problems of unique continuation arising in the study of the Ricci flow. This technique allows for short – and transparently quantitative – proofs of a class of global backwards-uniqueness results for the flow, and has further applications to some questions of asymptotic rigidity for self-similar solutions.

Speaker: **Brian Krummel** (University of Cambridge)
Title: *Structure of branch sets of harmonic functions and minimal submanifolds*
Abstract: I will discuss some recent results on the structure of the branch set of multiple-valued solutions to the Laplace equation and minimal surface system. It is known that the branch set of a multiple-valued solution on a domain in $\mathbb{R}^n$ has Hausdorff dimension at most $n-2$. We investigate the fine structure of the branch set, showing that the branch set is countably $(n-2)$-rectifiable. Our result follows from the asymptotic behavior of solutions near branch points, which we establish using a modification of the frequency function monotonicity formula due to F. J. Almgren and an adaptation to higher-multiplicity of a "blow-up" method due to L. Simon that was originally applied to "multiplicity one" classes of minimal submanifolds satisfying an integrability hypothesis. This is joint work with Neshan Wickramasekera.

Speaker: **Chikako Mese** (Johns Hopkins Univerisity)
Title: *Harmonic maps in rigidity problems*
Abstract: We discuss harmonic maps into non-positively curved metric spaces (NPC spaces). Of particular interest is the regularity issues for these maps into special classes of NPC spaces that include the Weil-Petersson completion of Teichmuller space. As an application of the regularity theory, we study rigidity questions.

Speaker: **Andrea Mondino** (ETH Zürich)
Title: *A frame energy for tori immersed in $\mathbb{R}^m$: sharp Willmore-conjecture type lower bound, regularity of critical points and topological applications*
Abstract: In the seminar I will present some recent results about the Dirichlet energy of moving frames on 2-dimensional tori immersed in the euclidean $3 \leq m$-dimensional space. This functional, called Frame energy, is naturally linked to the Willmore energy of the immersion and on the conformal structure of the abstract underlying surface. As first result, a sharp Willmore-conjecture type lower bound is established in arbitrary codimension. Smootheness of the critical points of the frame energy is proved after the discovery of hidden conservation laws and, as application, the minimization of the Frame energy in regular homotopy classes of immersed tori in $\mathbb{R}^3$ is performed. Joint work with Tristan Rivière.

Speaker: **Huy Nguyen** (University of Queensland)
Title: *Singular Willmore Spheres in $\mathbb{R}^4$*
Abstract: In this talk, we will extend a recent classification by Lamm-Nguyen of singular Willmore spheres in $\mathbb{R}^3$ to the setting of $\mathbb{R}^4$. We will show if the order of singularities is bounded then a singular Willmore sphere must be either an umbilic sphere, the Penrose twistor projection of an (anti)-holomorphic curve in CP^3 or the Möbius transformation of a complete non-compact minimal surface in $\mathbb{R}^4$. We will then give an application to the Willmore flow of spheres in $\mathbb{R}^4$.

Speaker: **Frédéric Robert** (Université Henri Poincaré Nancy)
Title: *Glueing of a Bubble to a degenerate metric*
Abstract: We investigate the existence of blowing-up solutions to scalar-curvature equations by glueing a Bubble to a given ground state. The main difficulty occurs when the ground-state is degenerate. Using analytic expansions, we prove that the construction can be carried out for isolated ground states. This is joint work with Jérôme Vétois (Nice).

Speaker: **Felix Schulze** (University College London)
Title: *A local regularity theorem for the network flow*

Abstract: The network flow is the evolution of a network of curves under curve shortening flow in the plane, where it is allowed that at triple points three curves meet under a 120 degree condition. We present here a local regularity theorem for the network flow, which is similar to the result of B. White for smooth mean curvature flow.

Speaker: **Natasa Sesum** (Rutgers University)
Title: *Ricci flow neckpinches without rotational symmetry*
Abstract: We study "warped Berger" solutions $\left(S^1 \times S^3, G(t)\right)$ of Ricci flow: generalized warped products with the metric induced on each fiber $\{s\} \times \mathrm{SU}(2)$ a left-invariant Berger metric. We prove that this structure is preserved, that these solutions develop finite-time neckpinch singularities, and that they asymptotically approach round product metrics in space-time neighborhoods of their singular sets, in precise senses. These are the first examples of Ricci flow solutions without rotational symmetry that develop local finite-time neckpinch singularities and locally become asymptotically rotationally symmetric. This is a joint work with James Isenberg and Dan Knopf.

Speaker: **Benjamin Sharp** (Imperial College London)
Title: *Interior and free boundary regularity for Dirac-harmonic maps, harmonic maps and related PDE*
Abstract: Since Hélein's celebrated proof of the regularity of weakly harmonic maps from surfaces to Riemannian manifolds there have been huge improvements and generalisations to the theory, with applications in many areas of analysis and geometry. Notably the work of Tristan Rivière has provided analytical insight to these problems leading to suitable generalisations. In this talk we will give an overview of some of these ideas and present new theorems leading to proofs (and hopefully some insight) for both new and classical results. Some of the work presented is joint with Peter Topping (Warwick), Miaomiao Zhu (MPI Leipzig) and Tobias Lamm (KIT).

Speaker: **Miles Simon** (University of Magdeburg)
Title: *Some local results for the Ricci flow*
Abstract: In this talk we present local results for the Ricci flow. In dimension two we generalise G.Perelman's "Pseudolocality" result to allow cone like regions (which are by definition not locally euclidean). We show that under certain assumptions one also obtains a local result (different from that of Perelman) in three dimensions.

Speaker: **Jeff Streets** (University of California at Irvine)
Title: *Singularities of the L^2 curvature flow*
Abstract: The L^2 norm of the Riemannian curvature tensor is a natural energy to associate to a Riemannian manifold, especially in dimension 4. A natural path for understanding the structure of this functional and its minimizers is via its gradient flow, the "L^2 flow." This is a quasi-linear fourth order parabolic equation for a Riemannian metric, which one might hope shares behavior in common with the Yang-Mills flow. We verify this idea by exhibiting structural results for finite time singularities of this flow resembling results on Yang-Mills flow. We also exhibit a new short-time existence statement for the flow exhibiting a lower bound for the existence time purely in terms of a measure of the volume growth of the initial data. As corollaries we establish new compactness and diffeomorphism finiteness theorems for four-manifolds generalizing known results to ones with minimal dependencies. These results all rely on a new technique for controlling the growth of distances along a geometric flow, which is especially well-suited to the L^2 flow.

Speaker: **Glen Wheeler** (University of Wollongong)
Title: *Curvature contraction of convex hypersurfaces by non-smooth speeds*
Abstract: In fully nonlinear curvature flow we typically aim to prove that suitably pinched initial data shrinks to a point in finite time, becoming asymptotically close to a self-similar solution as it does so. Theorems of this type trace back to Huiskens seminal contribution for the mean curvature flow of convex

hypersurfaces. Since then, the efforts of a number of researchers have established similar results for quite broad classes of (typically fully nonlinear) curvature flow with smooth velocity. Our present contribution is an analysis of the case where the speed of the flow, as a function of the eigenvalues of the Weingarten map, is not differentiable. Other requirements on the flow speed are reminiscent of Andrews early work from 1993 and 1994. Our main result is that up to necessary modifications, a Huisken-esque result holds.

In this talk I will describe the proof of this result, highlighting the essential new contributions, and mention possible future directions. This work is joint with Ben Andrews, Andrew Holder, James McCoy, Valentina-Mira Wheeler, and Graham Williams.

Speaker: **Eric Woolgar** (University of Alberta)
Title: *APEs, their close relatives, and their evolution*
Abstract: APEs are Asymptotically Poincaré-Einstein manifolds. I will first review the zoology of APEs and their relatives, which are various classes of Conformally Compactifiable manifolds. I will show that APEs have nice properties under the Ricci flow. Namely, if a manifold is initially APE, it remains APE under the flow, and if the Ricci curvature obeys the natural lower bound $Ric \geq -(n-1)$ initially, then this is preserved. The mass, when defined, is monotonic, as is the renormalized volume when the aforementioned Ricci curvature bound holds initially. This has a nice interpretation for the Hawking-Page phase transition in black hole physics. Time permitting, I will discuss the situation for more general classes of asymptotically hyperbolic manifolds, and perhaps discuss some open problems. This is based in part on joint work Eric Bahuaud and Rafe Mazzeo and with Tracey Balehowsky.

Speaker: **Yu Yuan** (University of Washington)
Title: *Self similar solutions for curvature flows*
Abstract: We talk about some rigidity of self similar solutions for mean curvature flow and Kahler Ricci flow.