Refined invariants in geometry, topology and string theory
Jun 2 - Jun 7, 2013

MEALS

Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	*Check-in begins @ Front Desk, Professional Development Centre*
17:30–19:30	*Buffet Dinner, Sally Borden Building*

Monday

7:00–8:45	*Breakfast*
8:45–9:00	*Introduction and Welcome by BIRS Station Manager*
9:00–10:00	Andrei Okounkov: M-theory and DT-theory
	Coffee
10.30–11.30	Sheldon Katz: Equivariant stable pair invariants and refined BPS indices
11:30–13:00	*Lunch*
13:00–14:00	*Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall*
14:00	*Group Photo; meet in foyer of TCPL*
14:10–15:10	Dominic Joyce: Categorification of Donaldson–Thomas theory using perverse sheaves
	Coffee
15:45–16.45	Zheng Hua: Spin structure on moduli space of sheaves on CY 3-folds
17:00-17.30	Sven Meinhardt: Motivic DT-invariants of (-2)-curves
17:30–19:30	*Dinner*

Tuesday

7:00–9:00	*Breakfast*
9:00–10:00	Alexei Oblomkov: Topology of planar curves, knot homology and representation theory of Cherednik algebras
	Coffee
10:30–11:30	Luca Migliorini: Support theorems for Hilbert schemes of families of planar curves
11:30–13:30	*Lunch*
14:00–15:00	Vivek Shende: Legendrian knots and constructible sheaves
	Coffee
15:45–16:45	Anatoly Preygel: Higher structures on Hochschild invariants of matrix factorizations
17:00–17:30	Andrew Morrison: Asymptotics of 3D partitions and refined invariants
17:30–19:30	*Dinner*
20:00–	*Jazz @ The Club, Theatre Complex*

Wednesday

7:00–9:00	*Breakfast*
9:00–10:00	Kentaro Nagao: Cut of a quiver with potential and the cohomological Hall algebra
	Coffee
10:30–11:30	Ben Davison: Purity of critical cohomology for graded potentials and quantum cluster positivity
11:30–12:00	Vittoria Bussi: On motivic vanishing cycles of critical loci
12:00–13:30	*Lunch*
	Free Afternoon
17:30–19:30	*Dinner*
20:00–	*Jazz @ The Club, Theatre Complex*

Thursday

7:00–9:00	*Breakfast*
9:00–10:00	Emanuel Diaconescu: Parabolic refined invariants and MacDonald polynomials
	Coffee
10:30–11:30	Emmanuel Letellier: Counting geometrically indecomposable parabolic bundles over the projective line
11:30–13:30	*Lunch*
13:30–14:30	András Szenes: Equivariant intersection theory of Higgs moduli spaces
	Coffee
15:15–16:15	Kai Behrend: Categorification of Lagrangian intersections via deformation quantization
16:30–17:30	Lotte Hollands: Surface defects and the superconformal index
17:30–19:30	*Dinner*
19:00–	*Exhibition tour "Bottles under the influence" @ Walter Phillips Gallery*
20:00–	*Jazz @ The Club, Theatre Complex*

Friday

7:00–9:00	*Breakfast*
9:00–10:00	Vincent Bouchard: Mirror symmetry for orbifold Hurwitz numbers
	Coffee
10:30–11:00	Dusty Ross: The gerby Gopakumar–Marino–Vafa formula
11:00–	Discussion
11:30–13:30	*Lunch*
By 12:00	**Checkout**

Participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3pm on Friday, but please check out of the guest rooms by 12 noon.

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Kai Behrend** (UBC)
Title: *Categorification of Lagrangian intersections via deformation quantization*
Abstract: We explain what the theory of deformation quantization has to say about Lagrangian intersections. Already the case of first order quantization gives a non-trivial result: namely the construction of a canonical Batalin–Vilkovisky differential on the sheaves of extensions between two Lagrangian half-densities. Extrapolating to the infinite order case, we propose a construction of a canonical twisted perverse sheaf on the intersection of two Lagrangians inside a complex symplectic manifold, which categorifies the intersection number if the intersection is compact. (Joint work in progress with B. Fantechi and J. Pecharich)

Speaker: **Vincent Bouchard** (Alberta)
Title: *Mirror symmetry for orbifold Hurwitz numbers*
Abstract: In recent years, it has been found that many enumerative geometric problems have a common feature: they have a mirror symmetric counterpart which is governed by a universal integral recursion formula due to Eynard and Orantin. The key ingredient to the mirror theory is the existence of a spectral curve (also known as "mirror curve" in this context). Once the spectral curve mirror to a given counting problem is determined, the integral recursion uniquely calculates all generating functions of the corresponding enumerative invariants. In this talk I will show that the mirror counterparts to orbifold Hurwitz numbers satisfy the integral recursion, with spectral curve given by the "r-Lambert curve". I will also argue that orbifold Hurwitz numbers can be obtained in the "infinite framing limit" of orbifold Gromov-Witten theory of $[\mathbb{C}^3/(\mathbb{Z}/r\mathbb{Z})]$, thus shedding some light on the appearance of the recursion for orbifold Hurwitz numbers and its relation with the so-called remodeling conjecture for Gromov–Witten theory on toric orbifolds. (Joint work with D. Hernandez Serrano, X. Liu and M. Mulase)

Speaker: **Vittoria Bussi** (Oxford)
Title: *On motivic vanishing cycles of derived critical loci*
Abstract: We prove that if (X, s) is an oriented algebraic d-critical locus in the sense of Joyce, there is a natural motive $MF_{X,s}$ in a modified version of the motivic Grothendieck ring defined by Denef–Loeser and Looijenga, such that if (X, s) is locally modelled on $\mathrm{Crit}(f\colon U \to \mathbb{A}^1)$, then $MF_{X,s}$ is locally modelled on the motivic vanishing cycle of f. Using results of Pantev, Toen, Vezzosi and Vaquie, this implies the existence of natural motives on moduli schemes of coherent sheaves on a Calabi–Yau 3-fold equipped with "orientation data", as required in Kontsevich and Soibelman's motivic Donaldson–Thomas theory, and on intersections of spin Lagrangians in an algebraic symplectic manifold. (Joint work with Joyce and Meinhardt)

Speaker: **Ben Davison** (Bonn)
Title: *Purity of critical cohomology for graded potentials and quantum cluster positivity*
Abstract: I will start by discussing the purity of the Hodge structure on the cohomology of vanishing cycles for certain $\mathbb{C}^*$-equivariant functions on smooth varieties. The applications of this result to Donaldson–Thomas theory occur when moduli spaces of objects being "counted" occur as critical loci of graded superpotentials. This turns out to be a fairly natural setup in the study of moduli spaces of modules for noncommutative resolutions of noncompact CY3s. The second part of this talk will focus on applications to a different area in which graded potentials commonly occur: the study of the cluster algebras of Fomin and Zelevinsky. I will go on to explain how these are defined, along with their quantum refinements, and how we prove a wide class of cases of the quantum positivity conjecture for these algebras using our Hodge-theoretic result, building on work of Efimov. (Joint work with Maulik, Schürmann, Szendrői)

Speaker: **Duiliu-Emanuel Diaconescu** (Rutgers)
Title: *Parabolic refined invariants and MacDonald polynomials*
Abstract: This is work in progress with Wu-yen Chuang, Ron Donagi and Tony Pantev building a string theoretic framework for the conjecture of Hausel, Letellier and Rodriguez-Villegas on the cohomology of character varieties with marked points. Their formula is identified with a Gopakumar–Vafa expansion in the refined stable pair theory of local orbifold curves, which is related by geometric engineering to K-theoretic invariants of nested Hilbert schemes. In particular MacDonald polynomials appear naturally in this framework via Haiman's geometric construction based on the isospectral Hilbert scheme. Supporting evidence is obtained by localization computations of parabolic refined invariants on the conifold via the equivariant index defined by Nekrasov and Okounkov.

Speaker: **Lotte Hollands** (Oxford)
Title: *Surface defects and the superconformal index*
Abstract: The superconformal index is a very fruitful tool to study four-dimensional $N = 2$ superconformal gauge theories. In this talk we will explain how to employ the superconformal index to study a general class of half-BPS surface defects in these theories. We will interpret these results in terms of a two-dimensional $N = (2, 2)$ supersymmetric gauge theory living on the support of the surface defect. Furthermore, we will make connections to a Verlinde algebra on the dual UV curve, and to four-dimensional $N = 2*$ instanton partition functions in the presence of line operators.

Speaker: **Zheng Hua** (Hong Kong)
Title: *Spin structure on moduli space of sheaves on CY 3-folds*
Abstract: Konstevich–Soibelman's orientation data is roughly speaking a consistent choice of spin structure on moduli spaces of sheaves on CY 3-folds. Such spin structure is necessary for defining any refined version of Donaldson–Thomas invariant. I will discuss some recent progress on this subject.

Speaker: **Dominic Joyce** (Oxford)
Title: *Categorification of Donaldson–Thomas theory using perverse sheaves*
Abstract: Pantev, Toen, Vezzosi and Vaquie introduced the notion of k-shifted symplectic structure on a derived scheme or derived stack, and proved that derived moduli stacks of (complexes of) coherent sheaves on a Calabi–Yau m-fold have a $(2-m)$-shifted symplectic structure. We prove a "Darboux Theorem" for k-shifted symplectic derived schemes for all $k<0$, which says in particular that a (-1)-shifted symplectic derived scheme (which includes moduli schemes of simple coherent sheaves on a Calabi–Yau 3-fold) is Zariski locally equivalent to the critical locus of a regular function on a smooth scheme. Next, we define "d-critical loci" (X,s), a classical scheme X with extra (classical) geometric structure s which records information on how X may be written locally as a critical locus. We construct a truncation functor from (-1)-shifted symplectic derived schemes to d-critical loci, and deduce that moduli schemes of simple coherent sheaves on a Calabi–Yau 3-fold are d-critical loci. A d-critical locus (X,s) has a "canonical bundle", which for moduli schemes is the determinant line bundle of the natural obstruction theory. An orientation is a choice of square root of this canonical bundle. We prove that an oriented d-critical locus (X,s) carries a natural perverse sheaf $P_{X,s}$ (also a D-module, and a natural mixed Hodge module), such that if (X,s) is locally modelled on $\mathrm{Crit}(f\colon U\to\mathbb{C})$ then $P_{X,s}$ restricts locally to the perverse sheaf of vanishing cycles of f. For a DT moduli scheme, the graded dimension of the hypercohomology $H^*(P_{X,s})$ is the corresponding Donaldson–Thomas invariant. Thus, this provides a categorification of Donaldson–Thomas invariants. There are also motivic and categorical variants of these results which may be discussed if time permits. (Joint work with O. Ben-Bassat, C. Brav, V. Bussi, D. Dupont, S. Meinhardt and B. Szendrői)

Speaker: **Sheldon Katz** (Illinois)
Title: *Equivariant stable pair invariants and refined BPS indices*
Abstract: A motivically-motivated refinement of the stable pair invariants of a local Calabi–Yau threefold is defined and shown to be equivalent to the equivariant index of Nekrasov and Okounkov. A conjectural product formula is given for the associated generating function in terms of refined BPS indices, agreeing with the known results for local $\mathbb{P}^1$. (Joint work with Jinwon Choi and Albrecht Klemm)

Speaker: **Emmanuel Letellier** (Caen)
Title: *Counting geometrically indecomposable parabolic bundles over the projective line*
Abstract: In this talk I will discuss a formula for counting (over finite fields) the number of isomorphism classes of geometrically indecomposable parabolic structures (of a given type) on a given vector bundle over $\mathbb{P}^1$. In the case of trivial bundles, we can use this kind of formulas to recover some motivic DT-invariants. Our final goal is to find an explicit formula for the Poincaré polynomial of the moduli space of stable parabolic Higgs bundles over the projective line. (This is work in progress)

Speaker: **Sven Meinhardt** (Wuppertal)
Title: *Motivic DT-invariants of (-2)-curves*
Abstract: In the first part of my talk I will gently introduce (-2)-curves and sketch how they show up in resolutions of singular 3-folds. After that, an alternative non-commutative resolution using quivers with potential is given. Finally, I will briefly introduce Donaldson–Thomas invariants and state the answer in our situation. (Joint work with Ben Davison)

Speaker: **Luca Migliorini** (Bologna)
Title: *Support theorems for Hilbert schemes of families of planar curves*
Abstract: Given a family of curves with at worst planar singularities over a base A, one can look at its associated relative Hilbert scheme of a fixed length. By Beilinson–Bernstein–Deligne, if the total space of the relative Hilbert scheme is nonsingular, the pushforward of the constant sheaf to A splits into a direct sum of intersection cohomology sheaves (plus shifts) and it is an interesting problem to determine them or at least their supports. If the curves in the family are all integral, a result independently due to Maulik–Yun and Migliorini–Shende ensures that the sheaves are all supported on A. This result, an analogue of Ngo's support theorem, leads to a generalization of the MacDonald formula, relating the cohomology of the Hilbert schemes of a singular curve to that of its compactified Jacobian. In the reducible case this is no longer true, as seen in easy examples, and I will discuss some new and interesting phenomena. I will also describe a slightly more general set-up to study the supports associated with a map, and introduce some loci associated with a map, which we call higher discriminants. These loci seem to play a big role in support-like theorems and possibly they have further applications. (Joint work in progress with V. Shende and F. Viviani)

Speaker: **Andrew Morrison** (Zurich)
Title: *Asymptotics of 3D partitions and refined invariants*
Abstract: Weighted counts of plane partitions give an elementary way to compute refined Donaldson–Thomas invariants. We discuss how these numbers, suitably normalized, have a Gaussian distribution as limit law. Our technique is to use the Hardy–Littlewood circle method to analyze the bivariate asymptotics of a q-deformation of MacMahon's generating series.

Speaker: **Kentaro Nagao** (Nagoya)
Title: *Cut of a quiver with potential and the cohomological Hall algebra*
Abstract: The algebraic structure of the cohomological Hall algebra of a quiver with potential would help us to understand its DT type theory. Unfortunately, we have not got any explicit description of the algebraic structure of CoHA so far. In this talk, I will provide an new approach for the algebraic structure of CoHA in the case when the QP has "cut".

Speaker: **Alexei Oblomkov** (UMass)
Title: *Topology of planar curves, knot homology and representation theory of Cherednik algebras*
Abstract: The talk is centered around the conjectures relating Khovanov–Rozansky homology of the link of a planar singularity to the homology of Hilbert schemes on the corresponding plane curve (due to Oblomkov, Rasmussen and Shende). In the case of the quasi-homogeneous singularity $x^p = y^q$ it is also expected that the homology of the link, which is a toric link, carry the action of Cherednik algebra (due to Gorsky, Oblomkov, Rasmussen and Shende). The goal of the talk is to explain the evidence for the conjectures.

Speaker: **Andrei Okounkov** (Columbia)
Title: *M-theory and DT-theory*
Abstract: We will discuss a conjectural evaluation of certain M-theoretic indices in terms of K-theoretic DT invariants of 3-folds. (Joint work with N. Nekrasov)

Speaker: **Anatoly Preygel** (Berkeley)
Title: *Higher structures on Hochschild invariants of matrix factorizations*
Abstract: The dg-categories of matrix factorizations categorify several linear algebraic singularity invariants: functions on the (derived) critical locus, and vanishing cycles (or some sort of local Fourier transform). After reminding the audience about Hochschild invariants of dg-categories and the various structures that they possess, we'll explain how the Hochschild invariants of matrix factorizations can be identified with classical linear-algebraic invariants via a formality theorem of Dolgushev–Tsygan–Tamarkin.

Speaker: **Dusty Ross** (Col State)
Title: *The gerby Gopakumar–Marino–Vafa formula*
Abstract: The Gopakumar–Marino–Vafa formula, proven independently in 2003 by Liu–Liu–Zhou and Okounkov–Pandharipande, expresses certain generating functions of Hodge integrals on moduli spaces of curves in terms of Schur functions. The identity can be interpreted as a special case of the Gromov–Witten/Donaldson–Thomas correspondence for Calabi–Yau 3-folds. We generalize the formula to express certain Hurwitz–Hodge integrals on moduli spaces of orbifold curves in terms of loop Schur functions. We use our formula to deduce the first class of examples of the orbifold Gromov–Witten/Donaldson–Thomas correspondence for targets with curve classes contained in the singular locus. (Joint work with Zhengyu Zong)

Speaker: **Vivek Shende** (MIT)
Title: *Legendrian knots and constructible sheaves*
Abstract: To a Legendrian knot at infinity in the cotangent bundle of a surface, we associate the subcategory of the Fukaya category consisting of Lagrangian branes ending on the knot. The Nadler–Zaslow dictionary relates these to constructible sheaves with singular support controlled by the knot diagram. When the surface is an annulus and the diagram is the closure of a positive braid, we determine the moduli spaces of 'rank n' objects in this category and show they are partial compactifications of the rank n character variety on a Seifert surface for the knot. These moduli spaces map to the space of local systems on a circle; the spectral sequence associated to the weight filtration on the pushforward of the constant sheaf recovers the HOMFLY homology of the knot. (Joint work with David Treumann and Eric Zaslow)

Speaker: **András Szenes** (Geneva)
Title: *Equivariant intersection theory of Higgs moduli spaces*
Abstract: In this talk, I will summarize what is known about equivariant intersection numbers of Higgs moduli under the rescaling action, and describe some of the applications of the emerging structures to understanding the ordinary cohomology of this space.