Banff International Research Station

for Mathematical Innovation and Discovery

Non-Gaussian Multivariate Statistical Models and their Applications

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Some Challenges in Portfolio Theory and Asset Pricing

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Introduction

Finance is a very large subject!

This is a personal view:

- Portfolio theory.
- Asset pricing.
- Modelling returns.
- A little option pricing theory.

Many exclusions - certain to be issues omitted which someone here finds important!

Structure

- Background – standard stuff.
- Background –skew-elliptical stuff.
- Skew- elliptical asset pricing models.
- Challenges – Skew-elliptical distributions.
- Challenges – Copulas and Stein's Lemma.
- Challenges – General.

Background – Standard Stuff

Asset returns have a multivariate normal distribution, $\mathbf{X} \sim N(\boldsymbol{\mu},\boldsymbol{\Sigma})$.

Portfolio return is an affine transformation, $X_p = \boldsymbol{w}^T\mathbf{X} \sim N(\boldsymbol{w}^T\boldsymbol{\mu},\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w})$.

Traditionally, investors maximise expected (quadratic) utility of return

$$U(\boldsymbol{w}^T\mathbf{X}) = qX_p - 1/2\left(X_p - m_p\right)^2.$$

This leads to Markowitz' efficient frontier and the Capital Asset Pricing Model:

$$m_p - r_f = b_p(m_m - r_f).$$

The conditional distribution of $\mathbf{X}$ given $X_p = x_p$ is *N()* and linear in x_p .

Background – Standard Stuff and ….

This in turn motivates, to some extent, **A**rbitrage **P**ricing **T**heory

$$X_i = \sum_{j=1}^{p} \beta_j F_{ij} + \varepsilon_i ; \quad \sum_{i=1}^{p} w_i \varepsilon_i \approx 0, E\left(\sum_{i=1}^{p} w_i \varepsilon_i \right) = 0.$$

Now………

Many "*quants*" seek to find better utility functions!

But thanks to Stein (1981), Liu (1984), Landsman and Nešlehová (2008):

The "standard stuff" works for all elliptically symmetric distributions and for all utility functions.

The efficient frontier, CAPM, APT, linear models.

Background – Skew-elliptical stuff

Is there a single mean-variance-skewness efficient surface for all expected utility maximisers?

Yes, under some skew-elliptical distributions: skew-normal, skew-Student.

And under distributions of the class introduced in finance by Simaan (1987, 1993).

$$\mathbf{X} = \mathbf{U} + \boldsymbol{\lambda} V.$$

$\mathbf{X}$ has a multivariate elliptically symmetric distribution and $V \geq 0$ has an unspecified distribution.

BTW, it very likely works for all skew-elliptical distributions. Zinoviy Landsman should be working on finer details of the proof!

Skew- elliptical asset pricing models

The conditional distributions are non-linear.

It is possible to have expected returns which are better or worse than the CAPM, solely because of skewness.

Residual risk in the APT model may be non-zero, which undermines it as a pricing model.

These are challenges in finance rather than statistics.

Challenges – Skew-elliptical distributions

How many hidden truncated variables? Some early evidence suggests more than one – work with Martin Eling and Nic Loperfido.

Temporal aggregation, leading to more than one truncated variable

Challenges in estimation and inference.

Distribution theory: how to approximate sum of truncated normal variables, for example.

Time series effects – ARMA-GARCH models.

Challenges – Copulas and Stein's Lemma

Under normal, Student, or any elliptically symmetric copula, utility function is

$$U(X_p) = U\left(\sum_{i=1}^{n} w_i X_i\right), X_i = h_i(Y_i), \boldsymbol{Y} \sim ellsym(\ ).$$

What happens to Stein's lemma?

Is it possible that some utility functions may be better than others?

Same question under skew-elliptical copulas…… any copula?

What is the conditional distribution of **AX** given **BX**?

Challenges – General

People build several models, compute some test statistics, then pick the best.

Better methods to compare models from different families.

For example, pick model A if it is significantly better than B, C and so on. Otherwise pick the simplest one!

Dealing with many variables, for example 500 in the S&P500.

Continuous time models - stochastic PDEs?

And Finally ………..

"*In the medium and long term, great innovations can have the effect of stifling future developments*"!

Are there other approaches to constructing multivariate models that we should not neglect?

From my point of view:

- Different marginal distributions.
- Tractable conditionals.
- Temporal aggregation & time series effects.
- Mean-variance-skewness surface(s).

Thank you very much.