Joint density of correlations in correlation matrix with chordal sparsity patterns

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Outline

- Main results of Joe 2006, Lewandowski et.al 2009

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- Parametrization of correlation matrices with set of partial correlations

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- Uniform distribution of correlations in correlation matrix with chordal sparsity patterns

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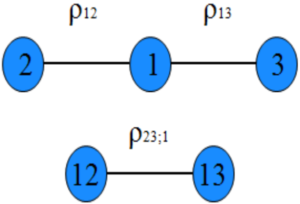
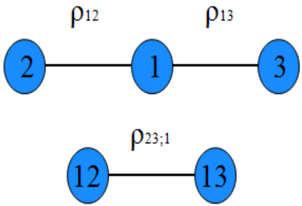
- Parametrization of correlation matrices with set of partial correlations
- Uniform distribution of correlations in correlation matrix with chordal sparsity patterns
- Volume of the set of correlation matrices with chordal sparsity patterns

Parametrization of correlation matrices in terms of partial correlations



Parametrization of correlation matrices in terms of partial correlations



$$det\left(\begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}\right) = (1-\rho_{12}^2)(1-\rho_{13}^2)(1-\rho_{23;1}^2)$$

Density of correlations - 3D Example

In case $d = 3$ the joint density f_3 of $(\rho_{12}, \rho_{13}, \rho_{23})$ is

$$f_3(r_{12}, r_{13}, r_{23}) = g_{12}(r_{12}) \cdot g_{13}(r_{13}) \cdot g_{23}(r_{23;1}) \times |J_3|.$$

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Since $\rho_{23;1} = \frac{\rho_{23} - \rho_{12}\rho_{13}}{\sqrt{(1-\rho_{12}^2)(1-\rho_{13}^2)}}$ then

$$J_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \neq 0 & \neq 0 & \frac{1}{\sqrt{(1-\rho_{12}^2)(1-\rho_{13}^2)}} \end{bmatrix}$$

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Consider a density of the form (denoted as $Beta(\alpha, \alpha)$):

$$g(u|\alpha) = \frac{1}{2^{2\alpha-1}B(\alpha, \alpha)}(1 - u^2)^{\alpha-1}, \ u \in (-1, 1)$$

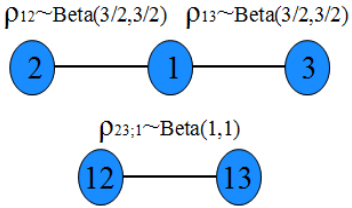
3D Example

Taking g_{12} and g_{13} to be $Beta\left(\frac{3}{2},\frac{3}{2}\right)$ and g_{23} as $Beta(1,1)$ we get

$$\begin{aligned} f_3(r_{12},r_{13},r_{23}) &= \left(\frac{1}{2^2 B\left(\frac{3}{2},\frac{3}{2}\right)}\right)^2 \cdot \frac{1}{2B(1,1)} \cdot \left[(1-r_{12}^2)(1-r_{13}^2)(1-r_{23;1}^2)\right]^0 \\ &= \frac{1}{2^5 B\left(\frac{3}{2},\frac{3}{2}\right)^2}\left[det\{(r_{ij})_{1\le i,j\le 3}\}\right]^0 = \frac{1}{\pi^2/2}. \end{aligned}$$

The normalizing constant $\pi^2/2$ is the volume of the set of three dimensional correlation matrices.

Vine partial correlation - 3D



Vine partial correlation - general

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The volume of the set of d dimensional correlation matrices in $\binom{d}{2}$ dimensional space is:

$$2^{\sum_{k=1}^{d-1} k^2} \prod_{k=1}^{d-1} \left[B\left(\frac{k+1}{2}, \frac{k+1}{2} \right) \right]^k .$$

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$2\sqrt{(1-r_{12}^2)(1-r_{13}^2)}$ is the volume of the space of the three dimensional correlation matrices with fixed (1,2) and (1,3) entries.

Graphs - Partially specified matrices

$$\begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} & r_{15} \\ & 1 & r_{23} & \Box & \Box \\ & & 1 & \Box & \Box \\ & & & 1 & r_{45} \\ & & & & 1 \end{bmatrix}$$

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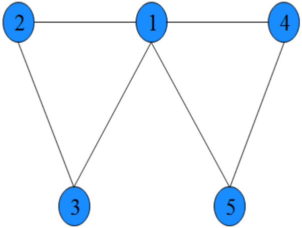

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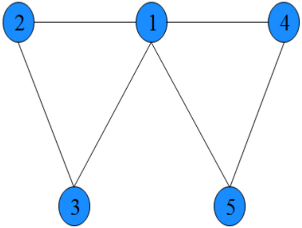

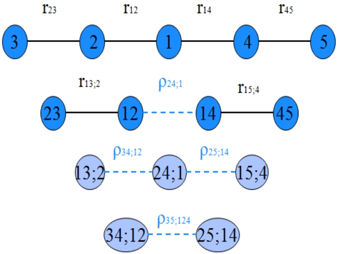

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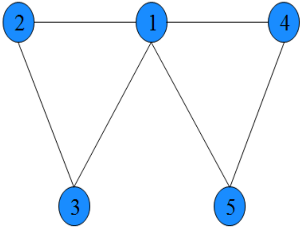
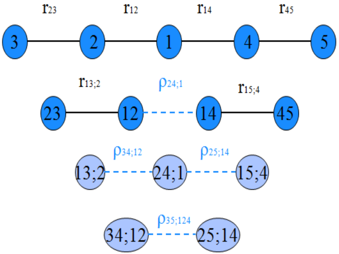
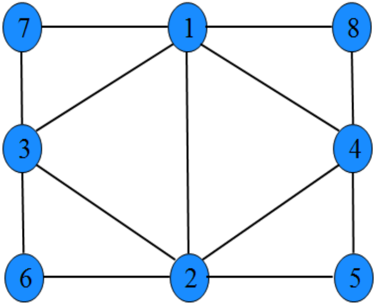


chordal graph



m saturated vine

Chordal Graph - not *m*-saturated vine



New parametrization of correlation matrix

Order variables $\{1, ..., d\}$. Let σ_k be permutation of $\{1, ..., k-1\}, k = 2, ..., d$.

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New parametrization of correlation matrix

Theorem

Let

$$\Omega_{\sigma_{2:d}} = \{\rho_{k,\sigma_k(k-j);\sigma_k(1)\ldots\sigma_k(k-j-1)} : 1 \leq j < k \leq d\}.$$

There is a one-to-one correspondence between the set of $d \times d$ full-rank correlation matrices and the set of partial correlations in $\Omega_{\sigma_{2:d}}$.
Partial correlations in $\Omega_{\sigma_{2:d}}$ are algebraically independent.

Distribution correlations in correlation matrix with chordal sparsity

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- Define

$$\sigma_k = (i_1^k, ..., i_{n_k}^k, j_1^k, ..., j_{k-1-n_k}^k)$$

where
$N(k) = \{i_1^k, ..., i_{n_k}^k\}$ neighbors of k and
$N'(k) = \{j_1^k, ..., j_{k-1-n_k}^k\}$ vertices not connected to k in $G(\{1, ..., k\})$

Distribution correlations in correlation matrix with chordal sparsity

Theorem

$$f_G\left(r_{k,j_t^k} : a \le k \le d, 1 \le t \le k-1-n_k\right) =$$

$$= \left[D(C_1)^{d-\#C_1} \prod_{i=1}^{u-1} \frac{D(C_{i+1})^{d-\#C_{i+1}}}{D(S_i)^{d-\#S_i}}\right]^{-\frac{1}{2}} \times \prod_{k=a}^{d} \prod_{t=1}^{k-1-n_k} \frac{g_{k,j_t^k}\left(r_{k,j_t^k;N(k),j_1^k,\ldots,j_{t-1}^k}\right)}{\left(1-r^2_{k,j_t^k;N(k),j_1^k,\ldots,j_{t-1}^k}\right)^{(d-1-n_k-t)/2}}$$

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Taking

$$g_{k,j_t^k}(r_{k,j_t^k;N(k),j_1^k,\ldots,j_{t-1}^k}) \sim Beta\left(\frac{d-n_k-t+1}{2}, \frac{d-n_k-t+1}{2}\right)$$

we get uniform distribution over the set of unspecified correlations.

Volume of the set of correlation matrices with sparsity pattern of G

$$c_G = \left[D(C_1)^{d-\#C_1} \prod_{i=1}^{u-1} \frac{D(C_{i+1})^{d-\#C_{i+1}}}{D(S_i)^{d-\#S_i}} \right]^{\frac{1}{2}}$$

$$\times \prod_{k=a}^{d} \prod_{t=1}^{k-1-n_k} 2^{d-n_k-t} B\left(\frac{d-n_k-t+1}{2}, \frac{d-n_k-t+1}{2} \right).$$

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If $d=4$ and r_{12}, r_{23}, r_{34} are specified then the volume is:

$$(1-r_{12}^2)(1-r_{23}^2)(1-r_{34}^2)\cdot 2^5 \cdot B\left(\frac{3}{2}, \frac{3}{2}\right)^2 \cdot B(1,1) = (1-r_{12}^2)(1-r_{23}^2)(1-r_{34}^2)\frac{\pi^2}{2}.$$

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- Can this be extended to other than chordal patterns of unspecified correlations?