Multifractal Analysis: From Theory to Applications and Back
February 23-28, 2014

Organizers: P. Abry (CNRS, ENS Lyon, France), S. Jaffard (Univ. Paris Est, Créteil, France), U. Molter (Universidad Buenos Aires, Argentina), V. Pipiras (Univ. North Carolina, USA).

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:50	Murad Taqqu: *Self-Similarity beyond Gaussian processes: Hermite processes and more*
9:50–10:40	Jun Kigami: *Diffusions on inhomogeneous media and multifractal analysis*
10:40–11:10	Coffee Break
11:10–12:00	Ka-sing Lau: *Spectral property of self-similar sets and measures*
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:10	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:10–15:00	Julien Barral: *On the connection between large deviations, multifractals, and some attempt towards applications*
15:00–15:40	Coffee Break
15:40–16:30	Stephane Seuret: *p-exponents and p-multifractal spectrum of some lacunary Fourier series*
16:30–17:20	Herwig Wendt: *Wavelet leaders and p-leaders multifractal analysis: theory and practice for signals and images*
17:30–19:30	Dinner

Tuesday	
7:00–9:00	Breakfast
9:00–9:50	Shaun Lovejoy: *Rocks, clouds, anisotropic multifractals and the unity of geophysics*
9:50–10:40	Beatrice Vedel: *Directional regularity of random fields*
10:40–11:10	Coffee Break
11:10–12:00	Gustavo Didier: *Self-similarity, symmetry and anisotropy in the multivariate and multiparameter settings*
12:00–13:30	Lunch
	Informal discussions
15:15–16:00	Coffee Break
16:10–17:00	Poster session 1
17:00–17:50	Franklin Mendivil: *Some applications of fractal methods in imaging*
17:50–19:30	Dinner
Wednesday	
7:00–9:00	Breakfast
9:10–10:00	Yang Wang: *The Cantor set: generating IFS and self-similar subsets*
10:00–10:30	Coffee Break
10:30–11:20	Paul Balanca: *Oscillating singularities of Lévy processes*
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner
Thursday	
7:00–9:00	Breakfast
9:00–9:50	Edward Waymire: *On normalized multiplicative cascades under strong disorder*
9:50–10:40	Philippe Ciuciu: *Scaling phenomena in brain activity: review, evidences, analysis and impact*
10:40–11:10	Coffee Break
11:10–12:00	Michel Zinsmeister: *Multifractality of whole-plane SLE*
12:00–13:30	Lunch
	Informal discussions
15:15–16:00	Coffee Break
16:00–17:00	Poster session 2
17:00–17:50	Ken Kiyono: *Non-Gaussian characteristics of heart rate variability in health and disease*
17:50–19:30	Dinner
Friday	
7:00–9:00	Breakfast
9:10–10:00	Alain Arneodo: *From DNA sequence to genome structure and function*
10:00–10:30	Coffee Break
10:30–11:20	Patrick Flandrin: *Data-driven methods for scale invariance analysis*
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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TALK ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Alain Arneodo** (CNRS, Ecole Normale Supérieure de Lyon)
Title: *From DNA sequence to genome structure and function*
Abstract: Understanding how chromatin is spatially and dynamically organized in the nucleus of eukaryotic cells and how this affects genome functions is one of the main challenges of cell biology. Since the different orders of packaging in the hierarchical organization of DNA condition the accessibility of DNA sequence elements to trans-acting factors that control the transcription and replication processes, there is actually a wealth of structural and dynamical information to learn in the primary DNA sequence. In this review, we show that when using concepts, methodologies, numerical and experimental techniques coming from statistical mechanics and nonlinear physics combined with wavelet-based multi-scale signal processing, we are able to decipher the multi-scale sequence encoding of chromatin condensation?decondensation mechanisms that play a fundamental role in regulating many molecular processes involved in nuclear functions.

[1] A. Arneodo, C. Vaillant, B. Audit, F. Argoul, Y. d?Aubenton-Carafa, C. Thermes, Multi-scale coding of genomic information: From DNA sequence analysis to genome structure and function, Phys. Rep. 498 (2011) 45-188.

Speaker: **Paul Balança** (Ecole Centrale de Paris)
Title: *Oscillating singularities of Lévy processes*
Abstract: Jaffard has determined a few years back the multifractal spectrum of Lévy processes. However, it has been shown in the regularity literature that the pointwise Hölder exponent considered in the classic multifractal formalism poorly describes the oscillating singularities of a function or a stochastic process. In this presentation, we therefore investigate the existence of cusp and oscillating singularities on Lévy processes, showing in particular that the latter behaviour is not entirely characterized by the Blumenthal-Getoor exponent. In a second part, we focus on a specific class of Lévy processes, which includes α-stable processes, where the fine fractal structure of these oscillating singularities can be determined.

Speaker: **Julien Barral** (Université Paris 13)
Title: *On the connection between large deviations, multifractals, and some attempt towards applications*
Abstract: We will propose a survey of the connection between large deviations and multifractal analysis of measures and functions, distinguishing between conformal and non conformal situations, and giving an overview of recent progress. We will also show a possible approach to the numerical calculation of large deviations spectra of signals, an its application to certain cardiac signals, compared to the standard multifractal Mandelbrot cascades.

Speaker: **Philippe Ciuciu** (CEA/NeuroSpin & INRIA)
Title: *Scaling phenomena in brain activity: review, evidences, analysis and impact*
Abstract: The field of scale-free neural dynamics represents a renascent interest in the arrhythmic fluctuations of the brain signals, captured conspicuously by a power-law form of the power spectrum. To uncover the functional relevance of such scale-free dynamics and to account for local fluctuations in neural signals, a description that reaches beyond global *univariate* (analysis of one time series at a time) self-similarity is necessary.

In this talk, we first investigate *univariate* scale-free and multifractal properties of Human brain activity recorded with magnetoencephalography (MEG) while participants were trained on a visual perceptual learning task. Contrasting brain activity before and after learning showed intertwined modulations of self-similarity and multifractality in distinct cortical regions that were implicated in the task. Crucially, each individual's multifractality parameter converged towards an attractor value that was common to all individuals suggesting the existence of an asymptotic behavioral performance for all. Our results show that the distance of an individual's multifractality to the common attractor value predicts an individual's learning ability.

Second, functional connectivity, a multivariate analysis for investigating the interaction between regions, especially applied to resting-state functional magnetic resonance imaging (fMRI) signals, has grown into an enormous field of central importance in human brain imaging. This development has allowed neuroscientists and clinicians to assess the dynamic organization of brain networks under various behavioral and clinical states. Given that functional connectivity and scale-free dynamics are mainly concerned with the intrinsic organization of brain activity (one in the temporal domain, one in the spatial domain), an imperative question is whether they are related. Hence, we deployed a recently developed mathematical framework for addressing this exact question (in essence, whether scale-free dynamics in two different time series are related to each other). We found that scale-invariance exists beyond univariate temporal dynamics, manifesting also in the interaction between brain regions (i.e., "bivariate cross-temporal dynamics"). In the frequency domain, within the scale-free range, lower frequencies contribute more to inter-regional functional connectivity. Interestingly, we found that task modulates cross-temporal dynamics by reducing the contribution of lowest frequencies. In sum, our findings provide novel insights into the spatiotemporal organization of intrinsic brain activity.

Speaker: **Gustavo Didier** (Tulane University3)
Title: *Self-similarity, symmetry and anisotropy in the multivariate and multiparameter settings*
Scaling phenomena, or self-similarity, are pervasive in nature and in data, and have been the subject of decades of research in probability and statistics. In higher dimension, self-similarity presents new challenges. These include the theoretical consequences of matrix-scaling, anisotropy, non-identifiability, and their impact on inferential pursuits. In this talk, we will give a broad view of related probabilistic and inferential issues in multidimensional settings. We will describe recent developments for multivariate, multiparameter Gaussian self-similar random fields, the so-named operator fractional Brownian fields (OFBFs). The analysis will draw upon harmonizable integral representations; the latter will allow us to characterize the symmetry groups and anisotropy of OFBFs. We will also discuss recent efforts on wavelet-based inference for multivariate self-similarity.

Speaker: **Patrick Flandrin** (CNRS, Ecole Normale Superieure de Lyon)
Title: *Data-driven methods for scale invariance analysis*
Abstract: The assessment and characterization of scale invariance relies basically on three steps, namely (*i*) choosing some multiresolution description of the observed data; (ii) identifying "straight lines in log-log plots" of suitably chosen "structure functions", and (*iii*) measuring slopes. This general scheme is usually implemented within some fixed, pre-determined framework (typically, wavelets and variations thereof), but it can also be revisited in a more flexible way, thanks to the recent development of data-driven techniques such as Empirical Mode Decomposition (EMD). The talk will review elements in such a direction, starting from a primer on EMD, then elaborating on built-in multiresolution features of the method, and finally commenting on some possibilities it offers for multiscaling analysis.

Speaker: **Jun Kigami** (Kyoto University)
Title: *Diffusions on inhomogeneous media and multifractal analysis*
Abstract: As is well known, a diffusion on homogeneous medium like the Brownian motions on Euclidean space exhibits uniform asymptotic behavior in time-space scaling. For example, for the Brownian motion, the expectation of displacement from the original point is proportional to the square root of time at every

point of the space. However, once you introduce inhomogeneity in the medium, asymptotic behavior of such a quantity sensitively depends on points and it has multifractal structure. In this talk, I will explain how we can change a medium and what kind of multifractal structure we have.

Speaker: **Ken Kiyono** (University of Osaka)
Title: *Non-Gaussian characteristics of heart rate variability in health and disease*
Abstract: Non-Gaussian fluctuations with temporal heterogeneity of local variance are observed in a wide variety of biomedical time series. To characterize such non-Gaussian time series, we introduce a multiplicative stochastic process in which an observed time series is assumed to be described by the multiplication of Gaussian and amplitude random variables [1]. In this framework, we propose an analysis method using log-amplitude cumulants and log-amplitude autocovariance [2], which is capable of characterizing non-Gaussian distributions and multifractal processes. As an application of this method, we will discuss non-Gaussian properties of human heart rate variability [3,4].

[1] K. Kiyono, Log-amplitude statistics of intermittent and non-Gaussian time series. Phys. Rev. E 79, 031129 (2009).
[2] K. Kiyono, Konno H. Log-amplitude statistics for Beck-Cohen superstatistics. Phys. Rev. E 87, 052104 (2013).
[3] K. Kiyono et al. Non-Gaussian heart rate as an independent predictor of mortality in patients with chronic heart failure. Heart Rhythm 5, 261-268 (2008).
[4] J. Hayano et al. Increased non-Gaussianity of heart rate variability predicts cardiac mortality after an acute myocardial infarction, Frontiers in physiology 2, 65 (2011).

Speaker: **Ka-sing Lau** (Chinese University of Hong Kong)
Title: *Spectral property of self-similar sets and measures*
Abstract: Let μ be a probability measure with compact support K on $\mathbb{R}^d$. μ is called a *spectral measure* if $L^2(\mu)$ admits an orthonormal basis of the form $\{e^{2\pi i\langle\lambda,\cdot\rangle}\}_{\lambda\in\Lambda}$ and if μ is the Lebesgue measure, then K is called a spectral set. In the 70's, Fuglede made a conjecture that K is a spectral set iff it is a translational tile. Although the problem was disproved eventually, it generates a lot of interesting questions. In this talk, we will give a discussion of the problem on the self-similar sets/measures.

Speaker: **Shaun Lovejoy** (McGill University)
Title: *Rocks, clouds, anisotropic multifractals and the unity of geophysics*
Abstract: Scaling processes abound in geophysics and this has important consequences for the statistics of the corresponding intensive and extensive geophysical variables. Classical scaling processes – such as in classical turbulence – are self-similar, they are characterized by exponents which are invariant under isotropic scale changes. However, the atmosphere and solid earth are strongly stratified so that we must generalize the notion of scale allowing for invariance under anisotropic zooms. When this is done, it is often found that scaling can apply over huge ranges, up to planetary in extent.

The generic scaling process is the multifractal cascade in which a scale invariant dynamical mechanism repeats (multiplicatively) from scale to scale. Anisotropic scale invariance is thus a symmetry principle that unifies geosystems – including rocks and clouds – by implying the existence of common underlying multifractal processes. The existence of multifractal universality classes – implies that multifractals are much relatively easy to handle since the infinite hierarchy of multifractal exponents is reduced to a small finite number (three). We demonstrate this on a wide variety of atmospheric and solid earth geophysics fields.

General (canonical) multifractal processes have nontrivial extremes. When developed over finite ranges of scale and analyzed at their smallest scale (the "bare" process), already have "long-tailed" distributions (e.g. the lognormal). However the small scale cascade limit is singular so that the integration/averaging of cascades developed down to their small scale limits leads to "dressed" properties characterized notably by (heavier) "fat-tails": power law probability distributions $\Pr(x > s) = s^{-q_D}$ where x is a random value,

s a threshold and q_D the critical exponent implying that the moments for $q > q_D$ diverge. For cascades averaged over scales larger than the inner cascade scale, the moments $q > q_D$ are no longer determined by the large scale finite by the small scale details: the"multifractal butterfly effect". The sampling properties of such processes can be understood with "multifractal phase transitions"; we review this as well as evidence for the divergence of moments in laboratory, atmospheric and climatological series, and in data from the solid earth and discuss implications (abrupt changes, etc.).

Speaker: **Franlin Mendivil** (Acadia University)
Title: *Some applications of fractal methods in imaging*
Abstract: Fractal (or multi-scale) methods have been used in image processing for quite some time, first starting with data compression techniques and then later continuing with denoising, edge detection, zooming and other common image processing tasks. In this talk we will first outline the basic block fractal-coding algorithm and discuss some of its extensions and variations. We will then review a few of the other applications of fractal methods in image processing, focusing in particular on denoising and zooming.

Speaker: **Stephane Seuret** (Université Paris Est Créteil)
Title: *p-exponents and p-multifractal spectrum of some lacunary Fourier series*
Abstract: We study the Fourier series $R_s(x) = \sum_{n\geq 1} \frac{\sin(\pi n^2 x)}{n^s}$ when $0 < s < 1$. In this range of parameters, it is easily seen that R_s does not convergence everywhere. We prove that the convergence of $R_s(x)$ depends on Diophantine properties of x. Then, at each point of convergence x, we compute the L^2-pointwise exponent of R_s, and we deduce its L^2-multifractal spectrum.

Speaker: **Murad Taqqu** (Boston University)
Title: *Self-Similarity beyond Gaussian processes: Hermite processes and more*
Abstract: We will start with a review of basic notions related to self-similarity and introduce the Hermite processes which are defined through Wiener-It multiple integrals. The first Hermite process is fractional Brownian motion, the second is the non-Gaussian Rosenblatt process. We will then focus briefly on the marginal distribution of the Rosenblatt process. We will also define generalized Hermite processes and state a multivariate limit theorem with generalized Hermite processes as limits

Speaker: **Beatrice Vedel** (Université de Bretagne Sud)
Title: *Directional regularity of random fields*
Abstract: The study of regularity via a multifractal analysis gives information on the roughness of textures. Nevertheless, in presence of anisotropy, a classical multifractal analysis fails to describe correctly the regularity properties of the texture. We will see how a multifractal analysis based on anisotropic regularity - and in practice on hyperbolic wavelet analysis - can help to recover the characteristics of some mono and multifractal random fields.

Speaker: **Yang Wang** (Michigan State University)
Title: *The Cantor set: generating IFS and self-similar subsets*
Abstract: The classical middle 3rd Cantor set is perhaps the best known fractal sets and to most researchers in fractal geometry there is probably very little that we don't understand about this set. But some of the open questions may surprise people. One such problem was raised by Mattila (1998): What are the self-similar subsets of it? In particular, are all such sets "derived" from the IFS that generates the Cantor set? The question turns out to be far more challenging than one may think. A related question is: Beside the standard IFS to generate the Cantor set, are than others that are not derived for the standard one? The second questions can also be asked for a general self-similar set, and we may ask whether there is always a "minimal" generating IFS such that every generating IFS is derived from it? In this talk, we shed lights on both questions.

Speaker: **Ed Waymire** (Oregon State University)
Title: *On normalized multiplicative cascades under strong disorder*

Abstract: Much of the mathematical theory of multiplicative cascades pertains to fine scale structure of the non-trivial limit of a sequence of T-martingales introduced by Kahane and Peyriere (1976) under conditions of weak disorder. The latter development was motivated by discussions with Benoit Mandelbrot on the nature of statistical turbulence models being discussed in the physics literature. The same T-martingale limit is trivial under conditions of strong disorder unless one normalizes the sequence to probabilities on a compact space. This talk will concern various conditions for existence of a weak limit of these probabilities and the structure of the limit so-obtained. This is based on joint work with Partha Dey at Warwick University and the University of Illinois at Urbana-Champaign.

Speaker: **Herwig Wendt** (CNRS, University of Toulouse)
Title: *Wavelet leaders and p-leaders multifractal analysis: theory and practice for signals and images*
Abstract: p-leaders are novel multiresolution quantities that have been recently introduced in the mathematical literature. In this talk, we will introduce its associated local regularity exponent, the p-exponent, and define and study the corresponding p-leaders multifractal formalism. We will establish connections with the Hölder exponent as well as with wavelet leader multifractal formalisms. We illustrate and compare the relative practical advantages and limitations of wavelet leaders and p-leaders for performing the multifractal analysis of discrete signals and images in applications with several numerical examples.

Speaker: **Michel Zinsmeister** (Université d'Orléans)
Title: *Multifractality of whole-plane SLE*
Abstract: Loewner introduced his famous differential equation in 1923 in order to prove the $n = 3$ case of Bieberbach conjecture. His theory associates to each slitted growth process a continuous function from $\mathbb{R}_+$ to the unit circle called the driving function, and vice-versa. In 1999 Oded Schramm had the idea of taking as driving functions Brownian motions, leading to the rapidly growing rich theory of SLE. In this work we revisit Bieberbach conjecture in the framework of SLE and apply the results to the study of the mean multifractal spectrum of whole-plane SLE. (Joint work with Bertrand Duplantier, Nguyen Thi Phuong Chi, Nguyen Thi Thuy Nga.)

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POSTER ABSTRACTS
(in alphabetic order by presenter surname)

Presenter: **Patrice Abry** (CNRS - ENSLyon)
Title: *Brueghel's Drawing under the Multifractal Microscope*
Abstract: Recently, a growing interest in the exploration of the potential of signal or image processing tools for the purposes of art analysis has emerged. The wavelet leader based multifractal analysis consists of a mathematical tool recently introduced in image processing for the characterization of homogeneous textures based on their regularity properties. Here, this novel tool is applied to a set of digitized versions of drawings, made available by the NY Metropolitan Museum of Art, consisting of authentic Bruegel drawings and several imitations. Multifractal attributes are estimated from several patches of each of these drawings, and their ability to discriminate authentic drawings from impostors is investigated by means of subspace projections and quadratic discriminant analysis. Besides showing very satisfactory performance, the achieved discrimination provides interesting insights into the differences between the regularity of the textures of authentic Bruegel drawings versus imitations, potentially relating the fractal properties of the drawings to the artist's drawing style. (Joint work with H. Wendt and S. Jaffard)

Presenter: **Shuyang Bai** (Boston University)
Title: *Fractional processes on Wiener Chaos and non-central limit theorems*
Abstract: By fractional processes, we mean self-similar processes with stationary increments. These processes are important because of their connection to the scaling limits of sum of stationary sequences. If the scaling limit is not a Brownian motion, this type of results are called non-central limit theorems. We focus here on some fractional processes defined on a Wiener chaos of a single order. In particular, we introduce a class of processes called generalized Hermite processes, which include the fractional Brownian motion, and more generally, the Hermite processes considered in the literature. We obtain new non-central limit theorems where the generalized Hermite processes arise as the scaling limits of some long-memory nonlinear stationary sequences.

Presenter: **Joan Bruna** (New York University)
Title: *Multifractal Analysis with Scattering Moments*
Abstract: Scattering moments provide non-parametric models of random processes with stationary increments. They are defined as expected values of a non-linear operator resulting from iteratively applying wavelet transforms and modulus non-linearities, which preserves the variance. First and second order scattering moments are shown to characterize intermittency and self-similarity properties of multiscale processes. Scattering moments of Poisson processes, fractional Brownian motions, Levy processes and multifractal random walks have characteristic decay which are explicitly calculated. The Generalized Method of Simulated Moments is applied to scattering moments to estimate data generating models. We show numerical applications to financial time-series and to energy dissipation of turbulent flows.

Presenter: **Céline Esser** (Université de Liège)
Title: *Revisiting S^ν spaces with wavelet leaders to detect non concave and non increasing spectra*
Abstract: A multifractal formalism is a formula which is expected to yield the spectrum of singularities of a function from quantities which are numerically computable. The most widespread of these formulas is

the so-called thermodynamic multifractal formalism, based on the Frish-Parisi conjecture [5]. It presents two drawbacks: it can hold only for spectra which are concave and it can only yield the increasing part of the spectrum. This first problem can be avoided using S^ν spaces [1,4]. The second one can be taken care using the wavelet leaders method [6].

In this poster, we present a new multifractal formalism based on a generalization of the S^ν spaces using wavelet leaders. It allows to detect non concave and non increasing spectra [2,3]. We compare this formalism with the S^ν method and the wavelet leaders method. It is based on joint works with F. Bastin, S. Jaffard, T. Kleyntssens and S. Nicolay.

[1] J.M. Aubry, F. Bastin and S. Dispa, *Prevalence of multifractal functions in S^ν spaces*, J. Fourier Anal. Appl., **13**, 2, 175–185, 2007.
[2] F. Bastin, C. Esser and S. Jaffard, *Beyond S^ν spaces with wavelet leaders*, preprint.
[3] C. Esser, T. Kleyntssens, S. Jaffard and S. Nicolay, *A multifractal formalism for non concave and non increasing spectra: the L^ν spaces approach*, preprint.
[4] C. Esser, T. Kleyntssens and S. Nicolay, *A multifractal formalism based on the S^ν spaces: from theory to practice*, submitted for publication.
[5] S. Jaffard, *On the Frisch-Parisi conjecture*, J. Math. Pures Appl., **79**, 6, 525–552, 2000.
[6] S. Jaffard, B. Lashermes and P. Abry, *Wavelet Leaders in Multifractal Analysis*, Wavelet analysis and applications, Appl. Numer. Harmon. Anal., 201–246, 2007.

Presenter: **Julien Hamonier** (Ecole Normale Superieure de Lyon)
Title: *Estimation of the Hurst functional parameter of Linear Multifractional Stable Motion*
Abstract: The Linear Multifractional Stable Motion (LMSM) is a symmetric α-stable stochastic process which was introduced by Stoev and Taqqu in [5,6] with a view to model some features of traffic traces on telecommunications networks, typically changes in operating regimes and burstiness (the presence of rare but extremely busy periods of activity). This process is obtained by replacing the constant Hurst parameter of the Linear Fractional Stable Motion (LFSM) by a function $H(\cdot)$. We will assume that $\alpha \in (1,2)$, the function $H(\cdot)$ takes values in $(1/\alpha, 1)$ and satisfies an uniform Hölder condition.

In the case of LFSM, the statistical issue of estimation of H has already been studied in several works by using wavelet coefficients or discrete variations of this process. (see e.g. [1,2,3,4]).

In the LMSM's case, the problem is harder since the Hurst parameter changes with time and the increments of the process are no longer stationary. Therefore, we propose to estimate $H(t)$ and $\min_{t\in I} H(t)$ (I is a fixed, non-empty, compact interval) thanks to discrete variations of LMSM. *A joint work with Antoine Ayache, University Lille1.*

[1] Abry, P. and Pesquet-Popescu, B. and Taqqu, M. S., *Estimation ondelette des paramètres de stabilité et d'autosimilarité des processus α-stables autosimilaires*, 17ème Colloque sur le traitement du signal et des images, FRA, 1999, GRETSI, Groupe d'Etudes du Traitement du Signal.
[2] Delbeke, L., *Wavelet based estimators for the Hurst parameter of a self-similar process*, Ph.D. thesis, KU Leuven, Belgium, 1998.
[3] Delbeke, L. and Abry, P., *Stochastic integral representation and properties of the wavelet coefficients of linear fractional stable motion*, Stochastic Processes and their Applications, 86, 2, 177–182, 2000.
[4] Stoev, S. and Pipiras,V. and Taqqu, M. S., *Estimation of the self-similarity parameter in linear fractional stable motion*, Signal Processing, 82, 1873–1901, 2002.
[5] Stoev, S. and Taqqu, M. S., *Stochastic properties of the linear multifractional stable motion*, Advances in applied probability, 36, 4, 1085–1115, 2004.
[6] Stoev, S. and Taqqu, M. S., *Path properties of the linear multifractional stable motion.* Fractals, 13, 2, 157–178, 2005.

Presenter: **Roberto Leonarduzzi** (Universidad Nacional de Entre Rios)
Title: *Scaling range automated selection in wavelet leader based multifractal analysis*

Abstract: Scale invariance and multifractal analysis constitute paradigms nowadays widely used for real-world data characterization. In essence, they amount to assuming power law behaviors of well-chosen multiresolution quantities as functions of the analysis scale. The exponents of these power laws, the scaling exponents, are then measured and involved in classical signal processing tasks. Yet, the practical estimation of such exponents implies the selection of a range of scales where the power law behaviors hold, a difficult task with yet crucial impact on performance. In the present contribution, a non parametric bootstrap based procedure is devised to achieve scaling range automated selection. It is shown to be effective and relevant in practice. Its performance, benefits and computational costs are assessed by means of Monte Carlo simulations. It is applied to synthetic multifractal processes and shown to yield robust and accurate estimation of multifractal parameters, despite various difficulties such as noise corruption or inter-subject variability. Finally, its potential is illustrated at work for the analysis of adult heart rate variability on a large database.

Presenter: **N. N. Leonenko** (Cardiff University)
Title: *RÉnyi functions for multifractal products of stationary processes and detecting multifractality under heavy-tailed effects*
Abstract: The first part is joint work with D.Denisov (Manchester University, UK).

Multifractal and monofractal models have been used in many applications in hydrodynamic turbulence, finance, computer network traffic, etc. (see, for example, [7]). There are many ways to construct random multifractal models ranging from simple binomial cascades to measures generated by branching processes and the compound Poisson process ([4]-[7]).

Anh, Leonenko and Shieh ([1]-[3]) and Leonenko and Shieh [9] considered multifractal products of stochastic processes as defined in [10], but they provide a new interpretation of the conditions on the characteristics of geometric stationary processes in terms of the moment generating functions.

We investigate the properties of multifractal products of geometric Gaussian processes with possible long-range dependence and geometric Ornstein-Uhlenbeck processes driven by Lévy motion and their finite and infinite superpositions. We present the general conditions for the $\mathcal{L}_q$ convergence of commulative processes to the limiting processes and investigate their q-th order moments and Rényi functions, which are nonlinear, hence displaying the multifractality of the processes as constructed. We also establish the corresponding scenarios for the limiting processes, such as log-normal, log-gamma, log-tempered stable or log-normal tempered stable scenarios.

The second part is joint work with D.Grahovac (University of Osijek, Croatia).

We provide rigorous proof that estimating the scaling function using the partition function can lead to nonlinear estimates under the presence of heavy tails. These results shed new light on many data sets that were claimed to be multifractal by using the partition function method. This is particularly important for financial data, which is generally accepted to possess heavy tails, thus can produce nonlinear scaling functions when there is no multiscaling. Scaling functions can be estimated correctly, but only when the range of finite moments is known. This makes multifractal definition based on moment scaling impractical. Results proved in the paper are concerned with processes with short range dependence properties. However, it is to expect that infinite moments produce similar behavior of the scaling function also in the case of long range dependence, with possible involvement of dependence parameter. It is known that processes such as multiplicative cascade and multifractal random walk have heavy-tails. It can thus be suspected that combined effect of dependence and heavy tails may produce nonlinear empirical scaling functions in these models ([8]).

N.N. Leonenko partially supported by projects MTM2012-32674 of the DGI, and P09-FQM-5052 of the Andalousian CICE, Spain.

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Presenter: **Peter Moerters** (University of Bath)
Title: *Emergence of condensation in Kingman's model of selection and mutation*
Abstract: We describe the onset of condensation in Kingman's model for the balance between selection and mutation in terms of a scaling limit theorem. Loosely speaking, this shows that there are three different possible shapes of the wave moving towards genes of maximal fitness: the shape can be that of an exponential, gamma or a normal distribution. The gamma distribution is the most common, and we conjecture that this wave shape is a universal phenomenon that can also be found in a variety of more complex models, well beyond the genetics context.

Presenter: **Stephanie Rendon de la Torre** (Universidad Nacional Autnoma de México)
Title: *Applications on financial markets*
Abstract: This poster is about interesting empirical information of some characteristics obtained after evaluating the dynamical behavior of 13 important and international markets (China, South Korea, Israel, Mexico, Brasil, India, Indonesia, Turkey, Germany, France, US(2), UK) indices and 10 foreign exchange markets (MXN, CAD, CNY, JPY, RUB, CFH, RLS, AUD, EUR, GBP all versus USD) by using fractal and multifractal tools such as spectral analysis, Hurst exponents (R/S, generalized and wavelets), Holder and Lyapunov exponents. The main purpose is to show how an alternative approach like the multifractalism might be very helpful on further understanding and describing financial markets behavior and how it is a helpful tool in asset and specially risk management.