Dynamics in Geometric Dispersive Equations and the Effects of Trapping, Scattering and Weak Turbulence

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1 Overview of the Field

One of the major success stories in analysis over the past couple of decades is the deep and detailed insight into the qualitative properties of solutions to nonlinear dispersive PDE from Mathematical Physics which has been gained through application of techniques from harmonic analysis, spectral theory, the calculus of variations, and dynamical systems. In this way, our understanding of the nonlinear waves which characterize the dynamics of various systems in quantum physics, general relativity, optics, and fluid mechanics (just to name a few) has increased enormously over a remarkably short period.

This understanding extends to questions of local and global well-posedness, low-regularity solutions, singularity formation, asymptotic behaviour, the existence and stability of special solutions (such as solitons, or various threshold solutions), and the role such special solutions play in the general dynamics. Progress has been such that it could be said, very roughly speaking, that our mathematical comprehension of nonlinear dispersive PDE which are (a) posed on Euclidean space, and (b) of a sub-critical (roughly, conserved quantities provide some natural control over the size of solutions), or even (due to groundbreaking advances of the last few years) critical nature, is now rather good.

On the other hand, it is just as reasonable to say that nonlinear wave equations outside of this category are still quite poorly understood, due to inherent new difficulties. For equations posed on compact domains – such as compact manifolds – dispersion takes on a very different character, since waves cannot escape to infinity, and so interact with each other indefinitely. On domains with smooth obstacles or boundary, the effects of trapping or scattering can profoundly impact the behavior of nonlinear waves, whereas in the case of boundaries with edges or corners, the effects of diffraction can also play a role. For equations with non-trivial domain geometry, or target geometry (i.e. for maps into curved spaces), the challenge is to reveal and quantify whatever dispersion inhibiting/enhancing, trapping, or focusing/defocusing effects the geometry introduces. In each of these circumstances, dynamical systems theory has played a major role in studying the underlying Hamiltonian dynamics, analyzing the spectrum of a domain, or characterizing special nonlinear solutions related to underlying symmetries of the operator and the geometry. For super-critical equations, the overriding difficulty is to operate in the absence of useful, natural conserved quantities.

As it happens, numerous PDE of this type arise naturally in applications, in such diverse areas as general relativity, plasma models, magnetics, optics, and water waves. Moreover, some of the key physical examples of super-critical equations, such as the Einstein equations, and the gauge theories of particle physics, are inherently geometric. For all of these reasons, the field seems to be at something of a turning point, with researchers just beginning en masse to explore some of these challenging new directions. In particular, there has been a noticeable recent movement toward problems with a geometric character, leading to remarkable contributions from dynamical systems theory.

2 Recent Developments and Open Problems

The workshop was broken up into several themed sessions covering the topics of nonlinear PDE on compact domains; nonlinear/linear wave equations stemming from the study of General Relativity; the equations of Water Waves; and lastly the interaction of statistics and PDE. Below will give an overview of each topic and highlight the advances presented at the workshop.

Deterministic PDE on compact domains

The workshop featured talks by Dave Ambrose, Sebastian Herr, Benoît Grébert and Piotr Bizoń about solutions to nonlinear PDE in various settings where wave interactions take on a very different character on long time scales due to the lack of dispersion.

- In the setting of periodic domains, Ambrose in recent work with J. Doug Wright has a result showing that a dispersive equation of the form

  $$u_t = Au + N(u)$$

  on a periodic domain can have a measure 0 set of temporally periodic solutions provided that the linear operator, A, is strongly dispersive and the nonlinearity, N, has few enough derivatives in it such that local smoothing estimates can still be employed in a periodic setting on the Duhamel term of the solution,

  $$\int_0^t e^{A(t-\tau)} N(u(\tau)) d\tau.$$

  Indeed, even a compact manifold like the torus has can have smoothing properties due to averaging. This connects to the works of [45], [37, 15, 16, 85] and can be read about in the pre-print [9]. The techniques are centered around those of small-divisors and Nash-Moser iteration, but connects nicely to the mapping properties of strongly dispersive equations. Right now, the authors can treat several model problems that look somewhat like fifth-order KdV,

  $$\partial_t u + \partial_x^5 u - u u_x = 0,$$

  among others. The goal will be to push the requirements on the smoothing properties of the operator as low as possible with future refinements to capture more and more interesting models for which people are interested in periodic solutions, such as water wave equations, but also increase the dimension.

- Following the early work of Bourgain [14, 18] and Burq-Gerard-Tzvetkov [22, 23, 24, 25] proving well-posedness properties of nonlinear dispersive equations on compact manifolds, Herr has been working for some time looking at critical nonlinear Schrödinger equations on compact domains and trying to determine the effects of the geometry (and in particular the natural spectrum of the Laplacian) impacts the long time well-posedness properties, see [56, 58, 57]. Recently, in joint work with his student Nils Strunk, Herr has managed to show that on $M = \mathbb{S} \times \mathbb{S}^2$, the energy critical Nonilnear Schrödinger equation,

  $$i\partial_t + \Delta_g u = \pm |u|^{p-1} u, u(0, \cdot) = u_0 \in H^1(M),$$

  is globally well-posed and that the argument boils down to a trilinear estimate of the form

  $$\| P_{N_1} e^{it\Delta} \phi_1 P_{N_2} e^{it\Delta} \phi_2 P_{N_3} e^{it\Delta} \phi_3 \|_{L^2(\tau_0 \times M)} \lesssim \left(\frac{\langle N_3 \rangle}{\langle N_1 \rangle} + \frac{\langle 1 \rangle}{\langle N_2 \rangle} \right)^{\delta} \langle N_2 \rangle \langle N_3 \rangle \Pi_{j=1}^3 \| \phi_j \|_{L^2(M)}$$

  for $N_1 \geq N_2 \geq N_3$, some $\delta > 0$. In the case of $M = \mathbb{S} \times \mathbb{S}^2$, the authors are able to reduce the estimate to a statement about exponential sums due to Bourgain, [16]. The result uses the atomic U^p, V^p space machinery that has made great headway in treating critical problems in recent years, following their development in for instance [70, 55, 71]. The spectrum of the Laplacian on compact manifolds is intimately linked to application of the trilinear estimate here, which is why so far it has only been verified in specific cases such as the sphere and the torus (though now including irrational tori thanks to recent work of Strunk [88]. Advances in spectral estimates on more general domains following the work of say, Sogge [86] will be crucial for the advances in understanding nonlinear interactions on compact domains.

- Grébert discussed recent works with Tiphaine Jézéquel and Laurent Thomann, where they study the stability of large homoclinic orbits in periodic Klein-Gordon equations, [49, 50]. In other words, as a nice connection to the results above by Ambrose studying non-existence and the small data results of Herr-Strunk, the authors construct KAM-torus ([48, 89, 10]) like closed solutions for a nonlinear wave equation of the form

$$\partial_t^2 - \Delta_g u - m^2 u + u^{2p+1} = 0$$

for M a volume 1 Riemannian surface without boundary in up to 3 dimensions with $m < \lambda_1$ the second(!) eigenvalue of Δ_g and $p \geq 1$ for $d = 2$, $p = 1$ for $d = 3$. Note that the mass here has an unusual sign for what most would consider for a Klein-Gordon type equation, which allows one to look for stationary solutions using an eigenvalue decomposition

$$u = a_0 e_0 + \sum_{k=1}^{\infty} a_k e_k, \;\; \dot{v} = a_0 e_0 + \sum_{k=1}^{\infty} a_k e_k.$$

Decomposing the natural energy of the equation gives an approximate Hamiltonian system for (a_0, b_0), which has elliptic points at $\pm m^{\frac{1}{p}}$ and immediately see that doing an eigenvalue decomposition allows one to show that they are stable under perturbation for at least long time scales. Such KAM-torus like solutions both highlight the difficulties in getting nonlinear equations to disperse in a meaningful sense, as well as give explicit ways to understand the collection or transfer of energy for PDE on compact manifolds. These will play a crucial role in the long-time and large-data understanding of dynamics on compact manifolds.

- The question of so-called weak turbulence for dispersive equations is an important ongoing investigation tied into the underlying statistical mechanics of a Hamiltonian PDE in a compact setting, see [17, 31, 46, 51, 52, 74, 53, 54]. The basic question is whether some form of transfer of energy to high frequency happens over long time scales. Many works have attempted to understand such a phenomenon and Bizoń in joint work with Patryk Mach and Maciej Maliborski presented some recent advances and numerical simulations related to toy models for the Einstein equations. It has been conjectured that there is a mechanism for instability of Anti-De Sitter space through a weakly turbulent transfer of energy to high frequencies, which with collaborators Bizoń has been exploring numerically using semilinear wave equations as leading order toy models. This talk could easily have fit into the machinery of geometric wave equations or statistical mechanics in many ways. However, in the end the strongest results presented showed simulations of numerical solutions to

$$u_{tt} - \Delta u + m^2 u + u^3 = 0$$

on a compact manifold. The goal is to understand the out of equilibrium dynamics for small solutions, which they studied using the power spectrum of the solution in higher Sobolev norms over long times. On the circle for instance, there is a resonant system of ODEs eliminating small divisors such that

$$\pm 2in\dot{a}_n^{\pm} = \epsilon^2 \sum_{j-k+m=n} a_j^{\pm} \bar{a}_k^{\pm} a_m^{\pm} + 2\epsilon^2 \left(\sum_k |a_k^{\mp}|^2 \right) a_n^{\pm}.$$

In such a case, an exponential decay appears to arise in the power spectrum preventing a long time motion towards high frequency, which is of course possibly meta-stable. A Yang-Mills wave-map type equation on $\mathbb{S}^3$ exists, which simplify to the form

$$W_{tt} = W_{xx} + \frac{W(1-W^2)}{\sin^2 x}$$

has a similar structure. An equivariant wave map from $\mathbb{R} \times \mathbb{S}^3 \to \mathbb{S}^3$ of the form

$$W_{tt} = W_{xx} + \frac{2\cos x}{\sin x} U_x - \frac{\sin(2U)}{\sin^2 x}$$

looks like it has good unstable behavior but does not have a resonant system. However, a $4d$ wave map from $\mathbb{R}\times$ (some model manifold) $\to$ AdS related to Bianchi Nine Models, which has trully resonant spectrum and can be written

$$W_{tt} = W_{xx} + \frac{3}{\sin x \cos x} U_x - \frac{F(U)}{\sin^2 x}$$

for $F(U) = \frac{4}{3}(e^{-2U} - e^{-8U})$ appears to have preliminary power-law spectrum cascades. This interesting and quite application oriented talk led to a great deal of discussions with people interested in nonlinear wave maps equations and GR throughout the workshop.

Fluids

The workshop featured talks by about various asects of fluids models by Thomas Alazard, Roberto Camassa, Benjamin Harrop-Griffiths, Mihaela Ifrim, Herbert Koch, Jon Wilkening, and Fabio Pusateri.

- The generalized Korteweg-de Vries equation

  $$\partial_t u + \partial_x^3 u + \partial_x(|u|^{p-1} u) = 0$$

  arises as a long-wave model for water waves ($p = 2, 3$) and in its general form as a model for plasmas for instance. However, for many years it has been mostly an excellent model for studying nonlinear dispersive phenomena because it has such a rich structure. The workshop featured two talks on these types of models, one showing the existence of self-similar blow-up profiles for slightly super-critical KdV problems ($p > 5$) and one showing modified scattering for the mKdV equation ($p = 3$). Both models have effective dynamics driven by in one case a modified elliptic equation solved through careful implicit function theorem techniques and another on the tools of modified scattering. In particular, Harrop-Griffiths observes that the long-time decay properties for mKdV are related to a Painlevé type ODE. See [69] for the blow-up result, with the pre-print by Harrop-Griffiths forthcoming but motivated by recent advances in modified scattering from [62, 63].

- The gravity wave equations have recently seen a great deal of progress as quasilinear techniques have become more readily available and the equations better understood. Through a combination of different menthods, various groups have made progress on the problem recently, especially for gravity-waves in $2d$ on global time scales. Reports on this progress were made by Thomas Alazard, Mihaela Ifrim, Alex Ionescu, and Fabio Pusateri. The gravity-capillary wave equations can be represented as

  $$\begin{cases} \partial_t h = |D|\,\psi + \{G(h)\psi - |D|\,\psi\} \\ \partial_t \psi = (\tau\Delta - g)\,h + \left\{ \tau\left(\operatorname{div}\left(\frac{\nabla h}{\sqrt{1+|\nabla h|^2}}\right) - \Delta h\right) \right. \\ \qquad\qquad \left. -\frac{1}{2}\,|\nabla\psi|^2 + \frac{(G(h)\psi + \nabla h\cdot\nabla\psi)^2}{2\left(1+|\nabla h|^2\right)} \right\}. \end{cases}$$

  for h the height of the fluid at the interface and ψ a trace of a related velocity field. These equations have a great deal of quaslinear structure, but when $\tau = 0$, breakthroughs have been made in the work of [90, 91, 47, 2, 3, 36]. Recently, our understanding of these equations has improved using a microlocal paradifferential approach in [4, 2, 3], a modified energy method in holomorphic coordinates in [61, 63] and the theory of space-time resonances with weights as in [64]. The directions moving forward include understanding the long-time effects of surface tension (discussed by Pusateri in a model problem), moving to higher dimensions, looking at interactions with internal waves (see the work of Camassa below), etc. There are some benefits to a choice of Eulerian coordinate versus Holomorphic coordinates and vice versa that were also discussed at length in the workshop. The work of Pusateri was motivated by the presentation of Ionescu relating to models for particles in a plasma, where the Euler equations are couple to an electromagnetic field. This Euler-Maxwell system can be treated with a similar modified energy technique and represents joint work with Y. Deng, Y. Guo and B. Pausader.

- Roberto Camassa and Jon Wilkening talked about various numerical and modeling results related to water waves. Wilkening discussed new families of time-periodic and quasi-periodic solutions of the free-surface Euler equations involving standing-traveling waves and collisions of solitary waves of various types. The new solutions are found to be well outside of the KdV and NLS regimes and a Floquet analysis shows that many of the new solutions are linearly stable to harmonic perturbations. Evolving such perturbations (nonlinearly) over tens of thousands of cycles suggests that the solutions remain nearly time-periodic forever. See [5, 6, 7, 8, 92, 93]. Camassa discussed models for internal waves. In particular, he focussed on one of the simplest physical setups supporting internal wave motion, which is that of a stratified incompressible Euler fluid in a channel. He discussed asymptotic models capable of describing large amplitude wave propagation in this environment, and in particular of predicting the occurrence of self-induced shear instability in the waves' dynamics for continuously stratified fluids. See [32, 33, 34]. Many experiments were also compared to the asymptotics and numerics. T. Alazard was submitting a paper about capillary waves around the same week and was able to connect some of their observations to the numerical experiments of Wilkening, [1].

Waves and Geometry

The workshop featured talks by about various aspects of wave equation models by Ben Dodson, Alex Ionescu, Andrew Lawrie, Sun-Jin Oh, Joachim Krieger, Jason Metcalfe, Paul Smith and Jacob Sterbenz.

- A recent asymptotic stability (see [73]) result for equivariant wave maps from $\mathbb{H}^2 \to \mathbb{S}^2$ or $\mathbb{S}^2$ was discussed by Lawrie and Oh as joint work with Sohrab Shashahani. The central equation is

$$\psi_{tt} - \psi_{rr} - \coth r \psi_r + \frac{(g(\psi) g'(\psi)}{\sinh^2 r} = 0$$

  for $g = \sin u$ for target $\mathbb{S}^2$ and $g = \sinh u$ for target $\mathbb{H}^2$. The result brings together many of the ideas from stability theory for wave equations as well as geometric analysis and dispersive equations. A main novelty of the approach addresses the spectrum of the linearized operator about various explicit equivariant wave maps and in particular for a one parameter family of such states. See [75] for a similar asymptotic stability result but in a more restrictive geometry.

- The nonlinear interactions of the wave equation have been a major open area in dispersive PDE for a long time now. At this workshop some new advances were introduced related to long-time behavior of nonlinear waves. In particular, several talks addressed solutions to the problem

$$u_{tt} - \Delta_g u + \mu u^p = 0$$

  in different settings. Metcalfe presented a new result with Chengbo Wang, Hans Lindblad, Chris Sogge and Mihai Tohaneanu proving the Strauss conjecture on black hole-backgrounds, see [77]. In particular, given the Schwarzschild metric or the Kerr metric in $3d$ (any metric that has local energy decay estimates and is asymptotically flat really) the authors prove that for $p > 1 + \sqrt{2}$, or the Strauss exponent, small data smooth solutions exist for all time. In a complimentary talk, Sterbenz talked about recent work he has done with Daniel Tataru and his student Jesus Oliver showing that local energy decay can be applied to give results in a large variety of geometric settings, in particular, refining Klainerman vector field techniques to treat black hole space times. See for instance [87] for some earlier work with the pre-print forthcoming otherwise. Using concentration compactness tools showing that the energy must push to 0, Dodson in collaboration with A. Lawrie has shown a conditional scattering result for radial initial data for $p = 3$ in $3d$ Euclidean space, see [38]. Namely, if the energy critical norm $\dot{H}^{\frac{1}{2}} \times \dot{H}^{-\frac{1}{2}}$ remains bounded, their result proves that it must scatter to a linear solution. Concentration compactness tools have appeared previously in works such as [65, 66, 67, 68, 39, 40, 41]. This connects back to some numerical work Bizon and collaborators have done in the past as well, see [13]. Finally, Krieger talked about joint work with Wilhelm Schlag on the strongly supercritical wave equation for $p = 7$ in $3d$, see [72]. They are able to prove that a class of large data solutions exist and are stable under certain types of perturbation in the critical space $H^{\frac{7}{6}} \times H^{\frac{1}{6}}$. These solutions are self-similar, but with weak decay due to the supercritical nature of the problem. For a slight shift, Paul Smith discussed work with Baoping Liu on the Chern-Simons-Schrödinger equation in an equivariant setting. This equation is more a nonlinear Schrödinger equation coupled to Magnetic Field, but can be reduced to to a system of the form

$$\begin{aligned} (i\partial_t + \Delta)\phi &= \frac{2m}{r^2} A_\theta \phi + A_0 \phi + \frac{1}{r^2} A_\theta^2 \phi - g|\phi|^2\phi, \\ \partial_r A_0 &= \frac{1}{r}(m + A_\theta)|\phi|^2, \\ \partial_t A_\theta &= r \mathrm{Im}(\bar{\phi}\partial_r \phi), \\ \partial_r A_\theta &= -\frac{1}{2}|\phi|^2 r, \\ A_r &= 0. \end{aligned}$$

This is a very nonlocal, nonlinear Schrödinger equation but nonetheless, using again concentration compactness like techniques, they are able to prove global existence and scattering in each equivariant class, which is related to a vortex solution for NLS. See [76].

The Interaction of PDE and Statistical Mechanics

The workshop featured talks by about various aspects of the study of statistics in dispersive PDE by Aynur Bulut, Natasa Pavlovic, Jonathan Mattingly, Andrew Nahmod, Tadahiro Oh, and Nicolas Burq.

- The use of Gibbs Measures in low regular PDE Existence Theory was explored by Burq, Bulut, Nahmod and Oh. In particular, Burq introduced work in progress with Laurent Thomann and Nikolay Tzvetkov defining a modified version of the Gibbs measure approach in dispersive PDE on compact manifolds, which they can possibly introduce for any domain in $\mathbb{R}^2$ with smooth enough boundary, regardless of knowing the spectrum explicitly. This is a major breakthrough in the field and connects strongly to the world of quantum field theory. Nahmod and Bulut both presented low-regularly well-posedness results in specific case of a critical nonlinear wave equation on a domain. For Bulut, it is in joint work with Bourgain on the energy critical wave and Schrödinger equations on the disc. For Nahmod, in joint work with Gigliola Staffilani on the $3d$ quintic nonlinear Schrödinger equation with data below H^1. Oh introduced ongoing work with A. Bényi and O. Pocovnicu to define a Gibbs measure for the nonlinear Schrödinger equation on non-compact spaces. See [11, 20, 21, 26, 27, 28, 29, 30, 79, 80, 81, 82, 83, 84]

- Mattingly presented a large number of examples from the setting of stochastic dynamics where noise assisted in the formation of either cascading energies or preventing a solution from blowing up deterministically. As of yet, his examples include small systems, but if ODE blow-dynamics or cascade dynamics can be found in PDE systems, there are interesting amounts of overlap related to noise-stabilization and cascades in PDE and weak turbulence theory. See [78] for work on the cascades and [59, 60] for noise stabilization.

- Pavlovic discussed recent work with Thomas Chen, Christian Hainzl, and Robert Sieringer on an application of the Quantum de Finetti theorem in the equations in many body quantum mechanics. See [35] for more details, but this is further progress towards being able to apply much of the developments for NLS to large systems of Bosons. As a result, a scattering argument arises for the defocusing Gross-Pitaevskii system under a now understood to be necessary exponential growth bound. De Finetti is a key theorem from probability theory that allows one to write density functionals in relation to a measure over an independent family of solutions. Namely, it says that exchangeable particles (like Bosons) can be written as a combination of independent particles in a measurable way. This also began a collaboration with S. Herr on multilinear estimates in many-body systems related to his studies on the sphere.

3 Scientific Progress Made

Many results were presented that made clear previously unknown connections between researchers. As a result, numerous collaborations and discussions took place between people in PDE and probability, numerical analysts and specialists in nonlinear PDE, etc. Also, a number of young people attended and were able to learn a great deal about pioneering areas in the field. Several ongoing collaborations were strengthened at the same time. In the end the meeting was a great success at bringing together many areas of dispersive PDE strongly impacted by geometry and nonlinearity, as well as introducing applications in dire need of such forms of analysis.

Participant list:
Alazard, Thomas - Ecole Normale Suprieure
Ambrose, David - Drexel University
Baskin, Dean - Northwestern University
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Dodson, Benjamin - University of California
Grébert, Benoît - Université de Nantes, France
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