Computational Contact Mechanics:
Advances and Frontiers in Modeling Contact
Sunday, February 16 to Friday, February 21, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

4:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
5:30–7:30	Buffet Dinner, Sally Borden Building
8:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:15	Opening remarks and introductions
9:15–10:30	*Sliding with infinite friction*, **Andy Ruina** (Cornell)
	Exploiting the complementarity structure: stability analysis of contact dynamics via sums-of-squares, **Michael Posa and Russ Tedrake** (MIT)
10:30–11:15	Coffee Break
11:15–12:15	Tutorial: *Differential Variational Inequalities and Mechanical Contact Problems*, **David Stewart** (University of Iowa)
12:15–1:30	Lunch
1:30–2:00	Free time
2:00–2:10	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
2:10–2:40	*Contact Models and the Role of Simulation in Robotics: Reflections on the DARPA Robotics Challenge and the National Robotics Initiative*, **Jeff Trinkle** (Program Director, NSF)
2:40–3:00	Coffee Break
3:00–4:45	*A Quadratic Programming Active-Set Method Based On Sequential Hot-Starts*, **Andreas Wächter** (Northwestern)
	PATHVI: a pathsearch method for variational inequalities, **Michael Ferris** (University of Wisconsin)
	Beyond Jacobi and Gauss-Seidel: A First Order Nesterov Method for Multibody Dynamics with Frictional Contact, **Hammad Mazhar** (University of Wisconsin-Madison)
4:50–5:30	Free time
5:30–7:30	Dinner

Tuesday

7:00–9:00 Breakfast

9:00–10:45 *Landslide earthquakes, granular flow experiments and erosion: some earth science questions ripe for contact mechanics*, **Colin Stark** (Columbia University)

Real-time models and algorithms of interacting deformable bodies for surgical simulation and robotics, **Christian Duriez** (INRIA Lille)

Grasping Control, Reduced Simulation, and Approximate Contact Modelling, **Paul Kry** (McGill University)

10:45–11:15 Coffee Break

11:15–12:15 Tutorial: *High Performance Computing in Multibody Dynamics*, **Dan Negrut** (University of Wisconsin)

12:15–1:30 Lunch

1:30–2:40 *An open question: How to efficiently solve 3D frictional contact problems?*, **Vincent Acary** (INRIA Rhône-Alpes)

A framework for problem standardization and algorithm comparison in multibody dynamics, **Ying Lu** (Rensselaer Polytechnic Institute)

2:40–3:00 Coffee Break

3:00–4:45 *Animating Cloth with Coupled Contact, a Quick and Dirty Approach*, **Robert Bridson** (Autodesk)

Asynchronous contact mechanics and its inspiration in animation, **Rasmus Tamstorf** (Disney Research)

Project Chrono: Gauging military vehicle mobility through many-body dynamics simulation, **Daniel Melanz** (University of Wisconsin)

4:45–5:30 Free time
5:30–7:30 Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:45	*Neither Rigid nor Elastic: a middle way*, **David Stewart** (University of Iowa)
	A compliant visco-plastic particle contact model based on differential variational inequalities, **Mihai Anitescu** (Argonne National Lab)
	Favorable mathematical properties of a frictional contact model with local compliance, **Jong-Shi Pang** (USC)
10:45–11:15	Coffee Break
11:15–12:30	*Convex and Analytically-Invertible Contact Dynamics*, **Yuval Tassa** (University of Washington)
	Contact Modeling and Inverse Problems for Visual and Auditory Display, **Ming Lin** (UNC Chapel Hill)
12:30–1:30	Lunch
1:30–2:40	*Extending Open Dynamics Engine for the Virtual Robotics Challenge*, **Steven Peters** (Open Source Robotics Foundation)
	Collision Detection and Resolution in Nucleus, **Jos Stam** (Autodesk)
2:40–3:00	Coffee Break
3:00–4:45	*Regularized contact conditions within a dual Lagrange mortar contact formulation*, **Barbara Wohlmuth** (TUM)
	Contact Representation in Multibody System Models: Formulations and Algorithms, **Jozsef Kovecses** (McGill)
	Reflections on Simultaneous Impact, **Breannan Smith** (Columbia)
4:45–5:30	Free time
5:30–7:30	Dinner
6:30–8:00	Curling at the Banff Curling Club

Thursday

7:00–9:00	Breakfast
9:00–10:45	*Volume-based Contact*, **Francois Faure** (INRIA, LJK-CNRS)
	Moving conforming contact manifolds and related numerical problems, **Kenny Erleben** (University of Copenhagen)
	A complementarity based contact model for physically accurate treatment of polytopes in simulation, **Jed Williams** (Rensselaer Polytechnic Institute)
10:45–11:15	Coffee Break
11:15–12:30	*Making contact: bridging the gap between discrete mechanics and contact modeling*, **Danny Kaufman** (Adobe Research)
	Finite-time Switching Dynamics of Contact Mechanical Systems: A DVI and Hybrid Systems Perspective, **Jinglai Shen** (University of Maryland Baltimore County)
12:30–1:30	Lunch
1:30–5:30	Free Afternoon
5:30–7:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–10:00	Breakout session
10:00–10:45	Coffee Break
10:45–11:30	Group discussion
11:30–1:30	Lunch

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Computational Contact Mechanics: Advances and Frontiers in Modeling Contact Sunday, February 16 to Friday, February 21, 2014

TALK ABSTRACTS

Speaker: **Vincent Acary** (INRIA Rhône-Alpes)
Title: *An open question: How to efficiently solve 3D frictional contact problems?*
Abstract: In this talk, we want to discuss possible numerical solution procedures for the following discrete frictional contact problem. Let $n_c \in \mathbb{N}$ be the number of contact points and $n \in \mathbb{N}$ the number of degree of freedom. Given a symmetric positive (semi-)definite matrix $M \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$, a vector $f \in \mathbb{R}^{\mathrm{n}}$, a matrix $H \in \mathbb{R}^{\mathrm{n}\times\mathrm{m}}$ with $m = 3n_c$, a vector $w \in \mathbb{R}^{\mathrm{m}}$ and a vector of coefficients of friction $\mu \in \mathbb{R}^{\mathrm{n_c}}$, find three vectors $v \in \mathbb{R}^{\mathrm{n}}$, $u \in \mathbb{R}^{\mathrm{m}}$ and $r \in \mathbb{R}^{\mathrm{m}}$ such that

$$\begin{gathered} Mv = Hr + f, \quad u = H^\top v + w, \quad \hat{u} = u + g(u), \\ K^\star \ni \hat{u} \perp r \in K, \end{gathered} \tag{1}$$

where $g(u)$ is a nonsmooth function and $K \subset \mathbb{R}^{3\mathrm{n_c}}$ is a Cartesian product of second order cone in $\mathbb{R}^3$. For each contact α, the unknown variables $u^\alpha \in \mathbb{R}^3$ (velocity or gap at the contact point) and $r^\alpha \in \mathbb{R}^3$ (reaction or impulse) are decomposed in a contact local frame $(O^\alpha, \mathsf{N}^\alpha, \mathsf{T}^\alpha)$ such that $u^\alpha = u^\alpha_{\mathrm{N}}\mathsf{N}^\alpha + u^\alpha_{\mathrm{T}}, u^\alpha_{\mathrm{N}} \in \mathbb{R}, \mathrm{u}^\alpha_{\mathrm{T}} \in \mathbb{R}^2$ and $r^\alpha = r^\alpha_{\mathrm{N}}\mathsf{N}^\alpha + r^\alpha_{\mathrm{T}}, r^\alpha_{\mathrm{N}} \in \mathbb{R}, \mathrm{r}^\alpha_{\mathrm{T}} \in \mathbb{R}^2$. The set K is the cartesian product of Coulomb's friction cone at each contact, that is

$$K = \prod_{\alpha=1\ldots n_c} K^\alpha = \prod_{\alpha=1\ldots n_c} \{r^\alpha, \|r^\alpha_{\mathrm{T}}\| \leq \mu^\alpha |r^\alpha_{\mathrm{N}}|\} \tag{2}$$

and $K^\star$ is dual. The function g is defined as $g(u) = [[\mu^\alpha \|u^\alpha_{\mathrm{T}}\| \mathsf{N}^\alpha]^\top, \alpha = 1 \ldots n_c]^\top$.

This problem is at the heart of the simulation of mechanical systems with 3D Coulomb's friction and unilateral constraints. It might be the result of the time–discretization by event–capturing time–stepping methods or event–detecting (event–driven) techniques of dynamical systems with friction or the result of a space–discretization (by FEM for instance) of the quasi-static problems of frictional contact mechanics. On the mathematical programming point of view, the problem appears as Second Order Cone Complementarity Problem (SOCCP). If the nonlinear part of the problem is neglected ($g(u) = 0$), the problem is an associated friction problem with dilatation, and by the way, is a gentle Second Order Cone Linear Complementarity Problem (SOCLCP) with a positive matrix $H^\top M^{-1} H$ (possibly semi–definite). When the non-associated character of the friction is taken into account through $g(u)$, the problem is non monotone and nonsmooth, and then very hard to solve efficiently.

In this talk we will recall a result for the problem in (1) which ensures that a solution exists. In this framework, we will list several algorithms that have been previously developed for solving the SOCCP (1) mainly based variational inequality and nonsmooth equations reformulations. On one hand, we will show that algorithms based on Newton methods for nonsmooth equations solve quickly the problem when they succeed, but suffer from robustness issues mainly if the matrix H has not full rank. On the other hand, the iterative methods dedicated to solving variational inequalities are quite robust but with an extremely slow rate of convergence. To sum up, as far as we know there is no option that combines time efficiency and robustness. To try to answer to this question, we develop an open collection of discrete frictional contact problems called FCLIB `http://fclib.gforge.inria.fr` in order to offer a large library of problems to compare algorithms on a fair basis. In this work, this collection is solved with the software Siconos and its component Siconos/Numerics `http://siconos.gforge.inria.fr` .

Speaker: **Mihai Anitescu** (Argonne National Lab)
Title: *A compliant visco-plastic particle contact model based on differential variational inequalities*
Abstract: This work describes an approach to the simulation of contacts with compliance and damping between three dimensional shapes using the framework of the Differential Variational Inequality (DVI) theory. Within the context of non-smooth dynamics, we introduce an extension to the classical set-valued model for frictional contacts between rigid bodies, allowing contacts to experience local compliance, viscosity and plasticization. Different types of yield surfaces can be defined for various types of contact, a versatile approach that contains the classic dry Coulomb friction as a special case. The resulting problem is a differential variational inequality that can be solved, at each integration time step, as a variational inequality over a convex set.

Speaker: **Robert Bridson** (Autodesk)
Title: *Animating Cloth with Coupled Contact, a Quick and Dirty Approach*
Abstract: Cloth simulation has become a bread-and-butter part of visual effects, but it still poses some very real challenges. For realistic stiff constraints on cloth stretching, an implicit and/or constrained integrator is practically necessary. Incorporating robust collision and contact processing in such an integrator, in an efficient and visually plausible (if not accurate) way, has not been well explored. The essence of the difficulty appears to be that the internal dynamics poses a large and fairly expensive problem to solve, albeit with fixed structure and relatively little complications from nonlinearity, whereas though collision and frictional contact may be simple in principle to solve once the active set of constraints is discovered (in most scenarios), finding the necessary constraints may require many iterations. Straightforward coupling makes for many iterations of large and expensive systems to solve. In this talk I will discuss some recent progress my student Zheng Wang and I made in trying to accelerate this, using uncoupled collision processing to estimate likely constraints, then regularization to gently couple them with internal dynamics.

Speaker: **Christian Duriez** (INRIA Lille)
Title: *Real-time models and algorithms of interacting deformable bodies for surgical simulation and robotics*
Abstract: This research work aims to provide real-time simulation tools for surgical interventions. To obtain a realistic or even predictive simulation of the procedure, we must take into account the deformation of anatomical structures and the mechanical interactions between devices and tissues. A key point is to adequately set the boundary conditions: the mechanical interactions between organs and/or with surgical devices are often difficult to account for. Yet, a bad modeling of these interactions can lead to a large source of errors. Our approach builds on non-smooth mechanics for modeling contact and friction between solids but also extends to other models of interaction (like needle insertion for instance). In this context, we also address the difficult problem of computing the compliance of deformable structures in real-time. Finally, we will show that these simulations could be used in the future for the control of surgical / deformable robots with a certain number of challenges.

Speaker: **Kenny Erleben** (University of Copenhagen)
Title: *Moving conforming contact manifolds and related numerical problems*
Abstract: The stability and robustness of a simulation is highly dependent on how "good" one can compute contact points representing the geometry of the contact manifolds. There is no given definition of what "good" means and no general approach has been found, further the compute budget for generating a good contact representation is limited. Hence heuristics and practical solutions are often applied in simulation software. Work does exist that approach this challenge differently. For instance continuous collision detection which guarantee penetration free geometry. However, this is tricky too as it is not obvious how to use this in a fixed-time-stepping scheme nor does it produce convenient tessellation of the contact surfaces. To get around a possible unpleasant tessellation of the contact surfaces a mortar mesh can be used to define a nice mapping of contact forces from one object to the other and vice versa. However, this leads to book keeping and the need for computing nodal weights going into the contact Jacobians. Inspired by our work on moving meshes in computational fluid dynamics this talk explores the idea of applying

moving meshes to contact mechanics. This will be a computational challenge, however it may offer many advantages such as conforming contact manifolds, continuous collision detection, simple contact Jacobians, and quality meshes all within the same framework. If time permits then num4lcp will be presented with focus on how some of the GPU solvers perform and work.

Speaker: **Francois Faure** (INRIA, LJK-CNRS)
Title: *Volume-based Contact*
Abstract: The increasing geometric complexity of simulated objects raises questions on how to set up contact equations that can be solved efficiently. We show that contacts between volumetric objects with complex shapes are more easily detected and solved based on intersection volume than penetration depth. Repulsion between arbitrarily complex objects can be modeled using a single scalar equation, with a straight forward extension to friction. The spatial resolution of the contact equations can also be easily tuned, independently of the surfaces. The equations have a standard Signorini-Coulomb form and can be solved using all the available solvers. We discuss efficient GPU implementations, including one with adaptive resolution.

Speaker: **Michael Ferris** (University of Wisconsin)
Title: *PATHVI: a pathsearch method for variational inequalities*
Abstract: We outline an implementation of a path search method for variational inequalities over general polyhedral sets based on a normal map formulation. The method is applicable to large scale instances, utilizes state-of-the-art simplex technology, and enhances speed using advanced basis information. The code is available in subroutine form, and within the GAMS modeling system in the EMP format. Some examples of its improved performance over PATH on VI's will be given. Joint work with Youngdae Kim.

Speaker: **Danny Kaufman** (Adobe Research)
Title: *Making contact: bridging the gap between discrete geometric mechanics and contact modeling*
Abstract: The program of mapping fundamental physical properties to discrete and thus computable analogues meets numerous challenges when confronting contacting systems. Specifically a simple counting argument suggests that broad invariant preservation is unlikely. In this talk I will survey a number of these challenges and discuss steps towards bridging the gap between coarse discretizations and continuous models of contact.

Speaker: **Jozsef Kovecses** (McGill)
Title: *Contact Representation in Multibody System Models: Formulations and Algorithms*
Abstract: Contact interaction is a basic phenomenon in mechanical systems. The principles of mechanics allow for two different possible representations of a contact interaction: (1) defining the contact forces via constitutive relations, or (2) giving specifications for the contact kinematics. This latter forms the basis for a subgroup of problems often referred to as rigid body contact problems. The presence of friction gives the most significant difficulties in the modelling, analysis, and simulation of contact. We will particularly deal with problems where the Coulomb friction model appears to be appropriate.

We will present a conceptual framework relying on first principles of mechanics, which allows the analyst to systematically formulate models that can then be used together with different numerical algorithms. Our focus will be rigid body contact models and systems where multiple contact interactions take place simultaneously. We will demonstrate how various formulations and related algorithms can be interpreted from a common mechanics framework.

The representation of friction gives the ground to group various approaches. Based on this, we will discuss linear, and so-called nonlinear complementarity formulations, and other possible methods that do not require the discretization of the friction cone. These alternative methods include projection-based and proximal-point methods. We present results of comparative studies for the functioning of different algorithms particularly from the point of view of the physical meaning and the validity of results. We will highlight open challenges that are the topic of our current work, and discuss some basic issues in friction modeling.

Impact, when the system experiences a sudden redistribution of momentum and energy, is an important problem within the area of contact mechanics. Impact with friction, so-called rough collision, still poses many open questions, especially in three-dimensional models. We will discuss some aspects of these problems.

We will also address some complex contact problems, such as the modeling and simulation of the interaction of a wheeled vehicle with soft, unstructured soil, and outline partial solutions as well as highlight challenging issues.

Speaker: **Paul Kry** (McGill University)
Title: *Grasping Control, Reduced Simulation, and Approximate Contact Modelling*
Abstract: I will first present method for learning control policies for one-handed, task-based manipulation of objects. Our approach uses a mid-level, multi-phase approach to organize the problem into three phases. This provides an appropriate control strategy for each phase and results in cyclic finger motions that, together, accomplish the task. The exact trajectory of the object is never specified since the goal is defined by the final orientation and position of the object. I will then discuss a method for the simulation of compliant, articulated structures using a plausible approximate model that focuses on modelling endpoint interaction. The structure's behaviour about a reference configuration, resulting in a first order reduced compliant system where approximate computation of the full structure's state may be parallelized. Finally I will describe new work on modelling contact between rigid objects using moments of the intersection volume. We compute a contact position from the first moment of the intersection volume, and approximate the extent of the contact patch from the second moment. The result is a 6D constraint what provides approximate limits on torques and forces to model frictional contact.

Speaker: **Ming Lin** (UNC Chapel Hill)
Title: *Contact Modeling and Inverse Problems for Visual and Auditory Display*
Abstract: Data provide us with representative samples of the various phenomena constituting the world around us. Such availability of data consequently has led to recent advances in data-driven modeling in many different fields. However, most of the existing example-based methods offer empirical models and data reconstruction that may not provide an insightful understanding of the underlying process or may be limited to a subset of observations. In contrast, classical physics based modeling offers well-defined analytical models and governing physical laws that explain many of these natural processes, except in the case of contact modeling where contact-related phenomena introduce numerous open issues for numerical simulations and modeling. Furthermore, parameters underlying the computational models are often unmanageably difficult to obtain. In this talk, I will discuss some case studies where we integrate physics-based modeling and data driven synthesis to start addressing some of these challenging research problems. The examples include performing simultaneous estimation of tissue deformation and elasticity parameters for soft bodies in contact, automatic extraction of intrinsic material parameters from sounding objects upon impact, and detecting "crowd turbulence" due to frictional contact among multiple moving individuals. Solutions to these problems can potentially offer new insights for medical diagnosis and cancer treatment, provide a more immersive multi-modal human-computer interaction, and enable robust, consistent simulation-based prediction for human stampedes. I conclude by discussing some possible future directions.

Speaker: **Ying Lu** (Rensselaer Polytechnic Institute)
Title: *A framework for problem standardization and algorithm comparison in multibody dynamics*
Abstract: The underlying dynamic model of multibody systems takes the form of a differential Complementarity Problem (dCP), which is nonsmooth and thus challenging to integrate. The dCP is typically solved by discretizing it in time, thus converting the simulation problem into the problem of solving a sequence of complementarity problems (CPs). Because the CPs are difficult to solve, many modeling options

that affect the dCPs and CPs have been tested, and some reformulation and relaxation options affecting the properties of the CPs and solvers have been studied in the hopes to find the "best" simulation method. One problem with the existing literature is that there is no standard set of benchmark problems to fairly compare the simulation methods.

We propose a framework of "Benchmark Problems for Multibody Dynamics (BPMD)? to support the fair testing of various simulation algorithms. We designed and constructed a BPMD database (stored using hierarchical data format 5 (HDF5)) and collected a set of solution algorithms. The data stored for each simulation problem is sufficient to construct the CPs corresponding to several different simulation design decisions. Once the CPs are constructed from the data, there are several solver options to choose from. Additionally, a user-friendly interface is provided to add customized models and solvers.

To understand the validity of the various simulation methods, we use data from physical planar grasping experiments. Validation of a chosen simulation method proceeds as follows. Using the input from a physical experiment to drive the simulation, uncertain parameters of the model are identified, for example, friction coefficients. This is repeated for different simulation methods and the error of the identification process serves as a measure of the suitability of each method to predict the observed physical behavior.

Speaker: **Hammad Mazhar** (University of Wisconsin-Madison)
Title: *Beyond Jacobi and Gauss-Seidel: A First Order Nesterov Method for Multibody Dynamics with Frictional Contact*
Abstract: This talk will discuss an accelerated solution of the many-body dynamics problem where the equations of motion include Differential Variational Inequalities for handling friction and contact. Simulating large systems with millions of objects relies on a methodology that solves a Cone Complementarity problem using a first order Nesterov method. The overall algorithm, which leverages parallel computing, implements adaptive step size, restart, and fall back, has been used to investigate the dynamics of densely packed granular systems and shown to lead to one order of magnitude shorter simulation times compared to solutions obtained with Gauss-Seidel. Convergence results will be discussed to gauge the efficiency; i.e. level of accuracy for given amount of computational effort, of the new solution. Several applications will be demonstrated that showcase the interaction of complex mechanical systems with cohesive granular material.

Speaker: **Daniel Melanz** (University of Wisconsin)
Title: *Project Chrono: Gauging military vehicle mobility through many-body dynamics simulation*
Abstract: This presentation describes how a modeling, simulation, and visualization framework is used to enable physics-based analysis of ground vehicle mobility. This computational framework, called Chrono, has been built to leverage parallel computing both on distributed and shared memory architectures. Two factors motivated the development of Chrono. First, there is a manifest need of modeling approaches and simulation tools to support mobility analysis on deformable terrain. Second, the hardware available today has improved to a point where the amount of sheer computer power, the memory sizes, and the available software stack (productivity tools and programming languages) support computing on a scale that allows integrating highly accurate vehicle dynamics and physics-based terramechanics models. Chrono is designed to support the solution of large systems of equations associated with the modeling of terrain via discrete bodies, where the number of interacting elements can be in the millions. The code requires a small number of model parameters, allows for numerical integration at large step-sizes, and robustly handles the discontinuities in velocities by posing the equations of motion as a collection of Differential Algebraic Equations and Differential Variational Inequalities. The increased numerical solution computational pressure is alleviated by the use of parallel hardware architectures that reduce run times for large simulation problems.

Speaker: **Jong-Shi Pang** (USC)
Title: *Favorable mathematical properties of a frictional contact model with local compliance*

Abstract: As an alternative to a rigid-body model, a complementarity-based, 3-dimensional frictional contact model with local compliance and damping was introduced in the Ph.D. thesis of Peng Song in 2002. It turns out that the local compliance contact model has several desirable mathematical properties that compensate for the modeling simplicity of the rigid-body paradigm. Specifically, by examining a variant of the local compliance model where there is no damping in the normal contact forces but there is coupled stiffness between the normal and tangential forces via body deformations, we derive a formulation of this frictional contact model as an ordinary differential equation with a boundedly Lipschitz continuous, albeit implicitly defined, semismooth right-hand side with global linear growth. From this basic formulation, we obtain the following properties of the local compliance model with frictional contact:

(a) existence and uniqueness of a continuously differentiable solution trajectory originated from an arbitrary initial state;
(b) finite contact forces that are semismooth functions of the system state;
(c) semismooth dependence of the trajectory on the initial state;
(d) convergence of a shooting method for solving two-point boundary problems.

These results are valid for both a dynamic model and a quasistatic model, the latter being one in which inertia effects are ignored, and for a broad class of friction cones that include the well-known quadratic Coulomb cone and its polygonal approximations. In the case of a polygonal friction law, we further establish the absence of Zeno states in the model.

Speaker: **Steven Peters** (Open Source Robotics Foundation)
Title: *Extending Open Dynamics Engine for the Virtual Robotics Challenge*
Abstract: The Virtual Robotics Challenge (VRC)[1] was a cloud-based robotic simulation competition. Teams competed by writing control software for a humanoid robot to perform disaster response tasks in real-time simulation. Simulating the physics and sensors of a humanoid robot in real-time presented challenges related to the trade-off between simulation accuracy and computational time. The Projected Gauss-Seidel (PGS) iterative solver was chosen for its performance and robustness, but it lacks the accuracy and the fidelity required for reliable simulation of task-level behaviors. This talk presents the modeling decisions and algorithmic improvements made to the Open Dynamics Engine (ODE) physics solver which improved PGS accuracy and fidelity without sacrificing its real-time simulation performance in the VRC.

A detailed introduction to ODE's iterative solver and constraint/contact models is presented. A discussion of modeling decisions includes the use of simple collision shapes to approximate contact meshes and the sensitivity of elastic contact parameters. Detailed analysis is provided for improvements to ODE?s algorithms added for the VRC. Solver stability was enhanced by position constraint error correction (split-impulse) and implicit viscous joint damping. Solver convergence was accelerated through inertia ratio reduction, warm starting of the iterative solver, constraint row reordering, and residual smoothing. These additions allowed for stable simulation regardless of user input during VRC, and supported reliable contact dynamics needed to complete VRC tasks without violating the near real-time requirement.

[1] Defense Advanced Research Project Agency (DARPA), "DARPA Robotics Challenge (DRC) and Virtual Robotics Challenge (VRC)." [Online]. Available: `http://theroboticschallenge.org/about`

Speaker: **Michael Posa and Russ Tedrake** (MIT)
Title: *Exploiting the complementarity structure: stability analysis of contact dynamics via sums-of-squares*
Abstract: Formal stability analysis of robotic systems can play an important role in advising motion planning and control design, however, many optimization-based methods for control and planning have traditionally been limited to analysis of continuous or hybrid systems. We are interested in exploiting the natural structure of contact dynamics, particularly of the complementarity formulation, to create algorithms for planning and control through contact that avoid the combinatorial complexity inherent in hybrid models. By leveraging the complementarity structure, we generate semialgebraic constraints

that define the admissible region of non-penetrating states and physical contact forces. The semialgebraic constraints of the contact model fit naturally with tools from numerical algebraic geometry and lead to new algorithms. In particular, we apply sums-of-squares (SOS) based methods for computation of Lyapunov certificates, which are powerful tools for analyzing the stability of continuous nonlinear systems. We will discuss such a method for verification of rigid body systems subject to Coulomb friction that undergo discontinuous, inelastic impact events. Our algorithm then explicitly generates formal certificates of stability and positive invariance over the set of admissible states and forces generated by the contact law. The approach is demonstrated on multiple robotics examples, including simple models of a walking robot and a perching aircraft. We will also present recent results on SOS methods for designing feedback controllers as well as preliminary work on state estimation and system identification of systems undergoing impacts.

Speaker: **Andy Ruina** (Cornell)
Title: *Sliding with infinite friction*
Abstract: Some people use "infinite friction" and "no sliding" as synonyms. Not so. As is known from from Lynch and Mason (1995), slip with infinite friction is possible.

This was presumably known and accepted in various 19th century problems including: dragging a stick, the simplest analysis of a wheel, and the simplest analysis of a pulley. All will be reviewed here.

We propose that the ideal world of rigid-object mechanics can be better 'closed', that is, made so that initial value problems are more likely to have unique solutions, by supplementing classical extreme contact models like sticking collisions, and frictionless sliding, with infinite-friction sliding.

Why? So simple models can be defined without adding new physical parameters.

A classic problem where such issues arise is the falling pencil of McGeer (1989). Simulating this problem seems to require frictional slip. Instead, however, the problem can be solved reasonably with infinite-friction slip, or with a new contact-release condition which we propose here: no-slip contact is released when such release would not cause immediate subsequent interpenetration.

Speaker: **Jinglai Shen** (University of Maryland Baltimore County)
Title: *Finite-time Switching Dynamics of Contact Mechanical Systems: A DVI and Hybrid Systems Perspective*
Abstract: A wide range of contact mechanical systems subject to unilateral constraints can be formulated as a differential variational inequality (DVI) or a differential complementarity system. Such a dynamical system demonstrates inherent nonsmooth, multimodal, and switching behaviors, and thus can be treated as a switching hybrid dynamical system. In this talk, we will review some recent developments in finite-time switching dynamics analysis of differential complementarity systems (or DVIs), which is critical to computation, numerical analysis, and control analysis of contact mechanical systems. In particular, we will focus on the non-Zeno property (i.e. the property of having finitely many switchings in finite time) of both Lipschitz and non-Lipschitz complementarity systems. We will also present results on robust non-Zeno property under system parameter and/or initial state perturbations, and its implications in contact mechanics. Future research issues will be discussed.

Speaker: **Breannan Smith** (Columbia)
Title: *Reflections on Simultaneous Impact*
Abstract: Resolving simultaneous impacts is an open and significant problem in collision response modeling. Existing algorithms in this domain fail to fulfill at least one of five physical desiderata. To address this we present a simple generalized impact model motivated by both the successes and pitfalls of two popular approaches: pair-wise propagation and linear complementarity models. Our algorithm is the first to satisfy all identified desiderata, including simultaneously guaranteeing symmetry preservation, kinetic energy conservation, and allowing break-away. Furthermore, we address the associated problem of inelastic collapse, proposing a complementary generalized restitution model that eliminates this source of nontermination. We then consider the application of our models to the synchronous time-integration of

large-scale assemblies of impacting rigid bodies. To enable such simulations we formulate a consistent frictional impact model that continues to satisfy the desiderata. Finally, we validate our proposed algorithm by correctly capturing the observed characteristics of physical experiments including the phenomenon of extended patterns in vertically oscillated granular materials.

Speaker: **Jos Stam** (Autodesk)
Title: *Collision Detection and Resolution in Nucleus*
Abstract: Nucleus is unified dynamics solver that handles solids, cloth, membranes, ropes and hair in a unified manner. This is achieved by modeling shapes as a simplicial complex. Collisions are detected and handled in space-time. All possible combinations of collisions between simplices are handled. Some extensions of Nucleus in an optimization framework will be discussed as well.

Speaker: **Colin Stark** (Columbia University)
Title: *Landslide earthquakes, granular flow experiments and erosion: some earth science questions ripe for contact mechanics*
Abstract: TBA

Speaker: **David Stewart** (University of Iowa)
Title: *Neither Rigid nor Elastic: a middle way*
Abstract: Rigid body models of impact are subject to problems with uniqueness, even after coefficients of restitution are included. Experiments by Hurmuzlu and others have demonstrated that the coefficient of restitution can be extremely variable, depending on the geometry of impact and the body as well as the material of the body in impact. Fully elastic impacts should, on the other hand, be modeled using PDEs for which there remain as yet unresolved existence and uniqueness issues. Computationally, CFL conditions make the task of tracking important elastic waves expensive. In this talk, a different approach is described based on Convolution Complementarity Problems which use an integral equation representation of the dynamics. Advantages and complications with this strategy are presented.

Speaker: **Rasmus Tamstorf** (Disney Research)
Title: *Asynchronous contact mechanics and its inspiration in animation*
Abstract: Animated feature films and many visual effects rely heavily on simulation of physical phenomena. A critical component in most of these simulations is proper handling of contact; yet this remains an area where many practical solution have shortcomings. In this talk I will give a brief overview of physical simulations in animation and use it to motivate a framework for asynchronous contact mechanics. By combining asynchronous variational integrators with conceptually infinite sums of barrier potentials this framework is able to guarantee that simulations of well-posed problems (a) have no interpenetrations, (b) obey causality, momentum- and energy-conservation laws, and (c) complete in finite time. A key to making this work is a method that can handle the infinite sum of barrier potentials. Originally this was achieved through kinetic data structures, but this was rather slow. By using speculative execution with rollback on failure much of the expensive bookkeeping can be avoided and the algorithm becomes more amenable to parallelization. Combined this yields well over two orders of magnitude speedup. Still, the algorithm is slower than what is desired for feature film production, which is currently inspiring additional research. The work presented here is joint work with Columbia University.

Speaker: **Yuval Tassa** (University of Washington)
Title: *Convex and Analytically-Invertible Contact Dynamics*
Abstract: I will present recent work from our group on complementarity-free contact dynamics with an analytic inverse. Contact impulses follow the Gauss principle and minimize kinetic energy subject to the friction cone. Dropping the complementarity requirement makes the optimization convex and a regularization term makes the solution unique and smooth. I will discuss the properties of the forward dynamics, focusing on the parameters which control the contact-smoothness in a well-defined manner.

Using the KKT conditions at the minimum, the inverse dynamics is shown to solve a dual problem which conveniently decomposes into small independent optimizations, one for each contact. These independent problems can furthermore be solved analytically, leading to an extremely fast inverse-dynamics computation. In our implementation in the MuJoCo physics engine, the inverse is roughly an order-of-magnitude faster than the forward dynamics.

Finally, I will present an application of these dynamics to state estimation of contact-rich domains. Exploiting the computation speed, we generalize the Extended Kalman Filter to multiple time-steps and multiple Gauss-Newton iterations, leading to a fixed-lag smoother which can accurately infer a robot's contact state, even without tactile sensors.

Speaker: **Jeff Trinkle** (Program Director, NSF)
Title: *Contact Models and the Role of Simulation in Robotics: Reflections on the DARPA Robotics Challenge and the National Robotics Initiative*
Over the past several decades, robotics research and applications focused on engineering robots and their work environments to maximize their productivity. Automotive assembly lines are quintessential examples. Assembly-line robots are heavy, powerful, and dangerous to humans, but fast and precise enough to improve product quality. Human safety is ensured by preventing people from getting near the workcells. Contact problems arise primarily in grasp acquisition and assembly tasks.

The National Robotics Initiative is shifting the focus of robotics research to human-robot collaboration. Robots are now seen as tools to expand the capabilities of human individuals and teams, which means that humans and robots will have to share their workspaces, and do so productively and safely. Robots should be inherently safe (by construction; hardware and software), understand people's actions, anticipate human intention, and determine a helpful course of action in real-time. Contacts problem between soft bodies and understanding of manipulation tasks now become more important.

The DARPA Robotics Challenge is a competition in which a single robot executes eight manipulation tasks such as walking across uneven terrain and connecting a hose to a standpipe. As of February 2014 two preliminary competitions have been conducted: the Virtual Challenge (entirely in simulation) and the Trials (the same competition using real robots). I will describe the most common approach to testing and controlling the robots, and the places where contact models failed, making testing ineffective.

Speaker: **Andreas Wächter** (Northwestern)
Title: *A Quadratic Programming Active-Set Method Based On Sequential Hot-Starts*
Abstract: In some applications, such as optimal control, it is necessary to quickly solve a sequence of similar quadratic programs (QPs). In this presentation, we present a new quadratic programming method that, for each new QP in the sequence, utilizes hot-starts that employ information computed by an active-set QP solver during the solution of the first QP. This avoids the computation and factorization of the full matrices for all but the first problem in the sequence.

The proposed algorithm can be seen as an extension of the iterative refinement procedure for linear systems to QPs, coupled with the application of an accelerated linear solver method that employs hot-started QP solves as preconditioners. Local convergence results are presented.

The practical performance of the proposed method is demonstrated on a sequence of QPs arising in nonlinear model predictive control and during the solution of a set of randomly generated nonlinear optimization problems using sequential quadratic programming. The results show a significant reduction in the computation time for large problems with dense constraint matrices, as well as in the number of matrix-vector products.

Speaker: **Jed Williams** (Rensselaer Polytechnic Institute)
Title: *A complementarity based contact model for physically accurate treatment of polytopes in simulation*
Abstract: We present a contact model for rigid-body simulation that considers the local geometry at points of contact between convex polyhedra in order to improve physical fidelity and stability of simulation. This model formulates contact constraints as sets of complementarity problems in a novel way, avoiding or

correcting the pitfalls of previous models. We begin by providing insight into the special considerations of collision detection needed to prevent interpenetration of bodies during time-stepping simulation. Then, three fundamental complementarity based contact constraints are presented which provide the foundation for our model. We then provide general formulations for 2D and 3D which accurately represent the complete set of physically feasible contact interactions in six unique configurations. Finally, experimental results are presented which demonstrate the improved accuracy of our model compared to four others.

Speaker: **Barbara Wohlmuth** (Technische Universität München)
Title: *Regularized contact conditions within a dual Lagrange mortar contact formulation*
Abstract: In this talk, we present an algorithm for solving quasi-static, non-linear elasticity contact problems in the context of rough surfaces. The dual mortar method is used to enforce the contact constraints weakly without increasing the algebraic system size.

Mortar methods have been recently quite popular for the variationally consistent discretization of contact problems between two deformable bodies. They allow non-matching meshes and pass the patch test. Optimal a priori estimates, fast non-linear iterative solvers and a posteriori error estimates in combination with adaptive refinement have been obtained in the past. Extensions to thermo-mechanical problems, large deformation, second order finite elements, plasticity effects have been successfully considered. Quite often the discretization in space is combined with a modified energy preserving time integration scheme. Due to the DAE character of the saddle point system, spurious oscillation in the contact stress can be possibly observed. It can be shown that a shift in the mass at the nodes in the contact area can stabilize the system without changing the rigid body properties. From the analytical point of view this mass lumping trick can be analyzed within the abstract framework of variational crimes as influence of quadrature formulas. Here we want to model the transition from soft to hard contact in case of rough surfaces on the micro-scale. Firstly we consider the case without friction.

The algorithm is deduced from a perturbed Lagrange formulation and combined with mass lumping techniques to exploit the full advantages of the duality pairing. This leads to a regularized saddle point problem, for which a non-linear complementary function and thus a semi-smooth Newton method can be derived. Secondly we extend the numerical scheme to friction including micro slip models.

Numerical examples demonstrate the applicability to industrial problems and show a good agreement to experimentally obtained results and to analytical benchmark solutions. This is joint work with Saskia Sitzmann and Kai Willner.

TUTORIAL ABSTRACTS

Speaker: **David Stewart** (University of Iowa)
Title: *Differential Variational Inequalities and Mechanical Contact Problems*
Abstract: Differential Variational Inequalities (DVIs) encompasses a wide range of dynamic processes that are useful for understanding mechanical impact and other contact problems (such as Coulomb friction). DVIs combine a differential equation with the solution of a variational inequality, which can be used to describe many non-smooth dynamical systems. A crucial issue for understanding DVIs is the index, which for impact problems is two, but for Coulomb friction is one. The concepts can be applied to partial differential equation models as well as for finite-dimensional ordinary differential equation models.

Speaker: **Dan Negrut** (University of Wisconsin)
Title: *High Performance Computing in Multibody Dynamics*
Abstract: This talk provides an overview of options available to speed up the simulation of dynamic systems comprising large numbers of components interacting through friction and contact in the presence of bilateral constraints. Several illustrative applications are briefly considered to demonstrate the need for fast computing in multibody dynamics. The very question of what fast computing means and how it can be achieved provides the backdrop for a discussion that touches on GPU, multi-core, and distributed

(multi-node) computing. Each of these three alternatives is scrutinized from the point of view of the underlying hardware organization and of the accompanying software stack that modulates the user?hardware interaction. The talk concludes with two observations that although common sense never hurt to be reinforced: (i) most often, chip-memory data movement, controlled by the solution algorithm selection and by software implementation choices, decides the performance of a program; and (ii) at this time, the advanced computing landscape is fluid to the point where the answer to the question "what are the right architecture and software ecosystem for my application?" is problem specific and depends on subjective factors such as compile flags or familiarity of the software designer with the underlying hardware.