No slip vs Sliding with infinite friction

Special for Banff 2014

Comments on

Pulling by Pushing,
Slip With Infinite Friction,
and Perfectly Rough Surfaces
Kevin Lynch, Matt Mason
Int J of Robotics Research, 1995

Wobbling,
toppling,
and forces of contact
Tad McGeer
Am. J. of Physics, 1989

Matt Kelly,
Andy Ruina,
Gregg Stiesberg,
Mechanical Eng, Cornell U
" "
Physics, " "

Tuesday, February 18, 2014
Motivation/inspiration:

1) Mechanics (apparent) paradoxes are interesting
   a) friction and b) dynamics
2) Make a good robot \( \Rightarrow \) good simulation
   c) deal with these things, or
   d) do something worse

Conclusions:

3) Slip with \( \mu = \infty \) is possible
   e) In natural situations
   f) Is a useful model

4) No-slip BCs violates \( \vec{F} = m\vec{a} \) (or something, sometimes)
   g) In natural problems
   h) So \( \mu = \infty \) slip is better
Value system (in this talk):

1) Don’t violate $\vec{F} = m \vec{a}$, symmetries of space, etc.

2) Approximate constitutive laws are OK (like $F = \mu N$ etc).

3) The model should have a high-precision meaning.

4) No concern for computation speed.

5) 2D for simpler pictures etc.
The friction force

\[ \vec{F}_{\text{on } A \text{ from } B} = \frac{-\mu \vec{v}_{A/B}}{|\vec{v}_{A/B}|} \cdot N \]

The magnitude of the friction force

\[ |\vec{F}_{\text{on } A \text{ from } B}| \leq \mu N \]

Relative slip velocity.

\[ N \geq 0 \]

An upper bound on the friction force

During slip

During stationary contact

\[ \vec{F}_{\text{on } A \text{ from } B} \]

\[ |\vec{F}_{\text{on } A \text{ from } B}| \]

Dry friction forces are not small and thus cannot be sensibly neglected. As a simplification when we think friction is not important we sometimes neglect it by setting \( N = 0 \), and so we assume no tangential motion is allowed and that there is some unknown tangential correlation between smoothness and low friction. This equation, like many other simple equations, needs some descriptive context. We use the phrase "perfectly smooth" to mean perfectly non-frictional and perfectly rough to mean perfectly frictional. In the simplest renditions of mechanics problems involving sliding contact. The simplest model for friction force during stationary contact are very small compared forces of unlubricated contact. There is no contact they are not in real contact at all, a thin layer of liquid or gas separates the bodies. So a common mistake amongst beginning engineers is to use contact constants with confidence, as if accurate. Rather, all contact simple equations with confidence, as if accurate. Rather, all contact equations with confidence, as if accurate. Rather, all contact simple equations with confidence, as if accurate. Rather, all contact simple equations with confidence, as if accurate. Rather, all contact simple equations with confidence, as if accurate.
If you don’t like divide by zero etc:

\[ \tan \phi = \mu, \quad 0 \leq \mu \leq \infty, \quad 0 \leq \phi \leq \pi/2 \]

\[
\vec{F}_{tot} \cdot \vec{v} = -\sin \phi |\vec{F}_{tot}| v \quad \text{if } v \neq 0
\]
\[
\vec{F}_{tot} \cdot \vec{v} \leq |\sin \phi \vec{F}_{tot} v| \quad \text{if } v = 0
\]
Constitutive relation for friction

Coulomb friction
A surface in the space of
force angle $\theta$
sliding velocity $\nu$
force magnitude $|\vec{F}_{tot}|$

Single constitutive parameter:
the friction angle $\phi$\

(Nothing special about $\mu = \infty$)
Recall

**Conclusions:**

3) Slip with $\mu = \infty$ is possible
   e) In natural situations: 3 examples & demo
   f) Is a useful model

4) No-slip BCs violates $\vec{F} = m\vec{a}$ (sometimes)
   g) In natural problems
   h) So $\mu = \infty$ slip is better
Dragging stick  (similar to Lynch/Mason 1995)

\[ F = \frac{W}{2 \tan \theta} \]
Constitutive relation for friction

Coulomb friction
A surface in the space of
  sliding velocity \( \nu \)
  sliding force \( F \)
  normal force \( N \)

Single constitutive parameter: the friction angle \( \phi \)

Nothing special about \( \mu = \infty \) Surface becomes two quarter planes \( (\nu F \geq 0, N = 0) \)
and one half plane \( (N \geq 0, \nu = 0) \).
Dragging stick (cont’d)

Drag Force

Normal Force

\[ F = 0.5 \, W \]

\[ N = 0 \]

Nothing special happens when \( \mu \to \infty \).
Drag...
Infinite friction example 2: A wheel

The simplest wheel design uses a dry "journal" bearing consisting of a large bearing wheel. We neglect the wheel's weight because it is generally much less than the force we are trying to balance. To figure out the forces involved we draw a free body diagram of the wheel. We have as unknowns the horizontal forces cancel each other. With some mathematical manipulation we could solve the 4 scalar equations which will surely turn out to be negative for a cart moving to the right.

\[ \mu = \infty \Rightarrow F_x \approx \frac{r}{R} \]

\[ \mu = 0 \]

\[ \mu = \infty \]

A wheel is not just a lever.
Infinite friction example 3:  
**A pulley**

For all $\mu$ force at D is vertical

For $\mu \to \infty$

$$F = Mg \frac{1 + r/R}{1 - r/R} \approx Mg(1 - 2r/R)$$

A small axle makes an efficient pulley for arbitrarily large $\mu$
For a dragging stick and for journal bearings (wheel and pulley), infinite friction slip seems a reasonable worst case (biggest friction) model.
Recall

**Conclusions:**

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   e) In natural situations: 3 examples & demo
   f) Is a useful model

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Simulation of a falling pencil (McGeer 1989)

- Start nearly vertical at rest
- Tip is in non-slip infinite frictional contact with surface, equivalent to a pin joint constraint

(Usherwood video)
Simulation of a falling pencil (cont’d)

- Integrate the constrained EOM until the normal force vanishes

Release constraint when normal force gets to zero.
Simulation of a falling pencil (cont’d)

Released constraint when normal force went to zero.

Surprise!

Tip of pencil immediately accelerates through the floor.

Why?
Simulation of a falling pencil (cont’d)

A graphical proof that the tip must accelerate through the floor:

JUST BEFORE LIFTOFF

JUST AFTER LIFTOFF

Relaxing the constraint is equivalent to adding a tangential force at the tip that causes it to accelerate through the surface.

Conclusion: infinite non-slip friction is not a physically consistent contact model.
Simulation of a falling pencil (cont’d)

Three choices:

1) Allow slip (with infinite friction or whatever).

2) Allow the ground to suck.

3) Allow interpenetration.
The root of all evil:
Contact mass matrix relates force and acceleration at contact point

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix}
= [M]
\begin{bmatrix}
a_x \\
a_y
\end{bmatrix}
\]

(mass matrix video)
Constitutive relation for friction

Coulomb friction
A surface in the space of
  sliding velocity \( v \) (a)
  sliding force \( F \)
  normal force \( N \)

Single constitutive parameter: the friction angle \( \phi \)

*Nothing special about \( \mu = \infty \). Surface becomes two quarter planes (\( vF \geq 0, N = 0 \)) and one half plane (\( N \geq 0, v = 0 \)).*
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1) Mechanics (apparent) paradoxes are interesting
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2) Make a good robot  →  good simulation
   c) deal with these things, or
d) do something worse

Conclusions:

3) Slip with $\mu = \infty$ is possible
   e) In natural situations (e.g., our robot sims)
f) Is a useful model

4) No-slip BCs violates $\vec{F} = m\vec{a}$ (or something else)
g) In natural problems
h) So $\mu = \infty$ slip is better