| Time | | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|---|
| 9:00-9:50 | | Stephenson | van der Hofstad | Le Gall | Curien | Lawler |
| 10:00-10:30 | | Budd | Cerny | Sousi | Hutchcroft | Kortchemski |
| Coffee | | | | | | |
| 11:00-11:50 | | Sheffield | Albenque | Peres | Georgakopoulos | Haas |
| Lunch | | | | | | |
| 1:30-2:20 | | Miller | Gurel-Gurevich | | Chelkak | |
| 2:20-2:30 | | | Group Photo | | | |
| 2:30-3:20 | | Evans | Open problem session | | Ray | |

**Titles and Abstracts**

Speaker: Marie Albenque, École polytechnique

Title: Simple maps (also) converge towards the Brownian map

In the last years, numerous families of planar maps (embeddings of planar graphs in the sphere) have been shown to converge to the Brownian map introduced by Miermont and Le Gall. Here we prove that simple maps (that is maps without loops nor multiple edges) also converge to the Brownian map when their number of edges goes to infinity.

This work relies first on a bijection between simple maps with a triangular outer faces and oriented binary trees. Then the distance in the maps can be studied with the help of some canonical « leftmost paths », which behave well both in the map and in the tree. I'll emphasize the combinatorial constructions that play a major role in this work.

This is joint work with Olivier Bernardi, Gwendal Collet and Eric Fusy.

Speaker: Timothy Budd, University of Copenhagen

Title: First-passage percolation on random regular planar maps

Abstract: Recently two- and three-point functions have been derived for general planar maps with control over both the number of edges and number of faces. In the limit of large number of edges, the multi-point functions reduce to those for random cubic planar maps with random exponential edge lengths, and they can be interpreted in terms of either a First passage percolation (FPP) or an Eden model. We observe a surprisingly simple relation between the asymptotic first passage time, the hop count (the number of edges in a shortest-time path) and the graph distance (the number of edges in a shortest path). Using (heuristic) transfer matrix arguments, we show that this relation remains valid for random p-valent maps for any $p>2$.

Speaker: Dmitry Chelkak, ETH Zurich & Russian Academy of Sciences

Title: Discrete stress energy tensor in the O(n) loop model and the corresponding convergence result for O(1).

Abstract: For the critical O(n) loop model on the planar honeycomb lattice, we provide a combinatorial construction of a stress-energy tensor. The basic version of this "new lattice observable" describes a response of the partition function to a local change of the metric on a given edge (more sophisticated versions are available as well). We show that this observable satisfies some discrete holomorphicity relations (though, as for parafermions, these relations themselves are not enough to have a well-posed boundary value problem). Moreover, in the special case n=1 (i.e., the critical Ising model), we check a lattice analogue of the Conformal Field Theory (c=1/2) identity expressing the stress-energy tensor as a product of holomorphic fermions. In particular, this allows us to prove the convergence of our discrete stress-energy tensor to a scaling limit which has a Schwarzian-type covariance under conformal maps. Based on a joint ongoing project with Alexander Glazman and Stanislav Smirnov (Geneva & St. Petersburg).

Speaker: Jiří Černý, University of Vienna

Title: Macroscopic phase transition for the vacant set of random walk on a large torus

Abstract: It was conjectured that the vacant set of random walk on (some) finite graphs exhibits a phase transition similar to the Erdös-Rényi random graph. Up to now, this conjecture was proved only on the graphs that are "locally tree-like". The most interesting graph, the large d-dimensional discrete torus, remains open. In the talk, I will present the recent progress in proving such phase transition in this case.

This is a joint work with A. Teixeira (IMPA).

Speaker: Nicolas Curien, Université Paris-Sud

Title: Planar Stochastic Hyperbolic Triangulations

Abstract: Pursuing the approach of Angel & Ray we introduce and study a family of random infinite triangulations of the full-plane that satisfy a natural spatial Markov property. These new random lattices naturally generalize Angel & Schramm's Uniform Infinite Planar Triangulation (UIPT) and are hyperbolic in flavor. We prove that they exhibit a sharp exponential volume growth, are non-Liouville, and that the simple random walk on them has positive speed almost surely. We conjecture that these infinite triangulations are the local limits of uniform triangulations whose genus is proportional to the size.

Speaker: Steven N. Evans, University of California, Berkeley

Title: The fundamental theorem of arithmetic for metric measure spaces.

Abstract: A metric measure space is just a complete separable metric space equipped with a probability measure that has full support. Two such spaces are equivalent if they are isometric via an isometry that

maps the measure on one to the measure on the other.  Gromov and Vershik showed that an equivalence class is determined by the distribution of the random matrix of distances between the points of an i.i.d. sample, and so these objects are much ``tamer'' than complete separable metric spaces per se.  There is a natural way to ``multiply'' two of these objects: equip the Cartesian product with the sum of the metrics and the product of the probability measures.  In joint work with Ilya Molchanov (Bern), we show that any metric measure space has a unique factorization into a countable product of ``prime'' elements.  Moreover, we are able to characterize the class of infinitely divisible random metric measure spaces and the subclass of stable ones in a manner that is analogous to the characterization obtained by L\'evy, Hin\u{c}in and It\^o for real random variables.

Speaker: Agelos Georgakopoulos, University of Warwick

Title: The Poisson boundary of planar graphs via discrete Riemann maps

Abstract: Answering a question of Benjamini & Schramm, we show that the Poisson boundary of any planar, uniquely absorbing (e.g. one-ended and transient) graph with bounded degrees can be realised geometrically as a circle, which circle arises from "a square tiling", a discrete version of Riemann's mapping theorem. This also implies a conjecture of Northshield of similar flavour. When the graph is hyperbolic, the aforementioned circle coincides with the usual boundary.

Speaker: Ori Gurel-Gurevich, Hebrew University of Jerusalem

Title: Recurrence of planar graph limits

Abstract: We will show that any distributional limit of finite planar graphs in which the degree of the root decays exponentially is almost surely recurrent. As a corollary, we obtain that the uniform infinite planar triangulation/quadrangulation (UIPT/UIPQ) are almost surely recurrent, resolving conjectures of Angel, Benjamini and Schramm.
Joint work with Asaf Nachmias.

Speaker: Bénédicte Haas, Université Paris-Dauphine

Title: Scaling limits of k-ary growing trees

Abstract: For each integer $k \geq 2$, we introduce a sequence of k-ary discrete trees constructed recursively by choosing at each step an edge uniformly among the present edges and grafting on "its middle" $k-1$ new edges. When $k = 2$, this corresponds to a well-known algorithm which was first introduced by Rémy. Our main result concerns the asymptotic behavior of these trees as n becomes large: for all k, the sequence of k-ary trees grows at speed n^1/k towards a k-ary random real tree that belongs to the family of self-similar fragmentation trees. This convergence is proved with respect to the Gromov-Hausdorff-Prokhorov topology. We also study embeddings of the limiting trees when k varies. This talk is based on a joint work with Robin Stephenson.

Speaker: Remco van der Hofstad, Eindhoven University of Technology

Title: Hypercube percolation

Abstract: Consider bond percolation on the hypercube {0,1}^n at the critical probability p_c defined such that the expected cluster size equals 2^{n/3}, where 2^{n/3} acts as the cube root of the number of vertices of the n-cube. Percolation on the Hamming cube was proposed by Erd"os and Spencer (1979), and has proved to be substantially harder than percolation on the complete graph. In this talk, I will describe the phase transition for percolation on the hypercube, and show that it shares many features with that on the complete graph.

In previous work with Borgs, Chayes, Slade and Spencer, and with Heydenreich, we have identified the subcritical and critical regimes of percolation on the hypercube. In particular, we know that for p=p_c(1+O(2^{-n/3})), the largest connected component has size roughly 2^{2n/3} and that this quantity is non-concentrated. In work with Asaf Nachmias, we identify the supercritical behavior of percolation on the hypercube, by showing that, for any sequence \epsilon_n tending to zero, but \epsilon_n being much larger than 2^{-n/3}, percolation at p=p_c(1+\epsilon_n) has, with high probability, a unique giant component of size (2+o(1))\epsilon_n 2^n. This also shows that the proposed critical value really is a correct one. Finally, we `unlace' the proof by identifying the scaling of component sizes in the supercritical and critical regimes without relying on the percolation lace expansion, a beautiful technique that is a major technical tool for high-dimensional percolation, but that is also quite involved and can have a disheartening effect on some.

Speaker: Tom Hutchcroft, University of British Columbia

Title: Reversible Hyperbolic Triangulations: Circle Packing and Random Walk

Abstract: For deterministic bounded degree triangulations, circle packing has proven a powerful tool for studying random walk via geometric arguments. In this talk, I will discuss extensions and analogues for *random* triangulations without the assumption of bounded degree.

Speaker: Igor Kortchemski, University of Zurich

Title: Scaling limits and influence of the seed graph in preferential attachment trees

Abstract: In this talk, we will be interested in the asymptotics of random trees built by linear preferential attachment (also known as Barabasi-Albert trees or plane-oriented recursive tree). We will first try to understand the influence of the initial tree (the seed) on the long-term behavior of this process. Then we will see that this problem is closely related to the existence of scaling limits of so-called looptrees associated with these trees. Roughly speaking, a looptree of a tree encodes the geometric structure of its nodes of large degree. This is joint work with Nicolas Curien, Thomas Duquesne and Ioan Manolescu.

Speaker: Greg Lawler, University of Chicago

Title: Conformal Invariance of Loop-Erased Random Walk Green's function

Suppose D is a simply connected domain in C containing the origin with conformal radius 1 (with respect to 0). Let a,b be boundary points and consider a loop-erased walk in D from (near) a to (near) b on a square lattice $n^{-1}(Z \; x \; iZ)$. We show that the probability that the walk uses the undirected edge 0,1 is

$$c n^{-3/4} [ \sin^3 \theta + O(n^{-u}) ]$$

Here c,u are absolute positive constants and $\theta$ is the argument of the origin with respect to a,b. This is the same as the prediction based on the $SLE_2$ Green's function. This is joint work with Christian Benes and Fredrik Johansson Viklund. I will also discuss the relevance of this work to the question of convergence of LERW to SLE in the natural parametrization.

Speaker: Jean-François Le Gall, Université Paris-Sud

Title: The harmonic measure of balls in random trees

We study properties of the harmonic measure of balls in typical large discrete trees. For a ball of radius $n$ centered at the root, we prove that, although the size of the boundary is of order $n$, most of the harmonic measure is supported on a boundary set of size approximately equal to $n^\beta$, where $\beta = 0.78...$ is a universal constant. To derive such results, we interpret harmonic measure as the exit distribution of the ball by simple random walk on the tree, and we first deal with the case of critical Galton-Watson trees conditioned to have height greater than $n$. The constant $\beta$ is expressed in terms of the asymptotic distribution of the conductance of large critical Galton-Watson trees. If time permits, we will also discuss recent related results of Shen Lin. This talk is based a joint work with Nicolas Curien.

Speaker: Jason Miller, MIT

Title: Quantum Loewner Evolution

Abstract: We will describe a new universal family of growth processes called “Quantum Loewner Evolution” (QLE) and explain how QLE can be used to relate $\sqrt{8/3}$-Liouville quantum gravity with the Brownian map. We will also explain how QLE is related to diffusion limited aggregation, first passage percolation, and the dielectric breakdown model.

Speaker: Yuval Peres, Microsoft Research

Title: Markov type for planar graphs and applications

Abstract: A metric space X has Markov type 2 if every reversible stationary Markov chain taking values in X escapes at most diffusively from its starting point. In 1992, Keith Ball showed that Hilbert space has Markov type 2, and this was extended to $L^p$ for p>2, to trees and hyperbolic spaces in [1]. In joint work [2] with Jian Ding and James Lee, we answered a question raised in [1], by proving that planar graph metrics also have Markov type 2. A crucial new ingredient in [2] is a local-to-global bound for Martingales. I will emphasize two applications of these ideas to random walks:

(a) A proof of a conjecture of Itai Benjamini, that random walk on an infinite planar graph G of bounded degree, either has positive speed, or is at most diffusive, i.e. for every t there exists a starting point so that E d(X_0,X_t)^2 is at most C(G)t;
(b) A sharp bound [3] on the rate of escape of simple random walk on subcritical dynamical percolation.

[1] Naor, A.; Peres, Y.; Schramm, O.; Sheffield, S. Markov chains in smooth Banach spaces and Gromov-hyperbolic metric spaces. Duke Math. J. 134 (2006) 165–197. http://arxiv.org/abs/math/0410422
[2] Ding, J.; Lee, J. R.; Peres, Y. Markov type and threshold embeddings. Geom. Funct. Anal. 23 (2013), 1207–1229. http://arxiv.org/abs/1208.6088
[3] Peres, Y., Stauffer, A. and Steif, J., Random walks on dynamical percolation: mixing times, mean squared displacement and hitting times. (2013). PTRF, to appear. http://arxiv.org/abs/1308.6193

Speaker: Gourab Ray, University of Cambridge

Title: Random walk on hyperbolic maps

Abstract: We consider the behaviour of random walks on half plane hyperbolic random triangulations. We prove that the random walk escapes from the boundary at a linear rate. Further we show that the return probability after n random walk steps scales like exp(-cn^{1/3}). Joint work with Omer Angel and Asaf Nachmias.

Speaker: Perla Sousi, University of Cambridge

Title: Total variation cutoff in a tree

Abstract: I will describe an example of a tree on which a lazy simple random walk exhibits total variation cutoff. This implies that for this tree the relaxation time and the mixing time are not of the same order. I will explain the main idea behind the construction: hitting times can be concentrated. I will also discuss robustness of mixing times and of cutoff on trees. This is based on joint work with Yuval Peres.

Speaker: Ken Stephenson, University of Tennessee

Title: Emergent conformal structure via circle packing

Abstract: The talk will concern geometrically random triangulations of plane regions G. These are "geometric" in the sense of being Delaunay triangulations T based on Poisson point processes, so they have a native conformal structure inherited from G. Each such T can also be embedded in the unit disc D via circle packing, giving it a second conformal structure. Experiments suggest that the discrete map from G to D associated with T is conformal in nature. In this talk I will conjecture and provide experimental evidence that such random maps converge to classical conformal maps with probability 1 as the complexity of T grows. In other words, the conformal structure on G is an emergent phenomenon.