

Alberta Number Theory Days VII (15w2198)

Nathan Ng (University of Lethbridge),
Manish Patnaik (University of Alberta),
Ander Steele (University of Calgary)

June 12–June 14, 2015

This conference is the unique annual opportunity for the Albertan Number Theory community to regroup and discuss the latest research progress in the field as well hear about the advances of Alberta's own researchers. The conference also provides an ideal environment for younger researchers to introduce their work and connect with other well-established researchers in the province and outside of their own universities.

This year we had 24 participants from Alberta, one from Montreal, two from China (visiting UBC), and two distinguished plenary speakers: William Casselman (UBC) and Julia Gordon (UBC). Moreover, nearly one third of the participants were female this year and two of nine invited speakers were female.

1 Overview of the Field

Number theory is a broad and central area of research with many connections and applications to other areas of mathematics and science. It is also an extremely active and diverse area of research. The subject may be divided roughly into several subdisciplines that range from pure mathematics, such as algebraic number theory, arithmetic geometry, analytic number theory, and automorphic forms and representation theory, to more applied areas such as computational number theory and mathematical physics.

Alberta Number Theory Days allows for face to face discussion between peers and facilitates collaboration between researchers within the province as well as distinguished out of province participants. It allows the community of Alberta number theorists to discuss the important recent advances in the field and in their own research. New connections are made and old associations are renewed, and these personal interactions often lead to the conception of new projects. The meeting aims to allow for the exchange of knowledge among researchers in number theory, which will improve the progress of current projects.

2 Presentation Highlights

There were many exceptional talks this year and numerous informal discussions throughout the friendly two day meeting.

Our first plenary speaker, **William Casselman** from the University of British Columbia, a founding figure in the field of automorphic forms and leading expert on many areas of mathematics, gave a fascinating talk on “Newton polygons and ramification.” In this talk, he revisited a classical (and increasingly important) topic of ramification and explained a new, elementary perspective on the topic focusing on the geometry of Newton polygons. While this work was inspired by a recent article of Jonathan Lubin (which in turn followed an old idea of Tate), it aimed to provide more elementary and conceptual proofs of certain somewhat inaccessible topics in the literature.

Our other plenary speaker **Julia Gordon** (UBC) is a young expert on the representation theory of p -adic groups and its connections with model theory. She explained how an old technique due to Langlands and Kottwitz relating orbital integrals to isogeny classes of elliptic curves could be used to give an explanation of recent surprising heuristics of Geckler on the number of rational points on elliptic curves over finite fields. The talk was very well received by the Alberta audience, several members of whom had worked on closely related topics in the past.

Anna Puskas, a new face on the Alberta number theory scene, is currently a postdoctoral fellow at the University of Alberta. She explained a connection between a famous formula of William Casselman and Joseph Shalika for Whittaker functions on p -adic groups and the theory of crystal bases which arise in combinatorial representation theory. It was a particular highlight to have her speak on a result of one of our distinguished plenary speakers, and Dr. Puskas benefitted a great deal from her interactions with Dr. Casselman during the weekend.

Fitting of the ever increasing purview of number theory, this year we invited the mathematical physicist **Thomas Creutzig** from the University of Alberta to deliver a lecture on “Logarithmic Hopf links and modular forms.” In his lecture, Dr. Creutzig first explained how the representations of a regular vertex operator algebra (or conformal field theory) form a modular semi-simple tensor category, so that the categorical $SL(2, \mathbb{Z})$ -action is generated by the Hopf link with twist corresponding to the elements S and T of the modular group. He then moved to the case of vertex operator algebras that have indecomposable but reducible modules, where nearly nothing is known about a similar modular story. In this setting, he introduced what he called logarithmic Hopf links and explained how they relate to representations of the corresponding fusion ring of the module category.

We were also fortunate this year to also have **Stephan Ehlen**, a postdoctoral fellow at McGill University, deliver a lecture on “On Two Arithmetic Theta Lifts”. Theta lifts are an important tool in the modern theory of automorphic forms but somewhat under-represented in the Alberta number theory community, so we were happy to have Dr. Ehlen explain how to generalize part of Stephen Kudla’s older work on central derivatives of Eisenstein Series and height pairings from the setting of Shimura curves to higher dimensional orthogonal and unitary Shimura varieties. He showed how to obtain the required Green’s currents in the theory as regularized theta lifts of certain ‘truncated’ Poincare series. He also explained one interesting application in arithmetic geometry, namely that his results (which were joint with Siddharth Sankaran) imply the modularity of the difference of two arithmetic generating series for Kudla-Rapoport cycles on unitary Shimura varieties.

Also on the subject of automorphic forms, Alberta’s own **Clifton Cunningham** from the University of Calgary delivered a lecture on “Lifting Hilbert modular forms to spin modular forms.” In this talk, he explained how Hilbert modular forms for totally real fields of degree n determine automorphic representations of forms of the group $GSpin(2n + 1)$ over \mathbb{Q} . Using examples, he explained how the automorphic representations obtained in this way may or may not be holomorphic. This was joint work with Lassina Dembele.

A veteran of many Alberta Number Theory days **Amir Akbary** from the University of Lethbridge delivered a lecture “On the greatest prime factor of some divisibility sequences”. If $P(m)$ denotes the greatest prime factor of m , Dr. Akbary first recalled a result of M. Ram Murty and S. Wong who proved, under the assumption of the ABC conjecture,

$$P(a^n - 1) \gg_{\epsilon, a} n^{2-\epsilon}$$

for any $\epsilon > 0$. He studied analogous results for the corresponding divisibility sequence over the function field $\mathbb{F}_q(t)$ and for some divisibility sequences associated to elliptic curves over the rational field \mathbb{Q} . We note here that divisibility sequences were a topic of last year’s meeting with one of the experts on this area Katherine Stange a distinguished plenary speaker during Alberta Number Theory Days 2014.

Another frequent participant of Alberta Number Theory Days past **Mark Bauer** from the University of Calgary delivered a lecture on “Cubic Irrationalities and a Ramanujan-Nagell Analogue.” In his talk, he considered a cubic analogue of the Ramanujan-Nagell equation and showed that by constructing explicit, restricted irrationality measures for the cube root of two that are, in some very real sense, better than expected, it was possible to derive meaningful and interesting bounds on the difference between the cube of an integer and powers of 2. These measures naturally yield results about certain Diophantine equations. This was joint work with Michael Bennett.

Finally, we mention that we had another postdoctoral fellow speak at our meeting: **James Parks** from the University of Lethbridge delivered a lecture on “The asymptotic constant for amicable pairs of elliptic curves.” If E is an elliptic curve over \mathbb{Q} with primes p, q of good reduction, he recalled the notion introduced

by Silverman and Stange, for a pair of such primes (p, q) to be *amicable*: $|E(\mathbb{F}_p)| = q$ and $|E_q(\mathbb{F}_q)| = p$, where $|E(\mathbb{F}_\ell)|$ denotes cardinality of the group of points of the reduction of E over the finite field of ℓ -elements \mathbb{F}_ℓ . He then considered a function that counts such pairs on average over a family of elliptic curves and discussed new results related to the constant obtained in the asymptotics of this function.

3 Objectives Achieved

This was the seventh edition of Alberta Number Theory Days. Previous conferences took place in Lethbridge (2008), Calgary (2009), and BIRS (2010, 2011, 2013, 2014). This friendly meeting gathers the number theorists of the Alberta Universities to interact and exchange ideas once a year. This year, the plenary speakers were William Casselman (UBC) and Julia Gordon (UBC). Professor Casselman is a leading authority on automorphic forms and representation theory and Dr. Gordon is a leading expert in the representation theory of p -adic groups and motivic integration.

During this conference there were a total of nine talks: three external speakers, two from Calgary, two from Lethbridge, and two from Edmonton. We have an increasing number of young female researchers and it was important to reflect this in both the schedule and the list of participants. As mentioned earlier, nearly one third of the participants this year were female and two of nine speakers were female. Another goal of the conference was to give the opportunity to young researchers to present their research. In particular, three of the talks were given by postdocs: Stephan Ehlen (McGill), James Parks (U. Lethbridge), and Anna Puskas (U. Alberta). Although this conference is mainly for Alberta researchers we have tried in recent years to have more outside participation. This year there were two participants from B.C., one from Quebec, and two from China. Although there were fewer talks than last year, participants had more opportunities to discuss and exchange ideas during the breaks. In particular, the Alberta researchers in automorphic forms and representation theory (Clifton Cunningham (Calgary), Manish Patnaik (Edmonton), and their students and postdocs) benefited greatly from interacting with the plenary speakers. We also tried to showcase some of the high quality research being done in Alberta, as evidenced by the talks by Mark Bauer (Calgary) and Amir Akbary (Lethbridge).