Littlewood-Richardson rules for symmetric skew quasisymmetric Schur functions

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Outline

1. The left Pieri composition poset
2. Composition tableaux
3. Skew quasisymmetric Schur functions
4. Left and right Littlewood-Richardson rules
5. Combinatorial classification of symmetric skew quasisymmetric Schur functions

Compositions

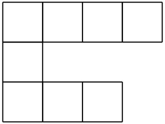
A composition $\alpha_1 \cdots \alpha_k$ of n is a list of positive integers whose sum is n: $2213 \vDash 8$.

$$\text{composition } \alpha = \alpha_1 \cdots \alpha_k \vDash n \leftrightarrow \text{subset } S = \{i_1, \ldots, i_{k-1}\} \subseteq [n-1]$$

$$\mathrm{comp}(S) \qquad 2213 \vDash 8 \leftrightarrow \{2, 4, 5\} \subseteq [7] \qquad \mathrm{set}(\alpha)$$

Composition diagrams

The composition diagram of $\alpha = \alpha_1 \cdots \alpha_k$, also called α, is the array of boxes with α_i boxes in row i from the top.



α=413

Algebra of quasisymmetric functions

Let **QSym** be the algebra of quasisymmetric functions

$$\mathbf{QSym} = \mathbf{QSym}^0 \oplus \mathbf{QSym}^1 \oplus \cdots \subset \mathbb{Q}[[x_1, x_2, \ldots]]$$

$$\mathbf{QSym}^n = \mathrm{span}_{\mathbb{Q}}\{F_\alpha \mid \alpha = \alpha_1 \cdots \alpha_k \vDash n\}$$

$$F_\alpha = \sum x_{i_1} x_{i_2} \cdots x_{i_n}$$

where the sum is over all n-tuples $(i_1, \ldots, i_n)$ of positive integers satisfying $1 \leq i_1 \leq \cdots \leq i_n$ and $i_j < i_{j+1}$ if $j \in \mathrm{set}(\alpha)$. The F_α are called the fundamental quasisymmetric functions.

Example

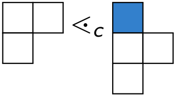
$$F_{13} = x_1^1 x_2^3 + x_1^1 x_3^3 + x_2^1 x_3^3 + \cdots + x_1^1 x_2^2 x_3^1 + x_1^1 x_2^2 x_4^1 + \cdots$$
$$+ x_1^1 x_2^1 x_3^2 + x_1^1 x_2^1 x_4^2 + \cdots + x_1^1 x_2^1 x_3^1 x_4^1 + x_1^1 x_2^1 x_3^1 x_5^1 + \cdots$$

The left Pieri composition poset

Given a composition β, cover relation $\lessdot_c$ given by

- adding a box in front to obtain $\alpha = \mathfrak{t}_1(\beta)$,

β



$\mathfrak{t}_1(\beta)$

- adding a box to the topmost part of size $i-1$ to obtain $\alpha = \mathfrak{t}_i(\beta)$ if $i \geq 2$.

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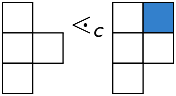

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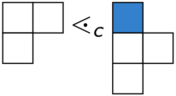

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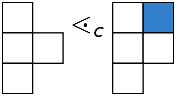

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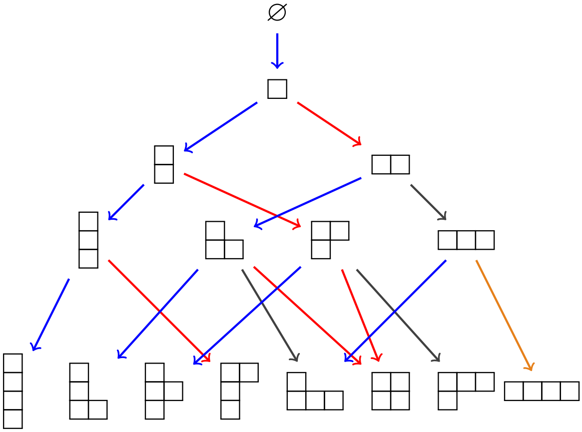
The $\mathfrak{t}_i$ are the box adding operators on compositions.

The left Pieri composition poset

The left Pieri composition poset $\mathcal{L}_c$ with partial order $<_c$: the poset on compositions obtained by taking the transitive closure of $\lessdot_c$.



Composition tableaux (CTs)

If $\alpha = \mathfrak{t}_{i_1} \cdots \mathfrak{t}_{i_k}(\beta)$ where $i_1 > i_2 > \cdots > i_k$, then $\alpha /\!\!/ \beta$ is called a horizontal strip.

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chains in $\mathcal{L}_c$	$\leftrightarrow$	composition tableaux
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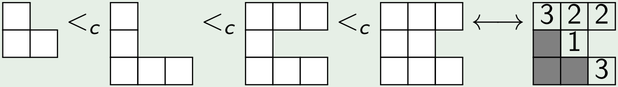

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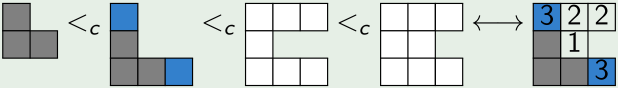

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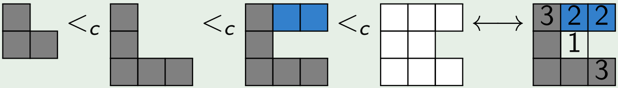

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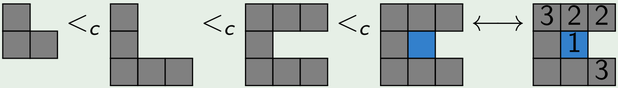

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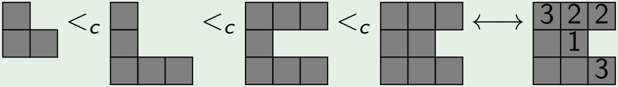


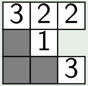
Skew quasisymmetric Schur functions

Given a CT τ, define the content $\mathrm{cont}(\tau)$ to be the list of nonnegative integers $\gamma_1 \cdots \gamma_k$ where γ_i counts the number of instances of i in τ and k is the maximum entry in τ.

Furthermore, let $x^\tau = x_1^{\#1s} x_2^{\#2s} x_3^{\#3s} \cdots$.

Example

$\tau =$



Then $\mathrm{cont}(\tau) = 122$ and $x^\tau = x_1 x_2^2 x_3^2$.

Skew quasisymmetric Schur functions

Given $\beta <_c \alpha$, the skew quasisymmetric Schur function $\mathcal{S}_{\alpha/\!\!/\beta}$ is defined as follows.

$$\mathcal{S}_{\alpha/\!\!/\beta} = \sum_{\tau \in \mathrm{CT}(\alpha/\!\!/\beta)} x^{\tau}$$

Example

$\mathcal{S}_{22/\!\!/1} = x_1^2 x_2 + x_1 x_2^2 + 2x_1x_2x_3 + \cdots$ from

1	1
	2

2	2
	1

3	2
	1

2	1
	3

$\ldots$.

Skew quasisymmetric Schur functions

1. If $\beta = \varnothing$, then $\mathcal{S}_{\alpha /\!/ \beta} = \mathcal{S}_{\alpha}$ is the quasisymmetric Schur function as defined by Haglund, Luoto, Mason and van Willigenburg.

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2. If β is a column, and $\ell(\beta) = \ell(\alpha)$, then $\mathcal{S}_{\alpha /\!\!/ \beta}$ is a Schur function.
3. If β is a partition, and $\ell(\beta) = \ell(\alpha)$, then $\mathcal{S}_{\alpha /\!\!/ \beta}$ is a skew Schur function.

Left Littlewood-Richardson rule

Given shape $\alpha /\!\!/ \beta$ and a partition ν, define the set of left Littlewood-Richardson composition tableaux $\mathsf{LRCT}^{\mathfrak{l}}_{\nu}(\alpha /\!\!/ \beta)$ as follows.

$$\mathsf{LRCT}^{\mathfrak{l}}_{\nu}(\alpha /\!\!/ \beta) = \left\{ \tau \in \mathsf{CT}(\alpha /\!\!/ \beta) \;\middle|\; \begin{array}{l} \mathrm{cont}(\tau) = \nu, \text{ and the } k\text{-th } i \\ \text{from the left is weakly left of} \\ \text{the } k\text{-th } i+1 \text{ from the left.} \end{array} \right\}$$

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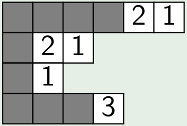
Example

Let $\alpha /\!/ \beta = 6324 /\!/ 4113$ and $\nu = 321$.



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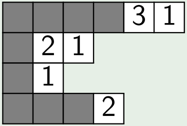

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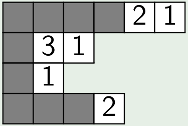

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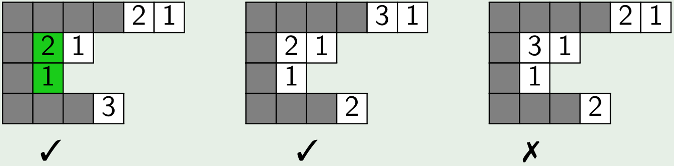

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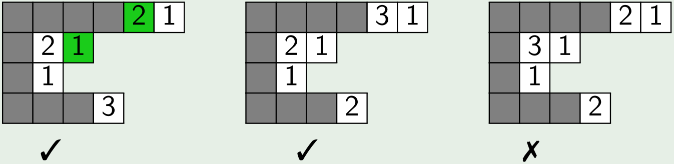

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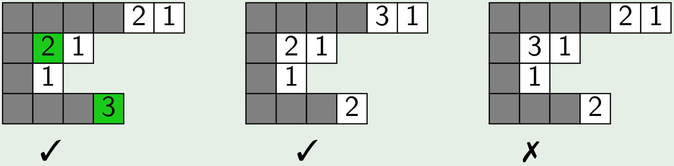

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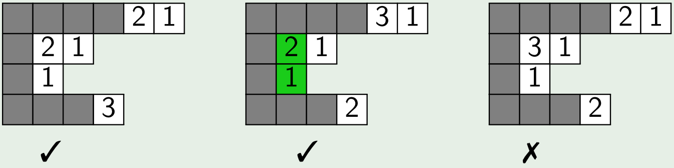

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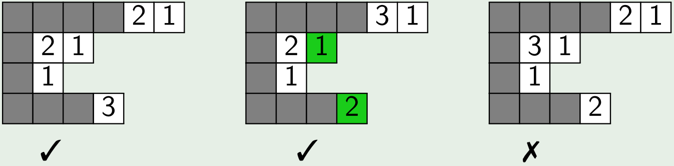

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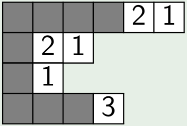

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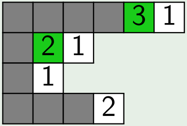

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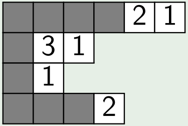

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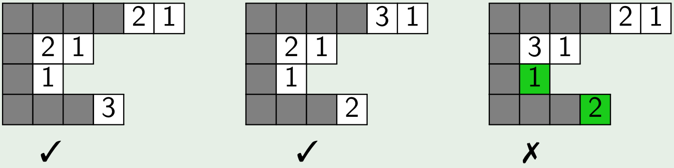

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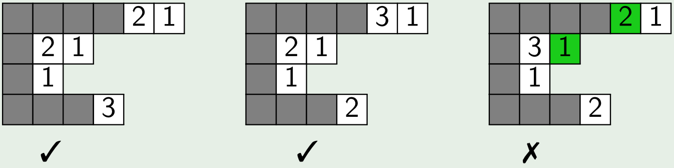

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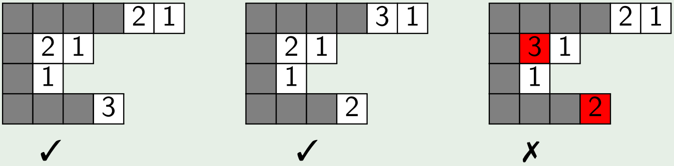


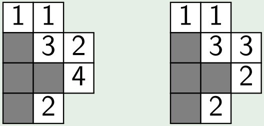
Left Littlewood-Richardson rule

Theorem (Bessenrodt-T.-van Willigenburg, 2014)

Given $\alpha /\!\!/ \beta$ such that $\mathcal{S}_{\alpha/\!\!/\beta}$ is symmetric, and a partition ν, the coefficient of s_ν in $\mathcal{S}_{\alpha/\!\!/\beta}$ is equal to the cardinality of the set $\mathrm{LRCT}^{\mathfrak{l}}_{\nu}(\alpha/\!\!/\beta)$.

Example

$\mathcal{S}_{2332/\!\!/121} = s_{2211} + s_{222}$ from



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Right Littlewood-Richardson rule

Given shape $\alpha /\!\!/ \beta$ and a partition ν, define the set of right Littlewood-Richardson composition tableaux $\mathsf{LRCT}^{\mathfrak{r}}_{\nu^r}(\alpha /\!\!/ \beta)$ as follows.

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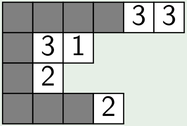

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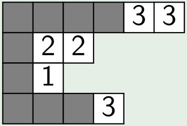

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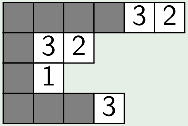

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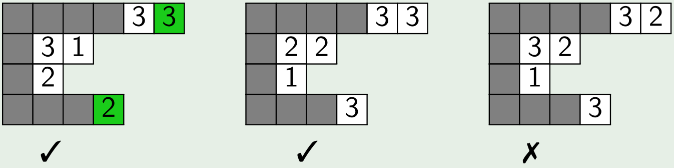

Right Littlewood-Richardson rule

$$\mathrm{LRCT}^{\mathfrak{r}}_{\nu^r}(\alpha/\!\!/\beta) = \left\{ \tau \in \mathrm{CT}(\alpha/\!\!/\beta) \;\middle|\; \begin{array}{l} \mathrm{cont}(\tau) = \nu^r, \text{ and the } \color{red}{k\text{-th } i+1} \\ \color{red}{\text{from the right is weakly right of}} \\ \color{red}{\text{the } k\text{-th } i \text{ from the right.}} \end{array} \right\}$$

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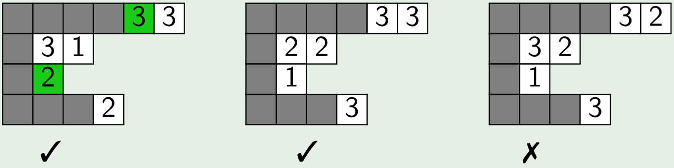

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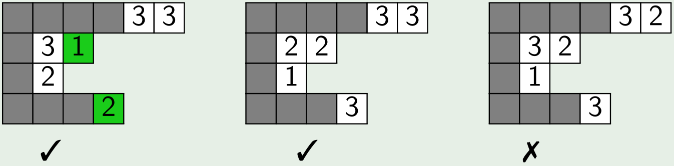

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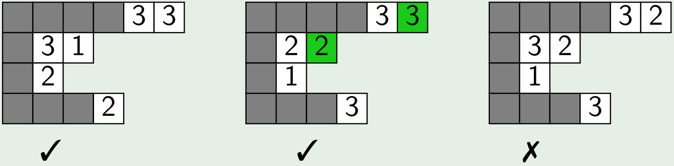

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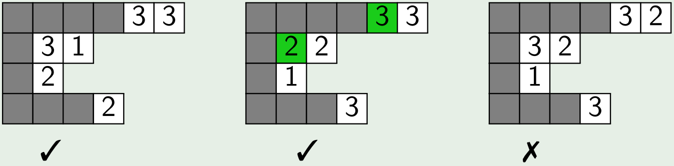

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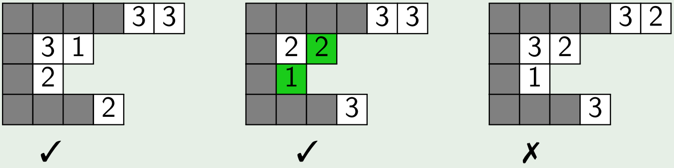

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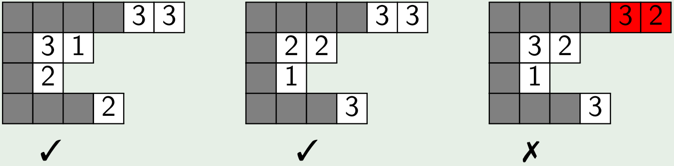


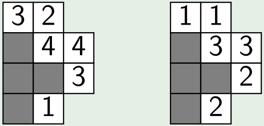
Right Littlewood-Richardson rule

Theorem (Bessenrodt-T.-van Willigenburg, 2014)

Given $\alpha /\!\!/ \beta$ such that $\mathcal{S}_{\alpha /\!\!/ \beta}$ is symmetric, and a partition ν, the coefficient of s_ν in $\mathcal{S}_{\alpha /\!\!/ \beta}$ is equal to the cardinality of the set $\mathrm{LRCT}^{\mathfrak{r}}_{\nu^r}(\alpha /\!\!/ \beta)$.

Example

$\mathcal{S}_{2332 /\!\!/ 121} = s_{2211} + s_{222}$ from



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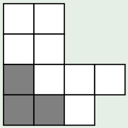
Symmetric skew quasisymmetric Schur functions

Let $\beta <_c \alpha$ and suppose further that $\ell(\beta) = k$ and $\ell(\alpha) = k + \ell$. Then we call the top ℓ rows of $\alpha /\!\!/ \beta$ the upper shape of $\alpha /\!\!/ \beta$.

If the upper shape of $\alpha /\!\!/ \beta$ is a rectangle, that is, $\alpha_1 = \cdots = \alpha_\ell$, then we say that $\alpha /\!\!/ \beta$ is uniform.

Example

The shape $2243 /\!\!/ 12$ is uniform.



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Theorem (Bessenrodt-T.-van Willigenburg, 2014)

If $\alpha /\!\!/ \beta$ is such that $\mathcal{S}_{\alpha /\!\!/ \beta}$ is symmetric, then $\alpha /\!\!/ \beta$ is uniform.

Thank you for listening!