Methods and Challenges in Extremal and Probabilistic Combinatorics
August 23–28 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honour system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00	Jacob Fox: *Packing problems*
9:40	Eyal Lubetzky: *Cutoff on all Ramanujan graphs*
10:20	Coffee Break, TCPL
10:40	Alex Scott: *Graphs of large chromatic number*
11:20	Dhruv Mubayi: *Hypergraph Ramsey numbers*
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:20	Po-Shen Loh: *Directed paths: from Ramsey to Ruzsa and Szemerédi*
15:00	Coffee Break, TCPL
15:40	Tibor Szabó: *Half-random Maker-Breaker games*
16:20	Asaf Ferber: *Online sprinkling and packing problems*
17:00	Choongbum Lee: *Ramsey numbers of degenerate graphs*
17:40–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00	David Conlon: *Rational exponents in extremal graph theory*
9:40	Hao Huang: *Digraphs of large girth with every small subset dominated*
10:20	Coffee Break, TCPL
10:40	Sasha Kostochka: *On the Corrádi-Hajnal Theorem and a question of Dirac*
11:20	Shagnik Das: *A removal lemma for nearly-intersecting families*
12:00	Asaf Shapira: *Decomposing a graph into expanding subgraphs*
12:40–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00	Nati Linial: *Random simplicial complexes - Progress report*
9:40	Hamed Hatami: *On the boundary of the region defined by homomorphism densities*
10:20	Coffee Break, TCPL
10:40	Boris Bukh: *Ranks of matrices with few distinct entries*
11:20	Mathias Schacht: *Forcing quasirandomness with triangles*
12:00–13:30	Lunch
13:40	Noga Alon: *Augmented trees with high girth*
14:20	Jacques Verstraete: *Full subgraphs*
15:00	Coffee Break, TCPL
15:40	Jozsi Balogh: *Triangle factors in graphs with small independence number*
16:20	Deryk Osthus: *Decompositions of large graphs into small subgraphs*
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00	Ehud Friedgut: *Entropy as a tool for proving analytical and geometrical inequalities*
9:40	Daniela Kühn: *Optimal path and cycle decompositions of random graphs*
10:20	Coffee Break, TCPL
10:40	Van Vu: *Anti-concentration inequalities for polynomials*
11:20	Sergey Norin: *The extremal function for 2-regular minors*
12:00	Oleg Pikhurko: *Supersaturation problem for colour-critical graphs*
12:40–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Informal Discussions
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Methods and Challenges in Extremal and Probabilistic Combinatorics
August 23–28 2015

ABSTRACTS
(in alphabetical order by speaker surname)

Speaker: **Noga Alon** (Tel Aviv University and IAS, Princeton)
Title: *Augmented trees with high girth*
Abstract: Let G be a graph consisting of a complete binary tree of depth h together with a back edge from each leaf connecting it to one of its ancestors. Suppose further that the girth of G exceeds g. What is the minimum possible depth $h = h(g)$ in such a graph?

This question is motivated by results in a joint paper with Kostochka, Reiniger, West and Zhu, where these graphs are used to provide simple explicit constructions of graphs and hypergraphs of high girth and high chromatic number, as well as tight examples of sparse high girth bipartite graphs with large list-chromatic number.

Speaker: **Jozsi Balogh** (Urbana–Champaign)
Title: *Triangle factors in graphs with small independence number*

Speaker: **Boris Bukh** (Carnegie Mellon)
Title: *Ranks of matrices with few distinct entries*
Abstract: Given a number λ, what is the maximum multiplicity of λ as an eigenvalue of an adjacency matrix of an n-vertex digraph? I will present a solution to this question, and describe general results of ranks of matrices with few distinct entries. The connection to the "linear algebra method in combinatorics" will also be explained.

Speaker: **David Conlon** (Oxford)
Title: *Rational exponents in extremal graph theory*
Abstract: Given a family of graphs $\mathcal{H}$, the extremal number $\mathrm{ex}(n, \mathcal{H})$ is the largest m for which there exists a graph with n vertices and m edges containing no graph from the family $\mathcal{H}$ as a subgraph. We show that for every rational number r between 1 and 2, there is a family of graphs $\mathcal{H}_r$ such that $\mathrm{ex}(n, \mathcal{H}_r) = \Theta(n^r)$. This solves a longstanding problem in the area of extremal graph theory.

Joint work with Boris Bukh.

Speaker: **Shagnik Das** (Freie Universität Berlin)
Title: *A removal lemma for nearly-intersecting families*
Abstract: We will present a simple removal lemma for large intersecting families, showing that a k-uniform set family on $[n]$ with close to $\binom{n-1}{k-1}$ sets and few disjoint pairs can be made intersecting by removing few sets. We shall then apply the lemma to resolve a question of Bollobás, Narayanan and Raigorodskii regarding transference of the Erdős-Ko-Rado theorem to sparse random Kneser subgraphs.

Joint work with Tuan Tran.

Speaker: **Asaf Ferber** (Yale)
Title: *Online sprinkling and packing problems*
Abstract: We present a new perspective of generating random structures referred to as "online sprinkling" (joint with Vu). Using this method, we then solve few packing problems in random graphs and hypergraphs for almost optimal probabilities, such as: Packing perfect matchings in hypergraphs (with Vu), packing

loose cycles (with Luh, Nguyen and Montealegre), packing arbitrarily oriented Hamilton cycles in directed graphs (with Long), and packing given spanning trees in $G_{n,p}$ (with Lee and Samotij).

Speaker: **Jacob Fox** (Stanford)
Title: *Packing Problems*
Abstract: Packing problems have been studied for more than four centuries, and have connections to diverse areas of pure and applied mathematics. In this talk, we focus on two well-studied combinatorial packing problems, Pippenger and Golumbic's graph inducibility problem from 1975 and Wilf's permutation packing problem from 1992. We discuss recent joint work with Hao Huang and Choongbum Lee which solves these problems in almost all instances.

Speaker: **Ehud Friedgut** (Weizmann Institute)
Title: *Entropy as a tool for proving analytical and geometrical inequalities*
Abstract: The Loomis-Whitney inequality, which bounds the volume of an n-dimensional body in terms of the volumes of its $(n-1)$-dimensional projections, and the Bollobás-Thomason inequalities which generalize it, are known [folklore] to follow from entropy considerations. We will see in this talk how to derive stability versions using this approach, together with Pinsker's information-theoretical inequality. (Joint with Ellis, Kindler, Yehudayoff). We will also see how to use entropy considerations to derive the famous Bomami-Gross-Beckner inequality, one of the most useful inequalities in analysis of Boolean functions.

Speaker: **Hamed Hatami** (McGill)
Title: *On the boundary of the region defined by homomorphism densities*
Abstract: The Kruskal-Katona theorem and a theorem of Razborov show that the closure of the boundary of the set of points defined by the edge and the triangle homomorphism densities of finite graph is a countable set of algebraic curves. In particular, it is almost everywhere smooth. This raises the question that whether the boundary is always as well-behaved even if one considers other graphs instead of an edge and a triangle. We construct examples which show that the (restrictions) of the boundary can have nowhere differentiable parts. This is based on a joint work with Sergey Norin.

Speaker: **Hao Huang** (Emory)
Title: *Digraphs of large girth with every small subset dominated*
Abstract: A conjecture of Daskalakis, Mehta and Papadimitriou states that there exist integers k and l, such that if a directed graph D satisfies that every subset of l vertices share a common in-neighbor, then D contains a directed cycle of length at most k. In this talk, I will discuss a counterexample to this conjecture and its connection with a well-known open problem on tournament coloring, and some applications in game theory. This is joint work with Anbalagan, Lovett, Norin, Vetta and Wu.

Speaker: **Sasha Kostochka** (Urbana–Champaign)
Title: *On the Corrádi-Hajnal Theorem and a question of Dirac*
Abstract: In 1963, Corrádi and Hajnal proved Erdős' conjecture that for all $k \geq 1$ and $n \geq 3k$, every (simple) graph G on n vertices with minimum degree $\delta(G) \geq 2k$ contains k disjoint cycles. This sharp result inspired a series of generalizations and refinements. In particular, the same year, Dirac described the 3-connected multigraphs not containing two disjoint cycles and asked the more general question: Which $(2k-1)$-connected multigraphs do not contain k disjoint cycles?

The goal of the talk is twofold. First, we describe all extremal (simple) graphs for the Ore-type version of the Corrádi–Hajnal Theorem (this version was proved by Enomoto and independently Wang). Then we use this result to answer Dirac's question in full.

Joint work with Hal Kierstead, Theodore Molla and Elyse Yeager

Speaker: **Daniela Kühn** (Birmingham)
Title: *Optimal path and cycle decompositions of random graphs*

Abstract: Motivated by longstanding conjectures regarding decompositions of graphs into paths and cycles, we prove optimal decomposition results for dense random graphs into (i) cycles and edges, (ii) paths and (iii) linear forests. There is also an interesting connection to the overfull subgraph conjecture on edge-colourings of graphs. We actually derive (i)-(iii) from quasirandom versions of our results. The results are joint work with Stefan Glock and Deryk Osthus.

Speaker: **Choongbum Lee** (MIT)
Title: *Ramsey numbers of degenerate graphs*
Abstract: The Ramsey number of a graph G is the minimum integer n for which every edge-coloring of the complete graph on n vertices with two colors admits a monochromatic copy of G. A graph is d-degenerate if all its subgraphs have a vertex of degree at most d. In this talk, we prove that for all d, there exists a constant c such that every d-degenerate graph G has Ramsey number at most $c|V(G)|$. This solves a conjecture of Burr and Erdős from 1973.

Speaker: **Nati Linial** (Hebrew University)
Title: *Random simplicial complexes - Progress report*
Abstract: About ten years ago Roy Meshulam and I introduced a model of random d-dimensional simplicial complexes which for $d = 1$ coincides with $G(n, p)$. The first accomplishment in this research project was to find the d-dimensional analog of the graph connectivity threshold at $p = \log n/n$. More recently we sought the counterpart of the phase transition that occurs at $p = 1/n$ in $G(n, p)$. In addition to technical challenges this raises several conceptual difficulties: (i) In dimension $d > 1$ there are at least two distinct and natural analogs of a forest, namely a simplicial complex that is (a) acyclic or (b) collapsible, and (ii) In searching the analog of the giant component - In dimension > 1 there is no natural notion of a connected component. We have recently completed this project in a series of several papers. My collaborators in this project were R. Meshulam, T. Luczak and my PhD students L. Aronshtam and Y. Peled. The talk will be self-contained and no background in topology is needed to follow the lecture.

Speaker: **Po-Shen Loh** (Carnegie Mellon)
Title: *Directed paths: from Ramsey to Ruzsa and Szemerédi*
Abstract: Starting from an innocent Ramsey-theoretic question regarding directed paths in tournaments, we discover a series of rich and surprising connections that lead into the theory around a fundamental problem in Combinatorics: the Ruzsa-Szemerédi induced matching problem. Using these relationships, we prove that every coloring of the edges of the transitive N-vertex tournament using three colors contains a directed path of length at least $\sqrt{N}e^{\log^* N}$ which entirely avoids some color. We also completely resolve the analogous question for ordinary monochromatic directed paths in general tournaments, as well as natural generalizations of the Ruzsa-Szemerédi problem which we encounter through our investigation.

Speaker: **Eyal Lubetzky** (Courant Institute)
Title: *Cutoff on all Ramanujan graphs*
Abstract: We show that on every Ramanujan graph, the simple random walk exhibits cutoff, and in fact its L^p-mixing time is optimal for all p in $[1, \infty]$. Our proof also shows that for every vertex x in a d-regular Ramanujan graph on n vertices, its distance from $n - o(n)$ of the vertices is asymptotically $\log_{d-1}(n)$. Moreover, if the girth is sufficiently large (e.g., the bipartite LPS expanders) then almost every pair x, y lies on a simple cycle of length $(2 + o(1)) \log_{d-1}(n)$.

Joint work with Yuval Peres.

Speaker: **Dhruv Mubayi** (University of Illinois at Chicago)
Title: *Hypergraph Ramsey numbers*
Abstract: We provide a quantitative relationship between multicolored versions of the hypergraph Ramsey number of an ordered tight path versus a clique and the diagonal Ramsey number. This has several consequences to classical questions about hypergraph Ramsey numbers. This is joint work with Andrew Suk.

Speaker: **Sergey Norin** (McGill)
Title: *The extremal function for 2-regular minors*
Abstract: Erdős and Gallai determined the minimum number of edges in a graph with given number of vertices necessary to guarantee existence of a cycle of length at least k. Justensen, settling a conjecture of Erdős and Pósa, determined the minimum number of edges necessary to force k disjoint cycles. We extend both of the results, proving a bound on the number of edges necessary to guarantee a given 2-regular graph as a minor. This settles a conjecture of Reed and Wood. Our results are applicable to general disconnected minors.

Joint work with E. Csoka, K. Edwards, I. Lo, H. Wu and L. Yepremyan.

Speaker: **Deryk Osthus** (Birmingham)
Title: *Decompositions of large graphs into small subgraphs*
Abstract: A fundamental theorem of Wilson states that, for every graph F, every sufficiently large F-divisible clique has an F-decomposition. Here G has an F-decomposition if the edges of G can be covered by edge-disjoint copies of F (and F-divisibility is a trivial necessary condition for this). We extend Wilson's theorem to graphs which are allowed to be far from complete. Our main contribution is a general 'iterative absorption' method which turns an approximate or fractional decomposition into an exact one (joint work with B. Barber, D. Kuhn, A. Lo).

Speaker: **Oleg Pikhurko** (Warwick)
Title: *Supersaturation Problem for Colour-Critical Graphs*
Abstract: Let F be an r-colour-critical graph, that is, its chromatic number is $r+1$ while the removal of some edge brings it down to r. Let n tend to infinity. As was shown by Simonovits, the maximum size of an F-free graph of order n is attained by the Turan graph $T_r(n)$, the balanced complete r-partite graph of order n.

Let $h_F(n,q)$ be the minimum number of copies of F in a graph with n vertices and $e(T_r(n))+q$ edges. We show that for $q=o(n^2)$ the asymptotic value of $h_F(n,q)$ can be obtained by solving a certain optimisation problem independent of n. Also, our method gives exact results in some cases. For example, we can determine the supremum of c such that the value of $h_F(n,F)$ for every $q<cn$ is obtained by adding q edges to $T_r(n)$, when F is an odd cycle, a clique with one edge removed, a complete bipartite graphs plus an edge, etc. (For cliques the supremum was determined by Lovász and Simonovits.)

Joint work with Zelealem Yilma.

Speaker: **Mathias Schacht** (University of Hamburg)
Title: *Forcing quasirandomness with triangles*
Abstract: It follows from a result of Simonovits and Sós that for any $d>0$ an n-vertex graph of density d is quasirandom iff every linear sized subset induces a subgraph with the triangle density approximately d^3. The orignal proof was based on the regularity lemma and we present a simple proof, which avoids the use of the regularity lemma and which also extends to other graphs than triangles. Similar results were obtained by Conlon, Fox and Sudakov.

We also show (using the regularity lemma again) that one may strengthen the result and, in fact, it suffices to assume that for every linear sized subset X of the vertices the number of triangles with at least two vertices in X is approximately $d^3|X|^2n$ to enforce that the given graph is quasirandom with density d. This is joint work with Christian Reiher.

Speaker: **Alex Scott** (Oxford)
Title: *Graphs of large chromatic number*
Abstract: Let G be a graph with large chromatic number. What induced subgraphs must it contain? It may contain a large complete subgraph, but what can we say if this is not the case? We shall discuss some old questions and new results on this topic. (Joint work with Maria Chudnovksy and Paul Seymour.)

Speaker: **Asaf Shapira** (Tel Aviv University)
Title: *Decomposing a graph into expanding subgraphs*
Abstract: A paradigm that was successfully applied in the study of both pure and algorithmic problems in graph theory can be colloquially summarized as stating that "any graph is close to being the disjoint union of expanders". Our goal in this paper is to show that in several of the instantiations of the above approach, the quantitative bounds that were obtained are essentially best possible. These results are obtained as corollaries of a new family of graphs, which we construct by picking random subgraphs of the hypercube, and analyze using (simple) arguments from the theory of metric embedding.

Joint work with G. Moshkovitz

Speaker: **Tibor Szabó** (Freie Universität Berlin)
Title: *Half-random Maker-Breaker games*
Abstract: We study Maker-Breaker positional games between two players, one of whom is playing randomly against an opponent with an optimal strategy. In both such scenarios, that is when Maker plays randomly and when Breaker plays randomly, we determine the sharp threshold bias of classical graph games, such as connectivity, Hamiltonicity, and minimum degree-k. The traditional, deterministic version of these games, with two optimal players playing, are known to obey the so-called probabilistic intuition. That is, the threshold bias of these games is asymptotically equal to the threshold bias of their random counterpart, where players just take edges uniformly at random. We find, that despite this remarkable agreement of the results of the deterministic and the random games, playing randomly against an optimal opponent is not a good idea: the threshold bias becomes significantly higher in favor of the clever player. An important qualitative aspect of the probabilistic intuition nevertheless carries through: for Maker to occupy a connected graph or a Hamilton cycle, the bottleneck is still the ability to achieve that there is no isolated vertex in his graph. The talk represents joint work with Jonas Groschwitz.

Speaker: **Jacques Verstraete** (UCSD and NSF)
Title: *Full subgraphs*
Abstract: Let G be a graph of density p on n vertices. Erdős, Łuczak and Spencer defined a subgraph H of G to be *full* if every vertex of H has degree at least $p(|V(H)|-1)$ in H. Let $f(G)$ denote the largest number of vertices in a full subgraph of G and let $f_p(n)$ denote the smallest value of $f(G)$ over all graphs G of density p with n vertices. Erdős, Łuczak and Spencer proved $\sqrt{2n} \leq f_{0.5}(n) \leq 2n^{2/3}(\log n)^{1/3}$ and also showed that the random graph $G_{n,p}$ has linear-sized full subgraphs asymptotically almost surely as $n \to \infty$. In this talk, we give a short proof that $f_p(n) = \Omega(n^{\frac{2}{3}})$ whenever $p(1-p) \geq n^{-\frac{2}{3}}$ and we show that this is tight up to constants for infinitely many p near the Turán densities $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ...$ The problem of determining $f(G)$ is related to results on judicious partitions and discrepancy of graphs, which are of independent interest. In conclusion, a number of interesting open questions will be presented.

Joint work with Victor Falgas-Ravry and Klas Markström.

Speaker: **Van Vu** (Yale)
Title: *Anti-concentration inequalities for polynomials*
Abstract: An anti-concentration inequality bounds the probability that a random variable F takes value in a fixed small interval. The first such result is the famous Littlewood-Offord inequality, strengthened by Erdős. Let ξ_i be iid Rademacher random variables and c_i be real coefficients with absolute value at least 1. Consider the linear function

$$F(\xi) = c_1\xi_1 + ... + c_n\xi_n.$$

ELO inequality asserts that for any fixed interval I of length 1, the probability that F belongs to I is $O(n^{1/2})$.

In this talk, we generalize this result to the case when F is a polynomial in terms of the variables ξ_i. Our bound is near optimal and improves significantly earlier estimates by Costello-Tao-Vu and Razborov-Viola.

As applications, we settle (up to an iterative logarithmic term) a problem of Razborov and Viola in complexity theory and prove a result concerning the number of copies of a small fixed graph H in $G(n,p)$. Time allows, I will also discuss several open questions.

(joint work with R. Meka and O Nguyen)