The Use of Linear Algebraic Groups in Geometry and Number Theory
September 14 - 18, 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	A. Merkurjev
10:00–10:20	*Coffee Break*
10:20–10:50	S. Baek
11:00–11:30	R. Pirisi
13:30–14:30	V. Chernousov
14:40–15:10	I. Rapinchuk
15:10–15:30	*Coffee Break*
15:30–16:30	M. Borovoi
16:40–17:10	D. Zywina

Tuesday

9:00–10:00	E. Bayer
10:00–10:20	*Coffee Break*
10:20–10:50	A. Auel
11:00–11:30	M. MacDonald
13:30–14:30	V. Suresh
14:40–15:10	N. Bhaskhar
15:10–15:30	*Coffee Break* and Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
15:30–16:30	J. Hartmann
16:40–17:10	A. Stavrova
17:20–17:50	A. Duncan

Wednesday

9:00–10:00	M. Brion
10:00-10:20	*Coffee Break*
10:20-10:50	C. Zhong
11:00-11:30	A. Neshitov
13:30–14:30	V. Popov
14:40–15:10	O. Haution
15:10–15:30	*Coffee Break*
15:30–16:30	G. Savin
16:40–17:10	B. Antineau

Thursday

9:00–10:00	M. Levine
10:00–10:20	*Coffee Break*
10:20–10:50	B. Williams
11:00–12:00	I. Panin
	Free Afternoon

Friday

9:00–10:00	D. Krashen
10:00–10:20	*Coffee Break*
10:20–11:20	N. Karpenko
	Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Benjamin Antieau** (University of Illinois at Chicago)
Title: *Prime decomposition in period-index problems via representation theory*
Abstract: I will report on joint work with B. Williams on the use of representations of projective general linear groups to extend known facts about the prime divisors of the period and index of Brauer classes that hold over fields to more general settings. The first result is that the primes dividing the period and index agree. This is proved using only exterior representations. The second result is that the index of a Brauer class is the product of the indices of each of its p-parts. This requires more complicated Young diagrams.

Speaker: **Asher Auel** (Yale University)
Title: *Algebras of composite degree split by genus one curves*
Abstract: There is an old question of whether every central simple algebra can be split by the function field of a genus one curve defined over the base field. So far, the answer is known for algebras of degree at most 5. I will propose a method for answering this question in the affirmative for algebras of composite degree, when the answer is known for the prime power factors. Some applications will be given.

Speaker: **Sanghoon Baek** (Korean Advanced Institute of Science and Technology)
Title: *Semi-decomposable invariants of degree 3*
Abstract: Recently, a new type of degree 3 invariant of a split semisimple group, called a semi-decomposable invariant was introduced by Merkurjev-Neshitov-Zainoulline. This invariant is locally decomposable and it was shown that there is no nontrivial semi-decomposable invariant of a split simple group. In this talk, we discuss semi-decomposable invariants of a split reductive group in terms of the torsion in the codimension 2 Chow groups of a product of Severi-Brauer varieties. In particular, we present a method to find nontrivial semi-decomposable invariants of an arbitrary split semisimple group of type A.

Speaker: **Eva Bayer-Fluckiger** (École Polytechnique Fédérale de Lausanne)
Title: *Rationally isomorphic hermitian forms and torsors of some non-reductive groups*
Abstract: Let R be a semi-local Dedekind domain. Under certain assumptions, we show that two (not necessarily unimodular) hermitian forms over an R-algebra with involution that are rationally isometric are in fact isometric. The result can be regarded as a first step towards a version of the Grothendieck-Serre conjecture for certain non-reductive group schemes over $Spec\, R$.

Speaker: **Nivedita Bhaskhar** (Emory University)
Title: *Reduced Whitehead groups of division algebras over function fields of p-adic curves*
Abstract: The question of whether every reduced norm one element of a central simple algebra is a product of commutators was formulated in 1943 by Tannaka and Artin in terms of the reduced Whitehead group $SK_1(D)$.

Merkurjev's breakthrough on Suslin's conjecture shows that there are always field extensions over which non square free index algebras acquire a nontrivial Whitehead group. However it is a theorem of Merkurjev/Rost that for central simple algebras of degree 4, the Whitehead group is trivial over fields of cohomological dimension ≤ 3. This is a consequence of an injection of $SK_1(D)$ into a sub-quotient of the degree 4 Galois cohomology group which led Suslin to ask whether the same was true for index l^2 algebras for any prime l over cohomological dimension 3 fields.

In this talk, we address this question for l torsion, degree l^2 algebras over function fields of p-adic curves where l is any prime not equal to p. The proof relies on the techniques of patching as developed by Harbater-Hartmann-Krashen and exploits the arithmetic of these fields to show triviality of the reduced Whitehead group.

Speaker: **Mikhail Borovoi** (Tel Aviv University)
Title: *Real Galois cohomology of semisimple groups*
Abstract: In a 2-page note of 1969, Victor Kac described automorphisms of finite order of simple Lie algebras over the field of complex numbers $\mathbb{C}$. He used certain diagrams that were later called Kac diagrams. In this talk, based on a joint work with Dmitry Timashev, I will explain the method of Kac diagrams for calculating the Galois cohomology set $H^1(\mathbb{R}, G)$ for a connected semisimple algebraic group G over the field of real numbers $\mathbb{R}$. I will use real forms of half-spin groups as examples. A half-spin group over $\mathbb{C}$ is a simple $\mathbb{C}$-group of type D_n for even $n > 4$ which is neither simply connected nor adjoint and not isomorphic to $SO(2n, \mathbb{C})$. No prior knowledge of Galois cohomology, Kac diagrams, or of half-spin groups will be assumed.

Speaker: **Michel Brion** (Institut Fourier)
Title: *Realizing algebraic groups as automorphism groups*
Abstract: We address the question of realizing a given algebraic group as the automorphism group of some algebraic variety. We show that every smooth connected group scheme over a perfect field is the connected automorphism group scheme of a normal projective variety.

Speaker: **Vladimir Chernousov** (University of Alberta)
Title: *Algebraic groups and their maximal tori*
Abstract: We will survey recent developments dealing with characterization of absolutely almost simple algebraic groups having the same isomorphism/isogeny classes of maximal tori over the field of definition. These questions arose in the analysis of weakly commensurable Zariski-dense subgroups. While there are definitive results over number fields, the theory over general fields is only emerging. We will formulate the existing conjectures, outline their potential applications, and report on recent progress. Joint work with A. Rapinchuk and I. Rapinchuk.

Speaker: **Alexander Duncan** (University of South Carolina)
Title: *Pairs of quadratic forms in characteristic 2*
Abstract: We consider smooth complete intersections of two quadrics in even-dimensional projective space. Over an algebraically closed field of characteristic not 2, it is well known that one can find a basis in which both quadratic forms are diagonal. However, this fails in characteristic 2. We present a normal form which applies over an arbitrary field of characteristic 2. Using this normal form we determine the automorphism groups of these varieties.

Speaker: **Julia Hartmann** (RWTH Aachen)
Title: *Obstructions to Local-Global Principles for Linear Algebraic Groups*
Abstract: Local-Global Principles are a very important subject in the theory of linear algebraic groups. We study such principles for groups defined over arithmetic function fields. In that case, some obstructions come from a collection of overfields associated with patching. These obstructions are reasonably well understood for rational linear algebraic groups, but interesting examples due to Colliot-Thélène, Parimala and Suresh show that the case of nonrational groups is more difficult. Recent results provide an explanation as to why these examples occur, via the geometry of a model for the function field. This is joint work with D. Harbater and D. Krashen.

Speaker: **Olivier Haution** (University of Munich)
Title: *Finite group actions on the affine space*

Abstract: We discuss the existence of fixed rational points for the action of a finite p-group on the affine space over a field.

Speaker: **Nikita Karpenko** (University of Alberta)
Title: *On* 16*-dimensional quadratic forms in* I^3
Abstract: There are many mysteries related to quadratic forms with Witt class in I^3. And 16 is the least (if not only) dimension for which we don't know:

- whether the forms can be can be parameterized by algebraically independent variables,
- if every form contains a proper subform with Witt class in I^2,
- how many 3-Pfister forms are needed to write the Witt class of ab arbitrary 16-dimensional form in I^3 (over an arbitrary field) as their linear combination (no upper bound at all is available).

In the talk we discuss and try to reveal some of the above mysteries.

Speaker: **Danny Krashen** (UGA)
Title: *The Clifford algebra of a finite morphism of schemes*
Abstract: In this talk we will define a Clifford algebra associated to a finite morphisms of schemes, generalizing the notion of the Clifford algebra of a homogeneous polynomial. We will then describe connections with relative Brauer groups and index reduction, with Ulrich bundles, and with the period-index problem for genus 1 curves. (joint work with Max Lieblich)

Speaker: **Marc Levine** (University of Essen-Duisburg)
Title: *On the geometric part of some oriented motivic theories*
Abstract: For an oriented motivic ring spectrum E in $SH(k)$, k a field of characteristic zero, there is a canonical map

$$\Omega^* \to E^{2*,*}$$

of oriented cohomology theories on Sm/k, in the sense of Levine-Morel. If E has associated formal group law (F, R), this map descends to

$$\Omega^* \otimes_L R \to E^{2*,*}$$

We describe a criterion which implies that this second map is an isomorphism of oriented cohomology theories on Sm/k. We then show that for a wide class of examples, including MGL, Landweber exact theories and their connective covers as well as certain quotients or localizations of MGL, such as truncated Brown-Peterson theories, Morava K-theories and connective Morava K-theory, satisfy this criterion. This covers joint work with S. Dai and with G. Tripathi.

Speaker: **Mark MacDonald** (University of Lancaster)
Title: *Reducing* E_7 *and the slice method*
Abstract: The motivation for this talk is to improve the known bounds on essential dimension of simple algebraic groups, as well as to address the (still open) question of whether or not $k(V)^G$ is stably rational, for a linear G-representation V. From joint work with R. Loetscher, I will explain how to use a versal action with a stabilizer in general position to obtain a reduction of structure map for torsors, including over fields of characteristic p, and even non-smooth groups. I will show how such a reduction can give new information for both questions, and finally I will mention the new upper bound $ed(E_7) \leq 11$.

Speaker: **Alexander Merkurjev** (University of California at Los-Angeles)
Title: *Suslin's Conjecture on the reduced Whitehead group of a simple algebra*
Abstract: I will report on the proof of Suslin's Conjecture on the generic non-triviality of the reduced Whitehead group of a simple algebra.

Speaker: **Alexander Neshitov** (University of Ottawa / Steklov Institute at St.Petersburg)
Title: *Motives of twisted flag varieties and representations of Hecke-type algebras*
Abstract: This is a report on the joint work in progress with N. Semenov, V. Petrov and K. Zainoulline. Following the motivic Galois group philosophy, we relate the category of (cobordism-) Ω-motives of twisted flag varieties for a semisimple linear algebraic group G with the category of integer (or modular) representations of the associated Hecke-type algebra $H = H(G)$.

The algebra H was introduced and studied recently in a series of papers by Calmés, Hoffnung, Malagon-Lopez, Savage, Zainoulline, Zhao, Zhong. It has two important properties: (i) its dual over $\Omega_T(pt)$, where T is a split maximal torus of G, gives the T-equivariant cobordism ring $\Omega_T(G/B)$ of the variety of Borel subgroups of G; (ii) its complete set of generators and relations is known and resembles those of an affine Hecke algebra.

As an application, we translate various motivic invariants (e.g. J-invariant of linear algebraic groups, Grothendieck groups of the respective motivic subcategories), results about indecomposable motives, etc, into the language of respective integer/modular representations of H. For instance, the generalized Rost motive corresponding to the Serre-Rost invariant $(3, 3)$ corresponds to an irreducible projective $H(F_4)$-module of rank 3 over the polynomial ring $\mathbb{Z}/3\mathbb{Z}[x_1, x_2, x_3, x_4]$.

Speaker: **Ivan Panin** (Steklov Institute at St.Petersburg)
Title: *A purity theorem*
Abstract: We will discuss the following

Conjecture. Let $\mathcal{O}$ be a regular local ring and K be its fraction field. Let $m\colon G \to C$ be a smooth $\mathcal{O}$-morphism of reductive $\mathcal{O}$-group schemes, with a torus C. Suppose additionally that the kernel of m is a reductive $\mathcal{O}$-group scheme. Then the following sequence

$$\{1\} \to C(\mathcal{O})/m(G(\mathcal{O})) \to C(K)/m(G(K)) \to \bigoplus_{ht(p)=1} C(K)/[C(\mathcal{O}_p) \cdot m(G(K))] \to \{1\}$$

is exact, where p runs over all height 1 primes of $\mathcal{O}$ and

$$res_p\colon C(K)/m(G(K)) \to C(K)/[C(\mathcal{O}_p) \cdot m(G(K))]$$

is the natural map (the projection to the factor group).

Theorem. The conjecture is true, if $\mathcal{O}$ is a regular local ring containing a field.

Remark. The exactness of that sequence in the middle term is used in the proof of the Grothendieck–Serre conjecture for regular local rings containing a field.

Speaker: **Roberto Pirisi** (University of Ottawa)
Title: *Cohomological Invariants for stacks of algebraic curves*
Abstract: Cohomological invariants are arithmetic analogues of characteristic classes in topology, in which singular cohomology is replaced with Galois cohomology, and topological spaces with spectra of fields.

Given an affine algebraic group G, a cohomological invariant for G is a way to functorially assign to each principal G-bundle over the spectrum a field k an element of the Galois cohomology of k. These invariants form a graded ring, which has been computed for many classes of algebraic groups by several authors.

In my talk I will show how to extend the classical theory to a theory of cohomological invariants for Deligne-Mumford stacks and in particular for the stacks of smooth genus g curves.

The concept of general cohomological invariants turns out to be closely tied to the theory of unramified cohomology, which was introduced by Saltman, Ojanguren and Colliot-Thélène and is widely used to study rationality problems.

I will also show how to compute the additive structure of the ring of cohomological invariants for the algebraic stacks of hyperelliptic curves of all even genera and genus three.

Speaker: **Vladimir L. Popov** (Steklov Institute, Moscow)
Title: *Simple algebras and algebraic groups*
Abstract: The following topics will be discussed:

(1) Given an algebraic group G, let V be a finite-dimensional algebraic G-module that admits a structure of a simple (not necessarily associative) algebra A for which $G = Aut(A)$. Then V admits a close approximation to the analogue of classical invariant theory.

(2) What are the groups G for which such a V exists?

(3) Given G, what are the G-modules V for which (1) holds?

Speaker: **Igor Rapinchuk** (Harvard University)
Title: *Division algebras with the same maximal subfields*
Abstract: In this talk, I will address the following question: Suppose D and D' are finite-dimensional central division algebras over a field K. What can be said about these algebras if they have the same maximal subfields? I will give an overview of some techniques used in the analysis of this problem and also describe recent results. This is joint work with V. Chernousov and A. Rapinchuk.

Speaker: **Gordan Savin** (Utah University)
Title: *Twisted Bhargava Cubes*
Abstract: In this talk I will discuss the problem of classifying rational orbits for pre homogeneous spaces. A classical example is $GL(n)$ acting on the space of symmetric matrices. In this case rational orbits are parameterized by isomorphism classes of quadratic spaces. For some pre homogeneous spaces arising from exceptional groups the orbit problem has an answer in terms of (twisted) composition algebras. This is a joint work with Wee Teck Gan.

Speaker: **Anastasia Stavrova** (St.Petersburg University)
Title: *Simple algebraic groups and structurable algebras*
Abstract: We present a uniform proof of the well-known correspondence between isotropic simple algebraic groups and simple structurable or Jordan algebras. This work is part of the forthcoming joint preprint with T. De Medts and L. Boelaert. Supported by the grant 6.50.22.2014 of St.Petersburg State University.

Speaker: **Venapally Suresh** (Emory University)
Title: *Rost invariant over function fields of p-adic curves*
Abstract: Let F be a field and G an absolutely almost simple simply connected algebraic group over F. Rost defined a functorial map $H^1(F, G) \to H^3(F, \mathbf{Q}/\mathbf{Z}(2))$. We discuss the injectivity of this map when F is the function field of a p-adic curve, with special reference to $G = SL_1(D)$, where D is a central simple algebra over F of index coprime to p.

Speaker: **Ben Williams** (University of British Columbia)
Title: *The topological index of period-2 Brauer classes*
Abstract: I will outline how one can use the homotopy theory of classifying spaces of linear groups to find obstructions to representing Brauer classes as the classes of Azumaya algebras of specific ranks. The talk will concentrate on the case of classes of period 2.

Speaker: **Changlong Zhong** (State University of New-York at Albany)
Title: *Equivariant oriented cohomology of flag varieties*
Abstract: In this talk I will explain algebraic construction of equivariant oriented cohomology of (partial

and full) flag varieties and of the push-pull morphisms between these cohomology groups. Moreover, the Schubert classes in these groups will be constructed.

Speaker: **David Zywina** (Cornell University)
Title: *l-adic monodromy groups for abelian varieties*