Abstracts of lectures for Banff meeting, Groups and Geometries, May 4-8, 2015

Michael Aschbacher: Quaternion fusion packets

Abstract I'll discuss work on a fusion theoretic version of the so-called Classical Involution Theorem for groups. The project is part of a program to simplify part of the proof of the theorem classifying the finite simple groups.

Pierre-Emmanuel Caprace, Nicola Radu: A compact space of isomorphism classes of simple locally compact automorphism groups of trees, I and II

Abstract Several important families of examples of non-discrete simple locally compact groups arise as automorphism groups of trees: simple algebraic groups of rank one over local fields, Kac-Moody groups of rank two over finite fields, as well as a large number of groups initially studied by Burger and Mozes and defined by prescribing their local action in terms of a given finite permutation group. The goal of the first part of this talk is to explain that a large set of isomorphism classes of simple locally compact groups acting on a given tree can be given a compact Hausdorff topology. Several questions arise then naturally, and will be addressed in the second part of the talk: Can we describe limits of known isomorphism classes? Which isomorphism classes form isolated points?

Andrew Chermak: Locally grouped spaces

Abstract That the finite simple groups and the simple algebraic groups have various structural properties in common need come as no great surprise since - as we now know - all but a small number of the finite simple groups are derivable from algebraic groups. But some degree of mystery remains, as is indicated by the proliferation of exotic fusion systems which, as it seems, share many of the same sorts of structural properties after the fact, but for which general organizing principles are largely absent. Why not start at the other end of the thread ? The talk will propose a scheme (pun intended) for unifying these various classes of groups and sort-of groups.

Arjeh Cohen: Lie algebras generated by extremal elements

Abstract Together with Gabor Ivanyos, simple Lie algebras generated by extremal elements were shown to have an embedded geometry that is the shadow of a building. In recent work, Cuypers, Fleischmann, Roberts and Shpectorov proved that the Lie algebras are uniquely determined by these geometries. I intend to give an overview of this work.

David Craven: Maximal subgroups of exceptional groups of Lie type

Abstract The maximal subgroups of the finite exceptional groups have been extensively studied over the last few decades, and much is known about them. If $G = G(q)$ is such a group, and H is a maximal subgroup of G, then either H is known or H comes from a list of almost simple groups, which of course can be separated into two classes: H of Lie type in the same characteristic as G, or everything else. In this talk I will describe recent work on the former case, which for G not of type E_8 yields almost a complete answer. We also describe some powerful statements that arise as corollaries from this work.

Nick Gill: Constructing groupoids using designs

Abstract John Conway famously used a "game" played on the projective plane of order 3 to construct a groupoid which he called M_{13}. This object can be thought of as a subset of S_{13} that extends the Mathieu group M_{12}. Conway's game can be played on other geometries and we discuss what happens when one plays the game on a supersimple $2-(v,4,\lambda)$ design: for instance, if we choose our design carefully, then we are able to construct the groups $Sp_{2m}(2)$ and $2^{2m}.Sp_{2m}(2)$ as groupoids á la M_{13}. The appearance of these groups is related to the fact that they are also 3-transposition groups and it is hoped that the classification of 3-transposition groups may be used to give a full classification of groupoids arising from designs.

Matthias Grueninger: Trees, Twinnings and Moufang sets

Abstract A Moufang set consists of a set X with $|X| > 2$ and a family $(U_x)\,(x \in X)$ of subgroups of $Sym(X)$ (called the root groups) such that for all $x \in X$ the group U_x acts regularly on $X \setminus x$ and that $(U_y)^g = U_{yg}$ holds for all $y \in X$ and $g \in U_x$. Moufang sets can be regarded as Moufang buildings of rank one. Examples of Moufang sets arise from sharply 2-transitive groups or from algebraic, classical or mixed groups of relative rank one. It is an open problem if every Moufang set is contained in one of these families. Let T be a tree and G a closed subgroup of $Aut(T)$. We say that (T,G) is boundary Moufang if for every end e of T there is a closed subgroup U_e of G such that $(T^\infty, (U_e)\,(e \in T^\infty))$ is a Moufang set and that $G^\dagger := \langle U_e : e \in T^\infty \rangle$ is normal in G. A typical example for this situation originates from an algebraic group of relative rank one over a field equipped with a discrete valuation. P.-E. Caprace and T. De Medts proved that if (T,G) is boundary Moufang with T locally finite and the root groups torsion free, then G is essentially an algebraic group over a p-adic number field for a prime p. In this talk we will

discuss the case that T is locally finite and the root groups are abelian and of exponent p. We will derive a similar result by constructing a twinning for T.

Max Horn: Generalized spin representations

Abstract String theorists used the theory of Clifford algebras in order to construct finite-dimensional representations of the maximal compact subalgebra of the real Kac-Moody algebra of type E_{10}. It turns out that such finite-dimensional representations exist for arbitrary symmetrizable type and are particularly easy to describe in the simply-laced case.

The resulting quotients are compact, whence reductive and often even semisimple. Cartan-Bott periodicity of Clifford algebras enables one to concretely determine the isomorphism types of these quotients for the E_n series. Since for E_6, E_7, E_8 the generalized spin representation is injective, one in fact obtains an alternative method of determining the isomorphism types of the the maximal compact subalgebras of these types.

Thanks to the Iwasawa decomposition and the simple connctedness of buildings of rank at least three, there exists a well-known amalgam presentation of the maximal compact subgroup of a real Kac-Moody group using rank 1 and rank 2 groups. This allows one to integrate the above representations to group level.

As in the finite-dimensional situation, these representations do not in general integrate to maximal compact subgroup, but instead to a spin cover. In the irreducible simply-laced case this spin cover is a two-fold central extension; in general it is a central extension by a finite (abelian) 2-group. The cardinality of this 2-group can be extracted easily from the combinatorics of the underlying diagram.

This spin cover contains the so-called spin-extended Weyl group which can be obtained from the Tits-Kac-Peterson extended Weyl group by relaxing the relation $x^4 = 1$ for the fundamental generators to $x^8 = 1$. The combinatorics of the underlying diagram allows one to predict which of these generators actually will have order 8 and which is these generators will nevertheless still have order 4.

Ellen Henke: Subcentric linking systems

Abstract Fusion systems are categories generalizing important features of fusion in finite groups. Attached to each fusion system $\mathcal{F}$ there is a finite p-group S such that the objects of $\mathcal{F}$ are all the subgroups of S. Broto, Levi and Oliver introduced linking systems to be able to study classifying spaces of fusion systems. Here a linking systems associated to $\mathcal{F}$ is a category whose objects are certain subgroups of S. In this talk I will introduce a more general notion of a linking system allowing a larger set of objects. This concept seems to be significant both from an algebraic and a homotopy theoretic perspective. In particular, it simplifies implementing the amalgam method in the context of fusion systems and can thus be expected to be

useful for translating results from the MSS-program to fusion systems.

Martin Kassabov: Hopf algebras and representations of $\mathrm{Aut}(F_n)$

Abstract We show that if H is a co-commutative Hopf algebra, then there is a natural action of $Aut(F_n)$ on H^n which induces an $Out(F_n)$ action on a quotient. In the case when $H = T(V_{2g})$ is the tensor algebra, we show that there is a surjection from cokernel of the Johnson homomorphism for the mapping class group of genus g to the top coholomogy groups of $Out(F_n)$ with coefficents in this representation.

The same constuction can be used to constuct representation of $Aut(\Gamma)$ for any finitely generated group Γ. In the case of $H = U(g)$ the resulting representations are related to the representation variety of Γ into G, where G is a Lie group with Lie algebra g.

Hendrik van Maldeghem: Groups of mixed type in the Freudenthal-Tits Magic Square

Abstract The geometries of the split and nonsplit version of the second row of the Freudenthal-Tits magic square have a common geometric characterisation, which induces a functor from a family of orthogonal geometries to Veronese representations (in the broad sense) of certain projective planes and parapolar spaces of diameter 2 related to buildings. We focus on the case where the orthogonal geometry is related to a group of mixed type and deduce some properties of the groups belonging to the corresponding Veronese representation.

Tom de Medts: Jordan algebras and 3-transposition groups

Abstract A 3-transposition group is a finite group G generated by a conjugacy class D of involutions, such that the product of any two elements of D has order at most 3. These groups have been studied by Fischer in the 1970s, who discovered three new sporadic groups in this process. Inspired by the theory of vertex operator algebras, Matsuo associated a commutative non-associative algebra to each 3-transposition group. Such an algebra is spanned by idempotents, and the eigenspaces for each of these idempotents satisfy very specific fusion rules, which happen to be the same as the (well-known) fusion rules for the Peirce decomposition in Jordan algebras. This led us to the question whether some of these Matsuo algebras are also Jordan algebras. We give a complete answer to this question; this led to some rather unexpected results. Along the way, we also obtain results about Jordan algebras associated to (arbitrary) root systems.

Luke Morgan: Locally semiprimitive arc-transitive graphs

Abstract An arc-transitive graph is locally semiprimitive if the stabiliser of each vertex induces a semiprimitive group on the neighbourhood of that vertex. Potocnik, Spiga and Verret conjecture that there is a function bounding

the order of a vertex stabilisers in such graphs in terms of the valency. Some recent progress on this conjecture will be presented.

Bob Oliver: Automorphisms of fusion and linking systems of finite groups of Lie type

Abstract For a finite group G with Sylow p-subgroup S, the fusion system of G at S is the category $F_S(G)$ whose objects are the subgroups of S, and whose morphisms are the homomorphisms induced by conjugation in G. Let $Aut(S, F_S(G))$ be the group of all automorphisms of S which preserve the fusion in G, and set $Out(S, F_S(G)) = Aut(S, F_S(G))/Aut_G(S)$ (i.e., dividing out by automorphisms induced by conjugation in $N_G(S)$). There is a natural homomorphism from $Out(G)$ to $Out(S, F_S(G))$, which in general is neither injective nor surjective.

In recent work with Carles Broto and Jesper Muller, we look at the special case of finite groups of Lie type. If G is such a group, of universal or adjoint type, and p is the defining characteristic, then with just two exceptions ($G \cong Sz(2)$ or $G \cong SL_3(2)$), $Out(G) \cong Out(S, F_S(G))$, and both are isomorphic to a certain outer automorphism group of the linking system $L^c_S(G)$ of G. When p is different from the defining characteristic, the situation is much more complicated, and the natural homomorphism from $Out(G)$ to automorphisms of the fusion or linking system can be very far from being an isomorphism. However, we have shown that in all cases, $Out(S, F_S(G))$ and $Out(L^c_S(G))$ can be described as quotient groups of $Out(G^*)$ for some other group G^* of Lie type whose p-fusion system is isomorphic to that of G. In particular, all fusion systems of finite groups of Lie type are "tame" in the terminology of an earlier paper of mine with Kasper Andersen and Joana Ventura.

Sejong Park: Cohomology of fusion systems

Abstract We extend Mislin's theorem on isomorphism of cohomology and control of fusion to fusion systems. We also conjecture that Dwyer's sharpness result on subgroup homology decomposition of the classifying space of a finite group extends to arbitrary Mackey functors and arbitrary saturated fusion systems and confirm the conjecture for some cases.

Cheryl Praeger: Classifying the finite 3/2-transitive permutation groups

Abstract Wielandt introduced the notion of a 3/2-transitive permutation group in his 1960 book on Finite Permutation Groups: these are transitive groups such that the stabilizer of a point x is nontrivial with all orbits apart from $\{x\}$ of equal length. Examples are 2-transitive groups, and Frobenius groups, and, indeed, every non-regular normal subgroup of a 2-transitive group is 3/2-transitive. The complete classification of the finite examples has recently been completed by Liebeck, Saxl and the speaker, building on work of Passman from the 1960s, and more recent classifications by teams including Bamberg, Giudici, Liebeck, Saxl, Tiep and the speaker.

Laci Pyber: Finite subgroups of diffeomorphism groups of a compact manifold

Abstract Motivated by Jordan's classical theorem, Ghys conjectured in 1997 that if M is a compact smooth manifold, then every finite subgroup G of $Diff(M)$ has an abelian subgroup H of index at most $f(M)$. In joint work with Csiko's and Szabo' we have disproved this conjecture. We have also shown that a weaker version , with H soluble (rather than abelian), is true.

Jeroen Schillewaert: Small maximal independent sets

Abstract We call a d-regular graph δ-sparse if the number of paths of length two joining any pair of vertices is at most $d^{1-\delta}$. Our main theorem shows that δ-sparse graphs have small maximal independent sets. This theorem has applications to a range of problems in finite geometry.

Yoav Segev: A non-split sharply 2-transitive group.

Abstract In a pair of landmark papers from 1936 Zassenhaus gave a complete classification of the FINITE sharply 2-transitive (hence-forth $s2t$) groups. The first of these papers shows that every such group can be identified with the group of all affine transformations of the form $\{x \to ax + b : a \in F^*, b \in F\}$, where F is a FINITE near-field; and this fact is equivalent to the assertion that every FINITE $s2t$ group SPLITS, i.e., it has a non-trivial abelian normal subgroup. The second classified all FINITE near-fields. The answer to the question of whether any INFINITE $s2t$ group splits defied the attempts of many mathematicians. We give (the first) example of an infinite non-split $s2t$ group. Indeed we show that ANY GROUP can be embedded into a non-split $s2t$ group.

Sergey Shpectorov: Axial algebras and groups of 3-transpositions

Abstract The class of axial algebras, recently introduced by Hall, Rehren, and the speaker, generalizes the class of Majorana algebras of A.A. Ivanov, of which the key example is the Griess algebra of the Monster sporadic simple group. Instead of identities, such as the associativity law or the Jacobi identity, the structure of an axial algebra is controlled by fusion rules describing multiplication of eigenvectors with respect to the adjoint action of certain idempotents, called axes. For the Monster algebra, the set of eigenvalues involved is $\{1, 0, \frac{1}{4}, \frac{1}{32}\}$. In the talk, we will focus on the meaning of the "magic" numbers $\frac{1}{4}$ and $\frac{1}{32}$. What happens if we instead allow arbitrary values $\alpha, \beta \notin \{1, 0\}$?

Ron Solomon: Recognizing abelian and nilpotent Hall subgroups from the character table

Abstract Richard Brauer raised numerous questions concerning information about a finite group G which may be deduced from its character table. I will report on the following joint theorem with Navarro and Tiep:

Theorem: Let G be a finite group, p a prime, and X the character table of G. Then G has abelian Sylow p-subgroups if and only if the following conditions hold:

1) For each column of X indexed by a p-element x, the class size of x is prime to p; and

2) For each row of X indexed by a character in the principal p-block, the character degree is coprime to p.

I will also mention related results concerning nilpotent and abelian Hall subgroups.

David Stewart: Maximal subalgebras of the exceptional Lie algebras in good characteristic

Abstract In the 50s, Dynkin classified the maximal subalgebras of simple finite-dimensional Lie algebras over the complex numbers. We initiate the study in positive characteristic. The main result I want to present, joint with Sebastian Herpel, says that amongst non-classical type simple subalgebras, one only finds copies of the first Witt algebra of dimension p in exceptional Lie algebras in good characteristic. We also say precisely when they are maximal. I will also mention some current work with Alexander Premet classifying exotic semidirect product subalgebras.

Gernot Stroth: Groups which are almost groups of Lie type in characteristic p

Abstract We will report on joint work of Chr. Parker, G. Pientka, A. Seidel and G. Stroth on the following setup. Let G be a group of local characteristic p and K be a subgroup of G, which is a group of Lie type in characteristic p and of Lie rank at least 2. If $H = N_G(K)$ contains a Sylow p-subgroup of G, what can be said about G? We will also address, what might happen if we change the assumption of local characteristic p to parabolic charactristic p, at least for $p = 2$, and the obstructions which arise because of the existence of exotic fusion systems.

Richard Weiss: Galois Involutions and Exceptional Groups

Abstract We will describe a combinatorial theory of descent in buildings and an application to the construction of forms of the exceptional groups of relative rank at least 2. We will also say a few words about the relative rank 1 case.