GROUPS AND GEOMETRIES

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1 Overview of the field, and recent developments

As groups are just the mathematical way to investigate symmetries, it is not surprising that a significant number of problems from various areas of mathematics can be translated into specialized problems about permutation groups, linear groups, algebraic groups, and so on. In order to go about solving these problems a good understanding of the finite and algebraic groups, especially the simple ones, is necessary. Examples of this procedure can be found in questions arising from algebraic geometry, in applications to the study of algebraic curves, in communication theory, in arithmetic groups, model theory, computational algebra and random walks in Markov theory. Hence it is important to improve our understanding of groups in order to be able to answer the questions raised by all these areas of application.

The research areas covered at the meeting fall into three main inter-related topics.

1.1 Fusion systems and finite simple groups

The subject of fusion systems has its origins in representation theory, but it has recently become a fast growing area within group theory. The notion was originally introduced in the work of L. Puig in the late 1970s; Puig later formalized this concept and provided a category-theoretical definition of fusion systems. He was drawn to create this new tool in part because of his interest in the work of Alperin and Broué, and modular representation theory was its first berth. It was then used in the field of homotopy theory to derive results about the p-completed classifying spaces of finite groups. Lately, fusion systems have been very successfully adopted by finite group theorists. When finite groups are considered in a category of (saturated) fusion systems, it turns out the proofs of some results in this field can be obtained in a more direct fashion.

Among all recent applications of the theory of fusion systems, the program laid out by Michael Aschbacher stands out as one of the most promising. The goal of this program is to simplify the existing proof of the classification of finite simple groups. A large part of the original proof of the Classification Theorem addresses the so-called finite simple groups of component type. One of the main difficulties that arises here is rooted in the possible existence of 'cores' of the 2-local subgroups of an ambient finite simple group. Aschbacher's program proposes instead to study saturated simple 2-fusion systems of component type, where the issues associated with cores do not exist. The aim is then to use this new classification to in turn obtain a novel, more straightforward method of classifying finite simple groups of component type.

1.2 Buildings and groups

Group theory studies the symmetries of objects of various kinds. Historically, one of the main sources for such objects are geometric and combinatorial structures. Indeed, in his famous *Erlanger Programm* Felix Klein proposed to investigate geometries via their group of symmetries. This principle has been most fruitful over the last century and led to deep insights. People then began to realise that it works also in the opposite direction: in order to understand a given abstract group it is helpful to construct combinatorial objects on which the group acts in a natural way. One of the most beautiful examples of an application of this reversed form of the Erlanger Programm is Tits' notion of a building. To any semi-simple, isotropic algebraic group Tits associates a building on which the group of rational points acts strongly transitively. For this reason 'most' of the finite simple groups act strongly transitively on buildings which explains the special link between the finite simple groups and this outstanding class of combinatorial objects. But this is by far not the only bridge between finite group theory and geometry. There are questions about infinite geometries which can only be settled by using deep results from finite group theory and in particular the classification of finite simple groups. Questions about generalizing a known result about finite groups to the infinite case often have a geometric flavor. In these cases one can hope that combinatorial techniques also apply in a more general context. Thus, there is already a long history of fruitful interactions between group theory and geometry, in particular buildings, which continues to provide new inspiration and challenges for both areas.

During the last decade there have been several major developments which are based on the interplay of the theory of buildings and finite groups. One of them is the investigation of Moufang sets. The classification of finite Moufang sets was accomplished by Hering, Kantor and Seitz in 1972 [20]. It is one of the most challenging questions whether all proper infinite Moufang sets are of algebraic origin. The fact that one has to require 'proper' was already suspected for a long time, but definitively confirmed only recently by the construction of non-split infinite sharply 2-transitive groups by Rips, Segev and Tent. Although there has been a lot of activity in the area, the question is still wide open. However, this problem inspired Caprace, De Medts and Grüninger to investigate Moufang sets from the perspective of locally compact groups which culminated in the result that a group which acts on a locally finite tree with a abelian Moufang set at infinity is provided by Bruhat-Tits theory and hence of algebraic origin.

Another mainstream topic in the area of buildings and groups is provided by Kac-Moody theory. Indeed, it was observed by Rémy [25] and independently by Carbone and Garland [6] that the buildings associated to Kac-Moody groups over a finite field provide most interesting examples of irreducible lattices in the automorphism group of the product of two locally finite buildings. Using this fact, Caprace and Rémy have been able to show that Kac-Moody groups over finite fields are simple groups, and it is still open whether this is also true over infinite fields. Currently this direction is a very lively area of research, because there many results about groups of Lie type which might be extendable to the Kac-Moody situation, but also new ones which are relevant for the theory of locally compact groups.

A somewhat different mainstream topic in the area over the recent decade has its origin in Serre's notion of complete reducibility in spherical buildings. Through this notion Tits' center conjecture from the 1960s became prominent and its proof was accomplished by Ramos-Cuevas in 2012. Ramos-Cuevas' arguments are based on a sophisticated analysis of buildings of exceptional type E_8 and the attempt to simplify it led to whole new understanding of exceptional Moufang buildings from geometric point of view. But there are also other important directions which aim for a better understanding of exceptional groups and their geometries. From a combinatorial point of view there is Van Maldeghem's program to study the algebraic varieties related to the Freudenthal-Ttis magic square, and from the point of view of Lie algebras there is Cohen's program to study Lie algebras via extremal elements.

1.3 Finite groups and related algebras

There are several strands of modern research involving finite groups. One is the theory surrounding the classification of finite simple groups, some of which was discussed in the previous sections. Others involve studying properties of the finite simple groups themselves, particularly their representations and their subgroup structure. For the latter, the maximal subgroups of the finite simple groups are of special interest, since any primitive permutation action of a finite group is just an action on the coset space of a maximal subgroup. Many applications of the classification of finite simple groups have been achieved using the theory of maximal subgroups, since automorphism groups of interesting combinatorial structures, Galois groups of interesting field extensions, and so on, can often be shown to act as primitive permutation groups.

Another fruitful way to study finite simple groups is via various algebras on which they act. For example, many of the groups of Lie type act as automorphism groups of simple Lie algebras. As for the 26 sporadic simple groups, 21 of them are contained in the largest one – the Monster – which was constructed as the automorphism group of the Monster Algebra, a non-associative real algebra of dimension 196883. A great deal of recent effort has gone into creating a general theory of such algebras, into which the Monster Algebra will fit as a particular case.

2 Presentation Highlights and Scientific Progress

2.1 Fusion systems and finite simple groups

There were a number of interesting talks given on the subject of fusion systems and their applications to the study of finite groups and on the continuing study of finite simple groups.

Michael Aschbacher talked about his ongoing project on the use of fusion systems to simplify the existing proof of the classification of finite simple groups. The focus of his lecture was a study and classification of the so-called *quaternion fusion packets*: the pairs $(\mathcal{F}, \Omega)$ where $\mathcal{F}$ is a saturated 2-fusion system and Ω is an $\mathcal{F}$-invariant set of subgroups that satisfy conditions that are quite similar to the hypotheses in his celebrated work on classical involutions [1, 2]. Among the examples of quaternion fusion packets are those coming from the fusion systems of finite groups of Lie type over the fields of odd order.

Ellen Henke discussed *linking systems*. This notion was originally introduced in the work of Broto, Levi and Oliver; it proved a useful tool to allow them to study the classifying spaces of fusion systems. The original notion was modified in the subsequent works of Broto, Castellana, Grodal, Levi and Oliver. In her talk Henke proposed a new notion of a linking system that allows her to show that there is a unique linking system associated to each fusion system whose objects are the subcentric subgroups, and that the nerve of such a subcentric linking system is homotopy equivalent to the nerve of the centric linking system. The existence of subcentric linking systems seems to be of interest for a classification of fusion systems of characteristic p-type. Linking systems also featured in Robert Oliver's lecture. In his presentation, he discussed his work on the automorphisms of fusion and linking systems of finite groups of Lie type.

Sejong Park talked about the cohomology of fusion systems. A celebrated theorem of Mislin shows that an isomorphism on mod-p cohomology implies control of p-fusion among compact Lie groups, and in particular among finite groups. Park discussed how this result can be generalised and proved in the setting of saturated fusion systems.

Gernot Stroth spoke about an application of his recent paper [24] with U. Meierfrankenfeld and R. Weiss. The highlight of the talk was showing that if a finite group G of parabolic characteristic 2 contains a subgroup H of odd index and $F^*(H) \cong \Omega_8^+(2)$, then either $F^*(G) \cong \Omega_8^+(2)$ or $\Omega_8^+(3)$.

2.2 Buildings and groups

The presentations concerning the interplay between buildings and groups can be roughly subdivided into three directions.

2.2.1 Kac-Moody groups and locally finite trees

As mentioned before, it is a prominent open question whether Kac-Moody groups over infinite fields are simple. A Kac-Moody algebra over the real numbers has a so-called 'compact form'. In [10] and [11] physicists discovered that the compact form of E_{10} admits a finite-dimensional representation, a result which was somewhat surprising. Max Horn explained this phenomenon in his talk. In fact, it is possible to produce finite-dimensional representations of the compact form of any symmetrizable real Kac-Moody algebra. As a special instance one can produce generalized spin representations for algebras of type E_n for any n. The resulting quotients are compact, whence reductive and often even semisimple. Cartan-Bott periodicity enables one to determine the isomorphism types of these quotients in this special case. As a result, this leads to a more conceptual approach to the representations discovered in the references above. It turns out that that Kac-Moody buildings play a key role in the development of the theory of spin representations. Indeed, one knows that there is a Curtis-Tits presentation for 2-spherical Kac-Moody groups and their compact forms. In order to construct the desired representations one only has to check the Levi-factors of rank smaller than 2. It turns out that one has to work with central covers of the Levi-factors in order to make things work and this is settled with Tits' theory of extended Weyl groups [27]. Thus the theory of these generalized spin representations is based on a local to global principle for Kac-Moody groups where one takes advantage of the fact that the local information is provided by the classical theory. A similar idea provides also the motivation of the theory of locally grouped spaces presented by Andrew Chermak. The fusion systems of finite groups of Lie type are well understood and therefore it is natural ask whether Kac-Moody groups over finite fields might provide other interesting examples of fusion systems which would provide new insights. Chermak's program aims for a better understanding of the unipotent radicals of the Borel subgroups in Kac-Moody groups. As already mentioned, Kac-Moody groups over finite fields provide interesting examples of irreducible lattices in the automorphism group of a product of two locally finite buildings. In this context, these unipotent radicals have been intensively studied but they still remain rather mysterious. Any new approach to improve our understanding of these groups is most welcome. The ideas presented by Chermak provide a new link between finite group theory and Kac-Moody theory which is very promising. Kac-Moody groups and their buildings played also a central role in the talk of Matthias Grüninger in which he presented a result about groups acting on locally finite trees which induce an abelian Moufang set at infinity. His result can be seen as an analogue of an earlier result in [5] of Caprace and De Medts. In [5] the characteristic 0 case was settled, and this work relies on heavy machinery from the theory of p-adic analytic groups. In his talk, Grüninger described a completely different strategy in positive characteristic. The idea is to produce an RGD-system inside the group in question. RGD-systems were introduced by Tits in [28] in order to describe the systems of root-groups inside a group of Kac-Moody type. In general there is a Moufang twin building associated to such an RGD-system which is a Moufang twin tree in the case considered here. The end-game now consists of checking which Moufang twin trees provide an abelian Moufang set at infinity. This is not easy but can be achieved by elementary arguments. Groups acting on trees where also considered in the the joint talk of Pierre-Emmanuel Caprace and Nicolas Radu from a different perspective. The main goal of their presentation is motivated by the structure theory of simple locally compact groups, of which a large class is provided by groups acting faithfully on locally finite trees. For a given tree they introduce a topology on a the set of isomorphism classes of a large family of simple groups acting on it and raise several natural questions. One among those is whether on can describe limit points of known isomorphism classes of such groups, and they give a beautiful answer to that question for rank 1 groups over local fields. It turns out that by increasing the ramification index in characteristic 0 one obtains the groups over the Laurent series as a limit.

2.2.2 Exceptional groups and algebras

The interplay between buildings and algebraic structures is particularly fruitful in the exceptional cases G_2, F_4, E_6, E_7, E_8. This is not at all surprising because the theory of buildings was created by Tits in order to have an additional tool for investigating the Lie algebras and groups of exceptional type. Several lectures given at conference underlined that the theory of buildings plays a central role for exceptional structures. Richard Weiss presented in his talk a uniform approach to several classes of exceptional Moufang buildings by constructing them as fixed point buildings of Galois-involutions of higher rank buildings. This is an application of a more general theory of descent for Moufang buildings. A remarkable aspect of this construction is the fact, that it provides a natural link between the theory of algebraic groups and the classification of Moufang polygons. More precisely, the existence proofs for the exceptional Moufang buildings that have been known up until now were based on Galois cohomology of algebraic groups on the one hand, and on explicit calculations in their coordinatizing structures described obtained in [29] on the one the other. Through this new approach to exceptional geometries one has a better understanding of how results from the theory of algebraic groups have to be interpreted in the context of Moufang buildings, and there is reasonable hope that these new insights will be useful in the investigation of Moufang sets. The geometries constructed in Weiss' talk actually represent the most interesting entries in the Freudenthal-Tits magic square. The latter was also considered in the talk of Hendrik Van Maldeghem from a different perspective. A couple of years ago he and his collaborators started a promising program aiming for a uniform characterization of several algebraic varieties related to the magic square. Crucial for this program is a functor which associates to a class of quadratic spaces over a field a Veronese representation. In his talk he focused on the case where the quadratic forms in question are defective. The outcome is that the functor yields several buildings of mixed type. The latter are no longer associated to reductive, but to pseudo-reductive groups which have been recently studied intensively in [7]. The Veronese representations in Van Maldeghem's program yield characterizations of several geometries of exceptional type. An alternative characterization of exceptional buildings is provided by the theory of root filtration spaces and a beautiful application of those has been provided in the talk of Arjeh Cohen. In his talk he considered Lie algebras generated by extremal elements. The latter play a central role in the classification of simple Lie algebras in positive characteristic $p \geq 5$. As an application of root filtration spaces he obtains a characterization of the classical Lie algebras among those which are generated by extremal elements. The idea is to construct out of simple Lie algebra a line-space whose points are the one-dimensional subspaces generated by extremal elements. If this space satisfies the axioms of a root filtration space, then one can reconstruct the building and deduce that the algebra is classical. Apart from the three contributions described so far, there were two further talks on groups and algebras of exceptional type given by David Craven and David Stewart which did not have any immediate connection to buildings and which will be mentioned in more detail in the following section.

2.2.3 Groups and graphs

As buildings can be interpreted as edge-colored graphs, the latter represent a more general class of combinatorial objects which provide a powerful tool to study groups. Indeed, graph-theoretical methods played a major role in the talks presented by Cohen, Van Maldeghem and Weiss which have been already mentioned. One prominent open question in the area of groups and graphs is a conjecture stated by Weiss in 1978 [30]. Luke Morgan gave an overview about the recent developments concerning Weiss' conjecture and presented a new result about locally semiprimitive arc-transitive graphs. A purely combinatorial result about graphs was the subject of Jeroen Schillewaert's talk. He described a probabilistic approach for proving the existence of certain substructures (partial spreads and ovoids) of classical geometries which are usually obtained by algebraic methods. The surprising aspect of his result is that – at least asymptotically – the bounds obtained by these methods are much better than the ones obtained so far.

2.3 Finite groups and related algebras

A wide range of topics involving finite groups was covered. First, as mentioned in Section 1.3, maximal subgroups of finite simple groups are a particular focus, and David Craven gave a lecture *Maximal subgroups of exceptional groups of Lie type*. The theory of maximal subgroups of the alternating and classical finite simple groups is is a reasonably complete state, but the same cannot be said for the exceptional groups of Lie type. For the latter, results of Liebeck and Seitz [22, 23] reduce the study to the maximal subgroups which are themselves simple groups, and moreover provide an absolute bound on the order of those potential simple subgroups that need to be considered. Craven announced some new results that determine many of these small maximal subgroups, the first substantial progress in this area for a number of years. Following the lecture, discussions with Praeger, Morgan, Giudici and Liebeck led Craven to the proof of an additional result that, together with a theorem of Lusztig, shows that the alternating group of degree 5 is never a maximal subgroup of an exceptional group, a result which has now been used in an application to the study of multiply arc-transitive graphs. Cheryl Praeger's lecture *Classifying the finite 3/2-transitive permutation groups* announced the completion of a long-term project to determine the 3/2-transitive groups – that is, the transitive permutation groups for which a point-stabilizer has orbits of equal size on the remaining points. This class of permutation groups includes Frobenius groups and 2-transitive groups, and the techniques for their classification again involve the theory of maximal subgroups, together with a substantial amount of representation theory.

As mentioned in Section 1, a fruitful way to study groups is via various structures and algebras on which they act, and there were quite a number of lectures along these lines. In his lecture *Finite subgroups of diffeomorphism groups of a compact manifold*, Laci Pyber discussed a 20-year old conjecture of Ghys (see [12]) which states that if M is a compact smooth manifold, then every finite subgroup of the diffeomorphism group $\mathrm{Diff}(M)$ has an abelian subgroup of index at most $f(M)$, where $f(M)$ depends only on the manifold M. The conjecture was motivated by the famous theorem of Jordan showing that it is true when M is complex n-space. Over the years it has been proved in several special cases, so it came as quite a surprise when Pyber announced a counterexample to the conjecture. This has led to some weaker positive results, and a modification of Ghys's conjecture which still remains open. Another variation on the theme of groups acting on combinatorial structures came in the lecture of Nick Gill, *Constructing groupoids using designs*. This built on a famous example of John Conway [9] in which he used the projective plane of order 3 to construct the Mathieu group M_{12} and also a related groupoid that he called M_{13}. By considering the same construction, replacing the projective plane by more general Steiner systems with blocks having 4 points, Gill showed how to construct whole families of groupoids that generalize M_{13} and are related to classical groups over small fields. Concerning the representation theory of finite groups, Ron Solomon (*Recognizing abelian and nilpotent Hall subgroups from the character table*) answered part of an old question of Richard Brauer, showing how the character table of a finite group can be used to determine whether or not it has abelian Sylow p-subgroups for some prime p.

The Monster sporadic group was constructed by Griess [19] as the automorphism group of a 196883-dimensional real algebra now known as the Monster algebra. In the last decade or so, Ivanov and others have introduced the theory of Majorana algebras [21] which attempts to develop a general theory of algebras in which the Monster algebra is a special case. In their lectures, Sergey Shpectorov (*Axial algebras and groups of 3-transpositions*) and Tom de Medts (*Jordan algebras and 3-transposition groups*) discussed variations and generalizations on this theme, showing the potential richness of this line of investigation by demonstrating beautiful connections with the theories of Jordan algebras and 3-transposition groups. On the more classical topic of Lie algebras, David Stewart (*Maximal subalgebras of the exceptional Lie algebras in good characteristic*) announced new results on simple subalgebras of exceptional Lie algebras in positive characteristic, showing that even though there are many non-classical types of simple Lie algebras in such characteristics (see [26]), only the Witt algebra among these can occur in an exceptional Lie algebra in good characteristic. This potentially opens the way to a new detailed study of subalgebras that previously seemed intractable. Arjeh Cohen's lecture *Lie algebras generated by extremal elements* provided a link with the theory of buildings, showing that simple Lie algebras generated by extremal elements have an embedded geometry that is the shadow of a building, and moreover that this geometry determines the Lie algebra uniquely.

3 Outcome of the Meeting

3.1 Fusion Systems

Fusion systems play an increasing role in our understanding of the finite simple groups. This was already visible in the Banff 'Groups and Geometries' conference in 2012 and was confirmed by the multiple talks on this subject given during this conference. Following the presentation of Aschbacher and of his new ground-breaking results, we can be optimistic that the study of fusion systems will lead to more straightforward proofs of substantial parts of the classification of finite simple groups.

The new results of Henke are a very promising fresh tool in the program introduced by Meierfrankenfeld, Stellmacher and Stroth, of which the goal is to classify the finite simple groups of characteristic p-type. Fusion systems are also of course interesting to study in their own right, and the talks of Oliver and Park demonstrated new developments in the area itself.

Chermak's program appears to be a promising new direction in the theory of fusion systems. This program focuses on the investigation the unipotent radicals of Borel subgroups in Kac-Moody groups over finite fields from the point of view of fusion systems. These unipotent radicals naturally belong to the theory of buildings. This program hints at the possibility of the use of buildings in a new context, thus providing a novel perspective on fusion systems associated with finite groups of Lie type.

A discussion of Stroth and and Parker followed as a development of Stroth's talk. It allowed both to improve the results presented in Stroth's talk. Furthermore, Aschbacher and Stroth had some discussions on a possible fusion system classification.

3.2 Revision of the classification

Now that the first generation proof of the classification of finite simple groups is complete (cf. [3]), there are currently two very active mainstream areas of research focused on producing new proofs of the classification theorem. One is the Gorenstein-Lyons-Solomon program (GLS): a number of the participants of this meeting are involved at various degrees in this effort, under the leadership of R. Lyons and R. Solomon. The aim of this program is to provide a self-contained proof of the classification in a series of eleven monographs. The first six volumes have been published in the AMS monograph series ([13], [14], [15], [16], [17], [18]), and substantial progress has been made towards the completion of several of the other volumes. Both Lyons and Solomon were present at the meeting: such interactions were extremely beneficial, particularly to the progress of volume 7. Some parts of this volume, and also some of the later ones, require numerous results on the identification of finite simple groups of Lie type. The most recent approach to this is the geometrically oriented recognition theorems of Phan-Curtis-Tits type. Numerous discussions involving Lyons and Shpectorov centred around results of this type took place during the meeting. The remaining "special even case" is currently under close scrutiny. Some partial results have already been published, notably the ones dealing with the subcase of finite simple groups of mixed characteristics. Volume 8 of the GLS-series is planned to be devoted to this part of the classification, and to contain this work. The remainder of the $e(G) = 3$ case is planned to be presented in volume 9 according to the current projections. The state of the art on this part of the project was discussed by Lyons and Solomon during the meeting together with other participants (I. Capdeboscq, C. Parker, K. Magaard).

3.3 Groups acting on trees

Bruhat-Tits buildings have been a focus of many previous conferences, in particular the Banff 2012 'Groups and Geometries' meeting. A new direction which was already initiated in the meeting of 2012 is the investigation of the one-dimensional case, i.e. the case of Bruhat-Tits trees. It is motivated by the attempt to improve our understanding of Moufang sets and important classes of locally compact groups. There is the prominent question of whether all proper Moufang sets are of algebraic origin. By the result presented in Grüninger's talk, it is reasonable to conjecture that this is indeed the case for any Moufang set at infinity of a locally finite tree, because it is known to be true if the root groups at infinity are abelian. Assuming that this conjecture holds one has the interesting phenomenon that the non-uniform irreducible lattices in the automorphism group of the product of two locally finite trees (see [25]) are arithmetic if and only if they induce a Moufang set at infinity. It would be most exciting to have a direct proof of this.

3.4 Geometric structures in positive characteristic

Several talks at the conference were concerned with a geometric approach to exceptional groups and algebras. The big advantage of using combinatorial arguments in this context is the fact that they apply equally well in positive characteristic, which often a major obstacle when using algebraic methods. The talks of Cohen, Van Maldeghem and Weiss highlighted this in context of the classification of simple Lie algebras, the 'degenerate cases' of the Freudenthal-Tits magic square and the construction of exceptional geometries. It would be most desirable and a perspective for future research to relate espcecially the latter two contributions with the work of Conrad, Gabber and Prasad on pseudo-reductive groups in [7] and [8].

3.5 Finite groups and algebras

Some very promising themes for future reaserch emerged from the talks of Shpectorov and de Medts. The theory of Majorana algebras [21] originated in vertex operator algebra theory, and was originally designed mainly to provide an axiomatic setting for the Monster algebra in which that group could be investigated systematically. By introducing the more general theory of axial algebras, and showing their surprising and beautiful connections with Jordan algebras and 3-transposition groups, Shpectorov, de Medts and their collaborators have uncovered a potentially very rich new area of research in the area of groups and algebras.

4 Final remarks

One of the foremost objectives of this meeting was to bring together junior and well established researchers working in the various different fields closely related to finite and algebraic simple groups. The structure of the conference was based on the assumption that developing, maintaining scientific exchanges between these connected areas was best achieved by creating new perspectives in research and stimulating scientific collaboration.

The lectures and the time schedule were designed by the organizers in accordance with these objectives. In the selection of the lectures, preference was given to subjects which offered the participants the possibility to learn about novel developments in an area. To foster productive interaction, the timetable was such that significant breaks were introduced between the talks.

The feedback of many participants to the organizers was very positive. The outstanding quality of the talks was often mentioned. Several new collaborations were started during the meeting, and ongoing ones found it the perfect setting to be continued. Beside the outstanding scientific level, the attendees particularly enjoyed the clarity of the lectures. The speakers paid special attention to explaining clearly the main ideas and avoiding technical details, which made these lectures profitable to all. It was also remarked positively, that there was a comparatively high number of young speakers and that all of them gave beautiful lectures.

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