

# Nichols Algebras and Their Interactions with Lie Theory, Hopf Algebras and Tensor Categories

## September 7 to 11, 2015

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

### SCHEDULE

#### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and a small assortment of snacks are available on a cash honor system.

#### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL

**9:00–10:00** Victor Ostrik, *From tensor categories to fusion categories*

**10:00–10:30** Coffee Break

**10:30–11:30** Christoph Schweigert, *Conformal field theory, tensor categories and module categories*

**11:30–13:30** Lunch

**13:30–14:30** Iván Angiono, *Nichols algebras of finite GK-dimension*

**14:30–15:00** Coffee Break

**15:00–16:00** Ellen Kirkman, *Reflection Hopf algebras*

**16:30–17:30** Round Table *Cohomology of finite-dimensional Hopf algebras* (Moderator: Julia Pevtsova)

**17:30–19:30** Dinner

**20:00–21:00** Round Table *Modular tensor categories* (Moderator: Eric Rowell)

#### Tuesday

**7:00–9:00** Breakfast

**9:00–10:00** Ken Goodearl, *Hopf algebras of finite GK-dimension*

**10:00–10:30** Coffee Break

**10:30–11:30** Dmitri Nikshych, *Brauer-Picard groups of finite tensor categories and Hopf algebras*

**11:30–13:00** Lunch

**13:15** Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

**13:30–14:10** Arkady Berenstein, *H-cross products*  
**14:20–15:00** Sonia Natale, *On composition series of finite dimensional Hopf algebras and weakly group-theoretical fusion categories*  
**15:00–15:30** Coffee Break  
**15:30–16:10** Chelsea Walton, *Semisimple Hopf actions on Weyl algebras*  
**16:30–17:30** Noah Snyder, *Exotic fusion categories and the classification of small index subfactors*  
**17:30–19:30** Dinner  
**20:00–21:00** Round Table *Hopf actions in noncommutative geometry* (Moderator: Chelsea Walton)

### Wednesday

**7:00–9:00** Breakfast  
**9:00–10:00** Leandro Vendramin, *Nichols algebras over non-abelian groups*  
**10:00–10:30** Coffee Break  
**10:30–11:30** Milen Yakimov, *Connected Hopf algebras of finite GK-dimension and finite Drinfeld quantizations*  
**11:30–13:30** Lunch  
**13:30–14:10** Shahn Majid, *Duality and Fourier theory for differentials on Hopf algebras*  
**14:20–15:00** Vladislav Kharchenko, *Explicit formula for coproduct*  
**15:00–15:30** Coffee Break  
**15:30–16:10** Akira Masuoka, *Hopf-algebraic techniques applied to super affine groups*  
**16:30–17:30** Round Table *CFT and tensor categories* (Moderator: Christoph Schweigert)  
**17:30–19:30** Dinner  
**20:00–21:00** Round Table *Classification problems of Nichols algebras* (Moderator: István Heckenberger)

### Thursday

**7:00–9:00** Breakfast  
**9:00–10:00** Ken Brown, *Homological properties of Hopf algebras*  
**10:00–10:30** Coffee Break  
**10:30–11:30** Richard Ng, *Frobenius-Schur indicators—Generalizations and applications*  
**11:30–13:30** Lunch  
 Free Afternoon  
**17:30–19:30** Dinner  
**20:00–21:00** Round Table *Hopf algebras of finite GK-dimension* (Moderator: James Zhang)

### Friday

**7:00–9:00** Breakfast  
**9:00–10:00** Hans-Juergen Schneider, *The braided Hopf algebra structure of reflected Nichols algebras*  
**10:00–10:30** Coffee Break  
**10:30–11:30** Final Round Table (Moderator: Nicolás Andruskiewitsch)  
**11:30–13:30** Lunch  
**Checkout by 12 noon.**

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

**Nichols Algebras and Their Interactions with Lie Theory, Hopf  
Algebras and Tensor Categories**  
**September 7 to 11, 2015**

**ABSTRACTS**

Speaker: **Iván Angiono** (Universidad Nacional de Córdoba, Argentina)

Title: *Nichols algebras of finite GK-dimension*

Abstract: This talk is based in a joint work with N. Andruskiewitsch and I. Heckenberger. We recall the definition of Nichols algebras and present an overview about the classification of Nichols algebras of finite GK-dimension over abelian groups. We present different examples and results about the classification. Finally we recall the relation of these Nichols algebras with pointed Hopf algebras (of finite GK-dimension), and present examples of pre Nichols algebras and liftings.

Speaker: **Arkady Berenstein** (University of Oregon, USA)

Title: *H-cross products*

Abstract: It frequently happens that an algebra  $C$  factors as  $C = AB$ , i.e., as a vector space,  $C$  is isomorphic to  $A \otimes B$  for certain sub-algebras  $A$  and  $B$ . If  $A$  and  $B$  commute in  $C$ , then  $C$  is simply their tensor product as an algebra. Conversely, an algebra structure on  $C = A \otimes B$  can be established if  $A$  and  $B$  do not commute in  $C$ , e.g., that of a cross product (when  $B$  is a bialgebra covariantly acting on  $A$ ) or, more generally, a braided tensor product.

The goal of my talk (based on joint work with Yuri Bazlov) is to generalize these constructions to what we call “ $H$ -cross product of  $A$  and  $B$ ” for any bialgebra  $H$  covariantly acting on  $A$  and coacting on  $B$ . It turns out that, under minor technical conditions, any factorization  $C = AB$  can be realized this way.

By exploring  $H$ -cross factorizations, one can (re)discover some famous Hopf algebras. For instance, if  $C$  is a graded affine Hecke algebra or a nil Hecke algebra of a Coxeter group  $W$ , then  $C$  is an  $H$ -cross product of the polynomials  $A = S(V)$  on the reflection  $W$ -module  $V$  and the group algebra  $B$  of  $W$ , while  $H$  can be taken to be the bosonized Nichols algebra attached to the reflections in  $W$ .

Speaker: **Ken Brown** (University of Glasgow, UK)

Title: *Homological properties of Hopf algebras*

Abstract: I will survey results and open questions on homological properties of Hopf algebras and of related algebras, focussing on algebras which are noetherian or have finite Gelfand-Kirillov dimension (or both).

Speaker: **Ken Goodearl** (University of California, Santa Barbara, USA)

Title: *Hopf algebras of finite GK-dimension*

Speaker: **Vladislav Kharchenko** (UNAM, México)

Title: *Explicit formula for coproduct*

Abstract: We develop an explicit coproduct formula for PBW generators of quantum groups of infinite series and discuss a possible generalization to the quantizations of exceptional simple Lie algebras. An inexplicit coproduct formula for PBW generators of Lusztig form appeared in [1]. Recently, a similar inexplicit formula was proven within a more general context [2].

1. S.Z. Levendorskii and Ya.S. Soibelman, Some applications of the quantum Weyl groups, J. Geom. Phys. 7 (1990), 241–254.

2. I. Heckenberger and H.-J. Schneider, Right coideal subalgebras of Nichols algebras and the Duflo order on the Weyl groupoid, Israel Journal of Mathematics, 197, N1(2013), 139–187.

Speaker: **Ellen Kirkman** (Wake Forest University, USA)

Title: *Reflection Hopf algebras*

Abstract: The Shephard-Todd-Chevalley Theorem states that when a finite group  $G$  acts linearly on a commutative polynomial ring  $A = k[x_1, \dots, x_n]$  over a field  $k$  of characteristic zero, the invariant subring  $A^G$  is a commutative polynomial ring if and only if  $G$  is generated by reflections. More generally, let  $H$  be a finite dimensional Hopf algebra that acts on an Artin-Schelter regular algebra  $A$  so that  $A$  is an  $H$ -module algebra, the grading on  $A$  is preserved, and the action of  $H$  on  $A$  is inner faithful. When  $A^H$  is Artin-Schelter regular we call  $H$  a reflection Hopf algebra for  $A$ . We discuss what is known about such pairs  $(A, H)$  when  $H$  is a finite group, a semisimple Hopf algebra, or the dual of a group algebra.

Speaker: **Shahn Majid** (Queen Mary University of London, UK)

Title: *Duality and Fourier theory for differentials on Hopf algebras* Abstract: We reconsider the construction of bicovariant differentials on a Hopf algebra from braided-exterior algebras of Nichols type, with braided coproduct in terms of braided binomial operators. This allows us to define a canonical braided exponential and, in nice cases, Fourier theory on the braided-exterior algebra, which we interpret geometrically.

Speaker: **Akira Masuoka** (University of Tsukuba, Japan)

Title: *Hopf-algebraic techniques applied to super affine groups*

Abstract: I will present recent results on super affine groups obtained by myself and coauthors, explaining how our familiar notions and techniques of Hopf algebras are applied.

Speaker: **Sonia Natale** (Universidad Nacional de Córdoba, Argentina)

Title: *On composition series of finite dimensional Hopf algebras and weakly group-theoretical fusion categories*

Abstract: We shall discuss the notions of composition series in the contexts described in the title and show that a Jordan-Hlder theorem holds in both cases. We shall then examine the corresponding factors and length arising from these notions and compare them in some classes of examples.

Speaker: **Richard Ng** (Louisiana State University, USA)

Title: *Frobenius-Schur indicators—Generalizations and applications*

Abstract: Frobenius-Schur indicators were introduced in the representation theory of groups a century ago. They have been recently generalized to Hopf algebras, conformal field theory and fusion categories. The latest generalization does not only reveal the invariance of Frobenius-Schur indicators under monoidal equivalences of fusion categories, but also natural actions of the full modular group  $SL(2, Z)$ . In this talk, we will discuss the definition of these indicators and some recent applications to the Cauchy theorem and the rank-finiteness for modular tensor categories.

Speaker: **Dmitri Nikshych** (University of New Hampshire, USA)

Title: *Brauer-Picard groups of finite tensor categories and Hopf algebra*

Abstract: Let  $C$  be a finite tensor category (e.g.,  $C$  can be the representation category of a finite dimensional Hopf algebra). The group of invertible  $C$ -bimodule categories is called the Brauer-Picard group of  $C$  and is denoted  $BrPic(C)$ . It generalizes the classical group of invertible bimodules over an algebra. Brauer-Picard groups are important, e.g., for classification of graded extensions of tensor categories. There is a canonical group isomorphism between  $BrPic(C)$  and the group of braided autoequivalences of the Drinfeld center  $Z(C)$  of  $C$ . The latter group is of geometric nature (for example, in the case when  $C$  is the representation category of an abelian group, it coincides with the orthogonal group). In this talk I will discuss various methods of computing Brauer-Picard groups and describe their actions on categorical Grassmannians and their representations related to pointed Hopf algebras.

Speaker: **Victor Ostrik** (University of Oregon, USA)

Title: *From tensor categories to fusion categories*

Abstract: I will review classical construction of quotient of a tensor category by the negligible morphisms which allows to construct semisimple tensor categories from arbitrary tensor categories, e.g. representation categories of non semisimple Hopf algebras. I will give some examples and discuss problems which arise from the following question: which fusion categories can be obtained in this way?

Speaker: **Hans-Juergen Schneider** (Universität München, Germany)

Title: *The braided Hopf algebra structure of reflected Nichols algebras*

Abstract: This is joint work with Istvan Heckenberger. The reflection operator is a basic construction in the theory of Nichols algebras of semisimple Yetter-Drinfeld modules over some Hopf algebra. We give a new and explicit description of the Hopf algebra structure of a reflected Nichols algebra (and of generalizations) based on an abstract isomorphism of braided monoidal categories. In particular, the Lusztig automorphism of quantum groups is understood as a Hopf algebra isomorphism when restricted to certain subspaces. We obtain a new and direct proof of the bijection between right coideal subalgebras in a Nichols algebra (or in generalizations) and in the reflected Nichols algebra. This is the main tool to prove the existence of a generalized PBW-decomposition of Nichols algebras extending Lusztig's PBW-basis of the plus part of quantum groups, and to establish the strong axioms of the Cartan graph of a semisimple Yetter-Drinfeld module.

Speaker: **Christoph Schweigert** (University of Hamburg, Germany)

Title: *Conformal field theory, tensor categories and module categories*

Abstract: We explain what full local conformal field theory is and why module categories over tensor categories are a crucial structure to construct such a theory. We sketch a general construction and its recent interpretation using surface defects in three-dimensional topological field theories of Reshetikhin-Turaev type.

Speaker: **Noah Snyder** (Indiana University, USA)

Title: *Exotic fusion categories and the classification of small index subfactors*

Abstract: Oftentimes the only way new exotic objects are found is by an exhaustive search, for example the classification of semisimple Lie algebras revealed the  $E_8$  Lie algebra and the classification of finite simple groups yielded the Monster group. One key source of exotic fusion categories is the classification of von Neumann subfactors of small index. To any finite index finite depth subfactor one can assign a purely algebraic invariant, called the standard invariant, which can be thought of as a division algebra object in a unitary fusion category. The index of the subfactor is then the quantum dimension of the division algebra. For index below 4, the Jones index theorem and subsequent developments due to Ocneanu and others yields a nice ADE classification, but for index above 4 things are more complicated. The current state of the art (based on pioneering work of Haagerup and continued by Afzaly, Asaeda, Bigelow, Bisch, Calegari, Izumi, Jones, Liu, Morrison, Penneys, Peters, and myself) is that we have a complete classification of finite depth subfactors of index up to 5.25. In this talk I will give a survey of this classification, with an emphasis on the new examples found through this search.

Speaker: **Leandro Vendramin** (Universidad de Buenos Aires, Argentina)

Title: *Nichols algebras over non-abelian groups*

Abstract: Nichols algebras appear in several branches of mathematics going from Hopf algebras and quantum groups, to Schubert calculus and conformal field theories. In this talk we review the main problems related to Nichols algebras and we discuss some classification theorems.

Speaker: **Chelsea Walton** (Temple University, USA)

Title: *Semisimple Hopf actions on Weyl algebras*

Abstract: I will discuss the study of actions of semisimple Hopf algebras  $H$  on Weyl algebras  $A$  over a field of characteristic zero. In joint work with Juan Cuadra and Pavel Etingof, we show that the action of  $H$  on  $A$  must factor through a group algebra; in other words, if  $H$  acts inner faithfully on  $A$ , then  $H$  is cocommutative. The techniques used include reduction modulo a prime number and the study of semisimple cosemisimple Hopf actions on division algebras.

Speaker: **Milen Yakimov** (Louisiana State University, USA)

Title: *Connected Hopf algebras of finite GK dimension and finite Drinfeld quantizations*

Abstract: The classification problem for Hopf Algebras of finite GK dimension has attracted a lot of interest in recent years. We will describe a new perspective to it via deformation theory. In 1983 Drinfeld constructed quantizations of all triangular  $r$ -matrices. We will show that the ones that are finite define connected Hopf algebras of finite GK dimension. The Hopf algebras constructed this way are isomorphic, as algebras, to universal enveloping algebras. This construction recovers almost all of the known connected Hopf algebras of finite GK dimension and leads to many new examples from the general point of view of quasi-Frobenius Lie algebras. This is a joint work with Jesse Levitt (LSU).