Hypercontractivity and Log Sobolev Inequalities in Quantum Information Theory (15w5098)
22-27 February 2015

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.
Monday	
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **William Beckner** (University of Texas at Austin)
Title: *Hypercontractivity and the Logarithmic Sobolev Inequality – from Quantum Field Theory to Geometric Inequalities*
Abstract: This talk will feature a quick historical survey of connections between the log Sobolev inequality and now classical problems in analysis, physics and probability.

Speaker: **Salman Beigi** (Institute for Research in Fundamental Sciences (IPM))
Title: Measuring quantum correlation via completely bounded norms Abstract: A hypercontractivity ribbon is a measure of bipartite correlation that is dened based on the operator norm of some linear map between Schatten classes that is associated to a given bipartite distribution. This measure of correlation has the intriguing property that the hypercontractivity ribbon of a bipartite distribution is equal to that of its i.i.d. repetitions. This talk is about the generalization of hypercontractivity ribbon for bipartite quantum states. We will discuss that to generalize the above property of the hypercontractivity ribbon to the quantum case we should use completely bounded norms. Then non-commutative vector valued Schatten spaces naturally appear in the study of quantum correlations. Finally we comment that these spaces also appear in the definition of a new generalization of Renyi (conditional) entropy to the quantum case.

Speaker: **Fernando G.S.L. Brandao** (Microsoft Research and UCL)
Title: *Rapid Mixing versus Clustering for Quantum Systems*
Abstract: In this talk Ill present recent results relating the time of preparation of thermal states by local quantum dissipative processes (aka Liouvillians) with static properties of the state, namely whether the state is clustering (i.e. has a finite correlation length). In particular I'll show that for commuting Hamiltonians the Davies master equation (a quantum analogue of Glauber dynamics) has a constant spectral gap if, and only if, a certain strong type of clustering holds. I'll also discuss the challenges of proving a similar statement with the log-Sovolev constant of the Liouvillian in place of the spectral gap

Our goal is to generalize to the quantum case a sequence of beautiful works – by Stroock, Zergalinski, Martinelli and others – in mathematical physics and statistical mechanics showing the equivalence of mixing in time (fast convergence of the Glauber dynamics – meaning a constant spectral gap or even a size-independent log-Sobolev constant) to mixing in space (finite correlation length in the Gibbs state) for classical models.

The talk will be based on joint work with Michael Kastoryano (http://arxiv.org/abs/1409.3435).

Speaker: **Harry Buhrman** ( )
Title: Multipartite entanglement in XOR games

Speaker: **Eric Carlen** (Rutgrs University and I.M.A.)
Title: *Quantitative uniform convexity, fermion hypercontractivity, and related topics*
Abstract: There is a natural non-commutative analog of the Mehler semigroup, namely the fermionic Mehler semigroup, or fermionic oscillator semigroup. This arises naturally in quantum field theory, and plays a role there for fermionic systems that is entirely analogous to the role played by the usual Mehler semigroup in Edward Nelson's approach to boson quantum fields.

This close analogy suggested to Leonard Gross that the fermionic Mehler semigroup might enjoy hypercontractivity properties exactly analogous to those of the usual Mehler semigroup, with the exact same constants. He was able to prove a hypercontractivity result, but not the optimal one he conjectured. Gross's conjecture was later proved by myself and Elliott Lieb. The proof is strongly geometric and relies on optimal uniform convexity bounds for the trace norms due to myself, Keith Ball and Elliott Lieb. The bounds have since found many other applications, and there are still some associated open questions relating to them.

In this talk I will explain the geometry of the optimal uniform convexity bounds and give several simple applications. I will then introduce the fermionic Mehler semigroup through its very simple and elegant Stinespring representation. (It is a semigroup of completely positive maps). I will then sketch a proof of optimal fermion hyper contractivity using these bounds. The talk will be entirely elementary, with no prerequisites beyond linear algebra, and though physical application will be mentioned, no familiarity with these will be assumed.

Speaker: **Ronald de Wolf** (CWI and University of Amsterdam)
Title: *The hypercontractive inequality in theoretical computer science*
Abstract: This expository talk describes and motivates the hypercontractive inequality for functions on the Boolean cube, which is the relevant setting for theoretical computer science. We will go over several important consequences for functions with small support or small degree. We then describe a few applications to illustrate its use, such as the unpredictability of random parities over a large set; the KKL theorem that every balanced Boolean function has an influential variable; and lower bounds on polynomial degrees.

bigskip Speaker: **Aram Harrow** (MIT)
Title: *Replacing hierarchies with nets*
Abstract: The SDP hierarchy due to Lasserre and Parrilo has recently been shown to yield quasipolynomial time algorithms for some optimization problems related to quantum information. One representative example is to maximize over unentangled states the acceptance probability of a measurement that is implemented with local operations and one-way communication (1-LOCC). There have been a series of proofs of this fact, starting with Brandao-Christandl-Yard in 2010, mostly based on quantum information theory.

In this work we show how to achieve roughly the same performance instead using a simple enumeration over epsilon nets. This net-based approach gives a new geometric perspective into the problem that helps explain why 1-LOCC measurements appear to be easier than general measurements. It also yields two improvements of the SDP-based results. First, it gives faster algorithms for special cases of the problem, including estimating the 2-¿s norm of a matrix for $s >= 2$. Second we give a similar algorithm for a generalization of the problem: estimating the injective tensor norm for a map between two Banach spaces, whose factorization norm through ℓ_1 is bounded. As a particular case we find the first PTAS for estimating the maximum output p-norms of entanglement breaking quantum channels for any p¿=1.

This is joint work with Fernando Brandao. A preliminary version is available at http://web.mit.edu/aram/Public/

Speaker: **Todd Kemp** (UC San Diego)
Title: *Strong Hypercontractivity and Log Sobolev Inequalities*
Abstract: In the 1970s, Gross showed that the log Sobolev inequality is equivalent (first in the Gaussian setting, then later in wide generality) to Nelson's hypercontracivity for the associated Markov semigroup. The sharp time to contraction $L^p \to L^q$ is controlled by the optimal log Sobolev constant c:

$$t_{\text{Nelson}} = \frac{c}{2} \ln \frac{q-1}{p-1}, \qquad 1 < p \leq q < \infty.$$

In 1983, Janson discovered that in the holomorphic subspaces of Gaussian L^p, the time to contraction is *shorter*, and the result holds for $0 < p, q \leq 1$ as well:

$$t_{\text{Janson}} = \frac{c}{2} \ln \frac{q}{p}, \qquad 0 < p \leq q < \infty.$$

In 1999, Gross showed that this so-called *strong hypercontractivity* actually follows from the *same* log Sobolev inequality, in a wide category (of sufficiently regular Dirichlet forms on complex manifolds). However, the reverse implication does not hold: one needs the log Sobolev inequality to hold for a wider class of test functions than the holomorphic ones.

I will give some detail about the history of strong hypercontractivity, and then discuss my work on this topic in the classical and quantum worlds. In the quantum case, I showed in 2004 that Janson's strong hypercontractivity holds for Fermions (and more general anyons), at least for restricted exponents $p = 2$ and q even. In the classical world, in recent joint work with Graczyk and Loeb, I showed that strong hypercontractivity holds beyond the holomorphic category, for the class of *logarithmically subharmonic functions*, and here there is an equivalence to a new log Sobolev inequality.

Speaker: **Christopher King** (Northeaster University)
Title: *Remarks on the origins of hypercontractivity in quantum field theory.*
Abstract: The inequality $\|e^{-tH_0}\|_{2\to 4} < \infty$ for $t \geq t_0$ appeared in Nelson's 1965 paper on quantum field theory in two space-time dimensions, where it was used as a technical tool to prove semiboundedness of an interacting Hamiltonian. This was an important breakthrough in the mathematical understanding of quantum fields, and motivated the development of the Euclidean approach to field theory. This talk will present an overview of the setting for Nelson's result, and discuss some of the follow-up work which led to the formulation of hypercontractivity and log Sobolev inequalities.

Speaker: **Robert Koenig** (Technical University Munich)
Title: *Entropy power inequalities*
Abstract: The classical entropy power inequality, originally proposed by Shannon, is a powerful tool in multi-user information theory. In this talk, I review some of the history of this inequality, as well as Shannons original application: such inequalities provide bounds on capacities of additive noise channels. I then introduce a quantum entropy power inequality which lower bounds the output entropy as two independent signals combine at a beamsplitter. In turn, such inequalities provide bounds on the classical capacity of additive bosonic noise channels.

Joint work with Graeme Smith.

Speaker: **Ashley Montanaro** (University of Bristol)
Title: *Applications of hypercontractivity in quantum information*
Abstract: The beautiful theory of hypercontractivity has historically been applied in areas as diverse as quantum field theory and theoretical computer science. So it is perhaps not surprising that in recent years hypercontractivity has also found a number of applications in the theory of quantum computation and quantum information. In this talk I will briefly motivate and discuss some of these applications, many of which are due to attendees of this workshop. These include bounding the bias of nonlocal games, separating quantum and classical communication complexity, and proving limitations on quantum random access codes.

Speaker: **Mathilde Perrin** (ICMAT)
Title: *Hypercontractivity in group von Neumann algebras*
Abstract: The phenomenon of hypercontractivity was discovered simultaneously and independently in the early 70's in quantum field theory and harmonic analysis. Since then it has been widely studied, mainly from a gaussian point of view for Ornstein-Ulhenbeck semigroups. In this talk, in the line of Bonami ($\mathbb{Z}_2$) and Weissler ($\mathbb{Z}$), we will consider instead the trigonometric point of view by studying some Poisson semigroups. Beyond the commutative case, given any discrete group G equipped with a length function ψ, we may define a Poisson-like semigroup acting on the group von Neumann algebra $\mathcal{L}$G by

$$\mathcal{P}_t\Big(\sum_{g\in \mathrm{G}} \widehat{f}(g)\lambda(g)\Big) = \sum_{g\in \mathrm{G}} e^{-t\psi(g)}\widehat{f}(g)\lambda(g).$$

We will first focus on the free group $\mathbb{F}_n$ equipped with the word length. Several approaches have been developed to attack this problem, leading to optimal hypercontractivity estimates for the free Poisson semigroup at some ranges. One of those relies on a probabilistic argument, by proving hypercontractivity results for Ornstein-Ulhenbeck semigroups defined on the free product of Clifford algebras. A combinatorial method has also been used to study the free group case, and can be applied to other situations. In particular, it can improve classical hypercontractivity results for the cyclic groups $\mathbb{Z}_n$. By transference, we obtain hypercontractivity estimates for a large class of semigroups defined on finite-dimensional matrix algebras $\mathbb{M}_n$. This provides new hypercontractive families of quantum channels. We will finally discuss general conditions to obtain hypercontractivity estimates (not necessarily optimal) for Poisson-like semigroups associated to arbitrary group/length.

Joint work with Marius Junge, Carlos Palazuelos, Javier Parcet and Éric Ricard.

Speaker: **David Perez-Garcia** (Universidad Complutense de Madrid)
Title: *Stability and area law for many-body quantum Markovian dynamics*
Abstract: Open quantum systems weakly coupled to the environment are modeled by completely positive, trace preserving semigroups of linear maps. The generators of such evolutions are called Lindbladians. In the setting of quantum many-body systems on a lattice it is natural to consider local or exponentially decaying interactions. In this talk we will restrict to systems for which the Linbladian has a mixing time that scales logarithmically with the system size, a condition implied by having bounded quantum log-Sobolev constants. We will show that those systems are stable against perturbations in the Linbladian and that their fixed points satisfy an area law for the mutual information.

Joint work with F.G.S.L. Brandao, T.S. Cubitt, A. Lucia and S. Michalakis

Speaker: **Oleg Szehr** (University of Cambridge) Title: Spectral Methods for quantum Markov chains
Abstract: Quantum Markov chains constitute a natural quantum-mechanical generalization of classical Markov chain and are used to model the evolution of certain quantum systems. Both in the classical and quantum context many applications require estimates for convergence time and stability of the chain. We develop a framework that yields such estimates in terms of the spectrum of the transition map. The methods employed are new even to the well studied theory of classical Markov chains. The key observation is to relate the original problem of bounding a function of a transition map of a Markov chain to a Nevanlinna-Pick interpolation problem in Wiener algebra. We provide an introduction to an operator theoretic approach to the Nevanlinna-Pick problem employing Andos Theorem and other strong results from the theory of Hilbert function spaces.

Speaker: **Thomas Vidick** (Caltech)
Title: *Quantum XOR Games*
Abstract: In this talk I will discuss quantum XOR games, a model of two-player one-round games that extends the model of XOR games by allowing the referee's questions to the players to be quantum states. I will motivate the study of these games by giving examples showing that quantum XOR games exhibit a wide range of behaviors that are known not to exist for standard XOR games, such as cases in which the use of entanglement leads to an arbitrarily large advantage over the use of no entanglement.
I will describe very interesting connections between optimal quantum strategies (resp. without and with entanglement) in quantum XOR games and two extensions of Grothendieck's inequality (resp. to C*-algebras and to operator spaces). Using this connection we can give constant-factor approximation algorithms for the best performance players can obtain in a given game (resp. without and with entanglement).

Based on joint work with Oded Regev.

Speaker: **Boguslaw Zegarlinski** ()
Title: *Framework, results and open problems for Log Sobolev Inequalities in noncommutative spaces*

POSTERS

Presenter: **Daniel Stilck França**(TU München)
Title: it Log-Sobolev inequalities for unital Liouvillians
Abstract: We obtain tensor-stable bounds on the Log-Sobolev-2 constant of unital Liouvillians using hypercontractivity w.r.t. completely bounded norms and interpolation inequalities. Thereby we improve previous results and simplify proofs found in the literature. Furthermore, we obtain LS-like inequalities for general unital quantum channels through the LS2 constant of the Liouvillian $L = T^*T - id$and a comparison theorem for the spectral gap.

Joint work with Alexander Müller-Hermes and Michael Wolf

Presenter:bf Alexander Müller-Hermes(TU Munich) Title: it Log-Sobolev inequalities of the depolarizing Liouvillians Abstract: We compute the Log-Sobolev-1-constants for d-dimensional depolarizing Liouvillians onto an arbitrary full-rank quantum state. As an application of these results we derive an improvement of concavity of the von-Neumann entropy. Here for a given convex combination of two states we obtain a correction term depending on the relative entropy of the two states and on the Log-Sobolev constant of a depolarizing channel depolarizing on one of them.

Joint work with Daniel Stilck França (attendee) and Michael Wolf

Presenter: **David Reeb** (Hannover University, Germany) Title: it Positivity of linear maps under tensor powers Abstract: On matrix algebras, we investigate maps that remain positive under tensoring with copies of itself, the paradigmatic examples being the completely positive and the completely co-positive maps. The existence of such "tensor-stable positive" maps outside the latter two families remains an open question, but we reduce the existence question to certain one-parameter families, and we exclude their existence in two dimensions. We furthermore show that any tensor-stable positive map that is not completely positive gives rise to an upper bound on the quantum channel capacity. For the transposition map this is the well-known "cb-norm bound", and we show that this bound is even a strong converse rate for quantum communication assisted by arbitrary LOCC operations.

Joint work with Alexander Müller-Hermes (attendee) and Michael Wolf

Presenter:: **John Wright** (CMU)
Title: *Quantum spectrum testing*
Abstract: In recent years, the area of property testing of probability distributions has produced a wide variety of interesting results. Here one is given sample access to an unknown distribution and asked whether it satisfies some property, e.g. is it the uniform distribution, does it have small entropy, and so forth. In this work, we study the natural quantum analogue of this problem, in which one is given many copies of an unknown mixed state and asked whether it satisfies a given property. We focus on properties which depend only on the mixed state's spectrum (hence the name Quantum Spectrum Testing).

Our paper gives several optimal algorithms for testing specific properties of mixed states, e.g. the property of being the maximally mixed state. This can be viewed as the quantum analogue of a result of Paninski. In addition, we show a nearly-tight lower bound for the "empirical Young diagram" algorithm which learns an unknown mixed state's spectrum.

Our main tool is Schur-Weyl duality, which leads us to studying a particular algorithm called weak Schur sampling. This algorithm is based on the representation theory of the symmetric group, and to analyze it we use various tools in this area, including Kerov's algebra of observables and a limiting theorem of Biane.

Joint work with Ryan O'Donnell.