UNIVERSITÄT ZU LÜBECK

Fully Dynamic Bin Packing Revisited

Published in APPROX/RANDOM 2015

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Fully Dynamic Bin Packing =
Online + Removal + Repacking

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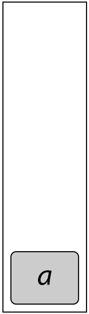
INS: $a/0.2$

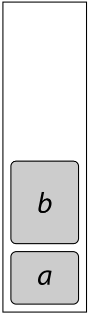
Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$



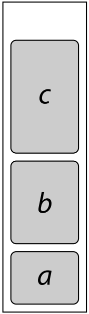
Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$, **INS**: $c/0.4$



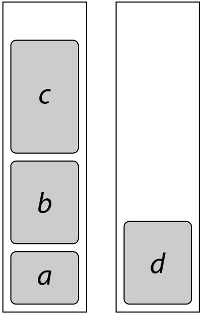
Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$, **INS**: $c/0.4$, **INS**: $d/0.2$



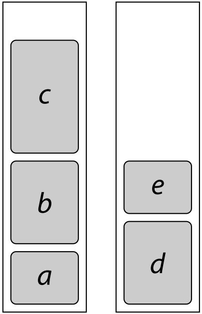
Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$, **INS**: $c/0.4$, **INS**: $d/0.2$, **INS**: $e/0.3$



Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: $a/0.2$, **INS**: $b/0.3$, **INS**: $c/0.4$, **INS**: $d/0.2$, **INS**: $e/0.3$, **REM**: c



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Fully Dynamic Bin Packing = Online + Removal + Repacking

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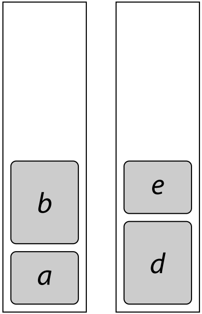
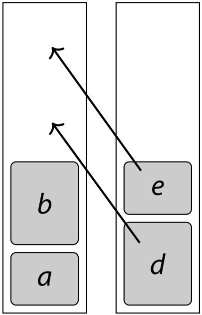


Fully Dynamic Bin Packing = Online + Removal + Repacking

INS: *a*/0.2, **INS**: *b*/0.3, **INS**: *c*/0.4, **INS**: *d*/0.2, **INS**: *e*/0.3, **REM**: *c*



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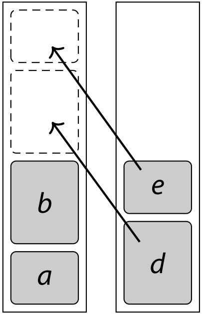

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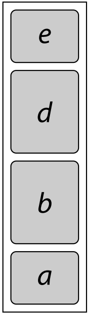

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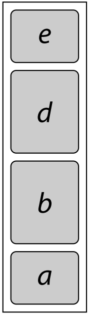
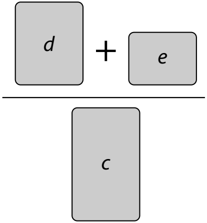


Migration Factor$_t$ =



$= \frac{\text{SIZE(moved)}}{\text{SIZE(new/removed)}}$

Fully Dynamic Bin Packing = Online + Removal + Repacking

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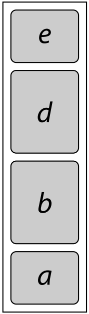

Migration Factor$_t$ $= \frac{d + e}{c} = \frac{\text{SIZE(moved)}}{\text{SIZE(new/removed)}}$

Shifting Moves = number of moved items

Lower Bound

Migration Factor of $\Omega(1/\varepsilon)$ is necessary for ratio $1+\varepsilon$

$L = 1/2 - 1/9$(Migration Factor), $S = 1/3$(Migration Factor)

INS: L, **INS**: L, **INS**: $L, \ldots$

Lower Bound

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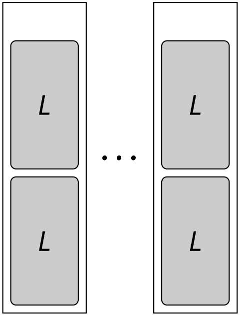
$L = 1/2 - 1/9(\text{Migration Factor})$, $\quad S = 1/3(\text{Migration Factor})$

INS: L, **INS**: L, **INS**: $L, \ldots$



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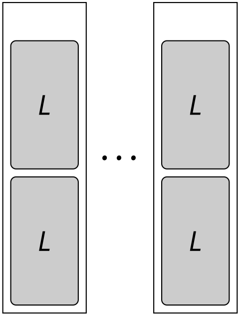
INS: L, **INS**: L, **INS**: $L, \ldots,$

INS: S, **INS**: S, **INS**: $S, \ldots$



Migration Factor of $\Omega(1/\varepsilon)$ is necessary for ratio $1+\varepsilon$

$L = 1/2 - 1/9(\text{Migration Factor})$, $S = 1/3(\text{Migration Factor})$

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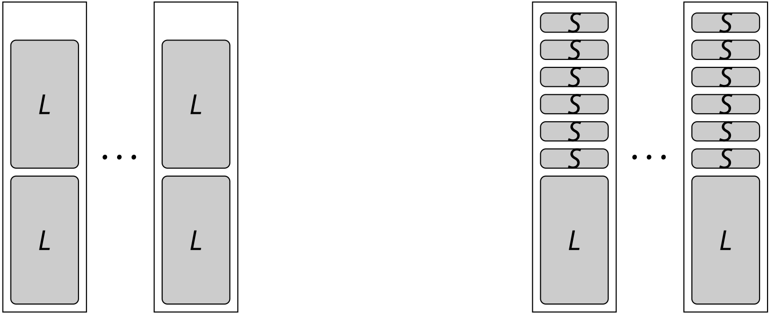
INS: S, **INS**: S, **INS**: $S, \ldots$



Known Results on upper bounds

Ratio	**REM**?	Shifting M.	Migration F.	Authors

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Known Results on upper bounds

Ratio	REM?	Shifting M.	Migration F.	Authors
3/2	✗	3	✗	Gambosi, Postiglione, Talamo (2000)
4/3	✗	7	✗	Gambosi, Postiglione, Talamo (2000)

Online Bin Packing with Repacking

Known Results on upper bounds

Ratio	**REM**?	Shifting M.	Migration F.	Authors
3/2	✗	3	✗	Gambosi, Postiglione, Talamo (2000)
4/3	✗	7	✗	Gambosi, Postiglione, Talamo (2000)
$1+\varepsilon$	✗	poly($\log n$) [am.]	✗	Ivković, Lloyd (1997)
$1+\varepsilon$	✗	$2^{\text{poly}(1/\varepsilon)}$	$2^{\text{poly}(1/\varepsilon)}$	Epstein, Levin (2006)
$1+\varepsilon$	✗	$\mathcal{O}(1/\varepsilon^4)$	$\mathcal{O}(1/\varepsilon^4)$	Jansen, Klein (2013)

Online Bin Packing with Repacking

Known Results on upper bounds

Ratio	**REM**?	Shifting M.	Migration F.	Authors
3/2	✗	3	✗	Gambosi, Postiglione, Talamo (2000)
4/3	✗	7	✗	Gambosi, Postiglione, Talamo (2000)
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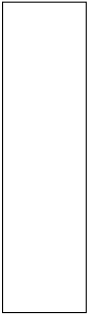
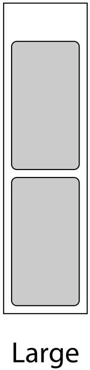
$1+\varepsilon$	✗	$2^{\text{poly}(1/\varepsilon)}$	$2^{\text{poly}(1/\varepsilon)}$	Epstein, Levin (2006)
$1+\varepsilon$	✗	$\mathcal{O}(1/\varepsilon^4)$	$\mathcal{O}(1/\varepsilon^4)$	Jansen, Klein (2013)
5/4	✓	poly($\log n$) [am.]	✗	Ivković, Lloyd (1998)
$1+\varepsilon$	✓	$\mathcal{O}(1/\varepsilon^4 \log(1/\varepsilon))$	$\mathcal{O}(1/\varepsilon^4 \log(1/\varepsilon))$	this work

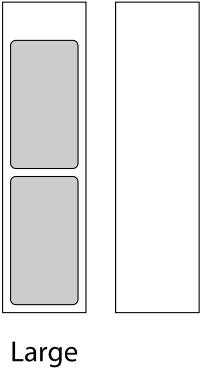
An Overview on the Packing



An Overview on the Packing



An Overview on the Packing



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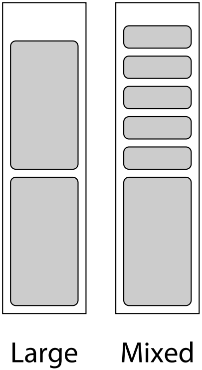

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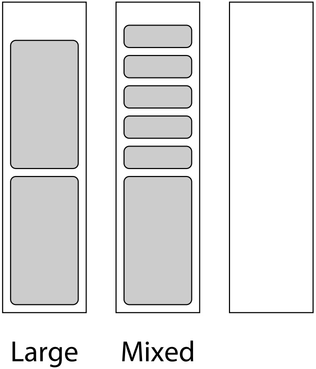

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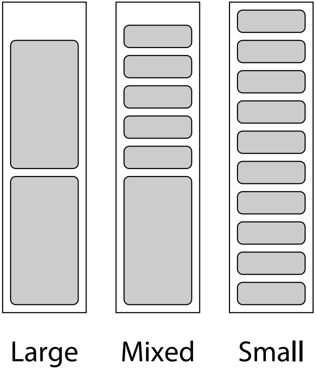

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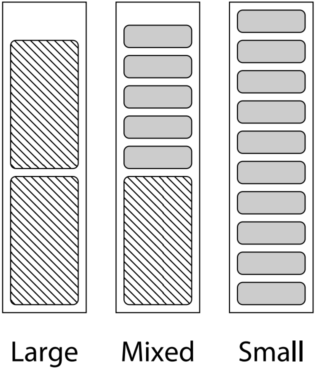
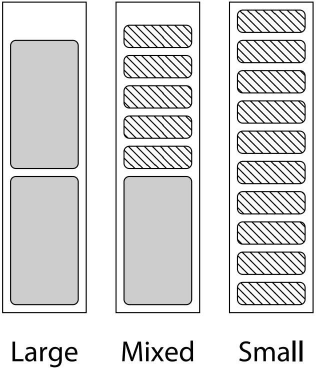


- Pack via LP

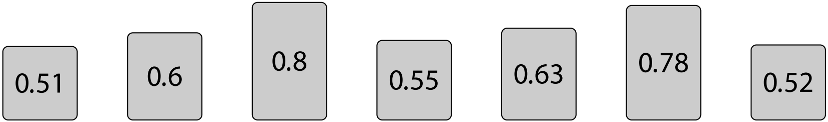
An Overview on the Packing



- Pack via LP
- Pack via “Sorting”

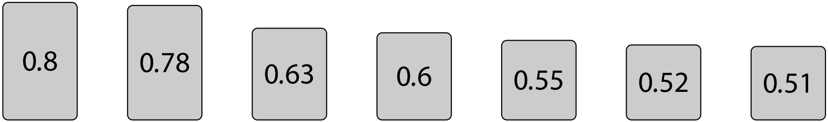
Rounding Large Items Geometrically

1. Find items of size (0.5, 1]



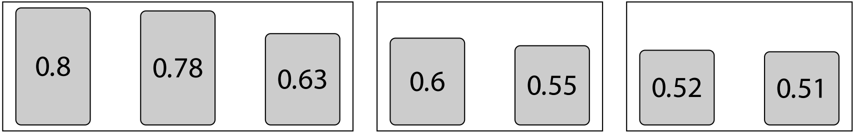
Rounding Large Items Geometrically

1. Find items of size (0.5, 1]
2. Sort items by size



Rounding Large Items Geometrically

1. Find items of size (0.5, 1]
2. Sort items by size
3. Group items in lists



Rounding Large Items Geometrically

1. Find items of size (0.5, 1]
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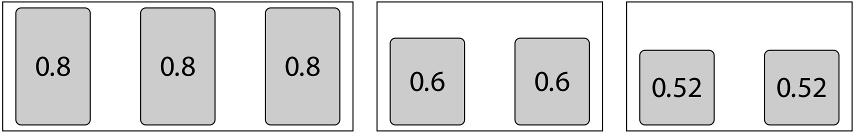
4. Round items



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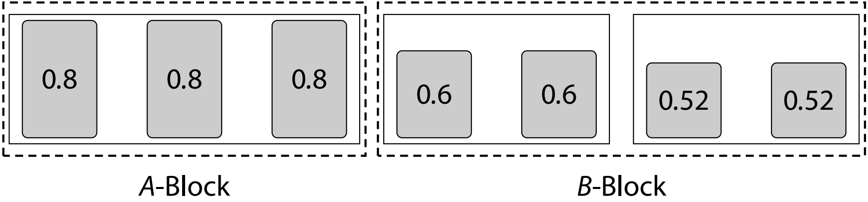
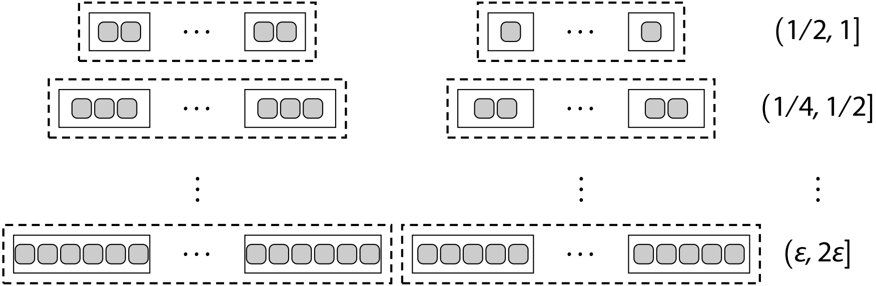
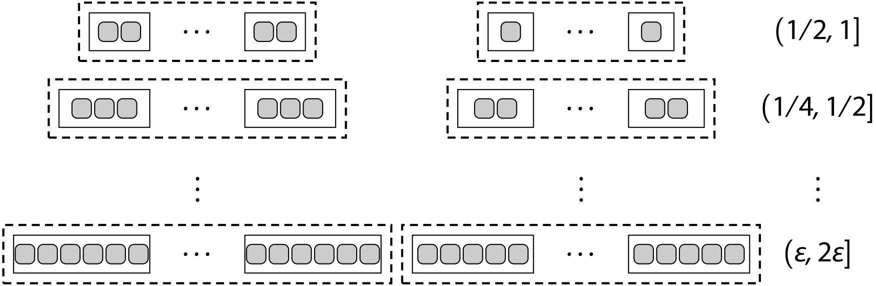


Packing Large Items



- Pack rounded items via LP

Packing Large Items



- Pack rounded items via LP
- Size of lists depends on volume of instance ⇒ Shifting

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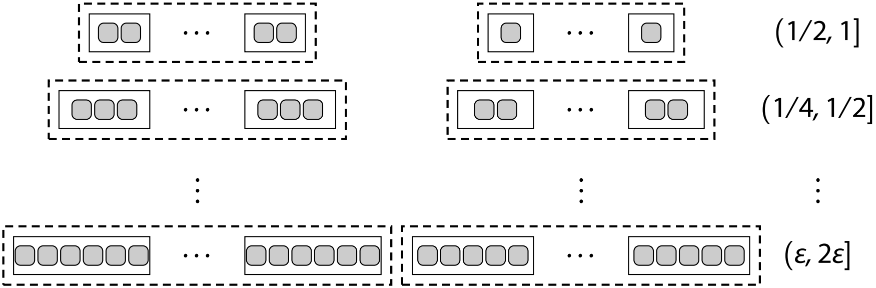
Packing Large Items



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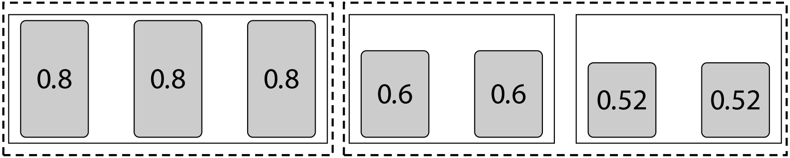

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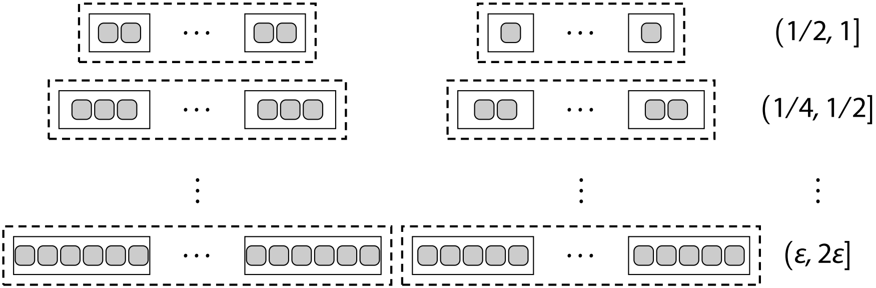

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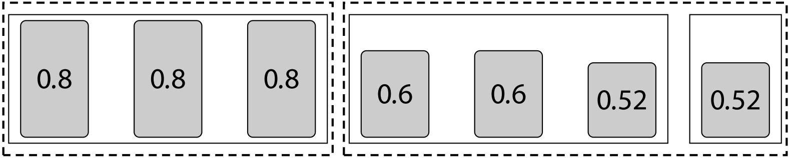

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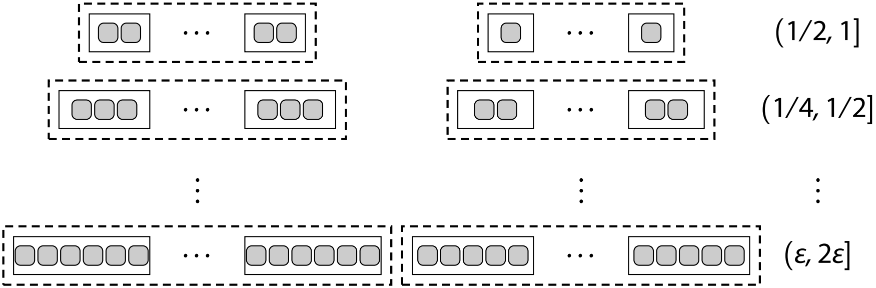

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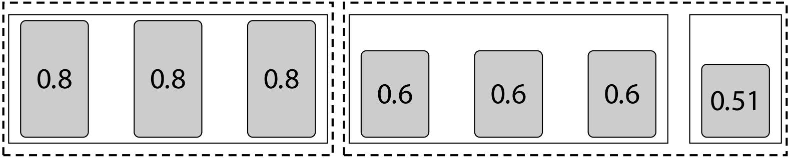

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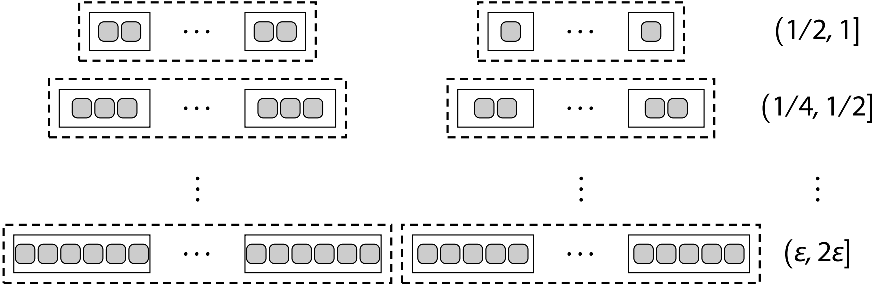

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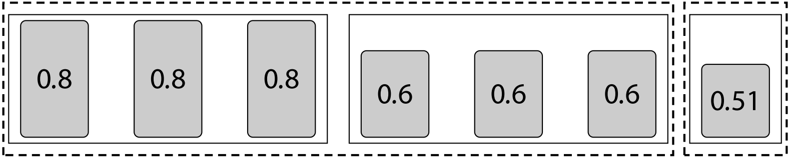

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Packing Small Items is Difficult

Greedy fails: $\varepsilon \gg L \gg S$

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$(\texttt{INS}: L, \texttt{INS}: S, \ldots, \texttt{INS}: S)^*$

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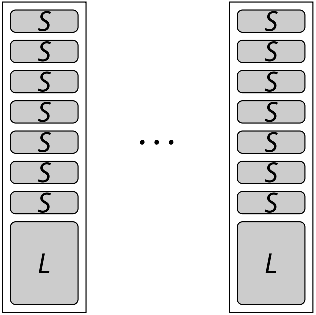

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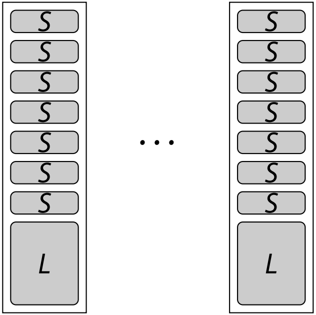

REM: S, **REM**: S, ...

Packing Small Items is Difficult

Greedy fails: $\varepsilon \gg L \gg S$

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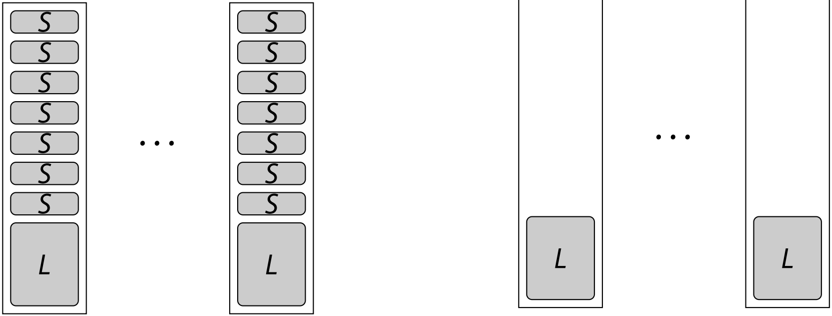

Packing Small Items via Sorting

Idea: "Sort" small items from left to right

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$$\texttt{INS}: L, \texttt{INS}: S, \ldots, \texttt{INS}: S,$$

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Idea: "Sort" small items from left to right

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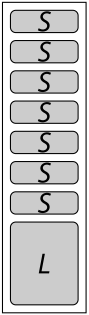
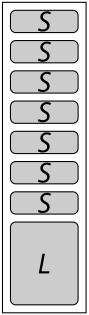


Packing Small Items via Sorting

Idea: “Sort” small items from left to right

INS: L, **INS**: S,. . . , **INS**: S, **INS**: L



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Idea: "Sort" small items from left to right

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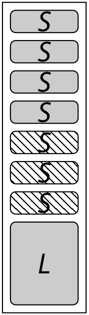

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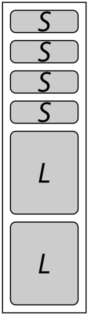

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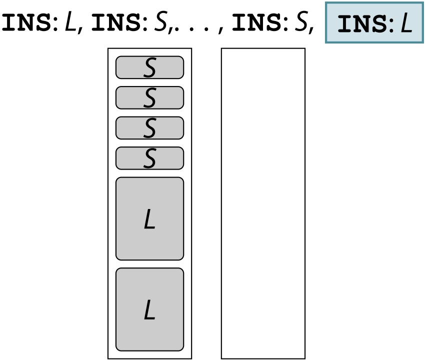

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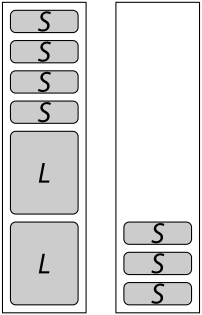

Packing Small Items via Sorting

Idea: "Sort" small items from left to right

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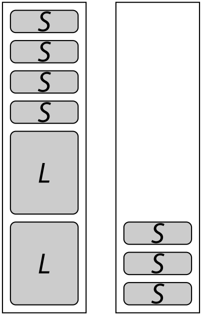
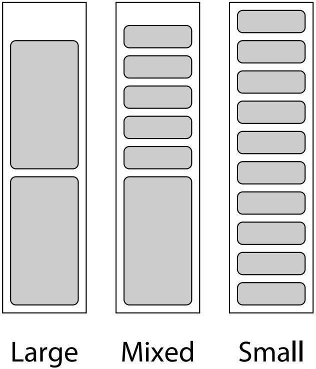


Stop at every $1/\varepsilon$-th bin (buffer bin) to bound Migration

An Overview on the Packing



- Pack large items via LP

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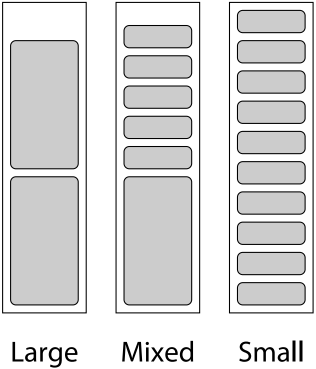

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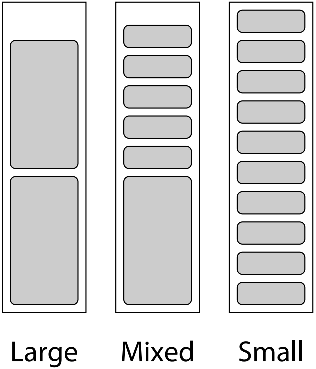

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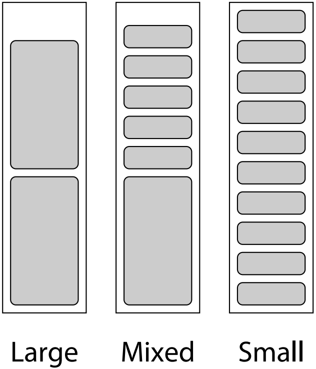
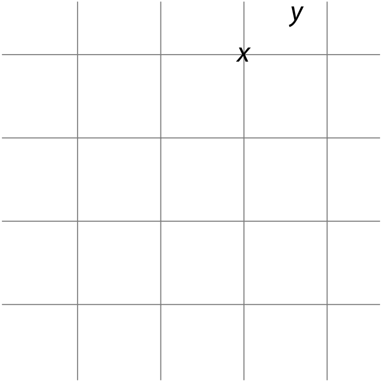


- Pack large items via LP
- Pack small items via “Sorting”
- Small bins $\Rightarrow$ little free space in other bins
- Relate nearly full/empty bins via potential function

Some Details

Theorem (Jansen and Klein '13)

[Some requirements] There is an algorithm that returns for a LP/ILP pair (y, x) an α-improved LP/ILP pair (x', y') with $||x'|| \leq ||x|| - \alpha$ and $||y'|| \leq ||y|| - \alpha$.



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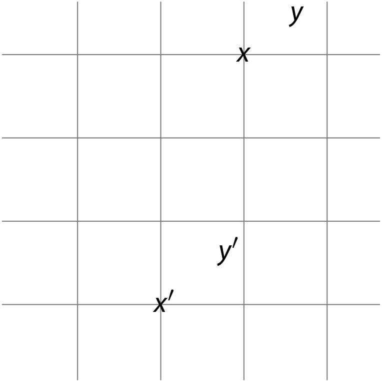

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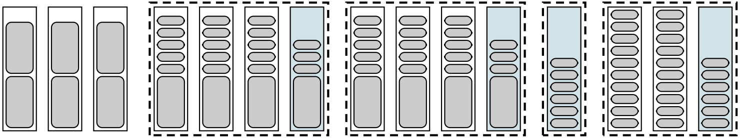

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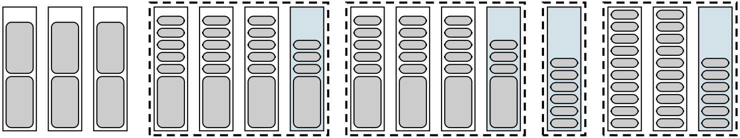
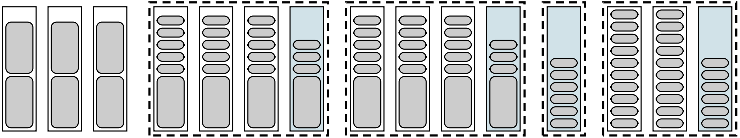


Invariants:

Some Details



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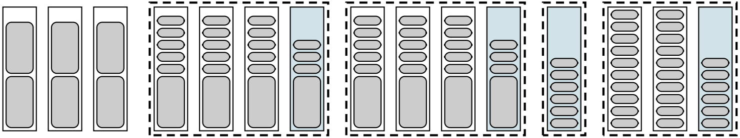

Invariants:

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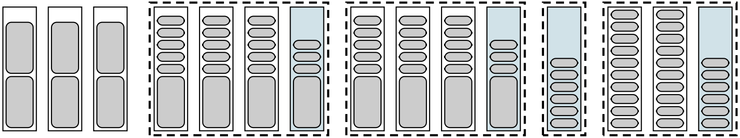
- Only buffer bins and large bins are non-full

Sebastian Berndt , Klaus Jansen , Kim-Manuel Klein

Some Details



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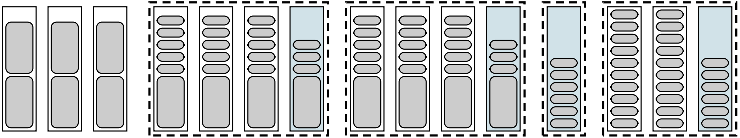

Invariants:

- Small Items are sorted from left to right
- Only buffer bins and large bins are non-full
- Length of each *queue* is in $[1/\varepsilon, 2/\varepsilon]$
- Number of non-filled large bins is $\approx$

$$\Phi = \sum_i r_i + \varepsilon \cdot \Delta + \ell$$

bins (↓ Δ)

fill-ratio of bb_i (↑ r_i)

mixed bins (↑ ℓ)

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- Can we adapt our techniques to other problems?