FPT Approximation Schemes for Shift Bribery

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Motivation

What is the "best" way to cope with NP-hard problems?

Some opinions:

Robert:	FPA	$\succ$	Rand	$\succ$	Avg	$\succ$	Heur
Jiehua:	Rand	$\succ$	FPA	$\succ$	Avg	$\succ$	Heur
(heretical) André:	Heur	$\succ$	Avg	$\succ$	Rand	$\succ$	FPA

FPA fixed-parameter approximation

Rand randomized algorithms

Avg average-case analysis

Heur heuristics

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(converted) André:	FPA	$\succ$	Heur	$\succ$	Avg	$\succ$	Rand

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Who wins the poll? **fixed-parameter approximation**

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- A set of candidates $C = \{c_1, \ldots, c_m\}$.
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Borda 1st: $m-1$ points, 2nd: $m-2$ points, …, m^{th}: 0 points.

Copeland$^\alpha$ Head-to-head contest between each two candidates.
Winner: 1 point, loser: 0 points, tie: α points each.

Shift Bribery

Setting:
An external agent may influence voter to shift some preferred candidate forward in the voters' preference list.

[Elkind, Faliszewski, and Slinko. Swap Bribery. SAGT '09]

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- each voter has a different price depending on the "shift length"

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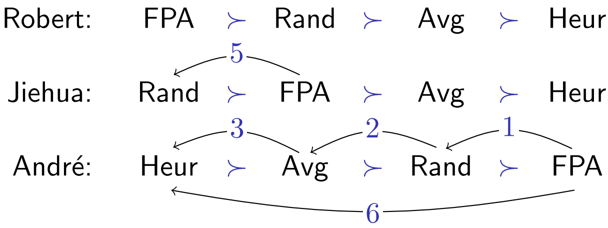

Shift Bribery - Problem Definition

$\mathcal{R}$ **Shift Bribery**

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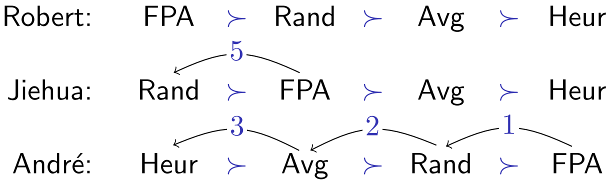

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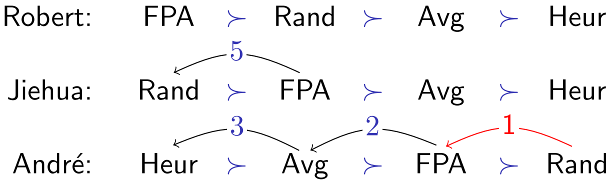

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- NP-hard for Borda, Copeland, and other voting rules
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[Schlotter, Faliszewski, and Elkind. Campaign management under approval-driven voting rules. AAAI '11]:

- Polynomial-time algorithm for Bucklin and Fallback voting rule
- Problem variant: **Support Bribery** (briber extends incomplete votes)

Our Results

[Bredereck, Chen, Faliszewski, N., and Niedermeier. Prices matter for the parameterized complexity of shift bribery. AAAI '14]:

- There is a factor-$(1+\varepsilon)$ approximation algorithm solving $\mathcal{R}$ **Shift Bribery** in time $O^*(\lceil n/\varepsilon + 1\rceil^n)$ for $\mathcal{R} \in \{$Copeland, Borda$\}$.

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Update: $\mathcal{R}$ **Shift Bribery** for $\mathcal{R} \in \{$Copeland, Borda$\}$ is fixed-parameter tractable with respect to the number m of candidates.

[Bredereck, Faliszewski, Niedermeier, Skowron, and Talmon. Elections with Few Candidates: Prices, Weights, and Covering Problems. ADT' 15]

Further Results

		$\mathcal{R}$ **Shift Bribery**				
parameter	$\mathcal{R}$	unit	convex	all	sort	$0/1$

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parameter	$\mathcal{R}$	unit	convex	all	sort	$0/1$
#shifts (t)	Borda Copeland$^{\alpha}$					
budget (B)	Borda Copeland$^{\alpha}$					
#candidates (m)	Borda/Copeland$^{\alpha}$					
#voters (n)	Borda Copeland$^{\alpha}$					

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Taks: Find a minimum-cost shift action that makes p a winner according to rule $\mathcal{R}$?

Parameter $n := |V|$.

Goal: $(1 + \varepsilon)$ approximation in $f(n, \varepsilon) \cdot \mathrm{poly}$

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Algorithm outline:

1. Rescale prices to the range $\{0, 1, \ldots, \lceil n/\varepsilon \rceil - 1, \lceil n/\varepsilon \rceil, \infty\}$.
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Observation: Above algorithm works for any voting rule where winner determination is fixed-parameter tractable with respect to n.

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Rescaling:

- Guess maximum cost $\pi_{\max}$ spend at a single voter.
- Set $K = \varepsilon \cdot \pi_{\max}/n$ and

for each $v_i \in V, j \in \{1, \ldots, m\}: \quad \pi'_i(j) = \begin{cases} \left\lceil \frac{\pi_i(j)}{K} \right\rceil & \text{if } \pi_i(j) \leq \pi_{\max} \\ \infty & \text{otherwise.} \end{cases}$

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Running time: Step 1: $O((nm)^2)$
Step 2: $O((\lceil n/\varepsilon \rceil + 1)^n \cdot \text{running time for winner determination})$
In total: $O^*((\lceil n/\varepsilon \rceil + 1)^n)$ for Copeland and Borda voting rule

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Approximation factor: algorithm solution: $\vec{s}'$ optimal solution: $\vec{s}$

(A solution is a vector $\in \{0, \ldots, m\}^n$ storing the number of shifts for each voter.)

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   for each $v_i \in V, j \in \{1,\ldots,m\}: \quad \pi'_i(j) = \begin{cases} \left\lceil \frac{\pi_i(j)}{K} \right\rceil & \text{if } \pi_i(j) \leq \pi_{\max} \\ \infty & \text{otherwise.} \end{cases}$
3. Try all possibilities to spend at most $\lceil n/\varepsilon \rceil$ per voter.
4. Output cheapest solution.

Approximation factor: algorithm solution: $\vec{s}'$ optimal solution: $\vec{s}$

(A solution is a vector $\in \{0,\ldots,m\}^n$ storing the number of shifts for each voter.)

cost of $\vec{s}'$: $\Pi(\vec{s}')$ cost of optimum solution: $\Pi(\vec{s})$

$$\Pi(\vec{s}') \leq K \cdot \Pi'(\vec{s}') \leq K \cdot \Pi'(\vec{s}) \leq \Pi(\vec{s}) + K \cdot n$$

Fixed-Parameter Approximation Scheme I: Details II

Algorithm outline:

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$$\begin{aligned} \Pi(\vec{s}') &\leq K \cdot \Pi'(\vec{s}') \leq K \cdot \Pi'(\vec{s}) \leq \Pi(\vec{s}) + K \cdot n \\ &= \Pi(\vec{s}) + \varepsilon \cdot \pi_{\max} \leq \Pi(\vec{s}) + \varepsilon \cdot \Pi(\vec{s}) = (1+\varepsilon)\Pi(\vec{s}) \end{aligned}$$

Fixed-Parameter Approximation Scheme I: Summary

We saw: There is a factor-$(1+\varepsilon)$ approximation algorithm solving $\mathcal{R}$ **Shift Bribery** in time $O^*(\lceil n/\varepsilon + 1\rceil^n)$ for $\mathcal{R} \in \{$Copeland, Borda$\}$.

Recall:

- Copeland **Shift Bribery** parameterized by the number n of voters is W[1]-hard for unit prices.
- There is a polynomial-time factor-m approximation algorithm for Copeland **Shift Bribery**. [Elkind and Faliszewski. Approximation algorithms for campaign management. WINE '10]

Open:
Is there a polynomial-time factor-$\log(m)$ (or better) approximation for Copeland **Shift Bribery**?

Fixed-Parameter Approximation Scheme II: Overview

$\mathcal{R}$ **Shift Bribery**

Input: An election $E = (C, V)$, a specific candidate $p \in C$, and a price function list $\Pi = (\pi_1, \ldots, \pi_n)$.

Taks: Find a minimum-cost shift action that makes p a winner according to rule $\mathcal{R}$?

Parameter $m := |C|$.

Goal: $(1 + \varepsilon)$ approximation in $f(m, \varepsilon) \cdot \mathrm{poly}$

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First Observations:

- at most $m!$ different preference orders
  $\rightsquigarrow$ at most $m!$ "blocks" of voters
- The prices give a sorting of the voters in one block.
  $\rightsquigarrow$ For each two voters of one block, one never shifts p farther in the preference order of the more expensive voter.

Fixed-Parameter Approximation Scheme II: Idea

Brief idea of the algorithm:

Step 1: Spent most of the budget optimally in $O^*(f(m, \varepsilon))$ time:
(exact part of the algorithm)

Step 2: Spent the remaining budget uniformly.
(approximation part of the algorithm)

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Brief idea of the algorithm:

Step 1: Spent most of the budget optimally in $O^*(f(m, \varepsilon))$ time:
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- $B \leftarrow$ available budget
- Repeat several times:

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- Repeat several times:
  - Guess the voter block where $B/(m \cdot m!)$ budget is spent.
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  - Guess how far p is shifted in this block with $B/(m \cdot m!)$ budget.
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  - $B \leftarrow B - B/(m \cdot m!)$

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Theorem

There is a factor-$(1 + 2\varepsilon + \varepsilon^2)$ approximation algorithm solving $\mathcal{R}$ **Shift Bribery** for sortable prices in time $O^*(M^{M \cdot \lceil \ln(M/\varepsilon) \rceil + 1})$ (where $M = m \cdot m!$) for $\mathcal{R} \in \{$Copeland, Borda$\}$.

Fixed-Parameter Approximation Scheme II: Remarks

- Algorithm relies on two observations:
  - At most $m!$ different preference orders (voter blocks).
  - Prices induce sorting on voters in one voter block.
- FPT algorithm with respect to parameter m uses the same two observations to construct a mixed integer programming.

Fixed-Parameter Approximation Scheme II: Remarks

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  - At most $m!$ different preference orders (voter blocks).
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Open questions:

- What is the parameterized complexity in the unrestricted price functions?
  (Known: **Shift Bribery** is in XP wrt. m)
- Is there a fixed-parameter approximation scheme for unrestricted price functions?

Conclusion

Mostly only polynomial-time approximation algorithms or only fixed-parameter algorithms for voting problems.

- $\rightsquigarrow$ Just few fixed-parameter approximation results for voting problems.
- $\rightsquigarrow$ Search for fixed-parameter approximation algorithms in social choice!

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Thank you!