Statistical Analysis of Image Reconstructed Fully-Sampled and Sub-Sampled fMRI Data

Daniel B. Rowe
Program in Computational Sciences
Department of Mathematics, Statistics, and Computer Science
Marquette University

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Outline

Introduction

Varied methods to produce fMRI images with varied properties.

3×

Reconstruction

Voxels are not directly measured (k-space). Reconstructed!

Processing

Images are processed for enhancement & artifact reduction.

Implications

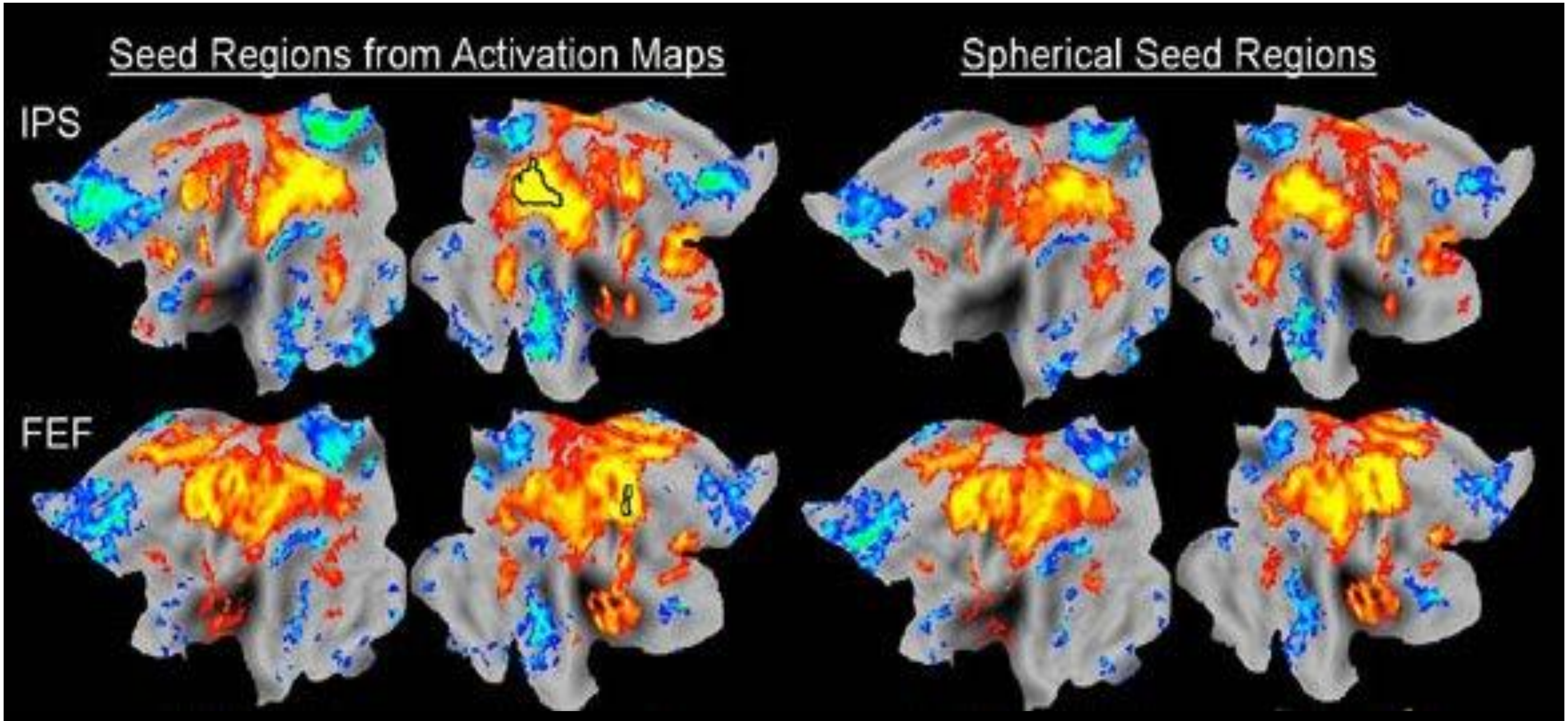
Effects of image reconstruction & processing? Mean, Var, Corr?

Discussion

How was our data was produced and what was done to it?

Introduction

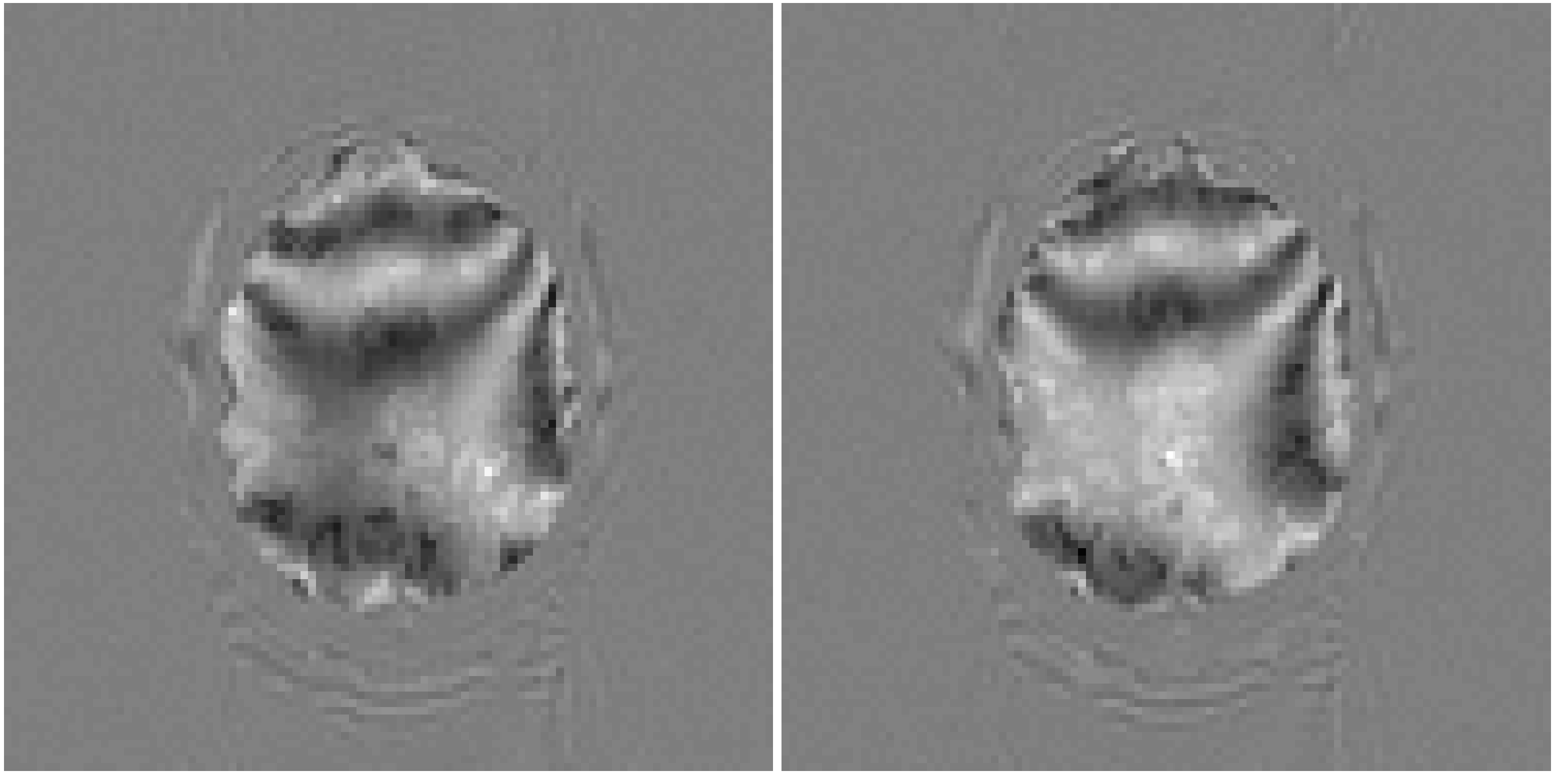
In fMRI and fcMRI, there has been an amazing amount of advanced analysis and interpretations presented, but little attention has been paid to what the data truly are.



Fox et al, PNAS, 2006.

Introduction

In general, reconstructed GRE EPI images look like below.
How do we get from the below to the previous activation?
And the below isn't even our original measurements.



96×96
240mm FOV
2.5 mm^2
In-Plane

Are we ahead of the data with our analyses and interpretations?

Reconstruction

In fMRI and MRI, the measurements taken by the machine are an array of complex-valued spatial frequencies.

This array of complex-valued spatial frequencies need to be reconstructed into an image for us to see, analyze, and interpret.

The array of complex-valued spatial frequencies are reconstructed into an image via the inverse Fourier transform.

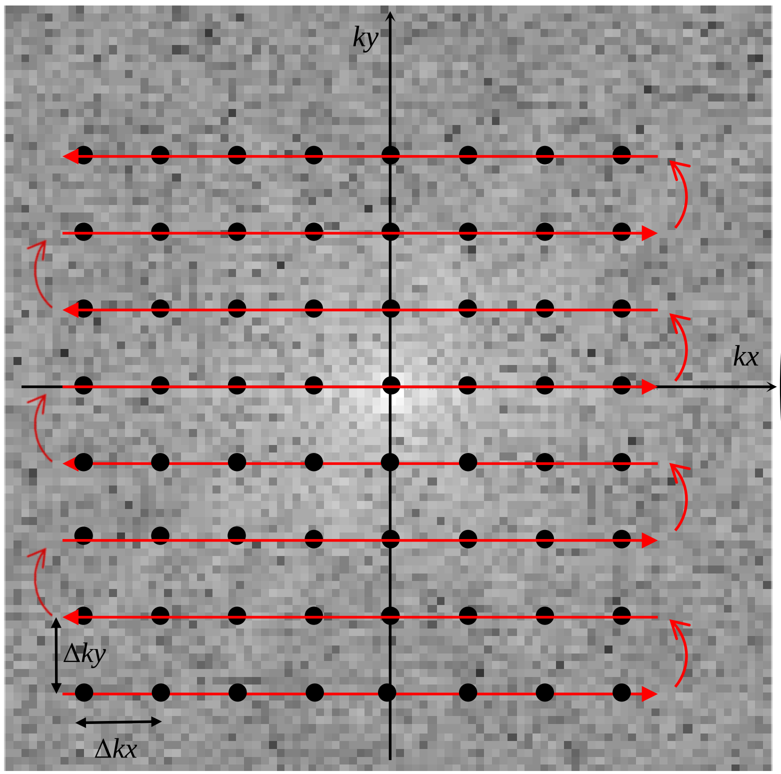
So lets briefly remind ourselves what the FT and IFT are.

Coil wraps around so uniform sensitivity.

Reconstruction

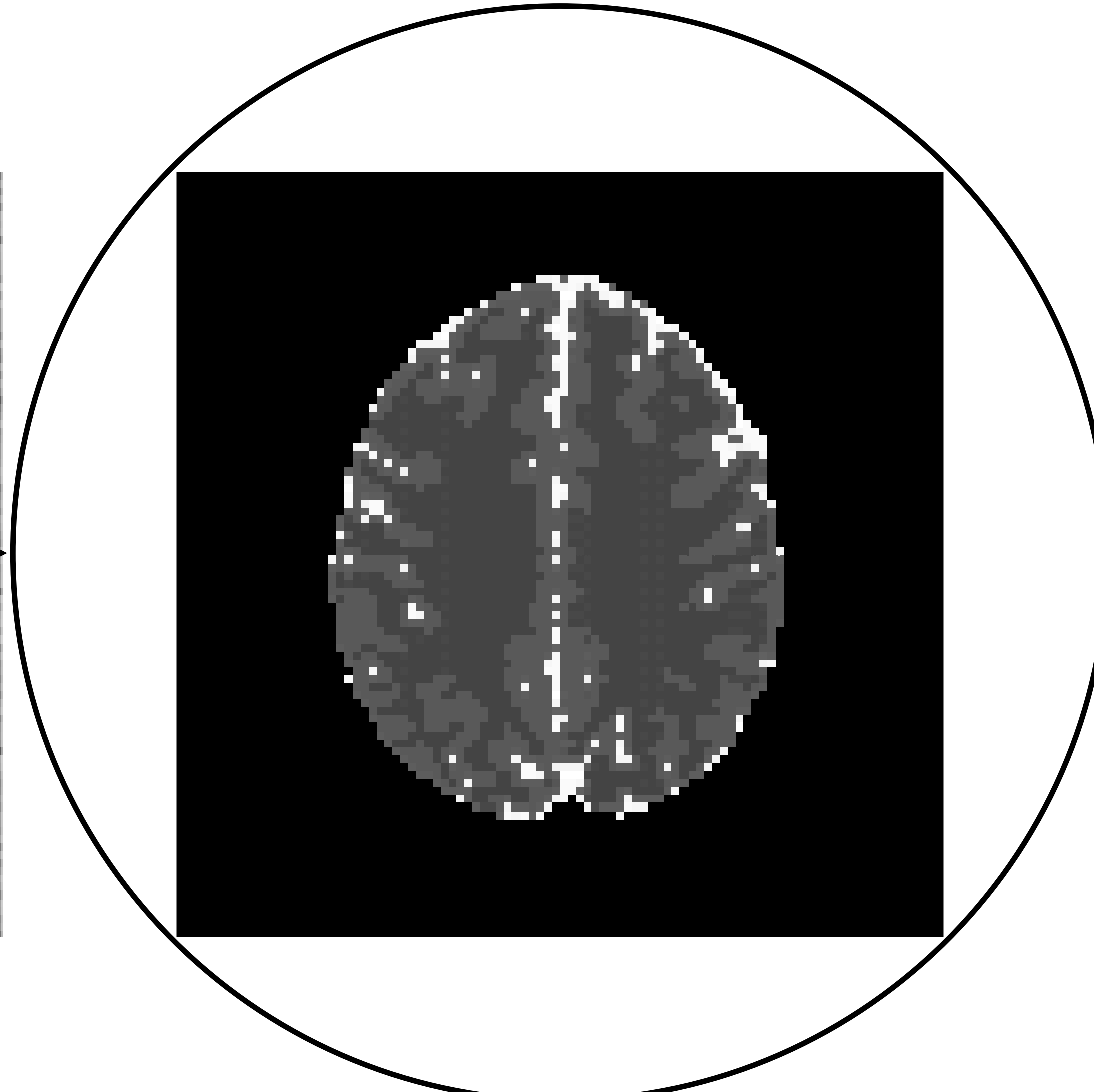
Single Coil Acquisition

Coil measures k-space.



Unaccelerated Acquisition (A=1).

Coil



Kumar et al: JMR, 18(1):69-83,1975.

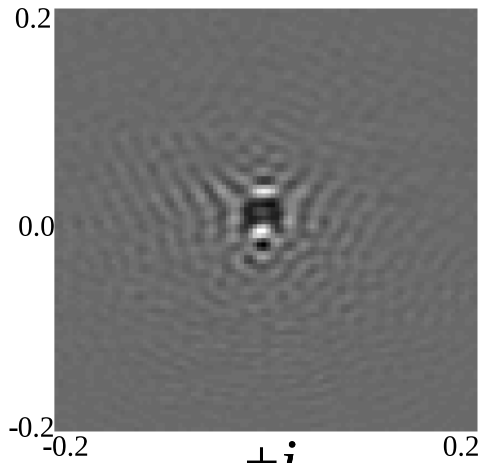
(FOV=240 mm)

Reconstruction

(n_x=n_y=96, Δx=Δy=2.5 mm)

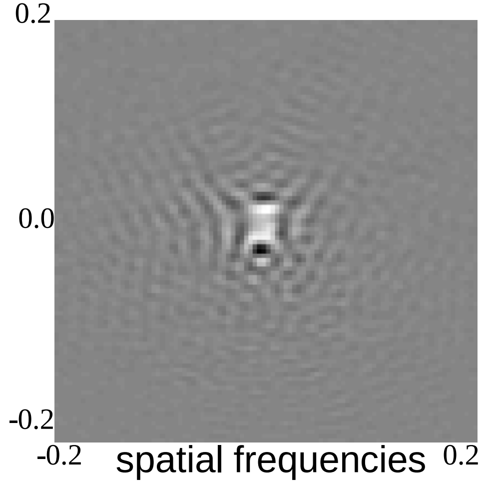
We inverse Fourier transform spatial freqs to generate image.

Real



$+i$

Imaginary



(FOV=240 mm)

Reconstruction

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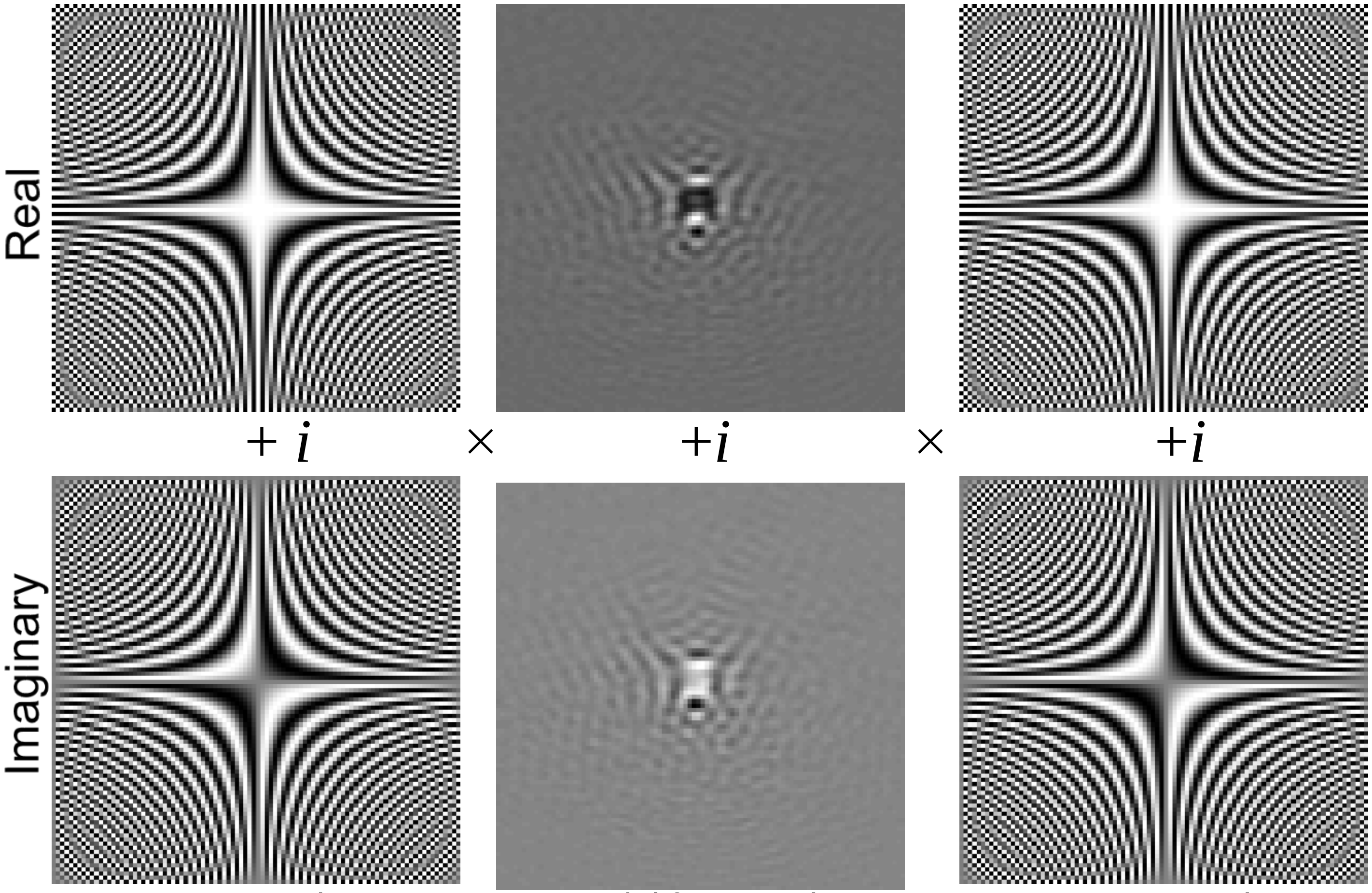

IFT matrix | spatial frequencies | IFT matrix

(FOV=240 mm)

($n_x = n_y = 96$, $\Delta x = \Delta y = 2.5$ mm)

Reconstruction

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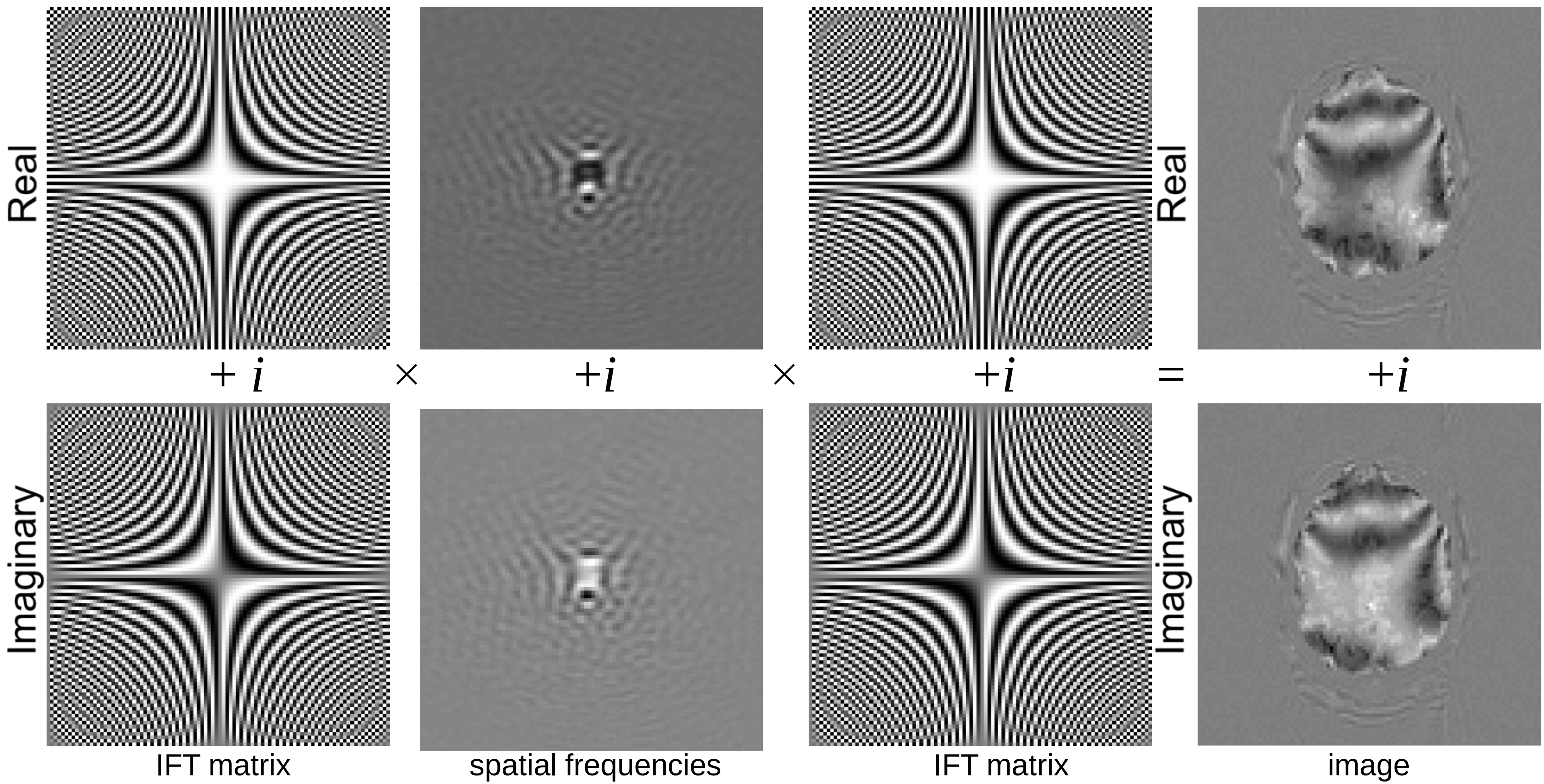

(FOV=240 mm)

($n_x=n_y=96$, $\Delta x=\Delta y=2.5$ mm)

Reconstruction

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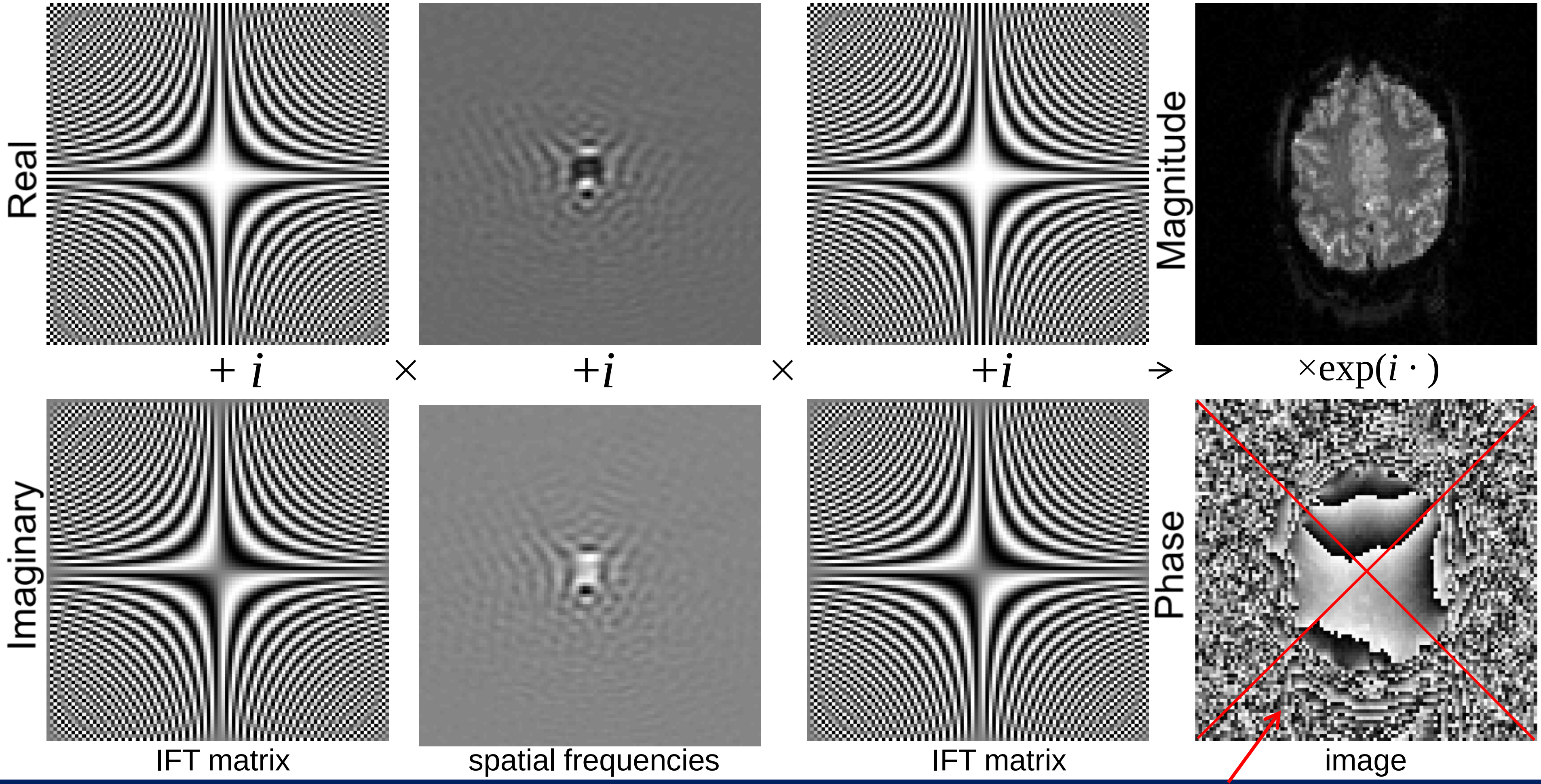
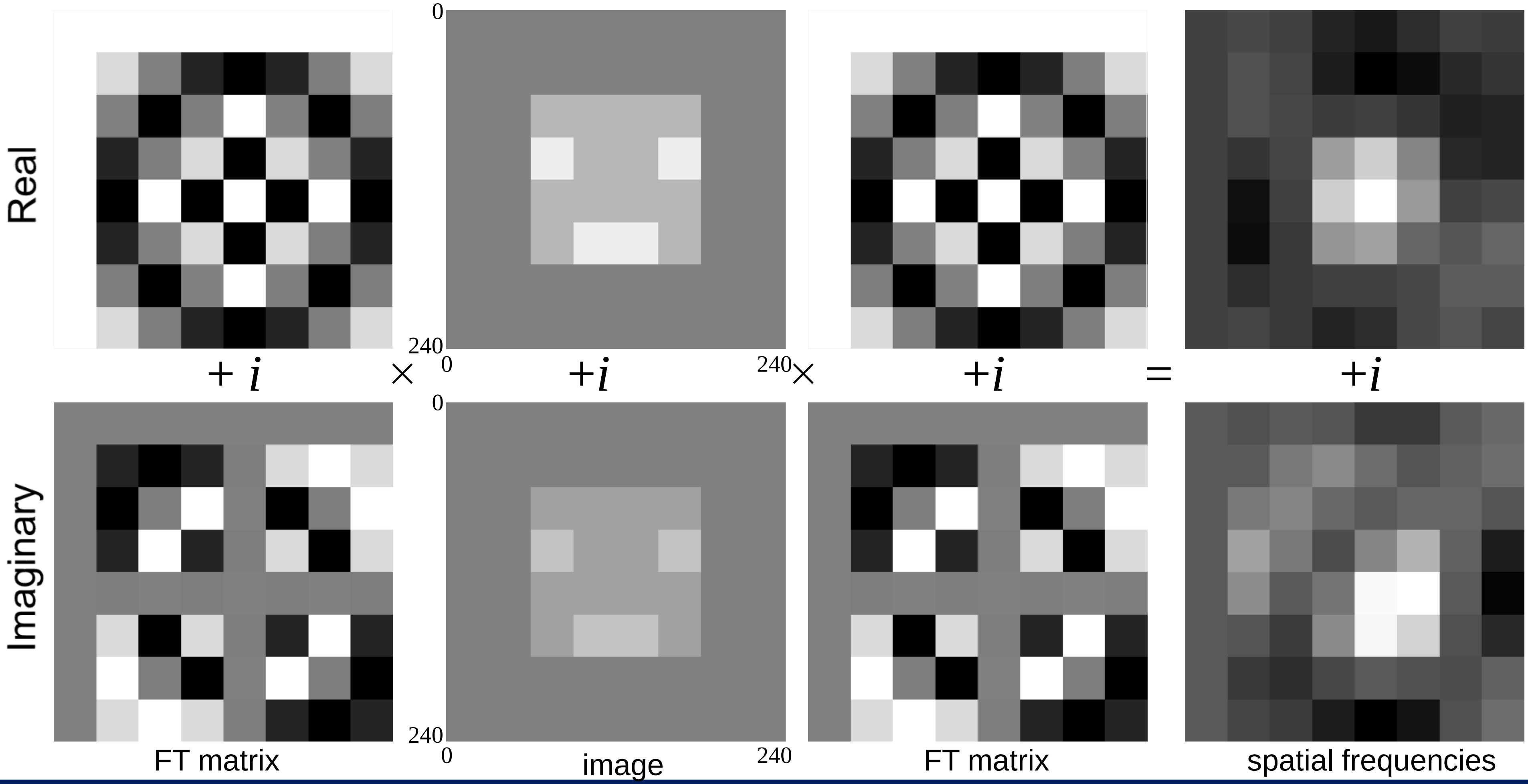


Phase discarded. Ask for the other ½ of YOUR data.

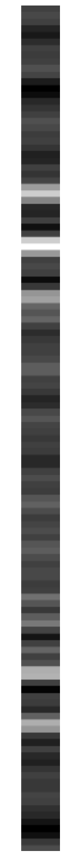
Reconstruction

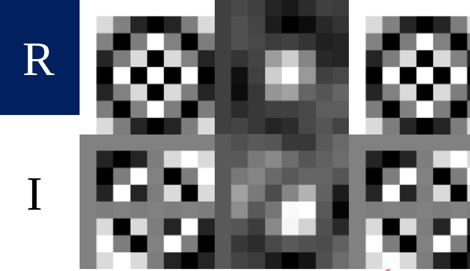
The machine Fourier encodes the image. Measure spatial freq.



Reconstruction

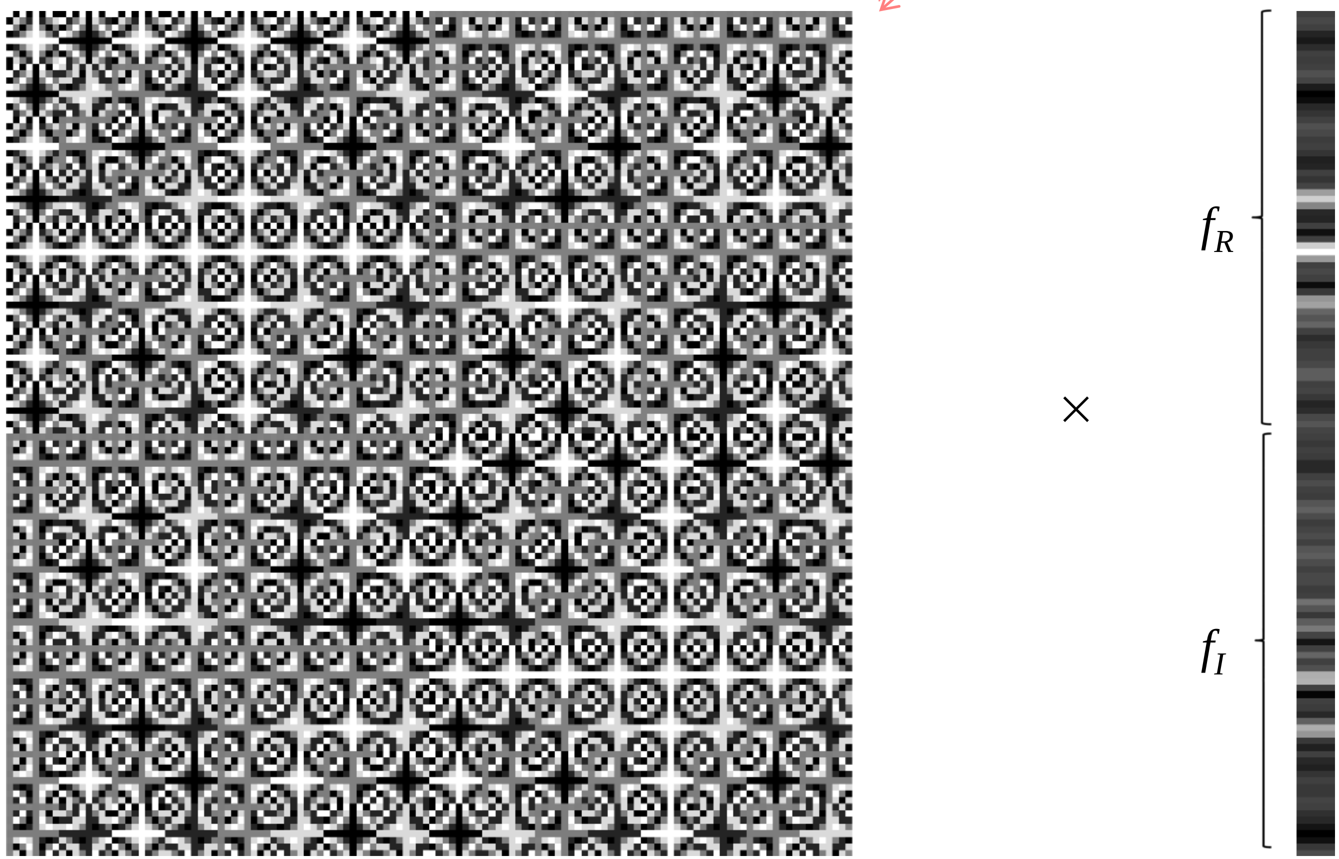
We can stack freq. rows of reals over rows of imaginaries,





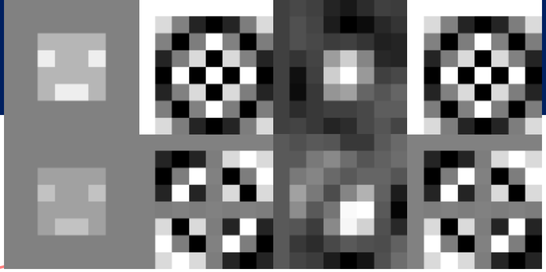
Reconstruction

We can stack freq. rows of reals over rows of imaginaries, make one IFT reconstruction matrix from the two,



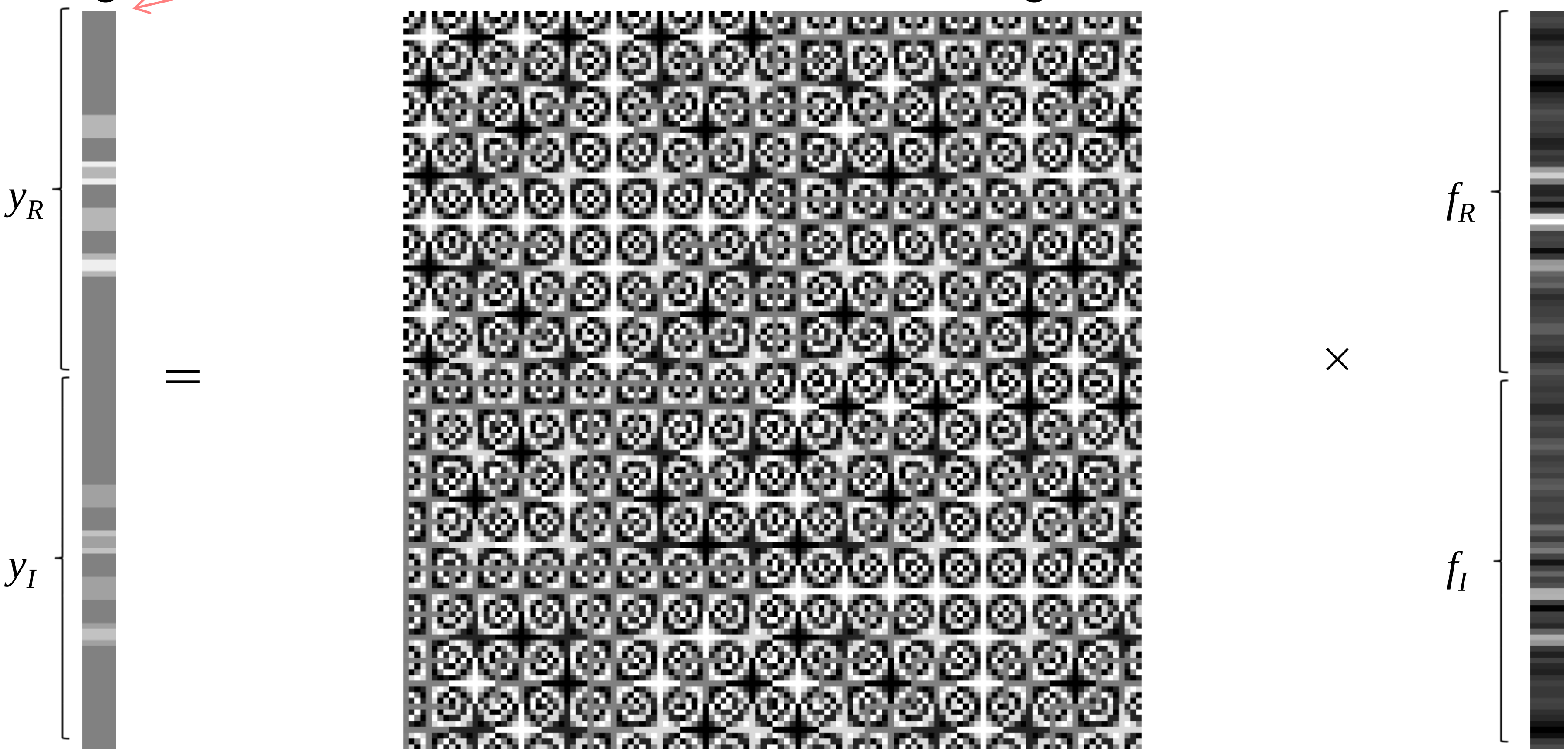
R

I



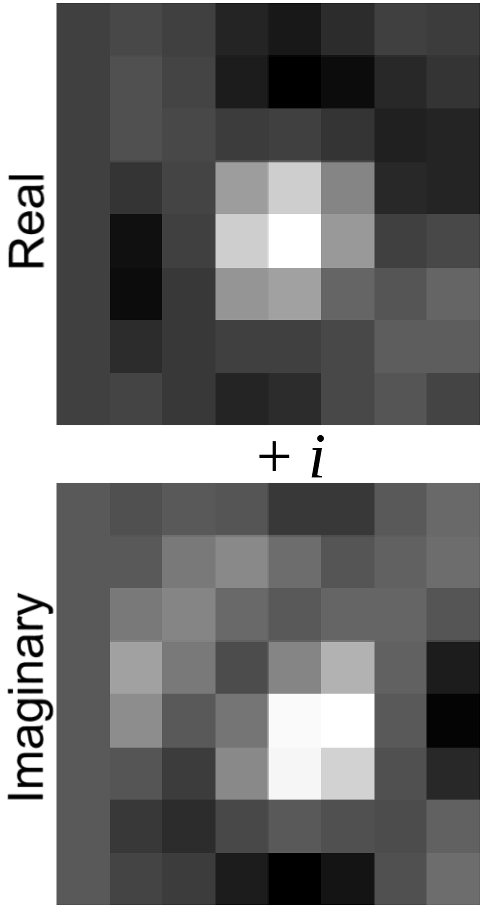
Reconstruction

We can stack freq. rows of reals over rows of imaginaries,
make one IFT reconstruction matrix from the two,
to get the rows of reals over rows of imaginaries.



Processing

Many processing operations are performed by the scanner, by physicists, and by engineers before statistical analysis.



<u>*k*-space Processing</u>

Nyquist Ghost Correction
Static B0 Field Correction
Zero Fill Interpolation
Non-Cartesian Interpolation
Ramp Sampling Interpolation
Homodyne Interpolation
Apodization
And many more…

<u>Image Reconstruction</u>

2D inverse Fourier transform
In-Plane SENSE/GRAPPA
Through-Plane SENSE

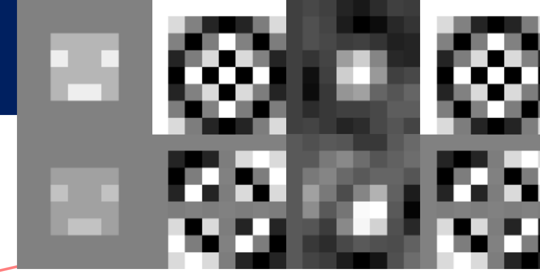
<u>Image Processing</u>

Image Smoothing
Global Normalization
Motion Correction
And many more…

<u>Time Series Processing</u>

Filtering
Smoothing
Dynamic B0 Correction
Slice Timing
And many more…

Show ones in blue.



Reconstruction

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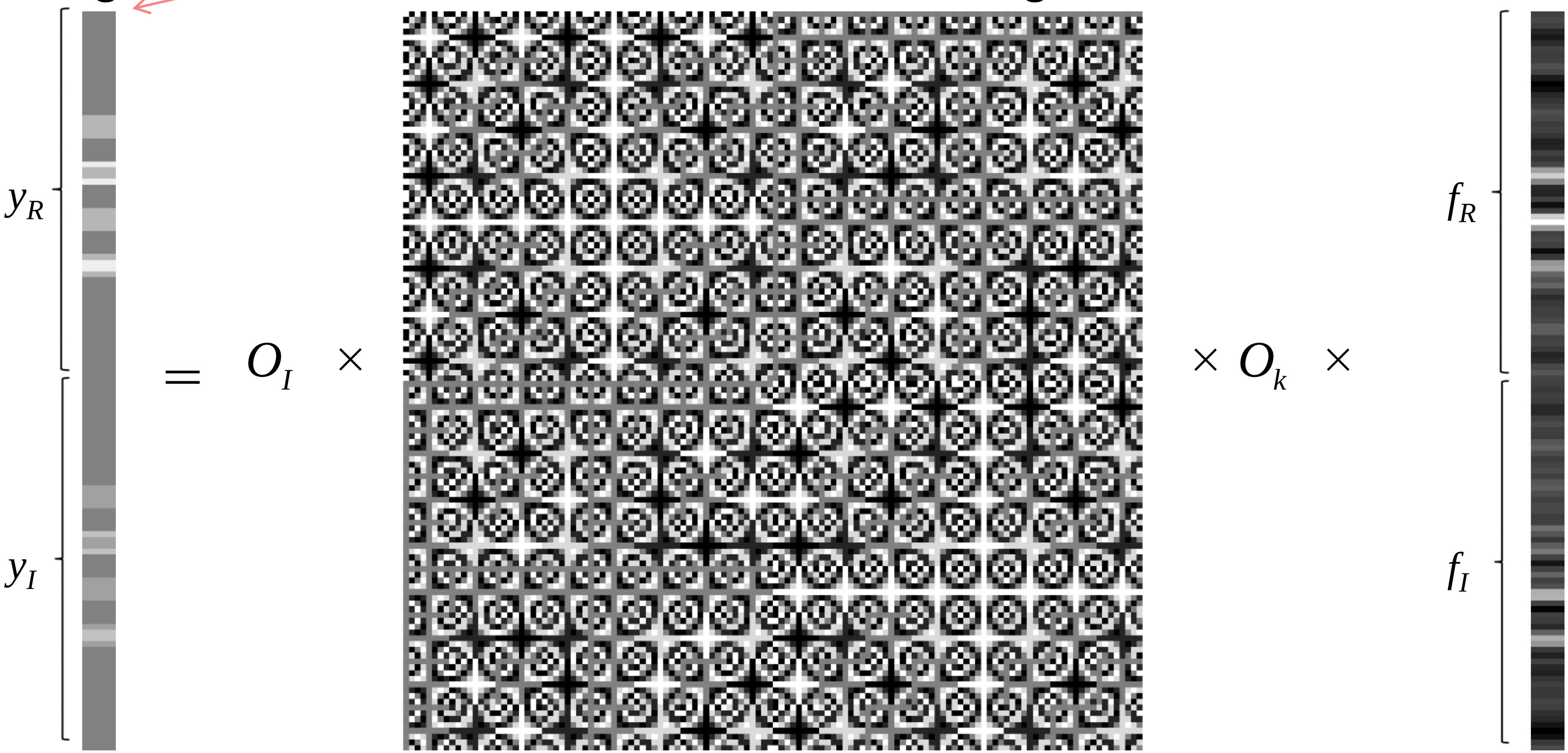
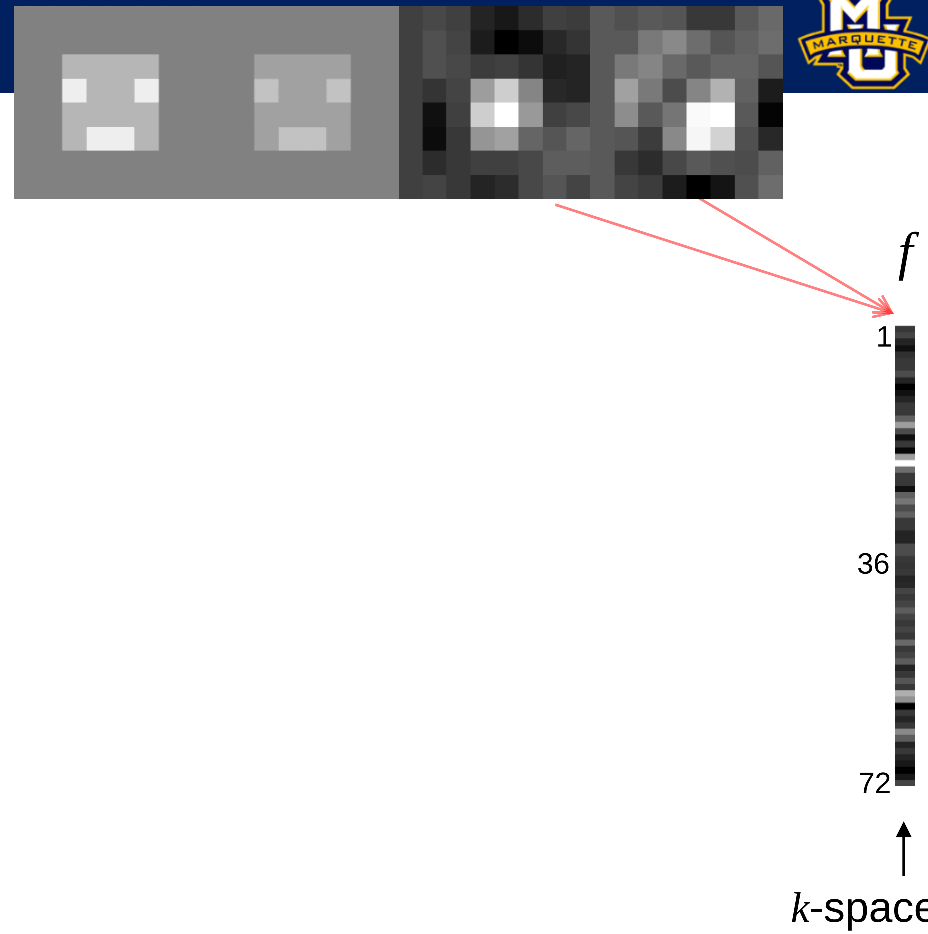
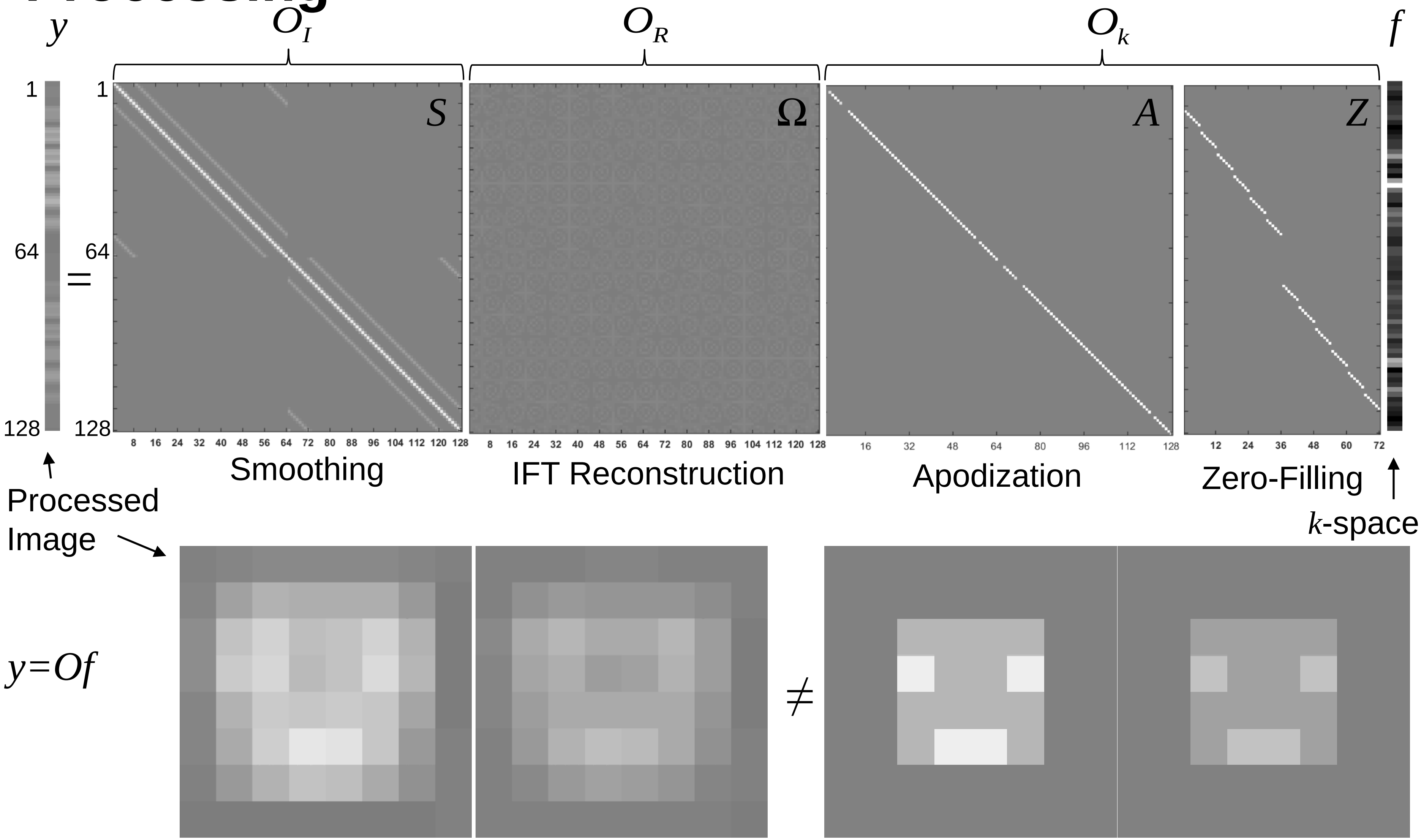
make one IFT reconstruction matrix from the two,
to get the rows of reals over rows of imaginaries.



Processing



Processing


y
O_I
O_R
O_k
f
1
64
128
=
S
Ω
A
Z
8 16 24 32 40 48 56 64 72 80 88 96 104 112 120 128
16 32 48 64 80 96 112 128
12 24 36 48 60 72
Smoothing
IFT Reconstruction
Apodization
Zero-Filling
Processed Image
k-space
y=Of
≠

Processing

We measure an array of complex-valued numbers, perform complex-valued image reconstruction to this array, to generate complex-valued images in real and imaginary, along the way, there is complex-valued image processing.

What are the implications of what was done to the data?

Implications

In statistics, we know the rule that says:

If a vector f has a mean δ, and a covariance Γ,

Then $y=Of$ has a mean $\mu=O\delta$, and a covariance $\Sigma=O\Gamma O^T$.

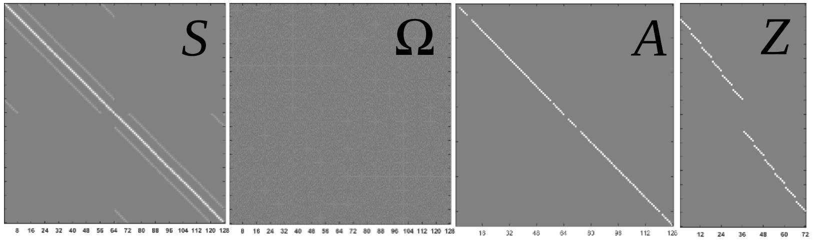
Then Σ can converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$.

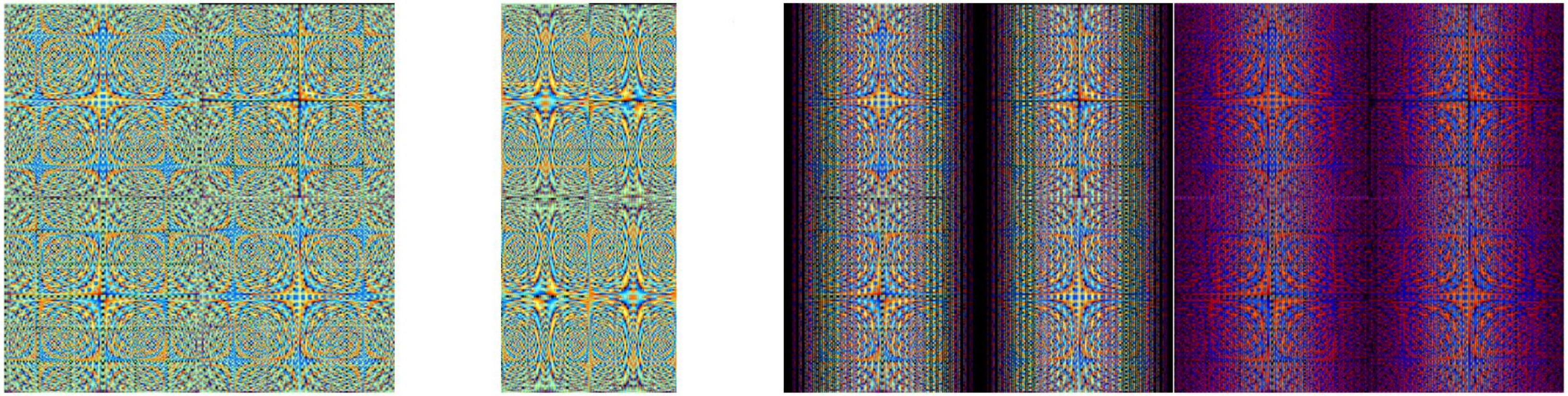
Where $D^{-1/2}=1/\sqrt{diag(\Sigma)}$.

Assume k-space measurements independent so $\Gamma=I$.

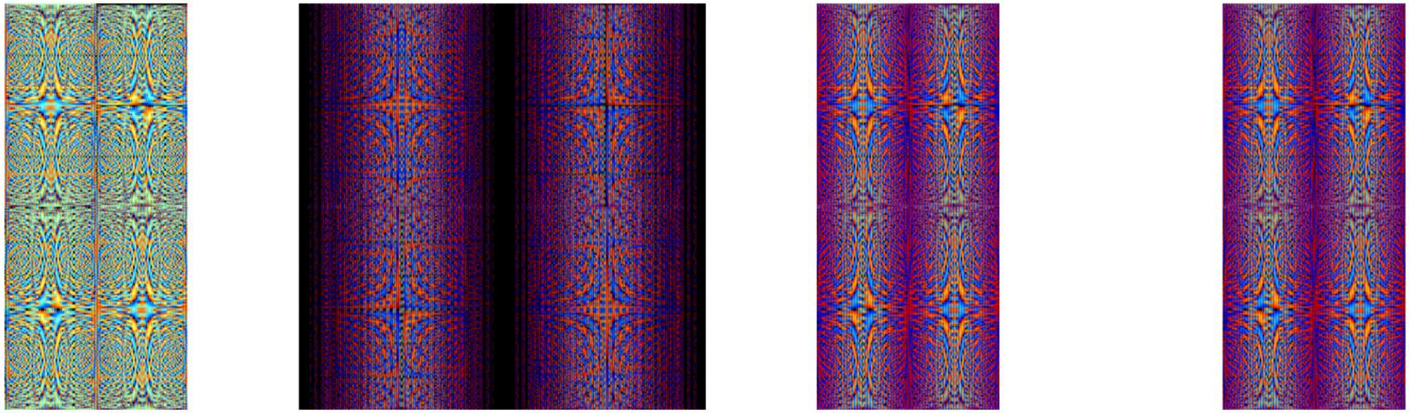
Implications

Operators, O.

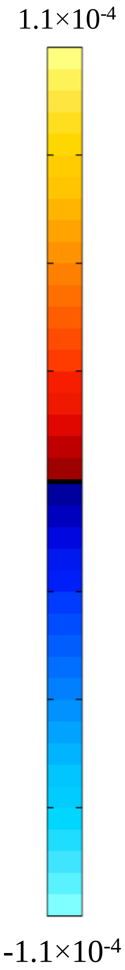




a) $O=\Omega$ b) $O=\Omega Z$ c) $O=\Omega A$ d) $O=S\Omega$



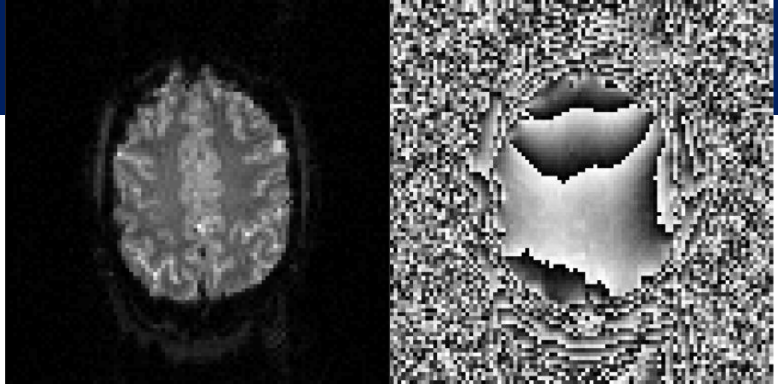
e) $O=\Omega AZ$ f) $O=S\Omega A$ g) $O=S\Omega Z$ h) $O=S\Omega AZ$



Implications

Mean, $\mu = Of$.

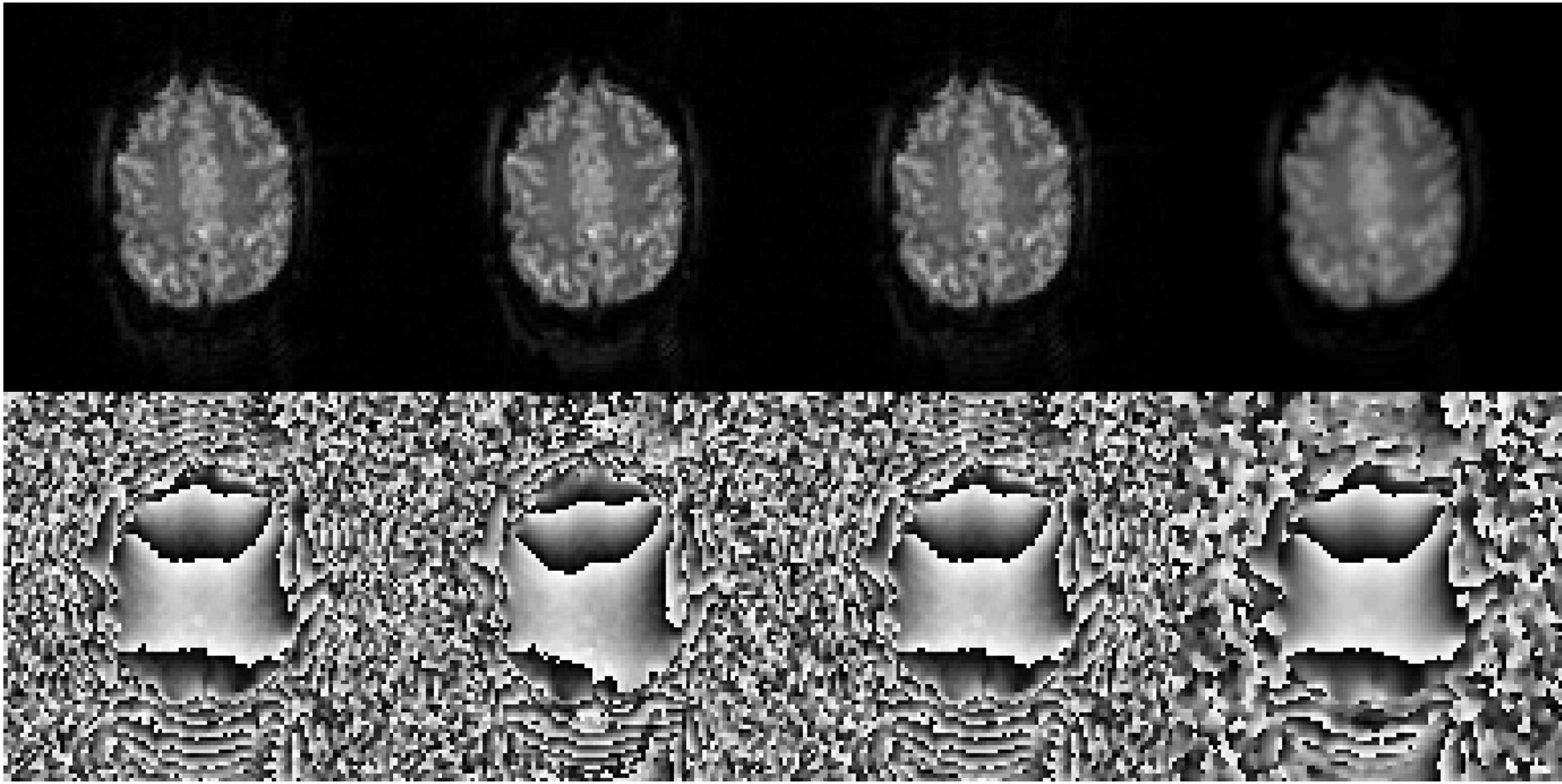
$O=\Omega$



$O=\Omega Z$ $O=\Omega A$ $O=\Omega AZ$ $O=S\Omega AZ$

M

P



Implications

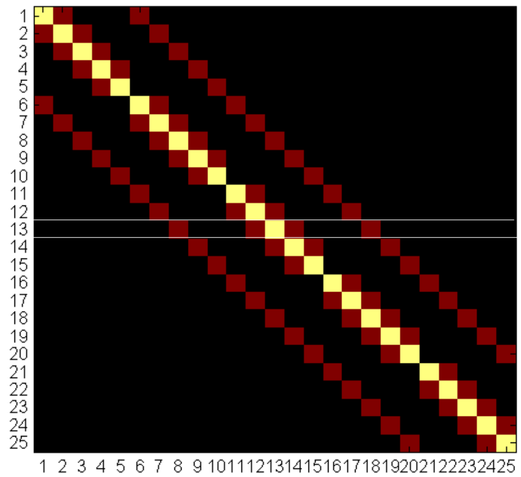
Correlation matrix and correlation image.

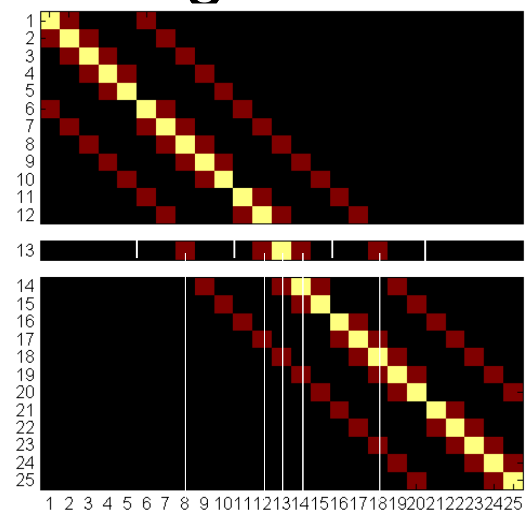
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

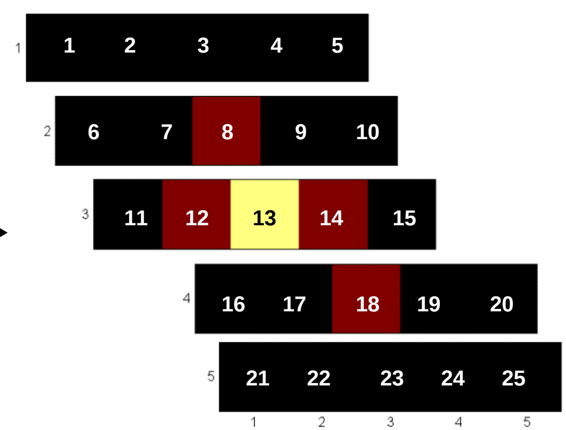
5×5 image

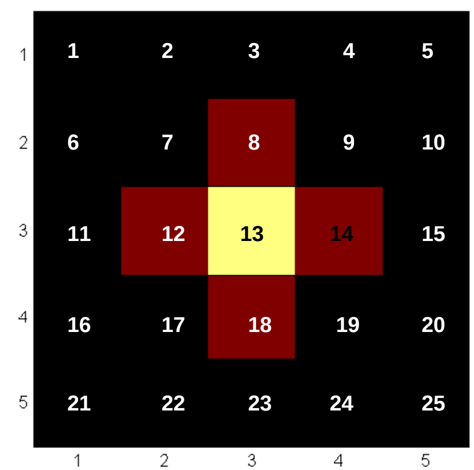
$\mathrm{cor}(y) =$

25×25 correlation matrix







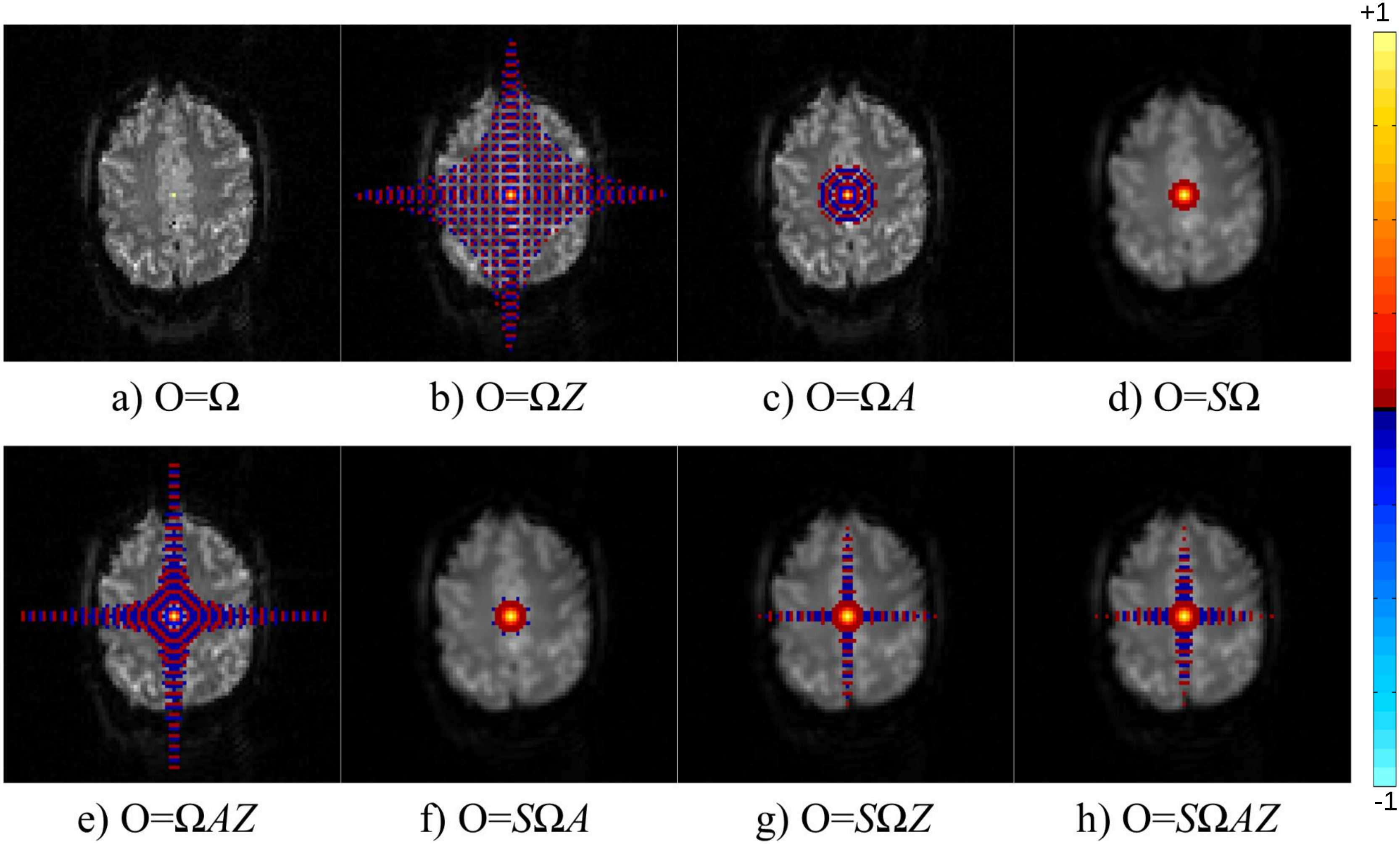


5×5 correlation image

Implications

Correlation, $R = D^{-1/2} \Sigma D^{-1/2}$.

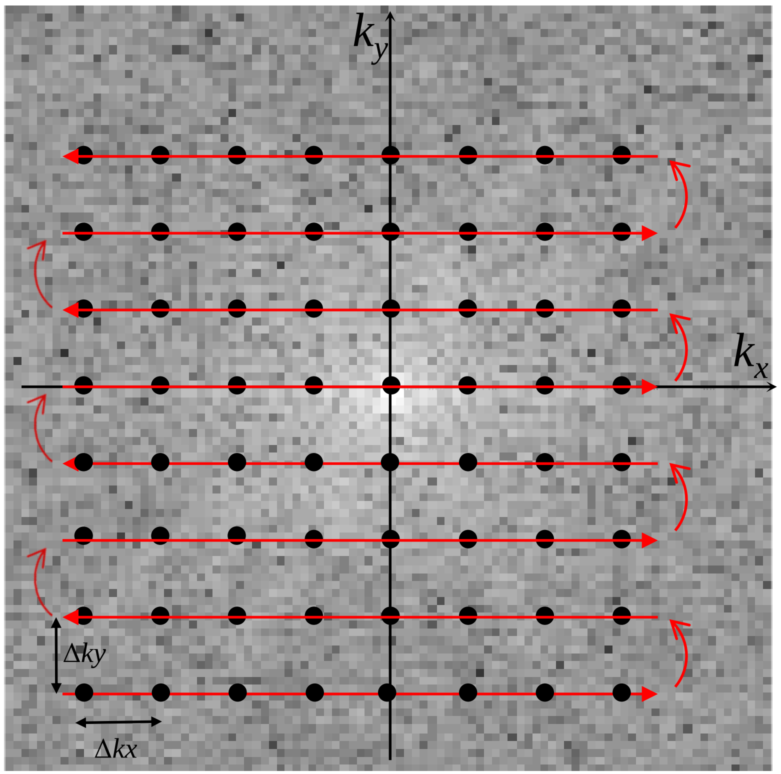
$$R = \begin{bmatrix} R_{RR} & R_{RI} \\ R_{IR} & R_{II} \end{bmatrix}$$

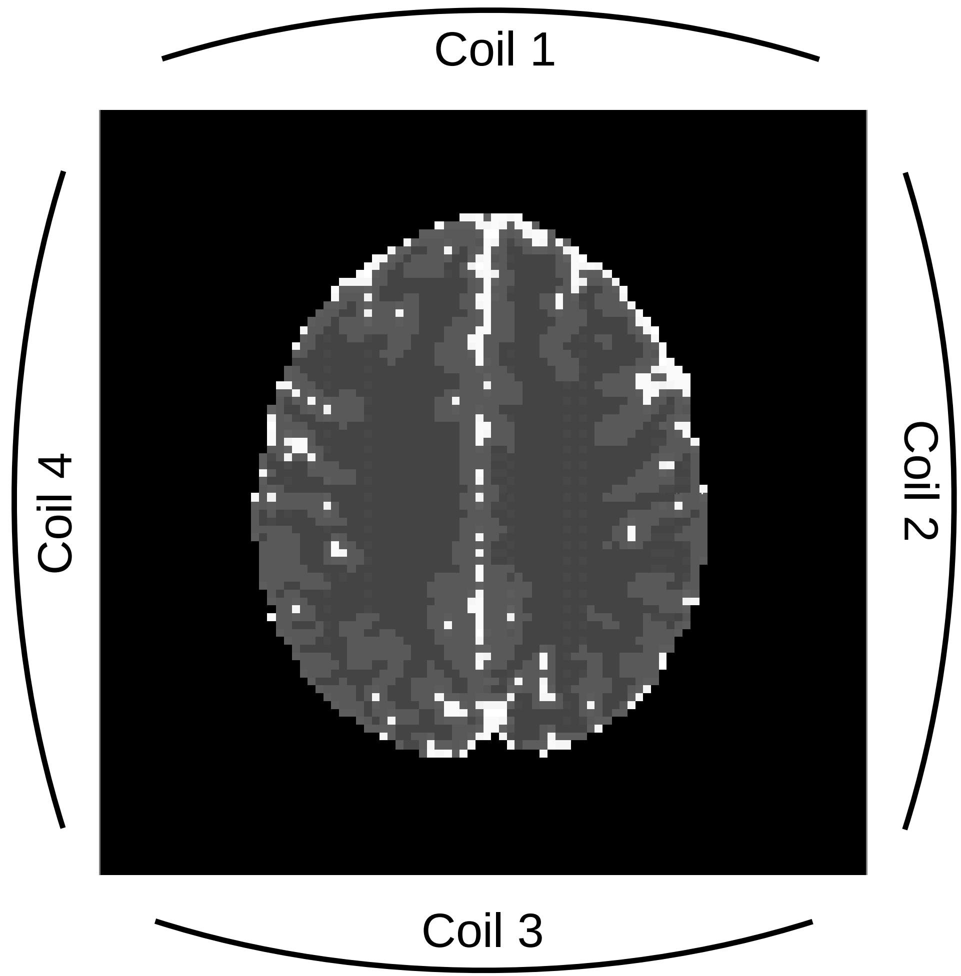


TH=.001

Reconstruction

Multi-Coil Acquisition





Each coil measures k-space.

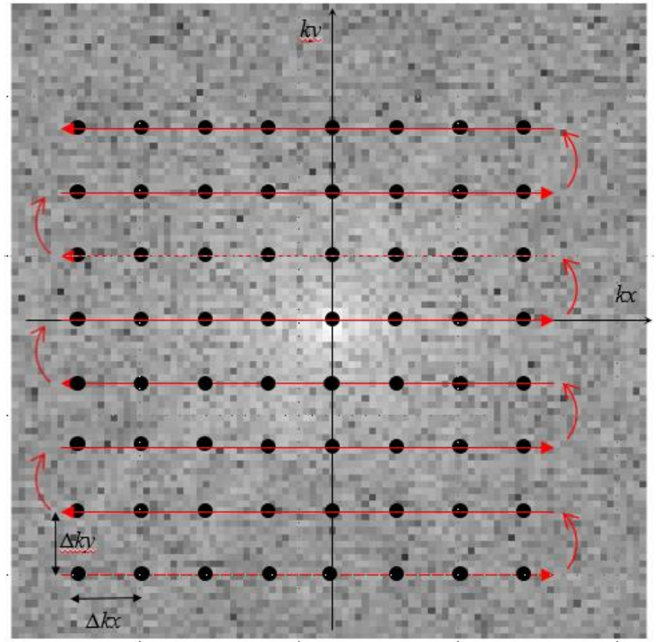
N_C=4, A=1

Hyde et al.: JMR, 70:512–517, 1986. Pruessmann et al.: Proc. ISRM, 579,1998.

Coil local so non uniform sensitivity.

Reconstruction

Multi-Coil Acquisition



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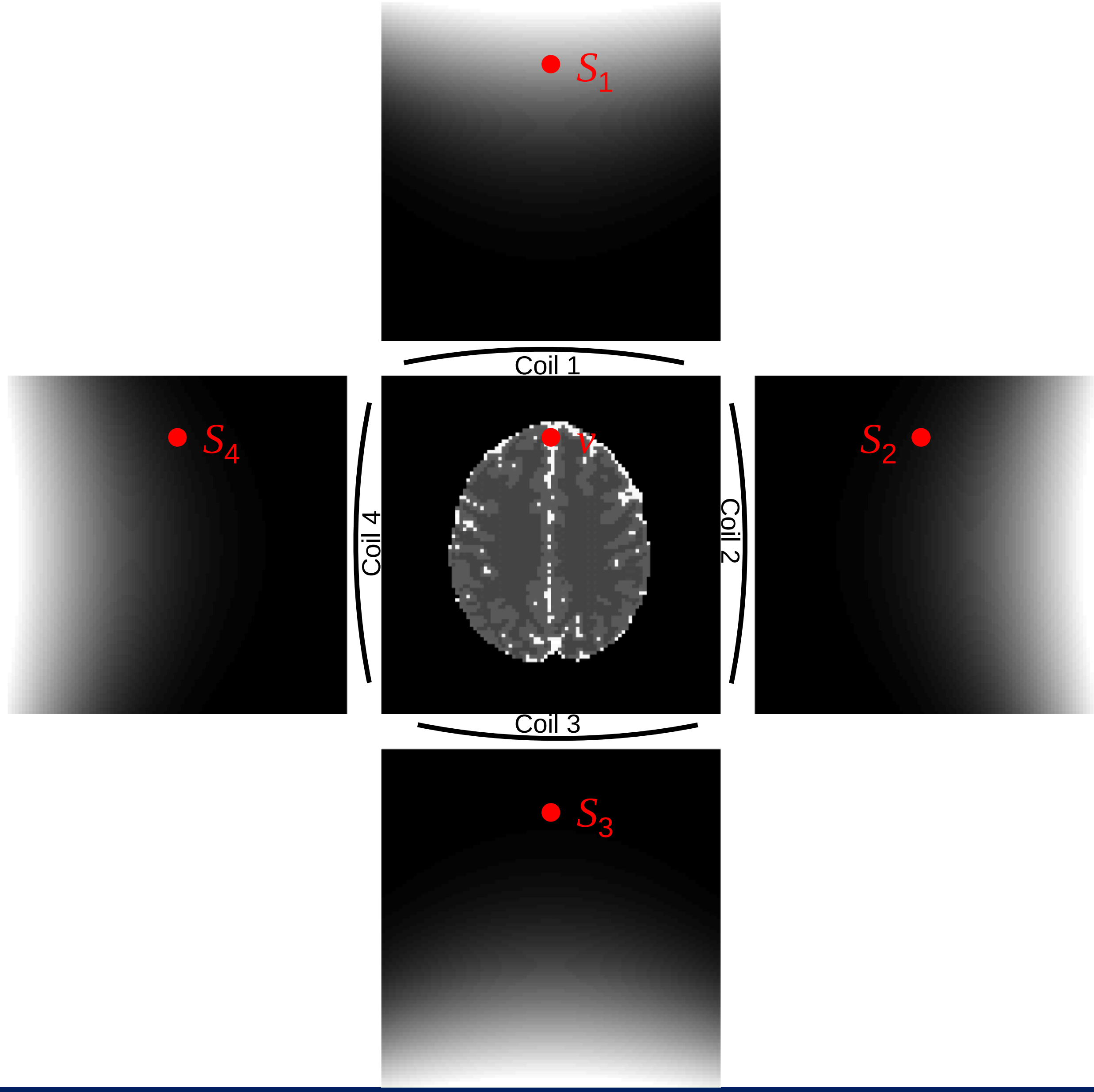

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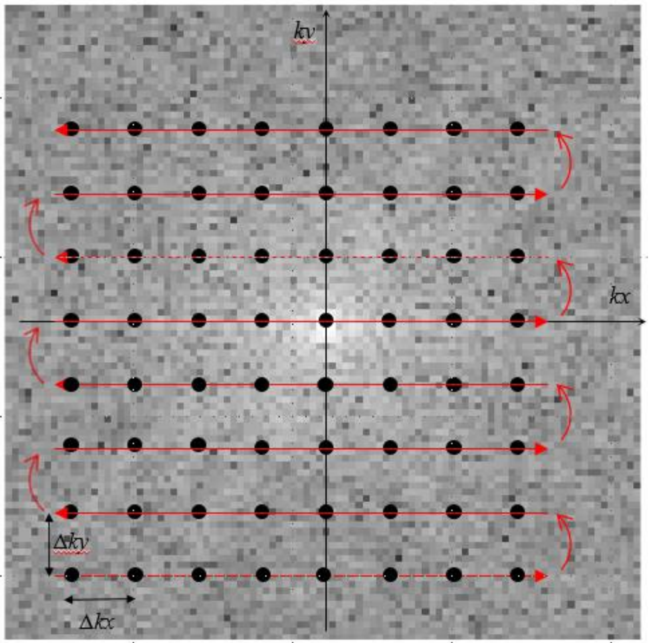
Reconstruction

Multi-Coil Acquisition



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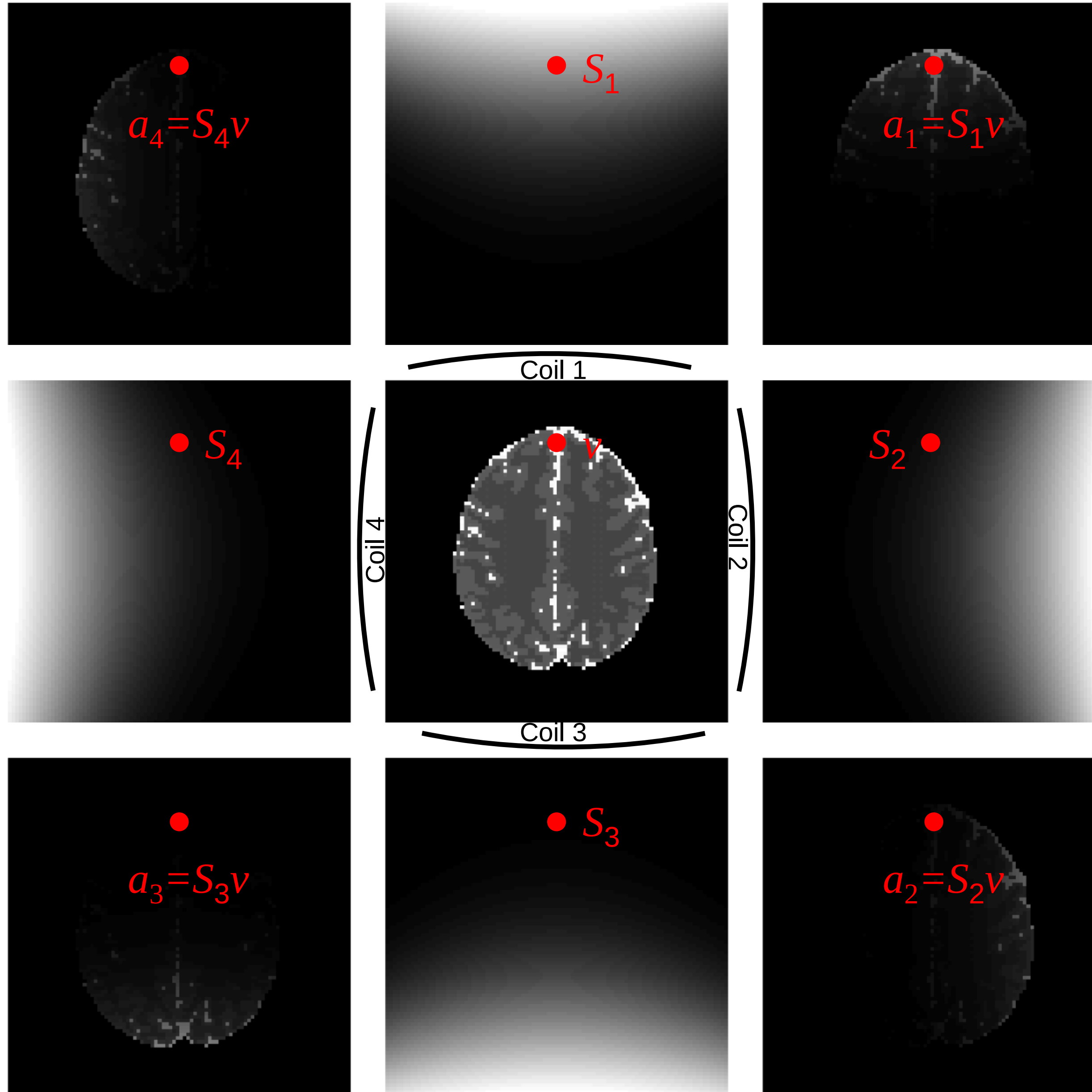
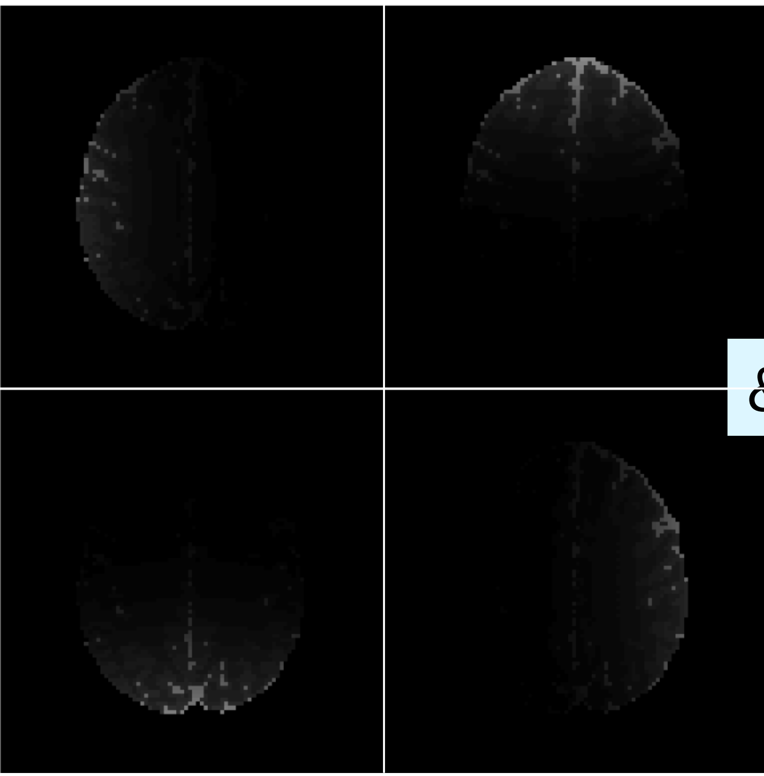


Reconstruction

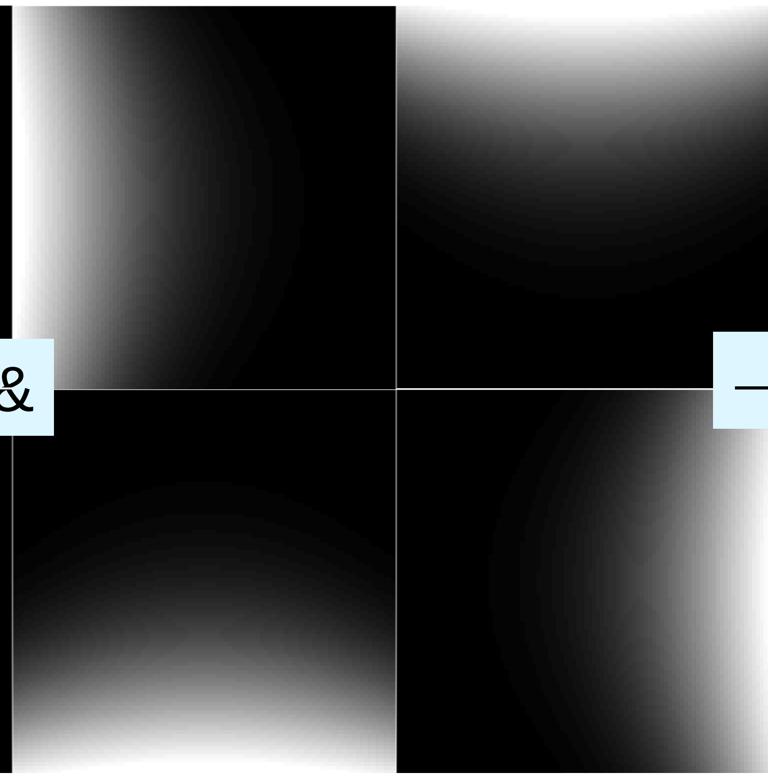
SENSE

Measured Coil Images



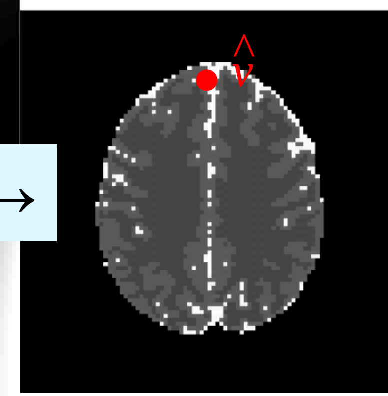
&

Estimated Sensitivities



→

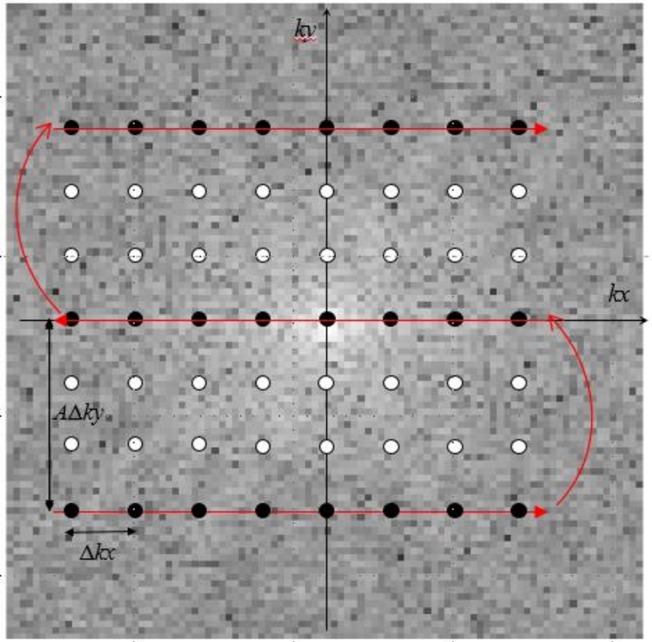
Combined Image



N_C=4, A=1

Reconstruction

Multi-Coil Acquisition



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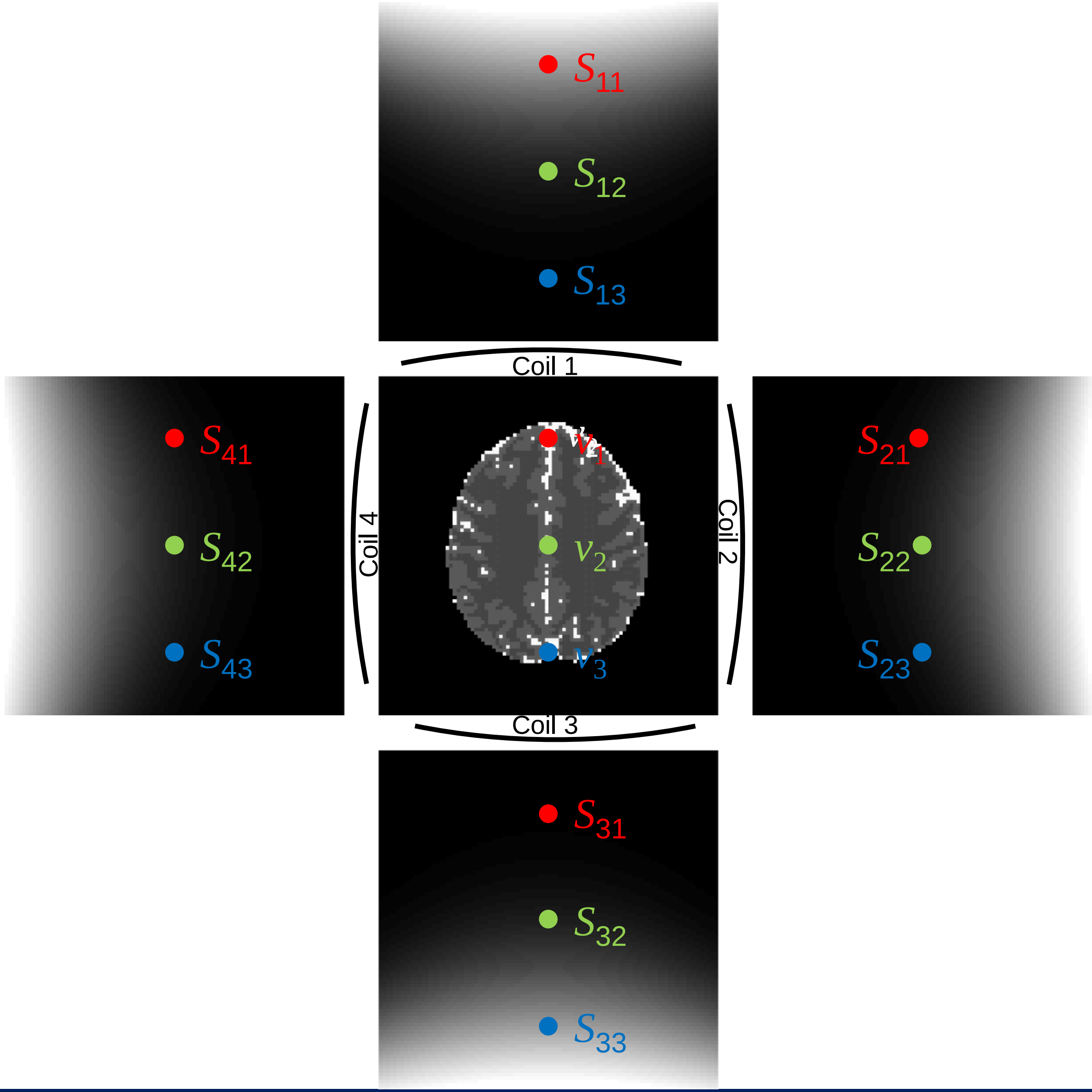
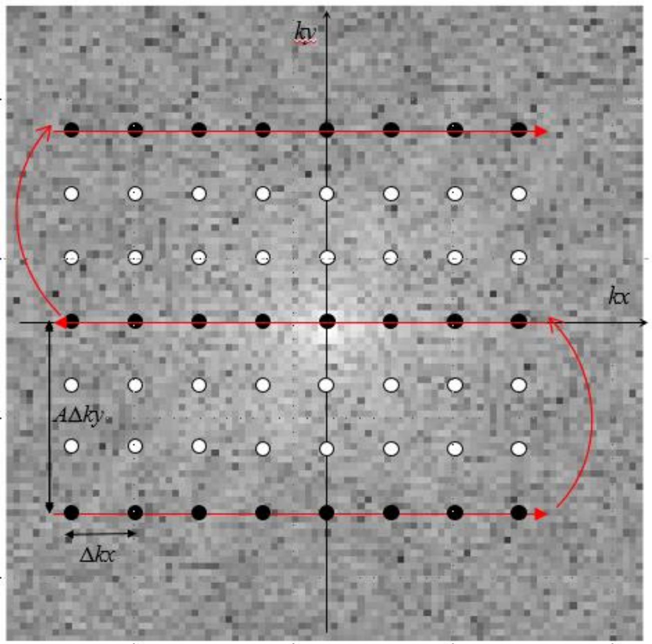


N_C=4, A=3

Coil local so non uniform sensitivity.

Reconstruction

Multi-Coil Acquisition



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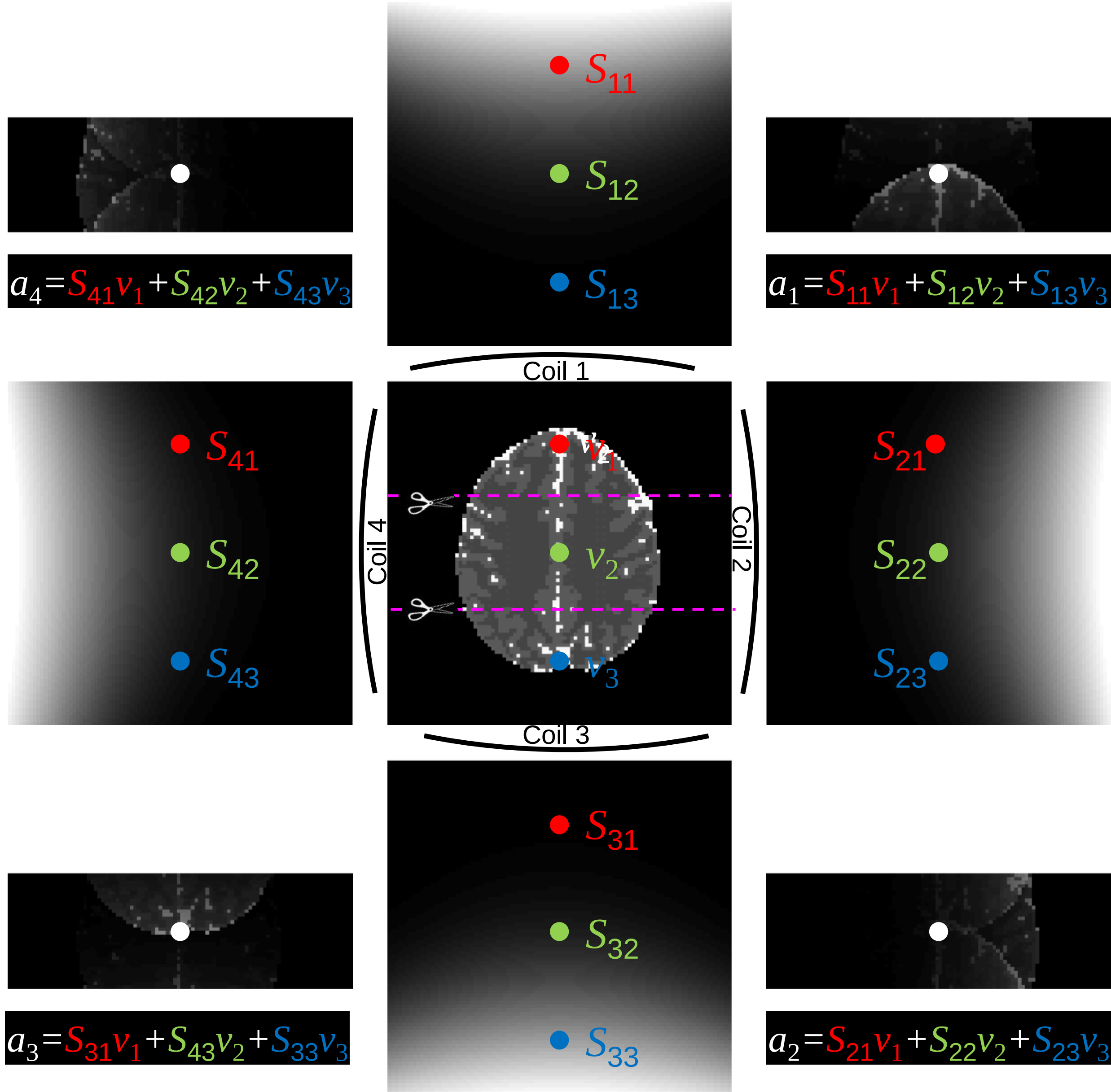
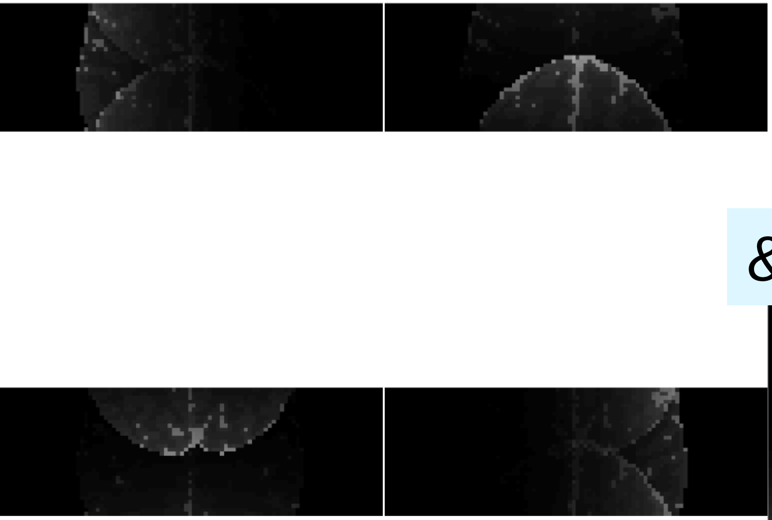
N_C=4, A=3



Reconstruction

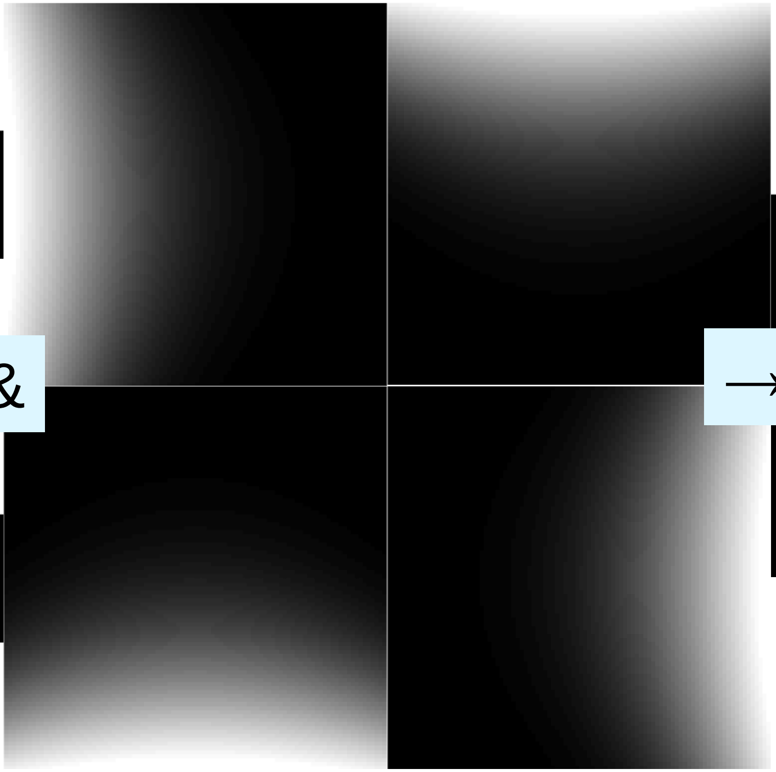
SENSE

Measured Coil Images



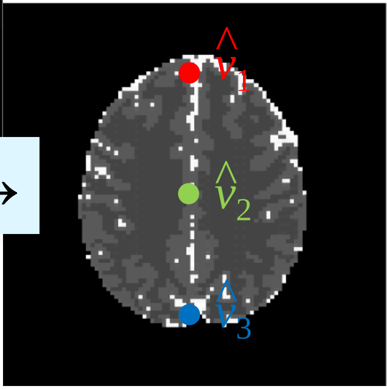
&

Estimated Sensitivities



→

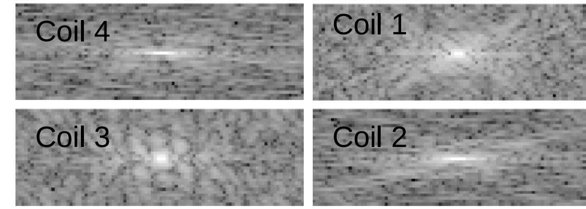
Separated Combined Image

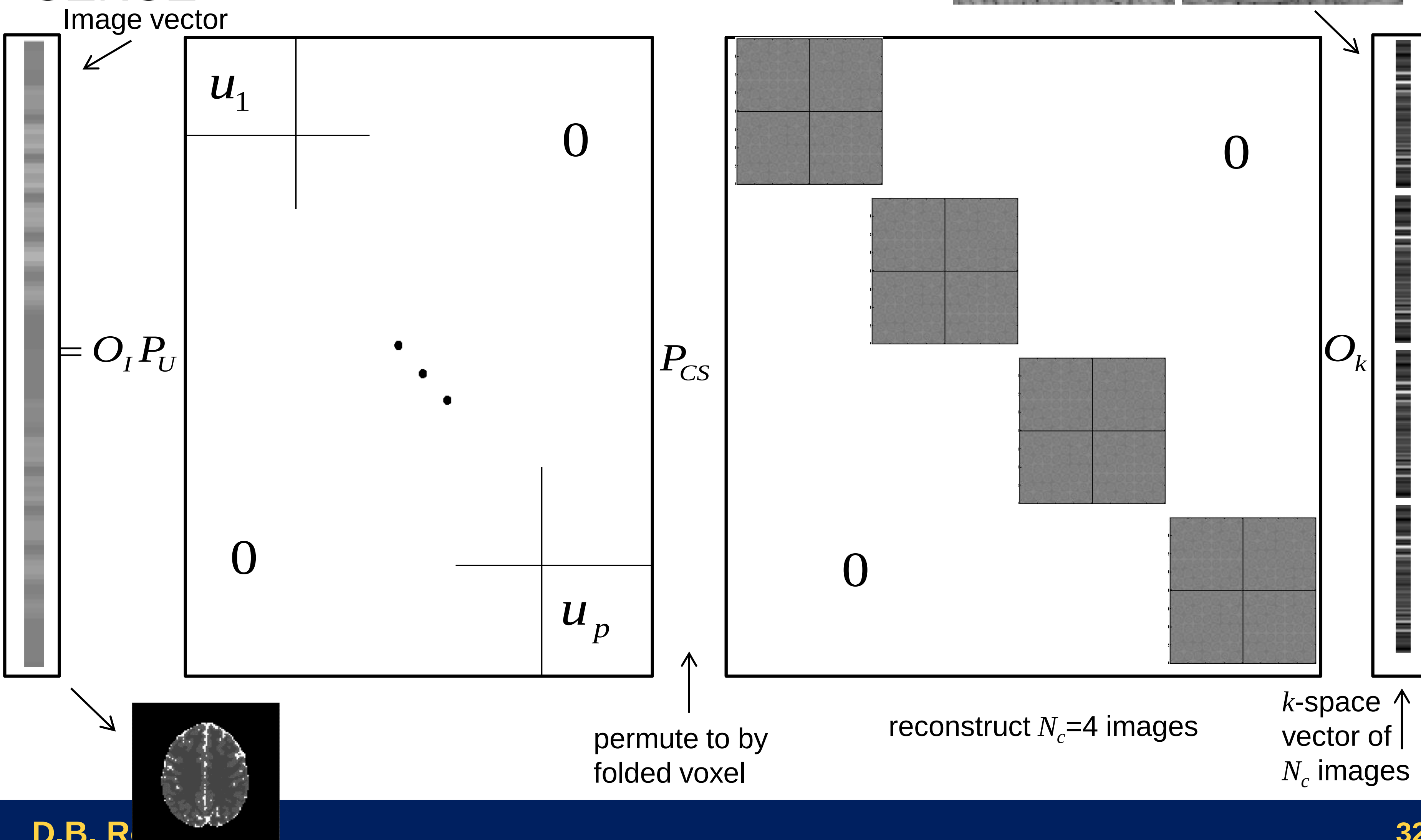


N_C=4, A=3

Reconstruction/Processing

SENSE





Implications

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Then Σ can converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$.

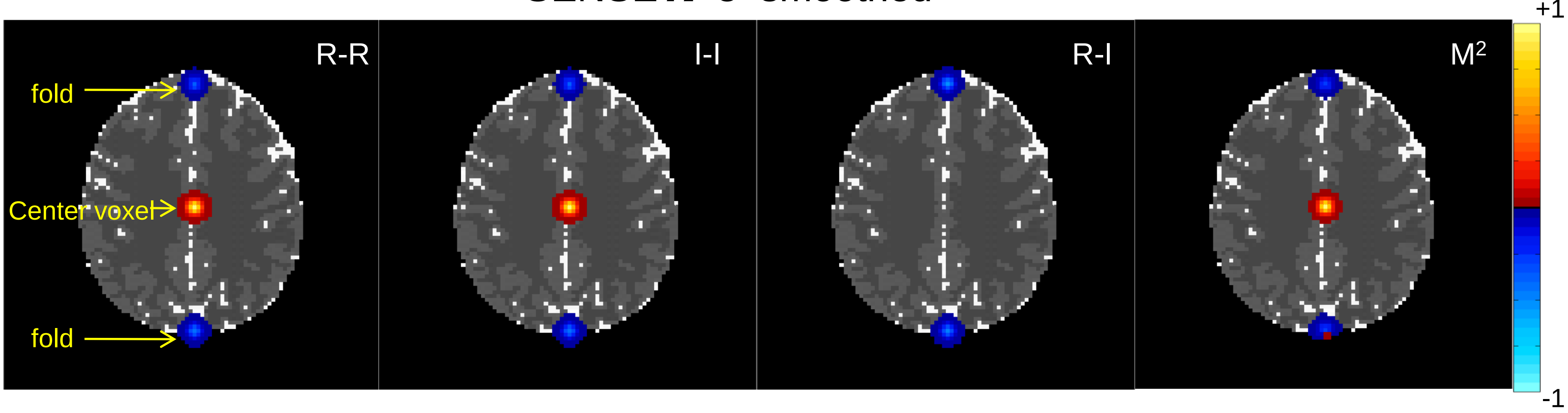
Where $D^{-1/2}=1/\sqrt{diag(\Sigma)}$.

Assume k-space measurements independent so $\Gamma=I$.

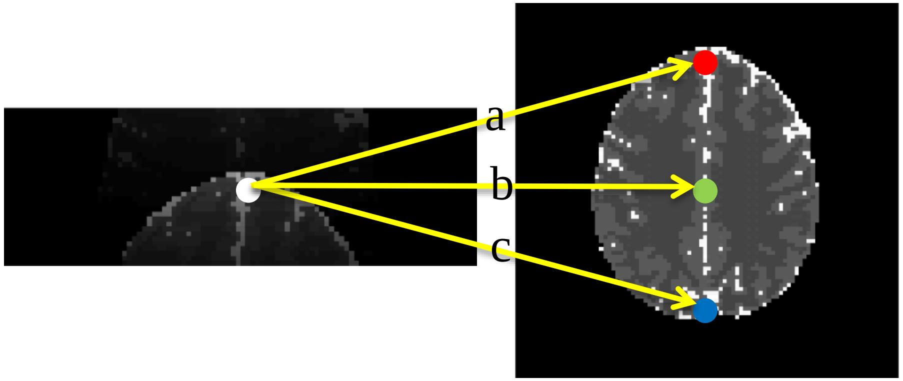
Implications

SENSE induces long-range in-plane correlation.

Theoretical Results SENSE A=3 smoothed



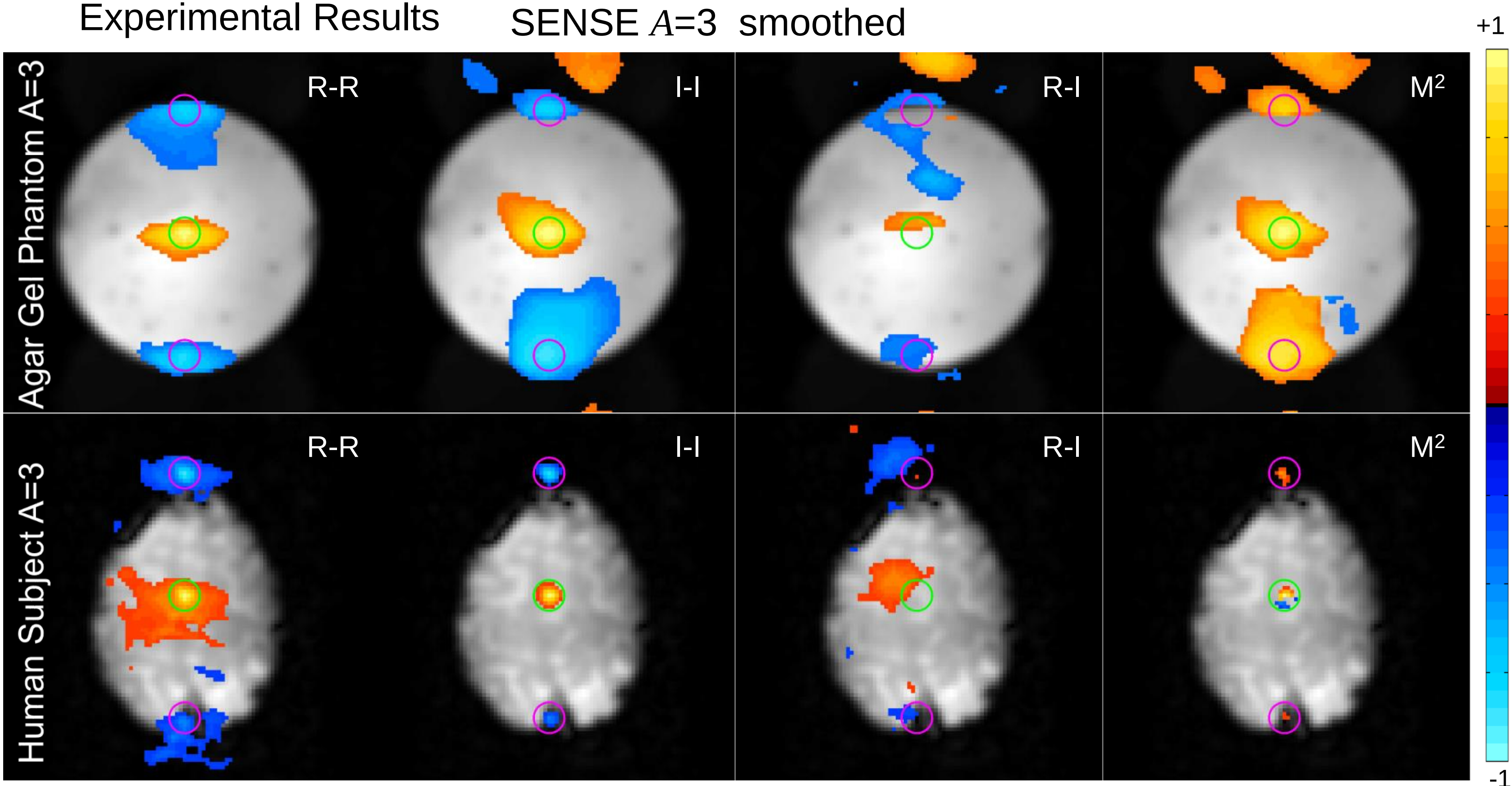
Basically multiplying voxel values a_t by same 3 numbers over time t to lead to correlated voxels.



Implications

SENSE Reconstruction induces long-range correlation.

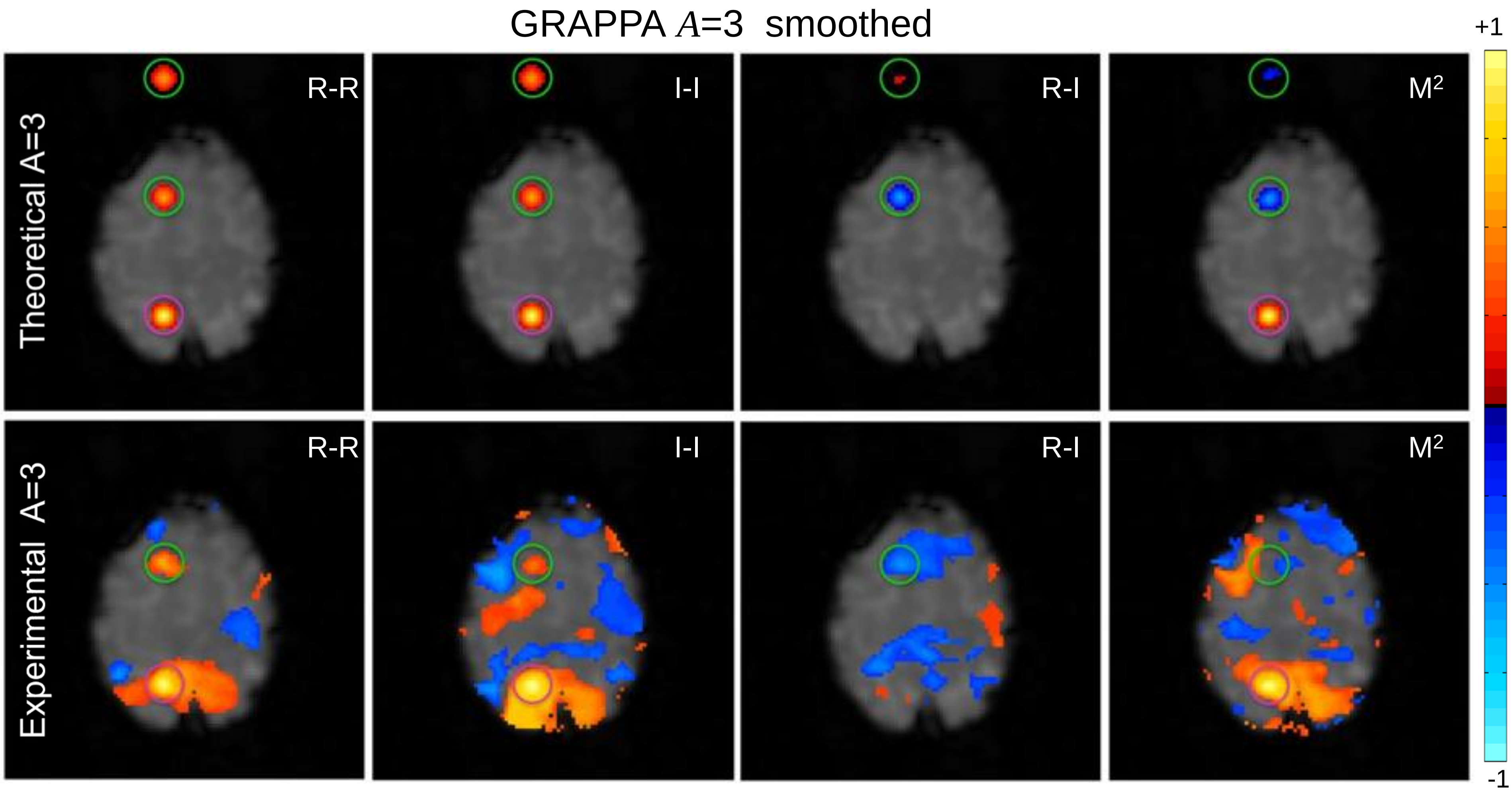
Experimental Results SENSE A=3 smoothed



Karaman,Nencka,Bruce,Rowe: Brain Connect, 4:649-661,2014.

Implications

GRAPPA reconstruction induces long-range correlation.

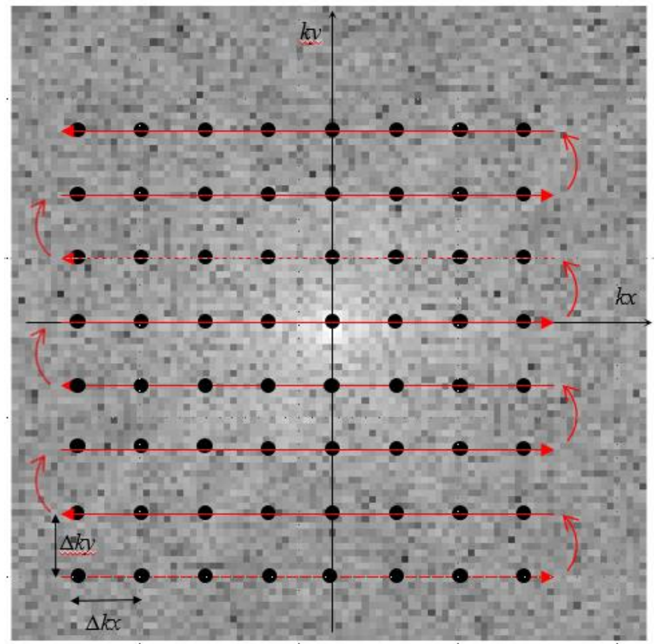


Bruce: PhD Dissertation, 2014.

Reconstruction

Multi-Coil Acquisition

Simultaneous Multi-Slice (SMS)



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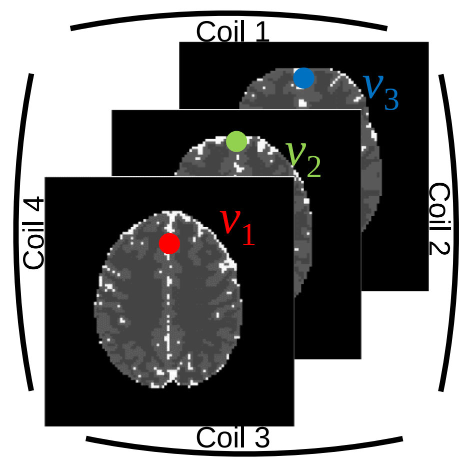


v_j

N_C=4, A=3 j=1:A

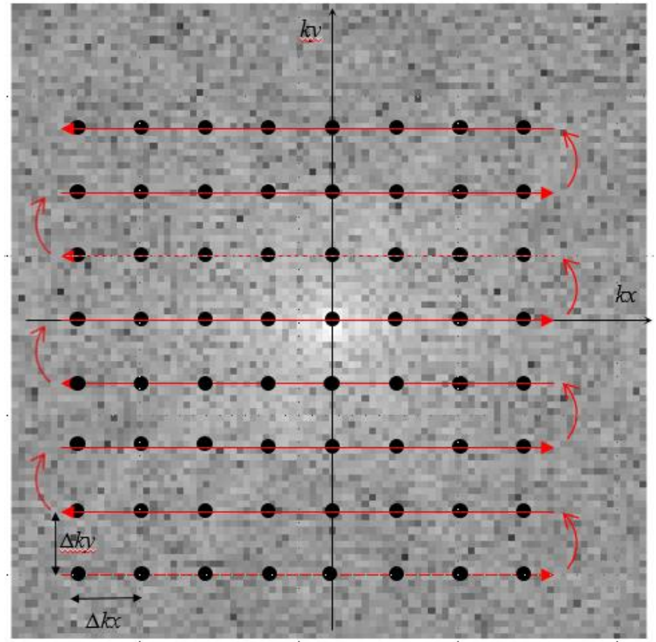
Larkman et al: JMRI, 13:313-317,2001.

Coil local so non uniform sensitivity.

Reconstruction

Multi-Coil Acquisition

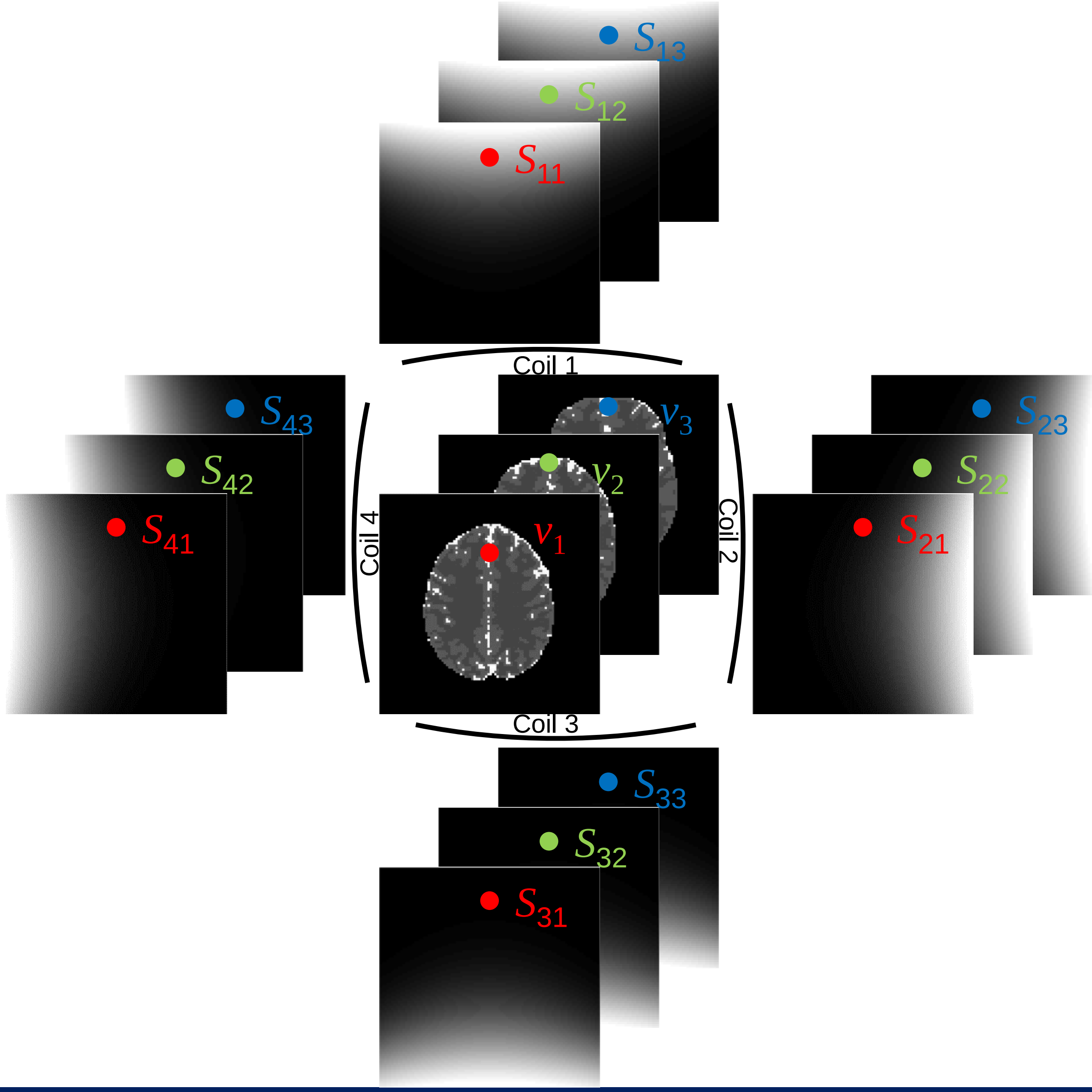
Simultaneous Multi-Slice (SMS)



Each coil measures k-space.

v_j, S_{kj}

$N_C=4$, $A=3$ $j=1:A$, $k=1:N_c$

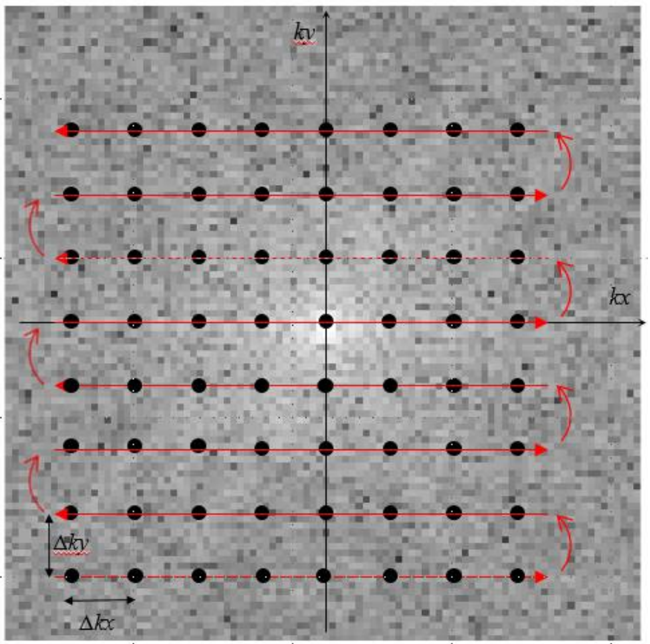


Coil local so non uniform sensitivity.

Reconstruction

Multi-Coil Acquisition

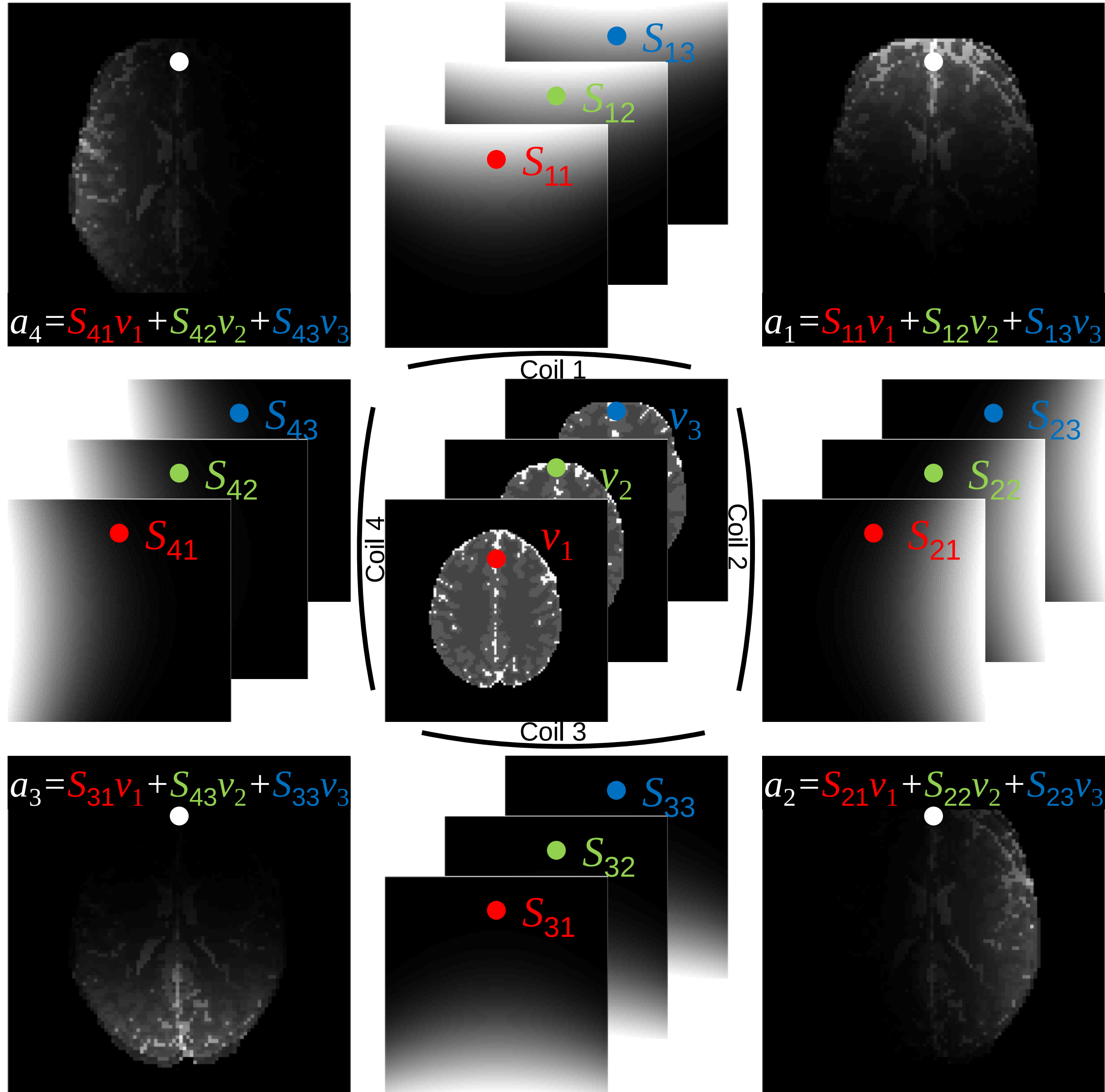
Simultaneous Multi-Slice (SMS)



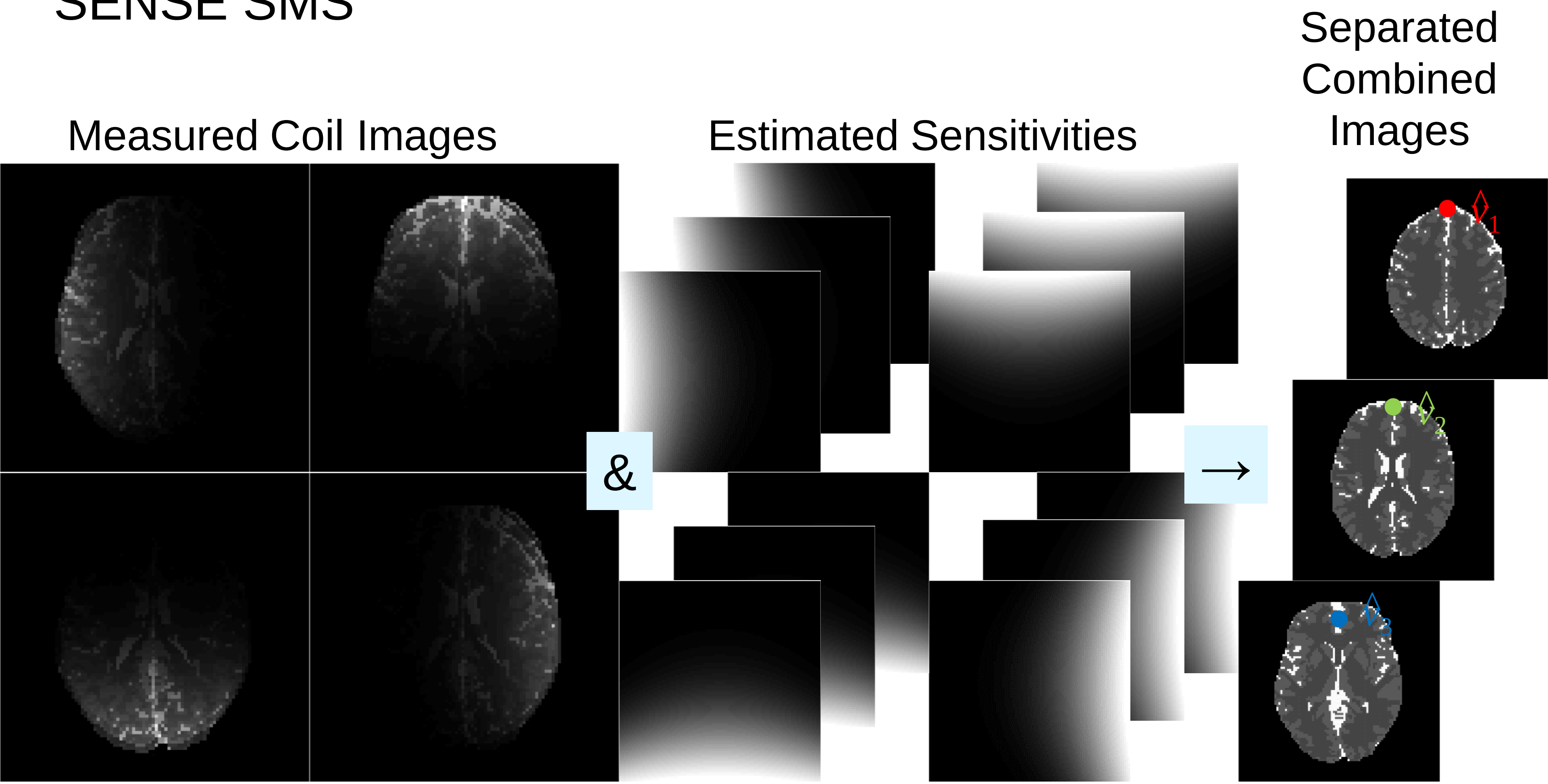
Each coil measures k-space.

v_j, S_{kj}, a_k

$N_C=4$, $A=3$ $\quad$ $j=1:A$, $k=1:N_c$



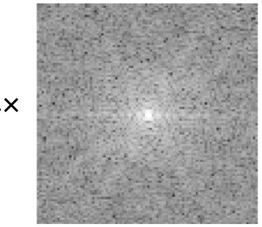
Reconstruction
SENSE SMS

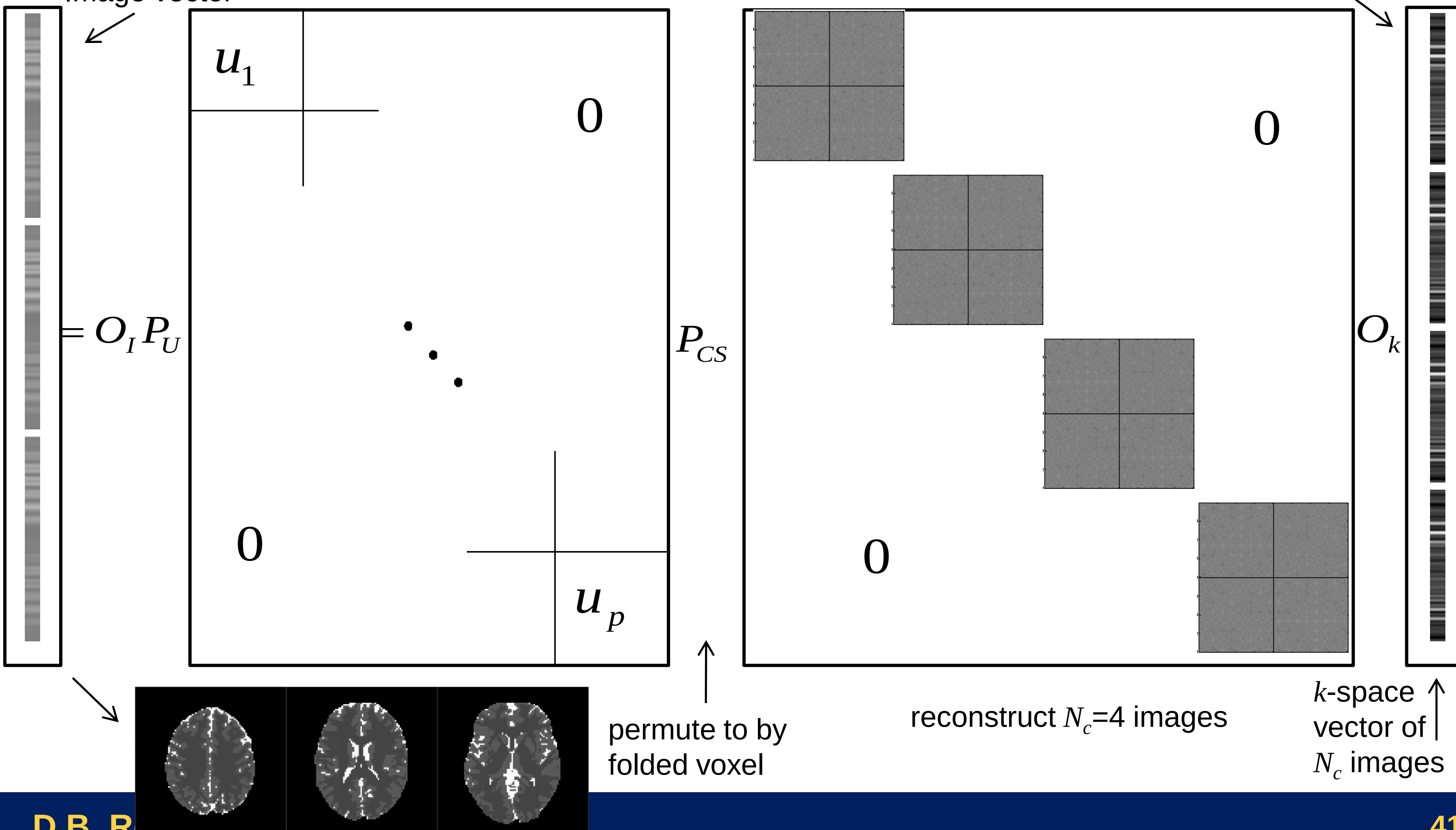


N_C=4, A=3

Reconstruction/Processing

SENSE SMS





Implications

In statistics, we know the rule that says:

If a vector f has a mean δ, and a covariance Γ,

Then $y=Of$ has a mean $\mu=O\delta$, and a covariance $\Sigma=O\Gamma O^T$.

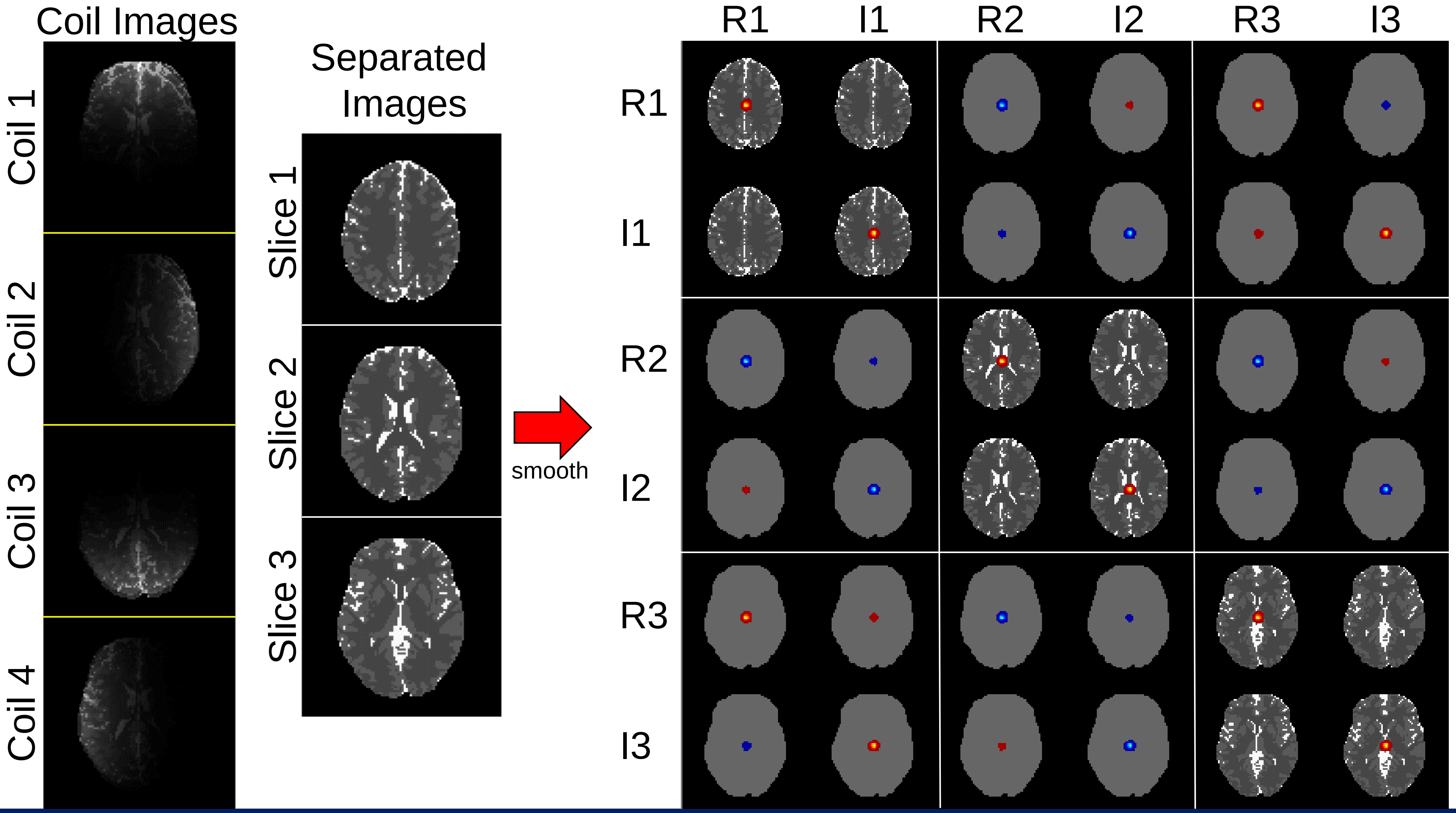
Then Σ can converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$.

Where $D^{-1/2}=1/\sqrt{diag(\Sigma)}$.

Assume k-space measurements independent so $\Gamma=I$.

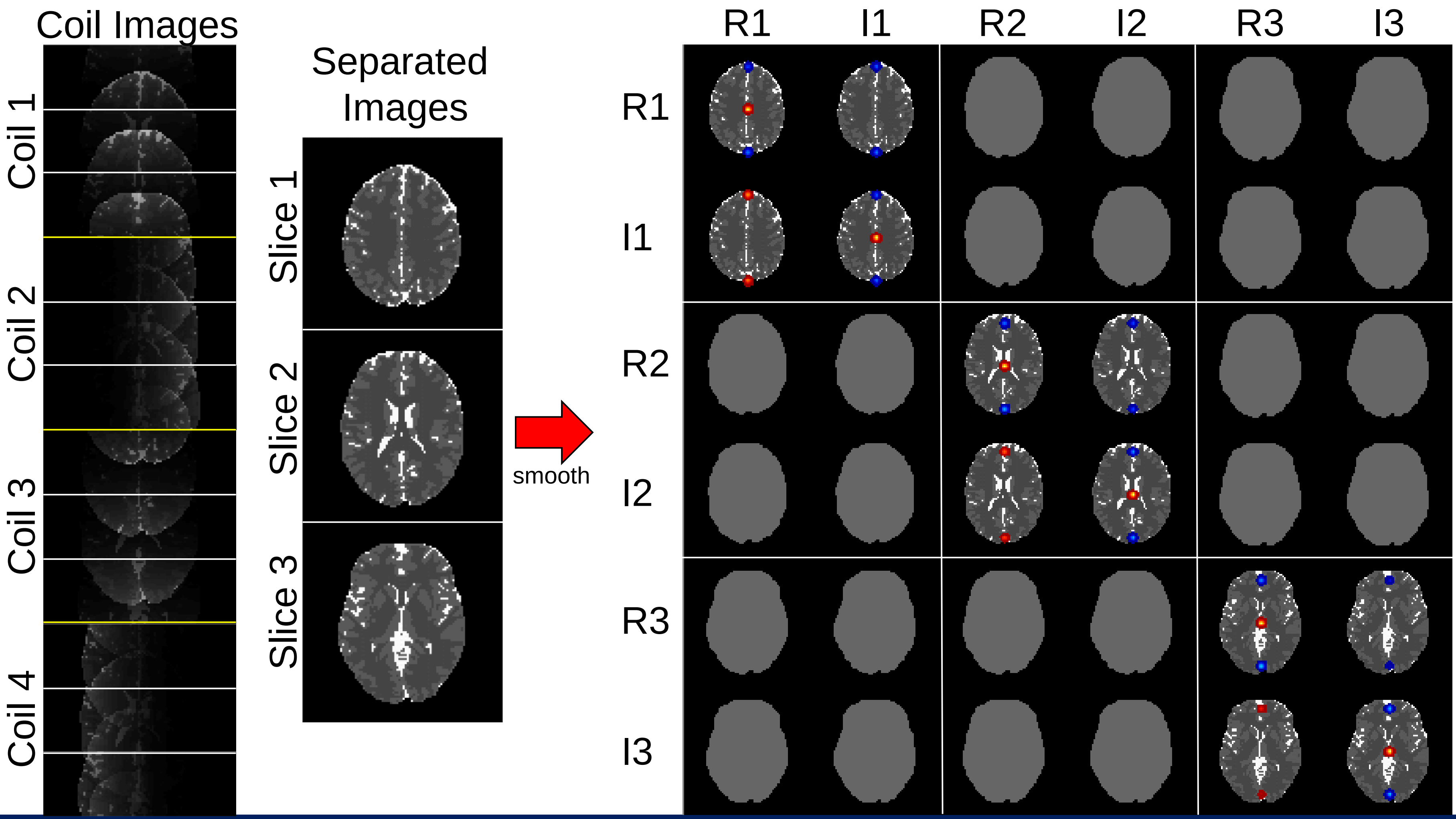
Implications

SENSE induces long-range through-plane correlation.



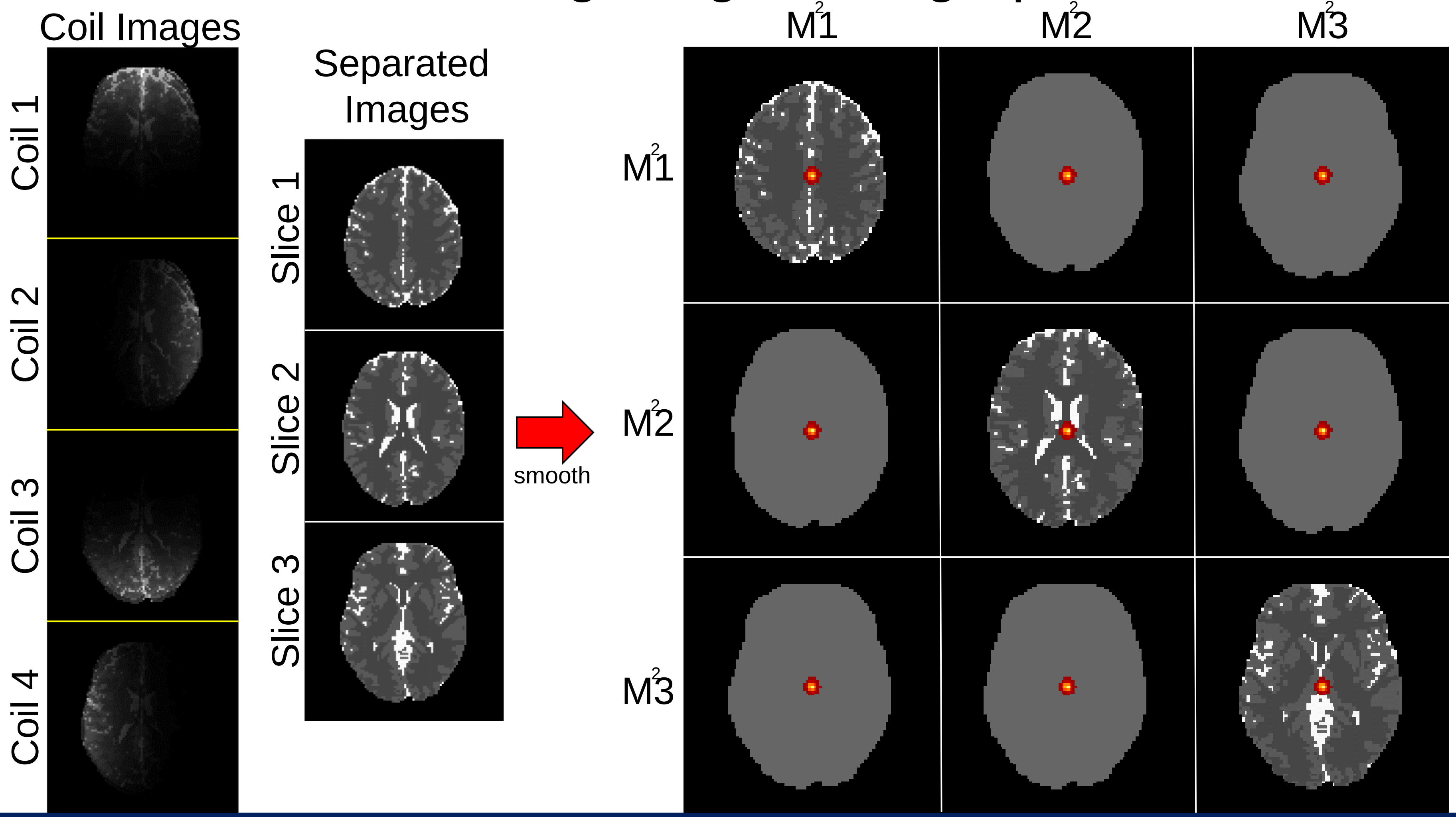
Implications

SENSE induces long-range in-plane correlation.



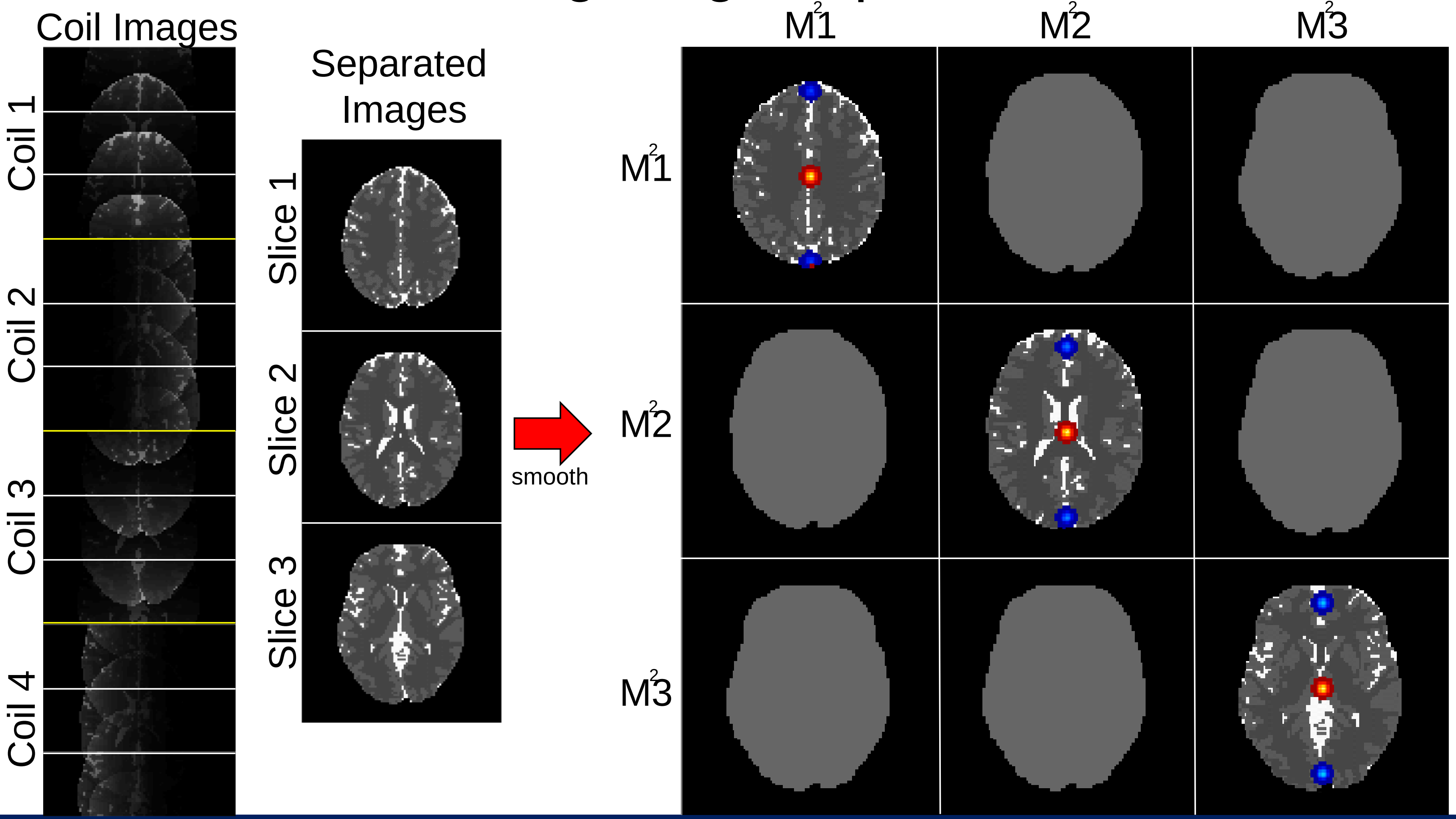
Implications

SENSE induces long-range through-plane correlation.



Implications

SENSE induces long-range in-plane correlation.



Discussion

Care needs to be taken when we obtain data.

We should get data in its originally measured form.

We should do any required processing ourselves.

We should develop models that incorporate processing.

We should use all of the data (magnitude and phase).

Thank You!

This work is joint with former & current PhD students:
Dr. Andrew S. Nencka, Medical College of Wisconsin
Dr. Andrew D. Hahn, University of Wisconsin-Madison
Dr. Iain P. Bruce, Duke University
Dr. M. Muge Karaman, University if Illinois-Chicago
Ms. Mary C. Kociuba, Marquette University
Mr. Kevin K. Liu, Marquette University