Least product relative error criterion based estimating equation approaches for the error-in-covariables multiplicative regression models

Qihua Wang

Academy of Mathematics and Systems Science, CAS

(Joint work with: Dahai Hu, University of Science and Technology of China)

Outline

Background

To handle the positive response, it is natural to consider the following multiplicative regression model,

$$Y_i = \exp(Z_i^T \beta_0)\varepsilon_i, \quad i = 1, \ldots, n, \tag{1}$$

where Y_i is a scalar response variable, Z_i is a random covariate vector with the first component being 1 (intercept), β_0 is the true regression parametric vector, and the error term ε is strictly positive.

Least Absolute Relative Errors (LARE)

Chen et al. (2010, JASA) consider the following two types of relative errors:

- $|Y_i - \exp(Z_i^T\beta)|/Y_i$;
- $|Y_i - \exp(Z_i^T\beta)|/\exp(Z_i^T\beta)$.

They proposed LARE criteria is to minimize

$$LARE_n(\beta) = \sum_{i=1}^{n} \left\{ \left| \frac{Y_i - \exp(Z_i^T\beta)}{Y_i} \right| + \left| \frac{Y_i - \exp(Z_i^T\beta)}{\exp(Z_i^T\beta)} \right| \right\}.$$

Denote the minimizer of $LARE_n(\beta)$ as $\hat{\beta}_{n,LARE}$.

Least Absolute Relative Errors (LARE)

- Advantage
  - Scale free and Robust ;
  - Relative error is concerned;
  - $LARE_n(\beta)$ is strictly convex in β under some regular conditions;
- Disadvantage
  - Nonsmooth;
  - Computation is complicated;
  - the limiting variance of $\hat{\beta}_{n,LARE}$ involves the density of the error

Least Product Relative Errors (LPRE)

To overcome the disadvantage of the LARE criteria, the product of the above two type relative errors are considered, namely,

$$\left|\frac{Y_i - \exp(Z_i^T\beta)}{Y_i}\right| \times \left|\frac{Y_i - \exp(Z_i^T\beta)}{\exp(Z_i^T\beta)}\right|.$$

- Wang et al. (2015, Test) developed a testing procedure to detect existence of the unknown change point and discussed a relative-based estimation of the change point.
- Chen et al.(2016, JMVA) study the least product relative error (LPRE) estimator.

Least Product Relative Errors (LPRE)

Advantage

- Smooth
- Convex

Our Focus

Existing Question:

- The aforementioned LPRE methods commonly assume that covariates are observed precisely.
- We usually encounter corrupted data in practice, where the covariates are measured with error.

Measurement Error Model

- $Z_i = (V_i^T, X_i^T)^T$
  - V_i : $q \times 1$ vector of explanatory variables that are precisely measured with the first component being 1 (intercept),
  - X_i : $p \times 1$ vector of error-prone explanatory variable
- Classical additive measurement error model:

$$W_{i,j} = X_i + U_{i,j}, \quad j = 1, \ldots, n_i, i = 1, \ldots, n,$$

  - U and $-U$ comes the same distribution
  - U is independent of (Z, ε)

LPRE without Measurement Error

When Z_i's are precisely measured, minimizing $LPRE_n(\beta)$ is equivalent to **solving the following estimating equation**

$$U_n(\beta) = \sum_{i=1}^{n} \psi(Z_i, Y_i, \beta) = 0 \tag{2}$$

where $\psi(Z_i, Y_i, \beta) = \left\{Y_i^{-1} \exp(Z_i^T \beta) - Y_i \exp(-Z_i^T \beta)\right\} Z_i$.

With the assumption $E[\varepsilon - 1/\varepsilon | Z] = 0$, it's easy to see that $E[\psi(Z_i, Y_i, \beta_0)] = 0$.

Conditional Mean Score Based Estimating Equation Approach

For simplification, denote the observed data $\mathcal{O}_{i,r} = (Y_i, V_i, W_{i,r})$ and let $\mathcal{U}_i = (Y_i, V_i, X_i)$ for $i = 1, \ldots, n$ and $r = 1, \ldots, n_i$.

The key point is to find a function $T^*(\mathcal{O}_{i,r}, \beta)$ such that

$$E[T^*(\mathcal{O}_{i,r}, \beta)|\mathcal{U}_i] = \psi(Z_i, Y_i, \beta).$$

If so, this leads to the following unbiased estimating equation,

$$\sum_{i=1}^{n} \left[\frac{1}{n_i} \sum_{r=1}^{n_i} T^*(\mathcal{O}_{i,r}, \beta) \right] = \mathbf{0}. \tag{3}$$

This general idea has also been used in Hu and Lin (2004, JASA) and Wu et al.(2015, JASA).

Construct $T^*(\mathcal{O}_{i,r}, \beta)$

Notation

- Take $\hat{Z}_{i,r} = (V_i^T, W_{i,r}^T)^T$ and $J = (0_{p\times q}, I_{p\times p})^T$.
  Then $\hat{Z}_{i,r} = Z_i + JU_{i,r}$.
- Denote $\varphi_0(\gamma) = E[\exp(U^T\gamma)]$ and $\varphi_1(\gamma) = E[U\exp(U^T\gamma)]$.
- Take

$$
\begin{aligned}
R_{i,r}^{(0)}(\beta) =& \varphi_0^{-1}(\gamma)\exp(\hat{Z}_{i,r}^T\beta),\\
R_{i,r}^{(1)}(\beta) =& \varphi_0^{-1}(\gamma)\exp(\hat{Z}_{i,r}^T\beta)\{\hat{Z}_{i,r} - J\varphi_0^{-1}(\gamma)\varphi_1(\gamma)\}.
\end{aligned}
$$

Construct $T^*(\mathcal{O}_{i,r},\beta)$

A simple algebraic manipulation yields

$$\exp(Z_i^T\beta)Z_i = E\left[R_{i,r}^{(1)}(\beta)|\mathcal{U}_i\right], \quad (4)$$

$$\exp(Z_i^T\beta) = E\left[R_{i,r}^{(0)}(\beta)|\mathcal{U}_i\right]. \quad (5)$$

Thus, the desired function $T^*(\mathcal{O}_{i,r},\beta)$ can be defined as

$$T^*(\mathcal{O}_{i,r},\beta) = Y_i^{-1}R_{i,r}^{(1)}(\beta) - Y_iR_{i,r}^{(1)}(-\beta).$$

However, $\varphi_0(\gamma)$ and $\varphi_1(\gamma)$ in $T^*(\mathcal{O}_{i,r},\beta)$ are unknown.

$\hat{\varphi}_0(\gamma)$ & $\hat{\varphi}_1(\gamma)$

Denote $\xi_i = I(n_i > 1)$ and $\tilde{n} = \sum_{i=1}^n \xi_i$. Then, $\varphi_k(\gamma), (k = 0, 1)$ can be estimated by

$$\hat{\varphi}_0(\gamma) = \left[\frac{1}{\tilde{n}} \sum_{i=1}^{n} \frac{\xi_i}{n_i(n_i - 1)} \sum_{r \neq s} \exp(\gamma^T (W_{i,r} - W_{i,s})) \right]^{1/2},$$

$$\hat{\varphi}_1(\gamma) = \frac{1}{2\tilde{n}\hat{\varphi}_0(\gamma)} \sum_{i=1}^{n} \left\{ \frac{\xi_i}{n_i(n_i - 1)} \sum_{r \neq s} (W_{i,r} - W_{i,s}) \exp(\gamma^T (W_{i,r} - W_{i,s})) \right\}$$

Construct $\hat{T}^*(\mathcal{O}_{i,r}, \beta)$

Substituting $\varphi_0(\gamma)$ and $\varphi_1(\gamma)$ in $R^{(0)}_{i,r}(\beta)$ and $R^{(1)}_{i,r}(\beta)$ with $\hat{\varphi}_0(\gamma)$ and $\hat{\varphi}_1(\gamma)$ yields $\hat{R}^{(0)}_{i,r}(\beta)$ and $\hat{R}^{(1)}_{i,r}(\beta)$. Thereafter, the resulting estimating equation is given by

$$\sum_{i=1}^{n}\left[\frac{1}{n_i}\sum_{r=1}^{n_i}\hat{T}^*(\mathcal{O}_{i,r}, \beta)\right] = \mathbf{0},$$

where $\hat{T}^*(\mathcal{O}_{i,r}, \beta) = Y_i^{-1}\hat{R}^{(1)}_{i,r}(\beta) - Y_i\hat{R}^{(1)}_{i,r}(-\beta)$. The solution of the above equation, $\hat{\beta}_{CMS}$ say, can be defined as estimator of β.

$\sqrt{n}$-consistency

Define $R_i^{(0)}(\beta) = n_i^{-1} \sum_{r=1}^{n_i} R_{i,r}^{(0)}(\beta)$, $\hat{R}_i^{(0)}(\beta) = n_i^{-1} \sum_{r=1}^{n_i} \hat{R}_{i,r}^{(0)}(\beta)$, $R_i^{(1)}(\beta) = n_i^{-1} \sum_{r=1}^{n_i} R_{i,r}^{(1)}(\beta)$ and $\hat{R}_i^{(1)}(\beta) = n_i^{-1} \sum_{r=1}^{n_i} \hat{R}_{i,r}^{(1)}(\beta)$. Define $\mathcal{A}_k = \{i : n_i = k, i = 1, \ldots, n\}$, $k = 1, \ldots m$.

Take

$$
\begin{aligned}
v_i =& Y_i^{-1} R_i^{(1)}(\beta_0) - Y_i R_i^{(1)}(-\beta_0), \\
r_i =& \frac{E(1/\varepsilon + \varepsilon)}{2(1-\rho_1)\varphi_0^2(\gamma_0)} \{h_i^{(1)}(\gamma_0) - 2\varphi_0^{-1}(\gamma_0)\varphi_1(\gamma_0) h_i^{(0)}(\gamma_0)\},
\end{aligned}
$$

where $\rho_1 = \lim |\mathcal{A}_1|/n$, $h_i^{(0)}(\gamma) = \frac{1}{n_i(n_i-1)} \sum_{r \neq s} \exp\{\gamma^T(W_{i,r} - W_{i,s})\}$ and $h_i^{(1)}(\gamma) = \frac{1}{n_i(n_i-1)} \sum_{r \neq s} (W_{i,r} - W_{i,s}) \exp\{\gamma^T(W_{i,r} - W_{i,s})\}$. Furthermore, define $V_0 = E[(1/\varepsilon + \varepsilon) Z Z^T]$.

$\sqrt{n}$-consistency

Theorem 1

Under Regularity Conditions, $\hat{\beta}_{CMS}$ *exists and is unique in a neighbourhood of* β_0 *with probability converging to 1 as* $n \to \infty$, *and* $\hat{\beta}_{CMS} \xrightarrow{p} \beta_0$. *In addition,*

$$\sqrt{n}(\hat{\beta}_{CMS} - \beta_0) \xrightarrow{D} N(0, \Gamma_{CMS}),$$

where $\Gamma_{CMS} = V_0^{-1} \Sigma_{CMS} V_0^{-1}$ *and* $\Sigma_{CMS} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} E(v_i - \xi_i J r_i)^{\otimes 2}$. Γ_{CMS} *can be estimated by plug-in method.*

Naive Method and Bias

Define $\bar{W}_{i,\cdot} = \frac{1}{n_i}\sum_{r=1}^{n_i} W_{i,r}$, and $\hat{Z}_i = (V_i^T, \bar{W}_{i,\cdot}^T)^T = Z_i + J\bar{U}_{i,\cdot}$, where $\bar{U}_{i\cdot} = \frac{1}{n_i}\sum_{r=1}^{n_i} U_{i,r}$. A naive computable estimating function $U_{nv}(\beta)$ can be obtained as follow

$$U_{nv}(\beta) = \sum_{i=1}^{n} \left\{ Y_i^{-1}\exp(\hat{Z}_i^T\beta) - Y_i \exp(-\hat{Z}_i^T\beta)\right\} \hat{Z}_i = \sum_{i=1}^{n} \psi(\hat{Z}_i, Y_i, \beta) \tag{6}$$

by replacing Z_i in (2) with $\hat{Z}_i$. Let β_{Naive} be the solution of $U_{nv}(\beta) = 0_{(p+q)\times 1}$. β_{Naive} is then the naive-LPRE estimator.

Naive Method and Bias

A simple algebraic manipulation leads to

$$
\begin{aligned}
&E[\psi(\hat{Z}_i, Y_i, \beta)|Y_i, Z_i] \\
=&\varphi_0^{n_i}(\frac{\gamma}{n_i})\psi(Z_i, Y_i, \beta) + J\left\{Y_i^{-1}\exp(Z_i^T\beta) + Y_i\exp(-Z_i^T\beta)\right\}\varphi_0^{n_i-1}(\frac{\gamma}{n_i})\varphi_1(\frac{\gamma}{n_i}) \quad (7) \\
:=&I_{1n}(\beta) + I_{2n}(\beta).
\end{aligned}
$$

- Bias: $E[\psi(\hat{Z}_i, Y_i, \beta_0)] = E[I_{2n}(\beta_0)]$, which might not $\mathbf{0}$, resulting in an biased estimating function;
- loss of efficiency: The factor $\varphi_0^{n_i}(\frac{\gamma}{n_i})$ in $I_{1n}(\beta)$.

Corrected estimating equation

In view of the bias and the loss of efficiency, we can construct an unbiased estimating function as

$$U^*(\beta) = \sum_{i=1}^{n} \tilde{\psi}_i$$

where

$$\tilde{\psi}_i = \{\varphi_0^{n_i}(\frac{\gamma}{n_i})\}^{-1}\Big\{\psi(\hat{Z}_i, Y_i, \beta) - I_{2n}(\beta)\Big\}.$$

However, X_i in $I_{2n}(\beta)$ can not be observed.

Corrected estimating equation

Note that

$$
\begin{aligned}
& E\left[Y_i^{-1}\exp(\hat{Z}_i^T\beta) + Y_i\exp(-\hat{Z}_i^T\beta)|\mathcal{U}_i\right] \\
=& \left\{Y_i^{-1}\exp(Z_i^T\beta) + Y_i\exp(-Z_i^T\beta)\right\}\varphi_0^{n_i}(\frac{\gamma}{n_i}).
\end{aligned} \tag{8}
$$

From (8), we have

$$
\begin{aligned}
& Y_i^{-1}\exp(Z_i^T\beta) + Y_i\exp(-Z_i^T\beta) \\
=& E\left[\varphi_0^{-n_i}(\frac{\gamma}{n_i})\left\{Y_i^{-1}\exp(\hat{Z}_i^T\beta) + Y_i\exp(-\hat{Z}_i^T\beta)\right\}|\mathcal{U}_i\right].
\end{aligned} \tag{9}
$$

Corrected estimating equation

Therefore, we can define ψ_i^* as follow,

$$\psi_i^* = \{\varphi_0^{n_i}(\frac{\gamma}{n_i})\}^{-1}\Big\{\psi(\hat{Z}_i, Y_i, \beta) \\ - J\big[Y_i^{-1}\exp(\hat{Z}_i^T\beta) + Y_i\exp(-\hat{Z}_i^T\beta)\big]\frac{\varphi_1(\gamma/n_i)}{\varphi_0(\gamma/n_i)}\Big\}$$

by replacing $Y_i^{-1}\exp(Z_i^T\beta) + Y_i\exp(-Z_i^T\beta)$ in $\tilde{\psi}_i$ with $\varphi_0^{-n_i}(\frac{\gamma}{n_i})\big\{Y_i^{-1}\exp(\hat{Z}_i^T\beta) + Y_i\exp(-\hat{Z}_i^T\beta)\big\}$.

Corrected estimating equation

However, $\varphi_0(\gamma)$ and $\varphi_1(\gamma)$ in ψ_i^* are unknown. Define

$$\begin{aligned}\hat{\psi}_i^* =& \{\hat{\varphi}_0^{n_i}(\frac{\gamma}{n_i})\}^{-1}\Big\{\psi(\hat{Z}_i, Y_i, \beta) \\ & - J\big[Y_i^{-1}\exp(\hat{Z}_i^T\beta) + Y_i\exp(-\hat{Z}_i^T\beta)\big]\frac{\hat{\varphi}_1(\gamma/n_i)}{\hat{\varphi}_0(\gamma/n_i)}\Big\},\end{aligned}$$

by replacing $\varphi_0(\gamma/n_i)$ and $\varphi_1(\gamma/n_i)$ in ψ^* with $\hat{\varphi}_0(\gamma/n_i)$ and $\hat{\varphi}_1(\gamma/n_i)$ given in the previous section, and we obtain an resultant estimating equation for β_0 as follow

$$\sum_{i=1}^{n} \hat{\psi}_i^* = \mathbf{0}.$$

Let $\hat{\beta}_{CEE}$ be the solution to the above estimating equation.

Corrected estimating equation

Denote $\tilde{R}_i^{(1)}(\beta) = \varphi_0^{-n_i}(\gamma/n_i)\exp(\hat{Z}_i^T\beta)\{\hat{Z}_i - J\frac{\varphi_1(\gamma/n_i)}{\varphi_0(\gamma/n_i)}\}$.

Let

$$\tilde{v}_i = Y_i^{-1}\tilde{R}_i^{(1)}(\beta_0) - Y_i\tilde{R}_i^{(1)}(-\beta_0),$$

$$\tilde{r}_{i,k} = \frac{E(1/\varepsilon+\varepsilon)}{2(1-\rho_1)\varphi_0^2(\gamma_0/k)}\{h_i^{(1)}(\gamma_0/k) - 2\varphi_0^{-1}(\gamma_0/k)\varphi_1(\gamma_0/k)h_i^{(0)}(\gamma_0/k)\},$$

where $h_i^{(k)}(\gamma)(k=0,1)$ are defined as the above section. Let $\rho_k = \lim_{n\to\infty}|\mathcal{A}_k|/n$.

$\sqrt{n}$-consistency

Theorem 2

Under regularity Conditions, $\hat{\beta}_{CEE}$ exists and is unique in a neighbourhood of β_0 with probability converging to 1 as $n \to \infty$, and $\hat{\beta}_{CEE} \xrightarrow{p} \beta_0$. In addition,

$$\sqrt{n}(\hat{\beta}_{CEE} - \beta_0) \xrightarrow{D} N(0, \Gamma_{CEE}),$$

where $\Gamma_{CEE} = V_0^{-1} \Sigma_{CEE} V_0^{-1}$ and $\Sigma_{CEE} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^n E\{\tilde{v}_i - \xi_i J \sum_{k=1}^m \rho_k \tilde{r}_{i,k}\}^{\otimes 2}$. Furthermore, Γ_{CEE} can be estimated by plug-in method.

Several Methods

- **CMS**:(Our proposal) Conditional Mean Score based estimating equation approach;
- **CEE**:(Our proposal) Corrected Estimating Equation approach;
- Full: LPRE with the true value of X;
- Naive: LPRE with the X replaced by the average of all the surrogate values

Simulation Models

- Model: $Y = \exp(c_0 + \alpha_0 V^* + \gamma_0 X)\varepsilon$.
  - $(c_0, \alpha_0, \gamma_0) = (1, 1, 2)$
  - (V^*, X): bivariate normal distribution with $Var(X) = Var(V^*) = 1$ and $\rho(X, V^*) = 0.5$
  - ε: the log-standard normal distribution
- Replication: 1000 times.
- Each of sample size: n=200, 300 or 500.

Measurement Error Model

- V^* : Measured precisely
- X : Classical additive model:

$$W_{i,j} = X_i + U_{i,j}, \quad i = 1, \ldots, n; j = 1, \ldots, n_i$$

- The distribution of U
  - $U \sim N(0, \sigma_u^2)$
  - the standardized $N(0,1)$ distribution truncated between $-c$ and c and scaled to have standard deviation of σ_u
  - $\sigma_u = 0.5$ or 0.75, inducing a signal-to-noise ratio of 0.8 or 0.64

Measurement Error Model

- n_i
  - **Case 1:** $n_i = 3$ for all subjects
  - **Case 2:** $1/3$ of the population: $n_i = 1$;
    $1/3$ of the population: $n_i = 2$;
    $1/3$ of the population: $n_i = 3$;

Scenario 1

- $n_i = 3$ for all the subjects
- $U \sim N(0, \sigma_u^2)$, $\sigma_u = 0.5$ or 0.75

Scenario 1

n	σ_u	method	$\hat{c}$			$\hat{\alpha}$			$\hat{\gamma}$		
			Bias	SE	MSE	Bias	SE	MSE	Bias	SE	MSE
200	0.50	Full	-0.0031	0.0752	0.0057	-0.0011	0.0872	0.0076	-0.0010	0.0899	0.0081
		Naive	-0.0034	0.0874	0.0076	0.1003	0.1025	0.0206	-0.1988	0.0997	0.0495
		CMS	-0.0042	0.0948	0.0090	-0.0048	0.1178	0.0139	0.0148	0.1397	0.0197
		CEE	-0.0030	0.0894	0.0080	-0.0036	0.1091	0.0119	0.0119	0.1168	0.0138
	0.75	Full	0.0036	0.0759	0.0058	0.0031	0.0870	0.0076	0.0001	0.0879	0.0077
		Naive	0.0041	0.1039	0.0108	0.2034	0.1182	0.0553	-0.3967	0.1095	0.1694
		CMS	0.0034	0.1452	0.0211	-0.0261	0.2074	0.0437	0.0686	0.3243	0.1099
		CEE	0.0040	0.1127	0.0127	-0.0173	0.1405	0.0200	0.0424	0.1646	0.0289
300	0.50	Full	-0.0017	0.0643	0.0041	-0.0016	0.0718	0.0052	-0.0006	0.0711	0.0051
		Naive	-0.0008	0.0755	0.0057	0.0995	0.0837	0.0169	-0.2034	0.0812	0.0479
		CMS	-0.0016	0.0800	0.0064	-0.0042	0.0965	0.0093	0.0069	0.1126	0.0127
		CEE	-0.0011	0.0767	0.0059	-0.0047	0.0885	0.0079	0.0053	0.0966	0.0094
	0.75	Full	-0.0009	0.0607	0.0037	0.0010	0.0695	0.0048	0.0008	0.0702	0.0049
		Naive	-0.0044	0.0896	0.0080	0.1996	0.0940	0.0487	-0.3979	0.0883	0.1661
		CMS	-0.0059	0.1143	0.0131	-0.0166	0.1752	0.0310	0.0308	0.2581	0.0676
		CEE	-0.0040	0.0950	0.0090	-0.0142	0.1106	0.0124	0.0252	0.1327	0.0182
500	0.50	Full	-0.0001	0.0482	0.0023	-0.0019	0.0554	0.0031	0.0008	0.0568	0.0032
		Naive	0.0016	0.0571	0.0033	0.0997	0.0664	0.0143	-0.1986	0.0642	0.0436
		CMS	0.0025	0.0624	0.0039	-0.0024	0.0782	0.0061	0.0068	0.0939	0.0089
		CEE	0.0021	0.0592	0.0035	-0.0021	0.0698	0.0049	0.0046	0.0745	0.0056
	0.75	Full	-0.0013	0.0505	0.0026	-0.0022	0.0537	0.0029	0.0006	0.0551	0.0030
		Naive	-0.0015	0.0674	0.0045	0.1956	0.0738	0.0437	-0.3992	0.0705	0.1643
		CMS	-0.0028	0.0932	0.0087	-0.0250	0.1577	0.0255	0.0331	0.2451	0.0612
		CEE	-0.0024	0.0703	0.0049	-0.0119	0.0865	0.0076	0.0121	0.1041	0.0110

Simulation Results for Scenario 1

- The naive estimators for γ_0 and α_0 are always seriously biased. Furthermore, the bias of the naive estimator is larger as σ_u becomes larger.
- The two proposed estimators $\hat{\beta}_{CMS}$ and $\hat{\beta}_{CEE}$ can effectively correct the biases caused by measurement error.
- The SE of $\hat{\beta}_{CMS}$ and $\hat{\beta}_{CEE}$ become smaller as the sample size n increases.
- The SE of $\hat{\beta}_{CMS}$ is larger than that of $\hat{\beta}_{CEE}$.

Scenario 2

- $n_i = 3$ for all the subjects
- U: the standardized $N(0,1)$ distribution truncated between $-c$ and c and scaled to have standard deviation of σ_u, where $c = 2$, $\sigma_u = 0.5$ or 0.75

Scenario 2

n	σ_u	method	$\hat{c}$			$\hat{\alpha}$			$\hat{\gamma}$		
			Bias	SE	MSE	Bias	SE	MSE	Bias	SE	MSE
200	0.50	Full	0.0019	0.0764	0.0058	-0.0007	0.0862	0.0074	0.0032	0.0866	0.0075
		Naive	0.0036	0.0898	0.0081	0.1006	0.0992	0.0199	-0.1984	0.0943	0.0483
		CMS	0.0021	0.0910	0.0083	-0.0015	0.1054	0.0111	0.0058	0.1101	0.0122
		CEE	0.0028	0.0902	0.0081	-0.0026	0.1053	0.0111	0.0082	0.1097	0.0121
	0.75	Full	0.0025	0.0760	0.0058	0.0016	0.0894	0.0080	-0.0019	0.0879	0.0077
		Naive	0.0047	0.0979	0.0096	0.2001	0.1178	0.0539	-0.4018	0.1068	0.1728
		CMS	0.0004	0.1031	0.0106	-0.0005	0.1298	0.0168	0.0087	0.1460	0.0214
		CEE	0.0036	0.1015	0.0103	-0.0096	0.1355	0.0184	0.0199	0.1561	0.0248
300	0.50	Full	-0.0011	0.0627	0.0039	-0.0004	0.0716	0.0051	-0.0011	0.0696	0.0048
		Naive	-0.0014	0.0713	0.0051	0.0972	0.0837	0.0165	-0.1988	0.0789	0.0457
		CMS	-0.0013	0.0729	0.0053	-0.0038	0.0861	0.0074	0.0045	0.0898	0.0081
		CEE	-0.0013	0.0726	0.0053	-0.0048	0.0861	0.0074	0.0063	0.0910	0.0083
	0.75	Full	0.0003	0.0634	0.0040	0.0035	0.0734	0.0054	0.0021	0.0732	0.0054
		Naive	0.0008	0.0859	0.0074	0.2038	0.0959	0.0507	-0.3941	0.0856	0.1627
		CMS	0.0009	0.0903	0.0082	-0.0009	0.1072	0.0115	0.0119	0.1199	0.0145
		CEE	0.0023	0.0890	0.0079	-0.0037	0.1099	0.0121	0.0206	0.1216	0.0152
500	0.50	Full	0.0005	0.0476	0.0023	0.0020	0.0540	0.0029	-0.0008	0.0553	0.0031
		Naive	0.0007	0.0555	0.0031	0.0999	0.0644	0.0141	-0.1978	0.0622	0.0430
		CMS	0.0003	0.0557	0.0031	-0.0024	0.0669	0.0045	0.0047	0.0702	0.0050
		CEE	0.0005	0.0559	0.0031	-0.0020	0.0680	0.0046	0.0048	0.0709	0.0050
	0.75	Full	-0.0001	0.0487	0.0024	-0.0034	0.0560	0.0032	0.0032	0.0562	0.0032
		Naive	-0.0030	0.0658	0.0043	0.1961	0.0700	0.0434	-0.3942	0.0653	0.1596
		CMS	-0.0024	0.0670	0.0045	-0.0065	0.0804	0.0065	0.0141	0.0928	0.0088
		CEE	-0.0023	0.0682	0.0047	-0.0083	0.0811	0.0066	0.0148	0.0954	0.0093

Simulation Results for Scenario 2

- Results for Scenario 2 shows similar patterns as results for Scenario 1 except that the SE of $\hat{\beta}_{CEE}$ is a little larger than that of $\hat{\beta}_{CMS}$.

Scenario 3

- $1/3$ of the population: $n_i = 1$;

  $1/3$ of the population: $n_i = 2$;

  $1/3$ of the population: $n_i = 3$;
- $U \sim N(0, \sigma_u^2)$, $\sigma_u = 0.5$ or 0.75

Scenario 3

n	σ_u	method	$\hat{c}$			$\hat{\alpha}$			$\hat{\gamma}$		
			Bias	SE	MSE	Bias	SE	MSE	Bias	SE	MSE
200	0.50	Full	-0.0031	0.0752	0.0057	-0.0011	0.0872	0.0076	-0.0010	0.0899	0.0081
		Naive	-0.0062	0.0993	0.0099	0.1702	0.1165	0.0426	-0.3440	0.1091	0.1302
		CMS	-0.0058	0.1084	0.0118	-0.0140	0.1461	0.0215	0.0287	0.1855	0.0352
		CEE	-0.0050	0.1036	0.0108	-0.0112	0.1341	0.0181	0.0227	0.1519	0.0236
	0.75	Full	0.0036	0.0759	0.0058	0.0031	0.0870	0.0076	0.0001	0.0879	0.0077
		Naive	0.0041	0.1271	0.0162	0.3300	0.1396	0.1283	-0.6454	0.1212	0.4312
		CMS	0.0064	0.1656	0.0275	-0.0119	0.2363	0.0560	0.0415	0.3422	0.1188
		CEE	0.0081	0.1500	0.0226	-0.0230	0.2000	0.0405	0.0640	0.2849	0.0853
300	0.50	Full	-0.0017	0.0643	0.0041	-0.0016	0.0718	0.0052	-0.0006	0.0711	0.0051
		Naive	-0.0001	0.0851	0.0072	0.1747	0.0932	0.0392	-0.3543	0.0885	0.1334
		CMS	-0.0029	0.0902	0.0082	-0.0136	0.1305	0.0172	0.0232	0.1803	0.0331
		CEE	-0.0011	0.0872	0.0076	-0.0092	0.1091	0.0120	0.0136	0.1286	0.0167
	0.75	Full	-0.0009	0.0607	0.0037	0.0010	0.0695	0.0048	0.0008	0.0702	0.0049
		Naive	-0.0036	0.1073	0.0115	0.3258	0.1088	0.1180	-0.6485	0.0969	0.4299
		CMS	-0.0068	0.1340	0.0180	-0.0155	0.2016	0.0409	0.0334	0.3147	0.1001
		CEE	-0.0031	0.1196	0.0143	-0.0253	0.1551	0.0247	0.0485	0.2265	0.0536
500	0.50	Full	-0.0001	0.0482	0.0023	-0.0019	0.0554	0.0031	0.0008	0.0568	0.0032
		Naive	0.0010	0.0651	0.0042	0.1756	0.0756	0.0366	-0.3479	0.0724	0.1263
		CMS	0.0016	0.0718	0.0052	-0.0046	0.1004	0.0101	0.0133	0.1443	0.0210
		CEE	0.0018	0.0692	0.0048	-0.0025	0.0847	0.0072	0.0087	0.1024	0.0106
	0.75	Full	-0.0013	0.0505	0.0026	-0.0022	0.0537	0.0029	0.0006	0.0551	0.0030
		Naive	-0.0041	0.0837	0.0070	0.3225	0.0876	0.1117	-0.6490	0.0804	0.4277
		CMS	-0.0022	0.1092	0.0119	-0.0245	0.1826	0.0340	0.0389	0.3002	0.0916
		CEE	-0.0048	0.0945	0.0090	-0.0205	0.1293	0.0171	0.0294	0.1875	0.0360

Simulation Results for Scenario 3

- Results for Scenario 3 shows similar patterns as results for Scenario 1.
- The SE of Scenario 3 is a litter larger than that of Scenario 1. The reason is that $1/3$ of the population has only one surrogate.

Scenario 4

- $1/3$ of the population: $n_i = 1$;
  $1/3$ of the population: $n_i = 2$;
  $1/3$ of the population: $n_i = 3$;
- U: the standardized $N(0,1)$ distribution truncated between $-c$ and c and scaled to have standard deviation of σ_u, where $c = 2$, $\sigma_u = 0.5$ or 0.75

Scenario 4

n	σ_u	method	$\hat{c}$			$\hat{\alpha}$			$\hat{\gamma}$		
			Bias	SE	MSE	Bias	SE	MSE	Bias	SE	MSE
200	0.50	Full	0.0019	0.0764	0.0058	-0.0007	0.0862	0.0074	0.0032	0.0866	0.0075
		Naive	0.0045	0.1002	0.0101	0.1688	0.1100	0.0406	-0.3356	0.1026	0.1231
		CMS	0.0026	0.1009	0.0102	-0.0053	0.1180	0.0140	0.0128	0.1283	0.0166
		CEE	0.0027	0.1006	0.0101	-0.0063	0.1188	0.0142	0.0148	0.1287	0.0168
	0.75	Full	0.0025	0.0760	0.0058	0.0016	0.0894	0.0080	-0.0019	0.0879	0.0077
		Naive	0.0021	0.1185	0.0141	0.3150	0.1334	0.1170	-0.6320	0.1130	0.4122
		CMS	0.0002	0.1236	0.0153	-0.0053	0.1604	0.0257	0.0166	0.1919	0.0371
		CEE	0.0007	0.1250	0.0156	-0.0123	0.1596	0.0256	0.0272	0.1910	0.0372
300	0.50	Full	-0.0011	0.0627	0.0039	-0.0004	0.0716	0.0051	-0.0011	0.0696	0.0048
		Naive	-0.0015	0.0810	0.0066	0.1674	0.0931	0.0367	-0.3378	0.0871	0.1217
		CMS	-0.0010	0.0831	0.0069	-0.0063	0.0966	0.0094	0.0102	0.1091	0.0120
		CEE	-0.0010	0.0829	0.0069	-0.0071	0.0979	0.0096	0.0123	0.1105	0.0124
	0.75	Full	0.0003	0.0634	0.0040	0.0035	0.0734	0.0054	0.0021	0.0732	0.0054
		Naive	-0.0050	0.1041	0.0109	0.3191	0.1065	0.1131	-0.6278	0.0913	0.4024
		CMS	-0.0026	0.1072	0.0115	-0.0051	0.1292	0.0167	0.0170	0.1581	0.0253
		CEE	-0.0030	0.1100	0.0121	-0.0094	0.1300	0.0170	0.0269	0.1575	0.0255
500	0.50	Full	0.0005	0.0476	0.0023	0.0020	0.0540	0.0029	-0.0008	0.0553	0.0031
		Naive	0.0026	0.0631	0.0040	0.1706	0.0731	0.0345	-0.3383	0.0670	0.1190
		CMS	0.0020	0.0631	0.0040	-0.0013	0.0793	0.0063	0.0032	0.0848	0.0072
		CEE	0.0026	0.0636	0.0040	-0.0021	0.0798	0.0064	0.0045	0.0837	0.0070
	0.75	Full	-0.0001	0.0487	0.0024	-0.0034	0.0560	0.0032	0.0032	0.0562	0.0032
		Naive	-0.0035	0.0799	0.0064	0.3123	0.0835	0.1045	-0.6264	0.0720	0.3975
		CMS	-0.0004	0.0790	0.0062	-0.0065	0.1013	0.0103	0.0147	0.1225	0.0152
		CEE	-0.0003	0.0808	0.0065	-0.0099	0.1013	0.0104	0.0203	0.1203	0.0149

Simulation Results for Scenario 4

- Results for Scenario 4 shows similar patterns as results for Scenario 2.
- The SE of Scenario 4 is a litter larger than that of Scenario 2. The reason is that $1/3$ of the population has only one surrogate.

Data Description

- **Data:** ACTG315 data, which is available at `https://www.urmc.rochester.edu/biostat/people/faculty/wusite/datasets/ACTG315LongitudinalDataViralLoad.cfm`
- **Aim:** The relationship between viral load and CD4+ cell counts of the first two days of treatment.
- **Response Y:** The average load of viral load of the first two days of treatment
- **Predictor X:** The average counts of CD4+ cell counts of the first two days of treatment.

Data Description

- **Measurement Error Model:**

$$W_{i,r} = X_i + U_{i,r}, \quad r = 1, \ldots, n_i, \quad i = 1, \ldots, 45$$

- **Model:**

$$Y_i = \exp(c_0 + \gamma_0 X_i)\varepsilon_i$$

Data Analysis

Table: Analysis of the ACTG315 data with LS(Least Square), CMS, and CEE

	LS		CMS		CEE	
	Est	p-value	Est	p-value	Est	p-value
c_0	12.217(0.436)	0	12.383(0.401)	0	12.424(0.416)	0
γ_0	-0.377(0.216)	0.040	-0.491(0.212)	0.010	-0.514(0.220)	0.010

Thank you!