TDBIE treatment of the wave equation with a nonlinear impedance boundary condition

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Banff, 19.1.2016

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Outline

Outline

Statement of the problem

Scattering with nonlinear impedance BC:

$$\begin{aligned} \ddot{u}^{\text{tot}} - \Delta u^{\text{tot}} &= 0 && \text{in } \Omega^+ \times (0, T), \\ \partial_\nu^+ u^{\text{tot}} &= g(\dot{u}^{\text{tot}}) && \text{on } \Gamma \times (0, T) \\ u^{\text{tot}} &= u^{\text{inc}} && \text{in } \Omega^+ \times \{t \le 0\}. \end{aligned}$$

- Ω-bounded Lipshitz domain
- $\Gamma = \partial\Omega$, $\Omega^+ = \mathbb{R}^d \setminus \overline{\Omega}$, ν- exterior normal
- $g(\cdot)$ - given nonlinear function, e.g., $g(x) = x + x|x|$.
- Incident wave u^{inc} satisfies

$$\ddot{u}^{\text{inc}} - \Delta u^{\text{inc}} = 0 \qquad \text{in } \Omega^+ \times \mathbb{R}$$

Motivation

- Acoustic (nonlinear) boundary conditions [Beale, Rosencrans '74, Graber '12]

$$
\begin{aligned}
\ddot{u} - \Delta u &= 0 && \text{in } \Omega, t > 0,\\
\dot{u} + m(x)\ddot{z} + f(x)\dot{z} + g(x)z &= 0, && \text{on } \Gamma, t > 0,\\
\partial_\nu u - g(\dot{u}) + h(x)\eta(\dot{z}) &= 0 && \text{on } \Gamma, t > 0.
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- Scattering of EM waves by nonlinear coatings [Haddar, Joly '01]
  - Nonlinear system in a thin layer:

    $$\dot{E} - \nabla \times H = 0, \qquad \dot{H} + \dot{M} + \nabla \times E = 0$$

    with M linked to H through a ferromagnetic law.
  - A nonlinear boundary condition obtained by a thin layer approximation.

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  - A nonlinear boundary condition obtained by a thin layer approximation.
- Coupling with nonlinear circuits (see talk of Michielssen).

Conditions on g

$$\begin{aligned} \ddot{u}^{\mathrm{tot}} - \Delta u^{\mathrm{tot}} &= 0 && \text{in } \Omega^+ \times (0, T), \\ \partial_\nu^+ u^{\mathrm{tot}} &= g(\dot{u}^{\mathrm{tot}}) && \text{on } \Gamma \times (0, T) \end{aligned}$$

Energy $E(t) = \frac{1}{2}\|\dot{u}^{\mathrm{tot}}\|^2_{L^2(\Omega^+)} + \frac{1}{2}\|\nabla u^{\mathrm{tot}}\|^2_{L^2(\Omega^+)}$ satisfies

$$E(t) = E(0) - \int_0^t \langle g(\dot{u}^{\mathrm{tot}}), \dot{u}^{\mathrm{tot}} \rangle_\Gamma \, d\tau.$$

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Conditions on g ensuring well-posedness [Lasiecka, Tataru '93, Graber '12]

- $g \in C^1(\mathbb{R})$,
- $g(0) = 0$,
- $g(s)s \geq 0, \ \forall s \in \mathbb{R}$,
- $g'(s) \geq 0, \ \forall s \in \mathbb{R}$,
- g satisfies the growth condition $|g(s)| \leq C(1 + |s|^p)$, where

$$\begin{cases} 1 < p < \infty & d = 2, \\ 1 < p \leq \frac{d}{d-2} & d \geq 3. \end{cases}$$

Outline

Semigroup setting

Definition

Let H be a Hilbert space, and $\mathcal{A} : H \to H$ be a (not necessary linear) operator with domain $\operatorname{dom}\mathcal{A}$. We call $\mathcal{A}$ maximally monotone if it satisfies:

(i) $(\mathcal{A}x - \mathcal{A}y, x - y)_H \leq 0 \qquad \forall x, y \in \operatorname{dom}\mathcal{A}$,

(ii) $\operatorname{range}(I - \mathcal{A}) = H$

Theorem (Komura-Kato)

Let $\mathcal{A}$ be a maximally monotone operator on a separable Hilbert space H with domain $\operatorname{dom}(\mathcal{A}) \subset H$.
Then there exists unique solution $u(t) \in \operatorname{dom}(\mathcal{A})$ of

$$\partial_t u - \mathcal{A}u = 0 \qquad u(0) = u_0 \in \operatorname{dom}(\mathcal{A})$$

and u is Lipschitz continuous on $[0, +\infty)$.

Let $\Delta t > 0$ and let $\partial_t^{\Delta t} u$ denote either 1st order $(k = 1)$ or 2nd order backward difference formula $(k = 2)$.

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Theorem ([Nevanlinna '78])

There exists unique solution $u_{\Delta t}^n \in \operatorname{dom} \mathcal{A}$ of

$$\partial_t^{\Delta t} u_{\Delta t} - \mathcal{A} u_{\Delta t} = 0,$$

assuming for starting values $u_j = u(j\Delta t)$ for $j \in 0, \dots k$.

Further for $N\Delta t \leq T$:

$$\max_{n=0,\dots,N} \| u(t_n) - u_{\Delta t}^n \| \leq C \| \mathcal{A} u_0 \| \left[\Delta t + T^{1/2} (\Delta t)^{1/2} + \left(T + T^{1/2} \right) (\Delta t)^{1/3} \right].$$

For $u \in C^{p+1}([0,T], H)$, p – order of the multistep method,

$$\max_{n=0,\dots,N} \| u(t_n) - u_{\Delta t}^n \| \leq C T \Delta t^p.$$

Setting

We will use the exotic transmission problem setting of [Laliena, Sayas '09].

- Closed sub-spaces $X_h \subseteq H^{-1/2}(\Gamma)$, $Y_h \subseteq H^{1/2}(\Gamma)$ (not necessarily finite dimensional).
- For $X_h \subseteq X$, the annihalator $X_h^\circ \subset X'$ is defined as

$$X_h^\circ = \{f \in X' \;:\; \langle x, f\rangle_\Gamma = 0 \;\forall x \in X_h\}.$$

Nonlinear semigroup setting

Setting $v := \dot{u}$ we get

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} v \\ \Delta u \end{pmatrix},$$

Consider the operator

$$\mathcal{A} := \begin{pmatrix} 0 & I \\ \Delta & 0 \end{pmatrix},$$

$$\operatorname{dom}(\mathcal{A}) := \Big\{ (u, v) \in BL^1 \times L^2(\mathbb{R}^d) : \Delta u \in L^2(\mathbb{R}^d \setminus \Gamma), v \in \mathcal{H}_h$$
$$[\![\partial_\nu u]\!] \in X_h, \partial_n^+ u - g([\![\gamma v]\!]) \in Y_h^\circ \Big\},$$

where

$$\mathcal{H}_h := \{ u \in H^1 \left(\mathbb{R}^d \setminus \Gamma \right) : [\![\gamma u]\!] \in Y_h, \gamma^- u \in X_h^\circ \}$$

and

$$BL^1 := \Big\{ u \in H^1_{loc}(\mathbb{R}^d \setminus \Gamma) : \| \nabla u \|_{L^2(\mathbb{R}^d \setminus \Gamma)} < \infty \Big\} / \ker \nabla.$$

Theorem

$\mathcal{A}$ is a maximally monotone operator on $\mathcal{X}$, and generates a strongly continuous semigroup which solves

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \mathcal{A} \begin{pmatrix} u \\ v \end{pmatrix}, \quad u(0) = u_0, v(0) = v_0.$$

Assume $u_0, v_0 \in \mathcal{H}_h$. Then, the solution satisfies:

(i) $(u, v) \in \operatorname{dom} \mathcal{A}$ and $u(t) \in \mathcal{H}_h$ and $v(t) \in \mathcal{H}_h$ for all $t > 0$.

(ii) $u \in C^{1,1}\big([0, \infty), H^1(\mathbb{R}^d \setminus \Gamma)\big)$,

(iii) $\dot{u} \in L^\infty\big((0, \infty), H^1\big(\mathbb{R}^d \setminus \Gamma\big)\big)$,

(iv) $\ddot{u} \in L^\infty\big((0, \infty), L^2(\mathbb{R}^d)\big)$.

Outline

Boundary integral potentials and operators

With the Green's function defined as ($\operatorname{Re} s > 0$)

$$\Phi(z; s) := \begin{cases} \frac{i}{4} H_0^{(1)}\left(is\,|z|\right), & \text{for } d = 2\,, \\ \frac{e^{-s|z|}}{4\pi|z|}, & \text{for } d \geq 3, \end{cases}$$

the single- and double-layer potentials:

$$\left(S(s)\varphi\right)(x) := \int_\Gamma \Phi(x - y; s) u(y)\, dy$$

$$\left(D(s)\varphi\right)(x) := \int_\Gamma \partial_{\nu(y)} \Phi(x - y; s) u(y)\, dy.$$

and their traces

$$\begin{aligned} V(s) &: H^{-1/2}(\Gamma) \to H^{1/2}(\Gamma), & V(s) &:= \gamma^{\pm} S(s), \\ K(s) &: H^{1/2}(\Gamma) \to H^{1/2}(\Gamma), & K(s) &:= \{\{\gamma S(s)\}\}\,, \\ K^t(s) &: H^{-1/2}(\Gamma) \to H^{-1/2}(\Gamma), & K^t(s) &:= \{\{\partial_\nu D(s)\}\}\,, \\ W(s) &: H^{1/2}(\Gamma) \to H^{-1/2}(\Gamma), & W(s) &:= -\partial_\nu^{\pm} D(s). \end{aligned}$$

Calderón operators

$$B(s) := \begin{pmatrix} sV(s) & K \\ -K^t & s^{-1}W(s) \end{pmatrix}$$

$$B_{\text{imp}}(s) := B(s) + \begin{pmatrix} 0 & -\frac{1}{2}I \\ \frac{1}{2}I & 0 \end{pmatrix}.$$

Lemma [LB, Lubich, Sayas '15, Abboud et al '11]

There exists a constant $\beta > 0$, depending only on Γ, such that

$$\operatorname{Re}\left\langle B_{\text{imp}}(s)\begin{pmatrix}\varphi\\ \psi\end{pmatrix}, \begin{pmatrix}\varphi\\ \psi\end{pmatrix}\right\rangle_\Gamma \geq \beta \min(1, |s|^2)\frac{\operatorname{Re}(s)}{|s|^2}\,\|(\varphi,\psi)\|_\Gamma^2,$$

where

$$\|(\varphi,\psi)\|_\Gamma^2 := \|\varphi\|^2_{H^{-1/2}(\Gamma)} + \|\psi\|^2_{H^{1/2}(\Gamma)}.$$

Scattered field

Scattered field $u = u^{\text{scat}} = u^{\text{tot}} - u^{\text{inc}}$ satisfies

$$\begin{aligned} \ddot{u} - \Delta u &= 0, \qquad \text{in } \Omega^+ \\ \partial_\nu^+ u &= g(\dot{u} + \dot{u}^{inc}) - \partial_\nu^+ u^{inc}, \quad \text{on } \Gamma \\ u(0) = \dot{u}(0) &= 0, \qquad \text{in } \Omega^+. \end{aligned} \tag{1}$$

Boundary integral formulation

$$B_{\text{imp}}(\partial_t) \begin{pmatrix} \varphi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 \\ g(\psi + \dot{u}^{inc}) \end{pmatrix} = \begin{pmatrix} 0 \\ -\partial_\nu^+ u^{inc} \end{pmatrix}. \tag{2}$$

(i) If $u := u^{scat}$ solves (1), then (φ, ψ), with $\varphi := -\partial_\nu^+ u$ and $\psi := \gamma^+ \dot{u}$, solves (2).

(ii) If (φ, ψ) solves (2), then $u := S(\partial_t)\varphi + \partial_t^{-1} D(\partial_t)\psi$ solves (1).

Time and space discretization

- Closed sub-spaces $X_h \subseteq H^{-1/2}(\Gamma)$, $Y_h \subseteq H^{1/2}(\Gamma)$ (not necessarily finite dimensional).
- $J_\Gamma^{Y_h} : H^{1/2}(\Gamma) \to Y_h$ a stable operator, e.g., Scott-Zhang.
- BDF1 or BDF2 based CQ with time-step $\Delta t > 0$.

Fully discrete problem

For all $n \in \mathbb{N}$, find $(\varphi^n, \psi^n) \in X_h \times Y_h$ such that:

$$\left\langle \left[B_{\mathrm{imp}}(\partial_t^{\Delta t}) \begin{pmatrix} \varphi \\ \psi \end{pmatrix} \right]^n, \begin{pmatrix} \xi \\ \eta \end{pmatrix} \right\rangle_\Gamma + \left\langle g(\psi + J_\Gamma^{Y_h} \dot{u}^{inc}), \eta \right\rangle_\Gamma = \left\langle -\partial_\nu^+ u^{inc}, \eta \right\rangle_\Gamma$$

for all $n \in \mathbb{N}, (\xi, \eta) \in X_h \times Y_h$.

Solution in Ω^+ given by representation formula

$$u^n := \left[S(\partial_t^{\Delta t}) \varphi \right]^n + \left[(\partial_t^{\Delta t})^{-1} D(\partial_t^{\Delta t}) \psi \right]^n .$$

Brief overview of basics of CQ

- Discrete convolution

$$\left[B_{\mathrm{imp}}(\partial_t^{\Delta t}) \begin{pmatrix} \varphi \\ \psi \end{pmatrix} \right]^n = \sum_{j=0}^{n} B_{n-j} \begin{pmatrix} \varphi_j \\ \psi_j \end{pmatrix}$$

- $\delta(z)$ generating function of BDF1 ($\delta(z) = 1 - z$) or BDF2 ($\delta(z) = 1 - z + \frac{1}{2}(1 - z)^2$)

$$B_{\mathrm{imp}}\left(\delta(z)/\Delta t\right) = \sum_{j=0}^{\infty} B_j z^j.$$

- In particular $B_0 = B_{\mathrm{imp}}(\delta(0)/\Delta t)$ and hence

$$\left\langle B_0 \begin{pmatrix} \varphi \\ \psi \end{pmatrix}, \begin{pmatrix} \varphi \\ \psi \end{pmatrix} \right\rangle_\Gamma \geq \beta_0 \Delta t \, \|(\varphi, \psi)\|_\Gamma^2$$

- $\partial_t^{\Delta t} u$ is the finite difference derivative, e.g., for BDF1

$$\partial_t^{\Delta t} u(t_n) = \frac{1}{\Delta t}(u(t_n) - u(t_{n-1})).$$

Proposition (Browder and Minty)

X a real separable and reflexive Banach space, $A : X \to X'$ satisfies

- $A : X \to X'$ is continuous,
- the set $A(M)$ is bounded in X' for all bounded sets $M \subseteq X$,
- $\lim\limits_{\|u\| \to \infty} \dfrac{\langle A(u), u \rangle_{X' \times X}}{\|u\|} = \infty$,
- $\langle A(u) - A(v), u - v \rangle_{X' \times X} \geq 0$ for all $u, v \in X$.

Then the variational equation

$$\langle A(u), v \rangle_{X' \times X} = \langle f, v \rangle_{X' \times X} \qquad \forall v \in X$$

has at least one solution for all $f \in X'$. If the operator is strongly monotone, i.e.

$$\langle A(u) - A(v), u - v \rangle_{X' \times X} \geq \beta \|u - v\|_X^2 \qquad \text{for all } u, v \in X,$$

then the solution is unique.

Well-posedness of the discrete system

Theorem

The fully discrete system of equations has a unique solution in the space $X_h \times Y_h$ for all $n \in \mathbb{N}$.

Proof:

- At each time-step we need to solve

$$\left\langle \left[B_0 \begin{pmatrix} \varphi^n \\ \widetilde{\psi}^n \end{pmatrix} \right]^n, \begin{pmatrix} \xi \\ \eta \end{pmatrix} \right\rangle_\Gamma + \langle g(\widetilde{\psi}^n), \eta \rangle_\Gamma = \left\langle f^n, \begin{pmatrix} \xi \\ \eta \end{pmatrix} \right\rangle_\Gamma$$

  with $f^n := -\partial_\nu^+ u^{inc}(t_n) - \sum_{j=0}^{n-1} B_{n-j} \begin{pmatrix} \varphi^j \\ \psi^j \end{pmatrix} + B_0 J_\Gamma^{Y_h} \dot{u}^{inc}$ and $\widetilde{\psi}^n := \psi^n + J_\Gamma^{Y_h} u^{inc}(t_n)$.
- Ellipticity of B_0 and $xg(x) \geq 0$ imply coercivity.
- Strong monotonicity follows from

$$\langle g(\eta_1) - g(\eta_2), \eta_1 - \eta_2 \rangle_\Gamma = \int_\Gamma g'(s(x))(\eta_1(x) - \eta_2(x))^2 \, dx \geq 0.$$

- Boundedness follows from properties of B_j and assumptions on g.

Equivalence of discretized PDE and BIE

Lemma

For all $n \in \mathbb{N}$, find $u^n_{\Delta t}, v^n_{\Delta t} \in \mathcal{H}_h$ such that:

$$\begin{aligned}
\left[\partial_t^{\Delta t} u_{\Delta t}\right]^n &= v^n_{\Delta t} \\
\left[\partial_t^{\Delta t} v_{\Delta t}\right]^n &= \Delta u^n_{\Delta t} \\
\partial_\nu^+ u^n_{\Delta t} - g\left(\llbracket \gamma v^n_{\Delta t} \rrbracket + J_\Gamma^{Y_h} \dot{u}^{inc}(t_n)\right) + \partial_\nu u^{inc}(t_n) &\in X_h^\circ, \\
\llbracket \partial_\nu u \rrbracket &\in X_h.
\end{aligned}$$

(i) If the sequences φ^n, ψ^n solve the fully discrete BIE then $u_{\Delta t} := S(\partial_t^{\Delta t})\varphi + \left(\partial_t^{\Delta t}\right)^{-1} D(\partial_t^{\Delta t})\psi$ and $v_{\Delta t} := \partial_t^{\Delta t} u_{\Delta t}$ solve the above.

(ii) If $u_{\Delta t}, v_{\Delta t}$ solves the above, then $\varphi := -\llbracket \partial_\nu u_{\Delta t} \rrbracket$, $\psi := \llbracket \gamma v_{\Delta t} \rrbracket$ solve the fully discretized BIE.

Convergence: time-discretization

Theorem

The discrete solutions, obtained by $u_{\Delta t} := S(\partial_t^{\Delta t})\varphi + (\partial_t^{\Delta t})^{-1} D(\partial_t^{\Delta t})\psi$ converge to the exact solution u, with the following rate:

$$\max_{n=0,\ldots,N} \|u(t_n) - u_{\Delta t}^n\| \lesssim T(\Delta t)^{1/3}.$$

If we assume, that the exact solution satisfies $(u, \dot{u}) \in C^{p+1}\left([0, T], BL^1 \times L^2\left(\mathbb{R}^d \setminus \Gamma\right)\right)$, then

$$\max_{n=0,\ldots,N} \|u(t_n) - u_{\Delta t}^n\| \lesssim T(\Delta t)^p.$$

Convergence results full discretization (low regularity)

With standard boundary element spaces X_h and Y_h.

Theorem (low regularity)

For the fully discrete scheme, we have

$$\begin{aligned} u_{\Delta t} + u^{inc} &\rightharpoonup u^{\text{tot}} \quad \text{pointwise a.e. in } (BL^1) \\ \partial_t^{\Delta t} u_{\Delta t} + \dot{u}^{inc} &\rightharpoonup \dot{u}^{\text{tot}} \quad \text{pointwise a.e. in } L^2(\mathbb{R}^d) \end{aligned}$$

If aditionally g strictly monotone and $|g(s)| \lesssim |s|^{\frac{d-1}{d-2}}$ for $d \geq 3$

$$\begin{aligned} u_{\Delta t} + u^{inc} &\to u^{\text{tot}} \quad \text{in } L^\infty\left((0,T); BL^1\right) \\ \partial_t^{\Delta t} u_{\Delta t} + \dot{u}^{inc} &\to \dot{u}^{\text{tot}} \quad \text{in } L^\infty\left((0,T); L^2(\mathbb{R}^d)\right). \end{aligned}$$

with a rate in time of $(\Delta t)^{1/3}$.

Convergence results (higher regularity)

Assumptions (regularity)

Assume, that the exact solution of has the following regularity properties:

1. $u \in C^2\big((0,T); H^1(\Omega^-)\big)$,
2. $\dot{u} \in C^2\big((0,T); L^2(\Omega^-)\big)$,
3. $\gamma^+ u, \gamma^+ \dot{u} \in L^\infty\left((0,T), H^m(\Gamma)\right)$,
4. $\partial_\nu^+ u, \partial_\nu^+ \dot{u} \in L^\infty\big((0,T), H^{m-1}(\Gamma)\big)$,
5. $\ddot{u} \in L^\infty\left((0,T), H^m(\Omega^-)\right)$,
6. $\gamma^+ \dot{u} \in L^\infty\big((0,T), H^{d-1}(\Gamma)\big)$,

for some $m \geq 1/2$.

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for some $m \geq 1/2$.

Theorem (high regularity)

Optimal rates in both space and time for BDF1.

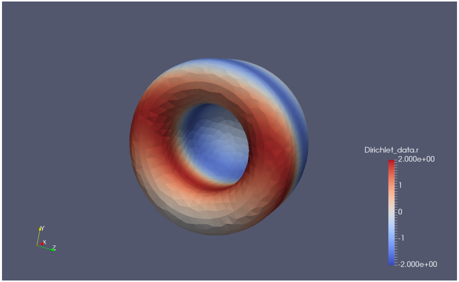
Outline

Comments on implementation

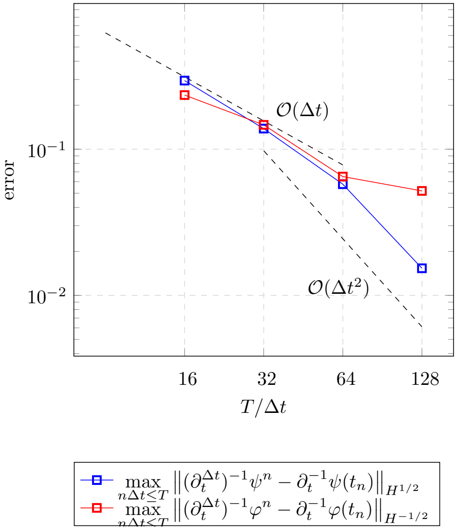
- Recursive, marching on in time implementation of CQ.
- Newton iteration in each step, with solution at previous step as initial guess.
- In practice, only a few steps of Newton needed.
- Main cost still the computation of history.
- Implementation in BEM++.

Setting

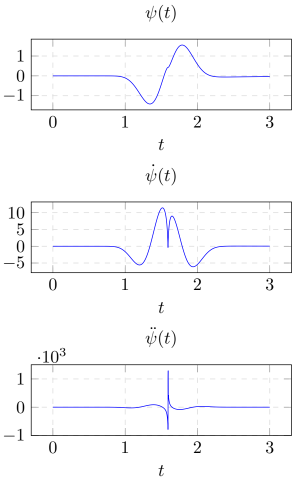
- $g(s) := s + |s|\, s$
- $u^{inc}(x,t) := F(x-t)$ with $F(s) := -\cos(\omega s) e^{-\left(\frac{s-A}{\sigma}\right)^2}$.
- The parameters were $\omega := \pi/2$, $\sigma = 0.5$, $A = 2.5$.

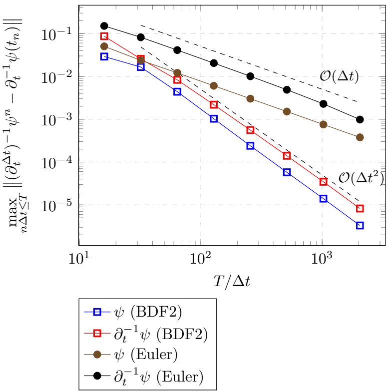


Convergence

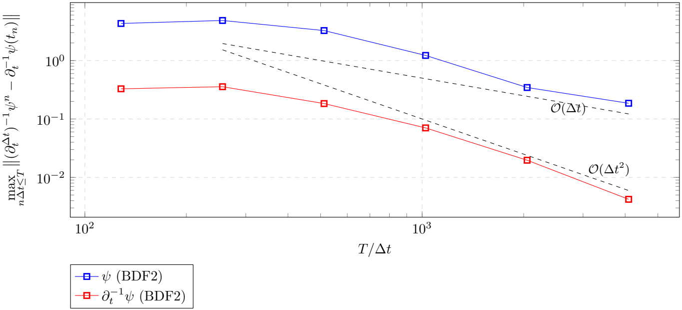


Space independent scattering by sphere: Exterior





Space independent scattering by sphere: Interior



Outline

Conclusions

This talk:

- A wave scattering problem with nonlinear damping.
- Convergence analysis of full CQ/Galerkin in space discretization.

Future work:

- More complex boundary conditions (DE on the boundary).
- Higher-order CQ, i.e., Runge-Kutta.
- Regularity of solution.