Structured matter vortex waves

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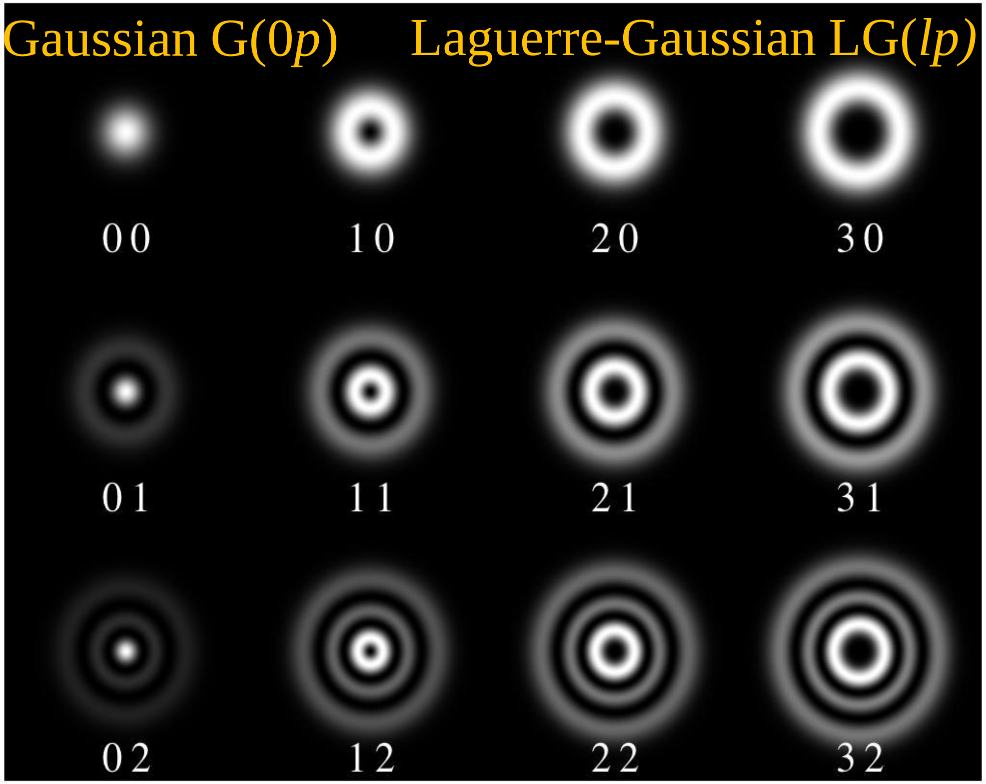
Aim: to create matter vortex waves



$$\Psi_{lp}(\rho,\phi,z) = N_{lp}\, \boldsymbol{E}_{lp}(k_{\perp}\rho)e^{ik_z z}e^{il\phi}$$

Introduction

- *Electron vortex waves: rapid and growing work (recent reviews: Lloyd et al and Bliokh et al)*
- *Work has begun on other matter vortex waves: atoms, positrons, neutrons and neutrinos*
- *How successful is current work?*
- *What are the practical limitations i.e. the feasibility of experimental work in the respective type of matter wave?*
- *What new physics is expected in each category and what new applications are envisaged .*

de Broglie: particle – wave duality

$$\lambda = \frac{h}{p}$$

Photons

$$p = \frac{E}{c} = \frac{\hbar\omega}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Particles

$$\lambda = \frac{h}{mv}$$

**Evidence:
Interference
and Diffraction**

This is a universal characteristic of ALL particles

de Broglie wavelength

$$\lambda_d = \frac{h}{\sqrt{2mE}} \propto (mE)^{-1/2}$$

Typically in electron microscope

$$E \approx 100\,\mathrm{keV} \qquad \lambda_d^{elec} \approx 3.7\,pm$$

Compare with visible light

$$\lambda_d^{phot} \approx 500\ \mathrm{n}m$$

Which particles? The standard model

Leptons					
First generation		*Second generation*		*Third generation*	
Name	*Symbol*	*Name*	*Symbol*	*Name*	*Symbol*
electron	e−	muon	μ−	tau	τ−
electron neutrino	ν e	muon neutrino	ν μ	tau neutrino	ν τ
Quarks					
First generation		*Second generation*		*Third generation*	
up quark	u	charm quark	c	top quark	t
down quark	d	strange quark	s	bottom quark	b

The Really Fundamental particles

Elementary Particles

	Types	Generations	Antiparticle	Colors	Total
Quarks	2	3	Pair	3	36
Leptons	2	3	Pair	None	12
Gluons	1	1	Own	8	8
W	1	1	Pair	None	2
Z	1	1	Own	None	1
Photon	1	1	Own	None	1
Higgs	1	1	Own	None	1
Total					61

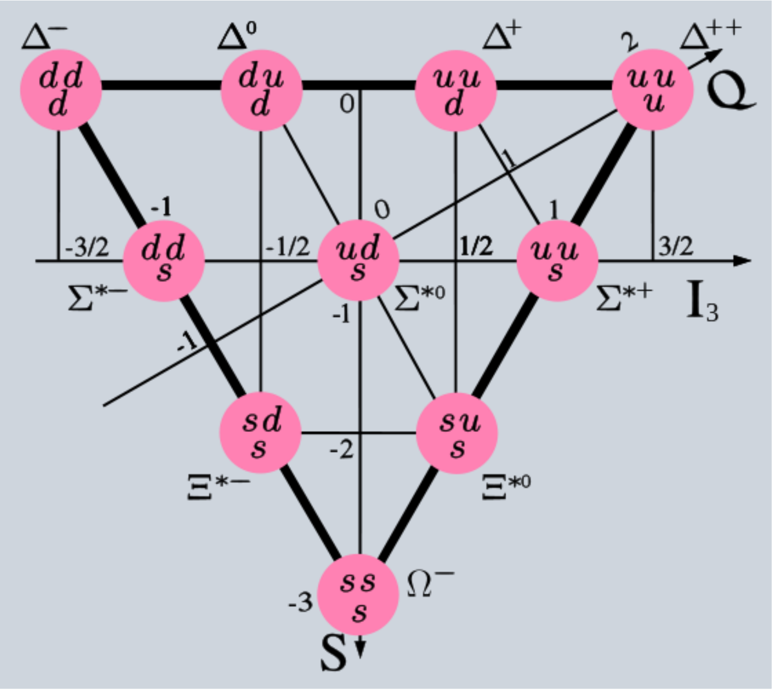
Mesons: quark-antiquark composites

Mesons $q\bar{q}$

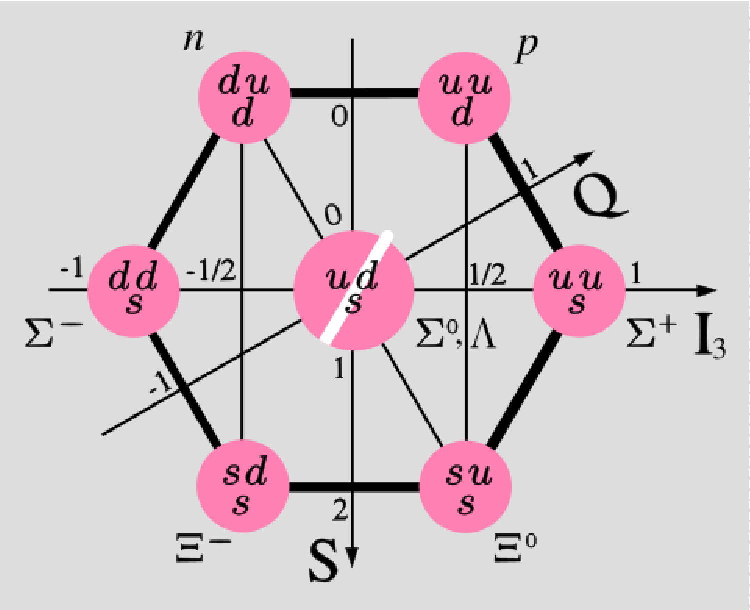
Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	$q\bar{q}$ 0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Baryons: 3-quark composites

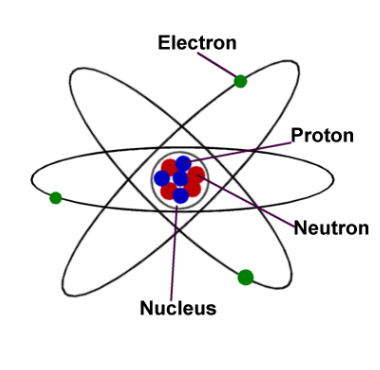


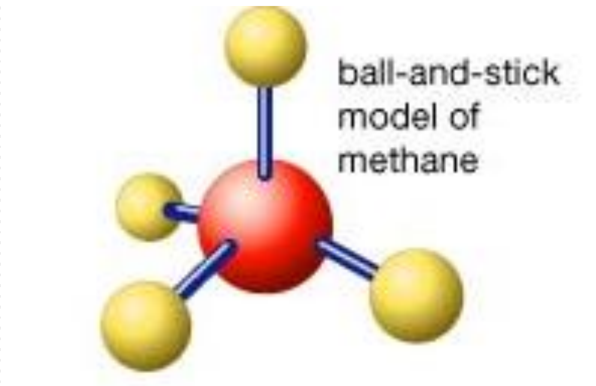
Combinations of three **u**, **d** or **s** quarks forming baryons with a spin-3/2 form the *uds baryon decuplet*



Combinations of three **u**, **d** or **s** quarks forming baryons with a spin-1/2 form the *uds baryon octet*

Bigger Composite particles : Atoms and molecules

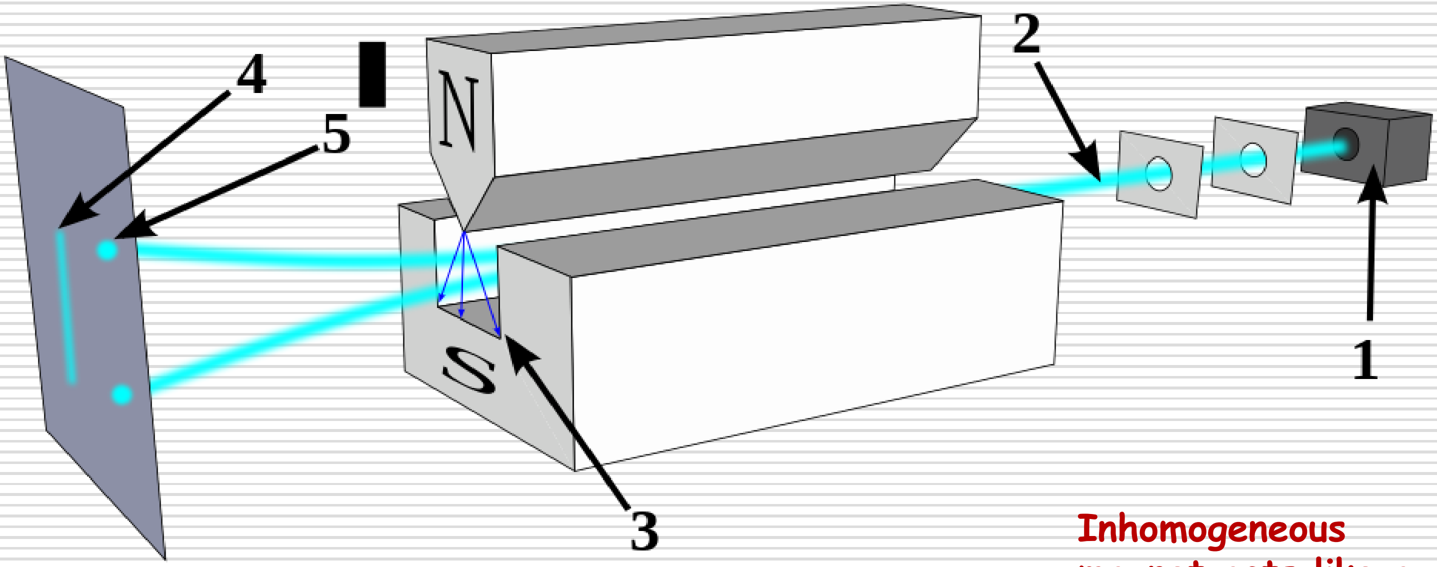




Experimentally relevant vortex properties

- Charge: own em fields: so far not featured much in electron vortex beams (strong field gradients: Lloyd et al PRL 2012,2013)
- Particle-particle interactions
- Spin: own spin magnetic moment
- Mass: responsible for linear momentum and OAM
- Dipole, quadrupole and higher moments, both electric and magnetic
- Hence: Effects of electric and magnetic fields
- Relativistic effects (Bialynicki-Birula et al, Barnett)

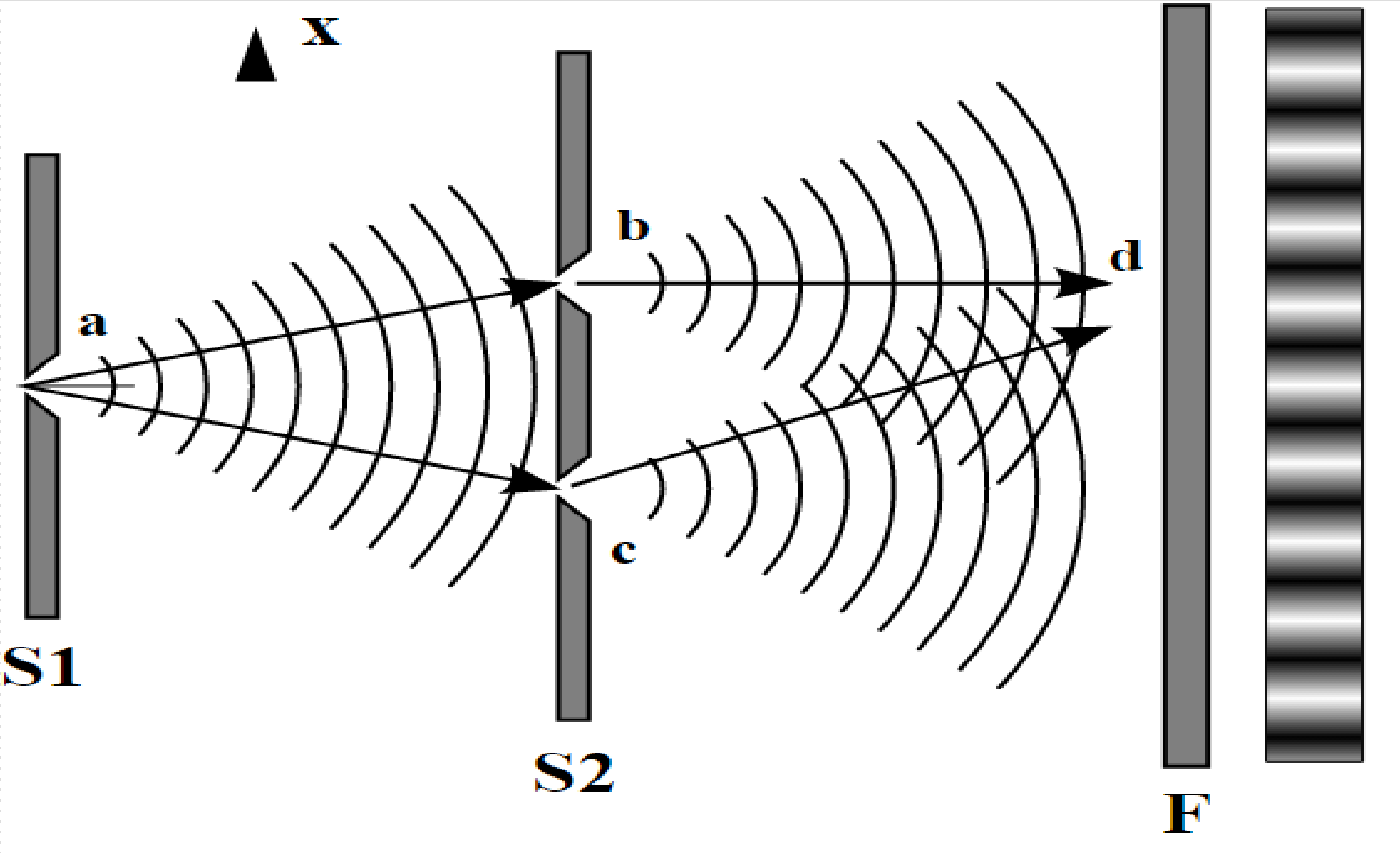
Stern-Gerlach experiment



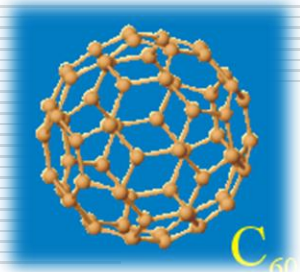
Stern-Gerlach experiment: silver atoms travel through an inhomogeneous magnetic field and are deflected up or down depending on their spin. 1: furnace. 2: beam of silver atoms. 3: inhomogeneous magnetic field. 4: expected result . 5: what was actually observed.

Inhomogeneous magnet acts like a diffractive element: property: spin magnetic moment

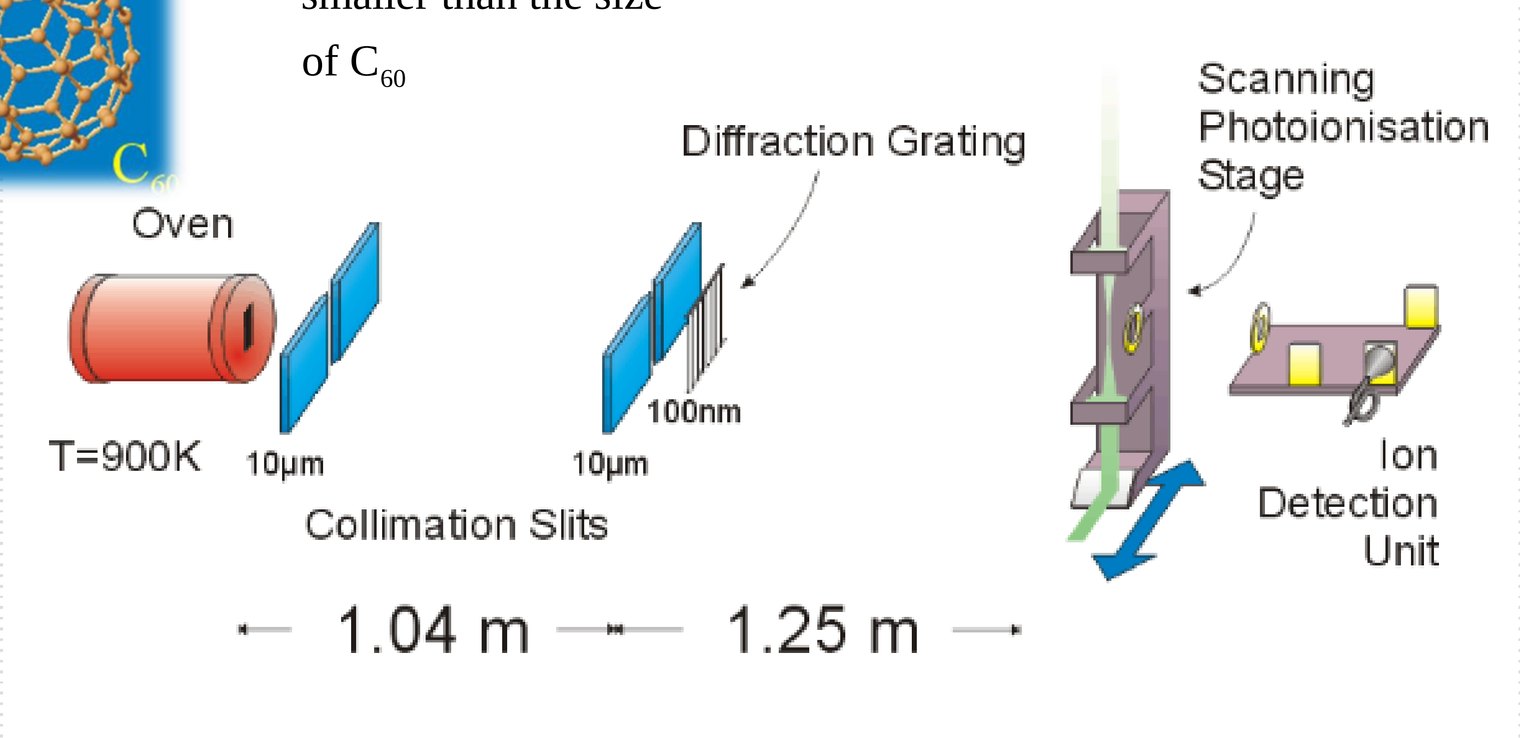
Double slit experiment



The Vienna experiments with fullerene

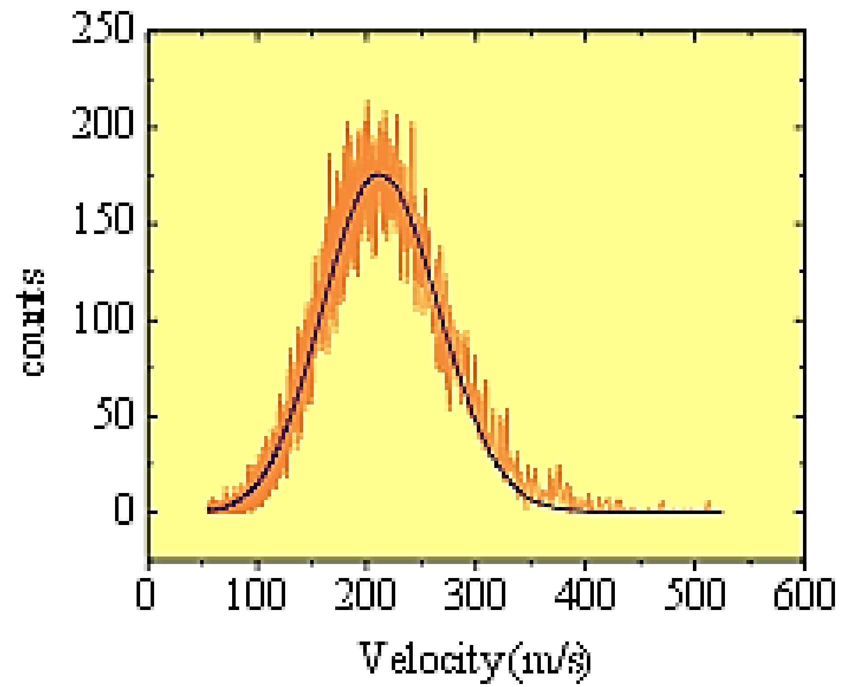


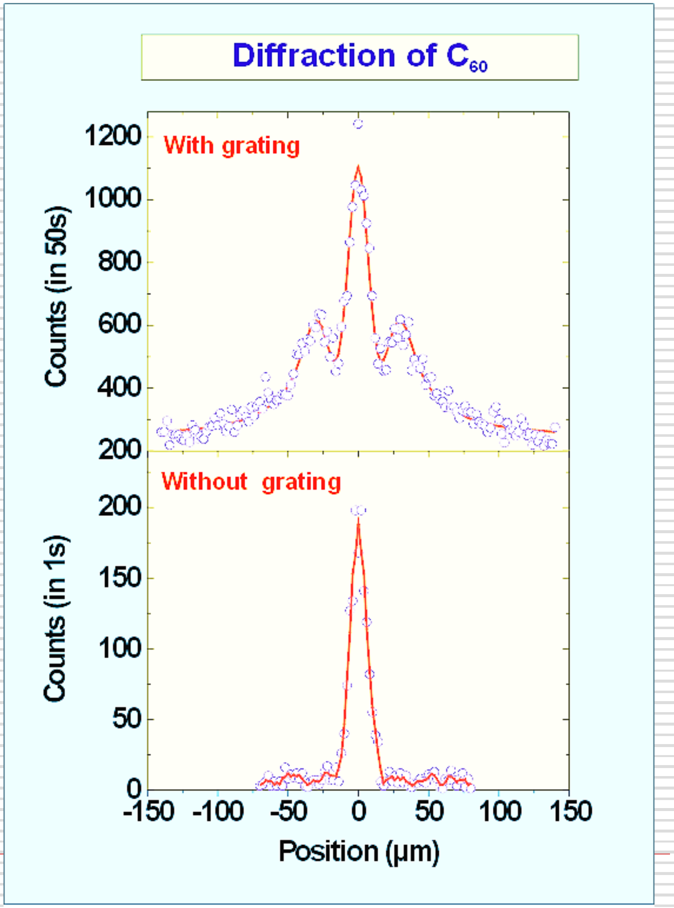
$\lambda = 2.5$ pm 400 times smaller than the size of C_{60}



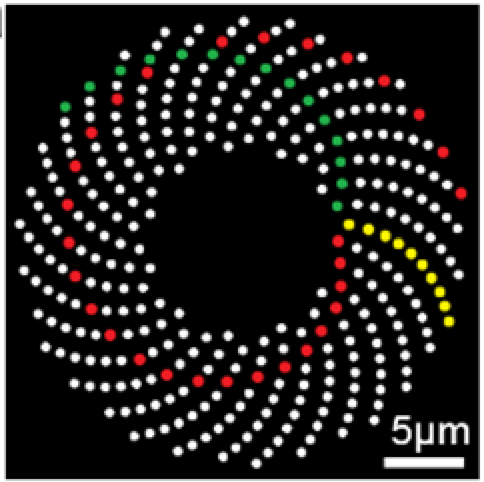
Evidence of diffraction

The most probable velocity of 210 m/s corresponds to a de Broglie wavelength for C60 of 2.5 pm !





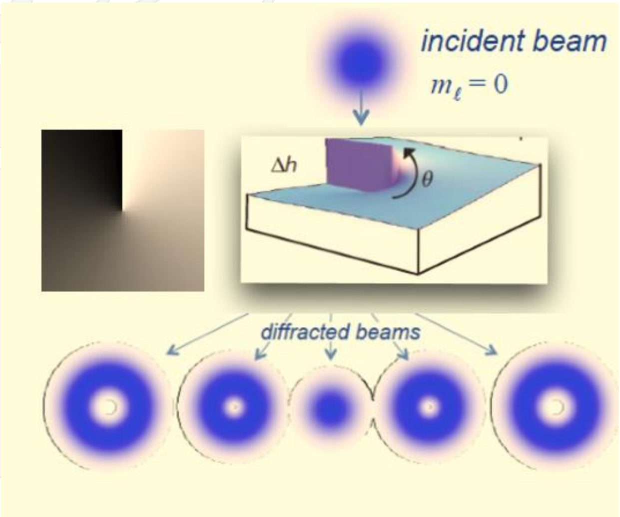
Particle vortex beam generation



Spiral sieve: exploits m-fold chiral rotational symmetry – real as well as accidental: m Fermat spirals

York group (2017)

$$\psi(\rho,\theta,z) = \begin{cases} mu(\rho,z,\theta), \ell = \mathrm{M}m \\ 0\,, \ell \neq \mathrm{M}m \end{cases}$$



Phase plate

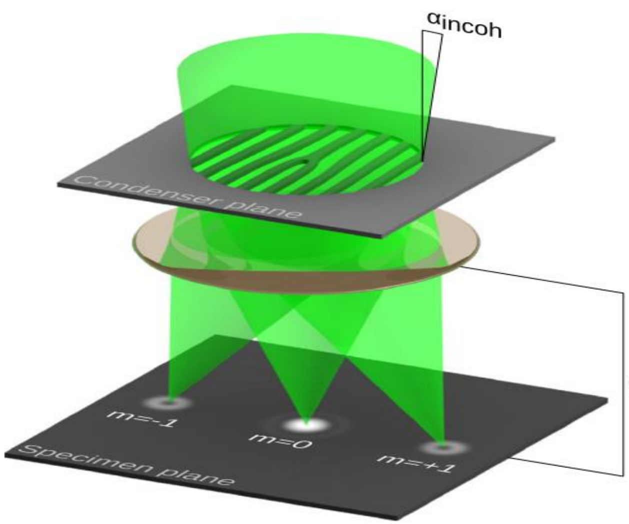


Fig. 1. Sketch of the experimental setup (not to scale). For simplicity, only the essential components are drawn. From the top, the electron beam hits the aperture with the screw dislocation (gray). There, it is diffracted into the central beam as well as side bands. These are focused by a lens (brown) on the object plane (black), where the typical donut shapes ($|m| \geq 1$) and Airy disk ($m = 0$) are well separated. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

CGH

Aperture set of modes (York group)

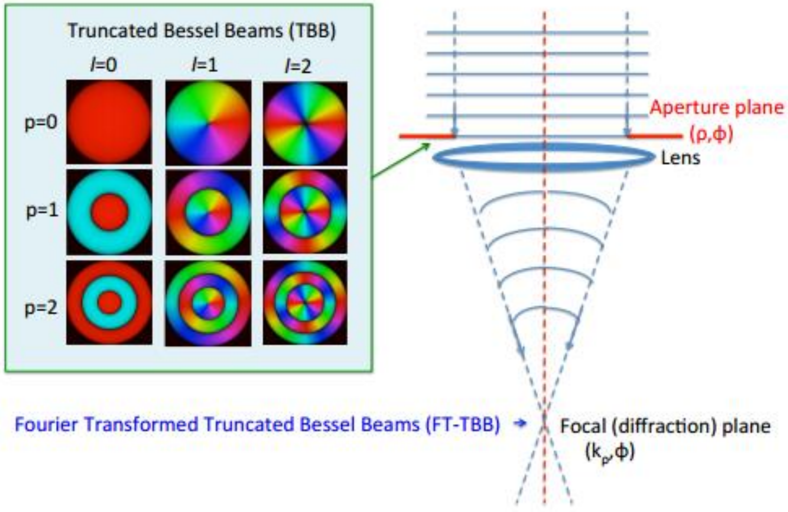


FIG. 6: (Color online) The intensity and phase distribution of the transverse wavefunctions of FT-TBB at $z = 0$. Image after ref. (Thirunavukkarasu *et al.*, 2017) and are plotted for the same relative scale.

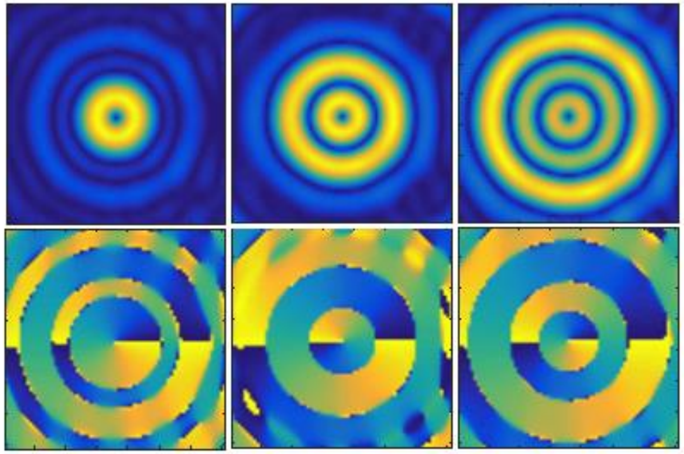
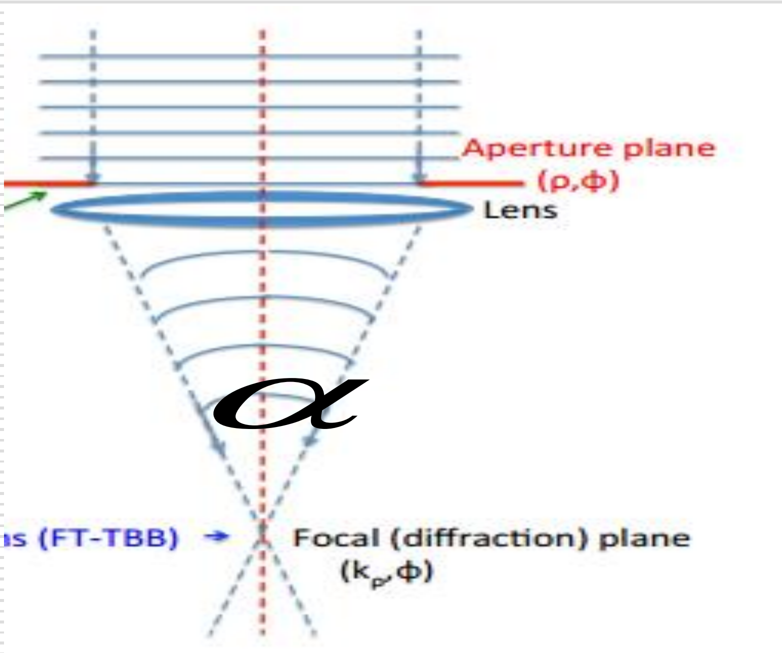


FIG. 7: (Color online) The Truncated Bessel Beam (TBB) is produced by illuminating a phase plate with a truncated plane wave, as shown in the aperture plane. The resulting transverse wavefunction is shown in the insert with the phase represented by the rainbow pseudocolor spectrum and the amplitude by the intensity. The diffraction of TBB can be recorded at the focal plane of the lens as FT-TBB. Image after ref. (Thirunavukkarasu *et al.*, 2017)

Size of a vortex doughnut



Maximum size linear momentum transferred by diffraction is

$$p = \alpha \hbar k_0$$

Angular momentum

$$L_z = |\boldsymbol{\rho} \times \mathbf{p}| \simeq l\hbar$$

Therefore estimated beam size is

$$\langle \rho \rangle \approx \frac{l\hbar}{\alpha \hbar k_0} = \frac{l\lambda}{2\pi\alpha}$$

SLAC experiment

Hemsing et al, Nature Physics (2013)

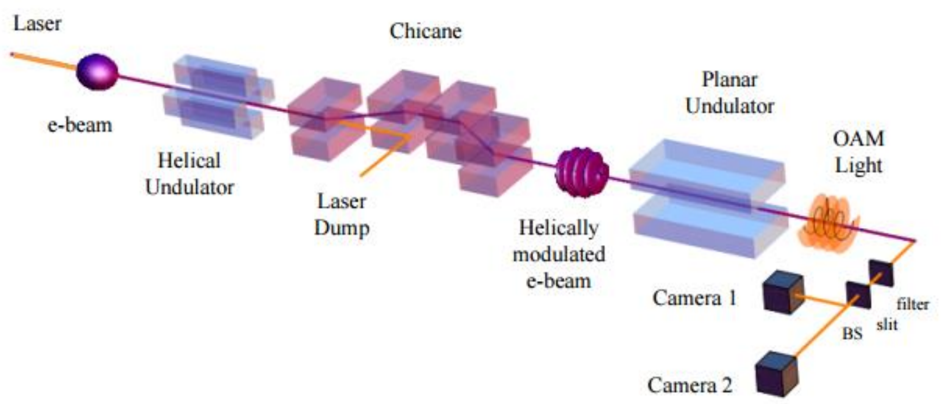


FIG. 1: Illustration of the experiment (not to scale). The unmodulated relativistic electron beam interacts with a linearly polarized laser in a helical undulator, which gives the electrons an energy kick that depends on their position in the focused laser beam. The e-beam then traverses a longitudinally dispersive chicane that allows the electrons with higher energy to catch up to those with lower energy (momentum compaction). The result is a "helically microbunched" beam that then radiates light with OAM at the fundamental frequency in the planar undulator.

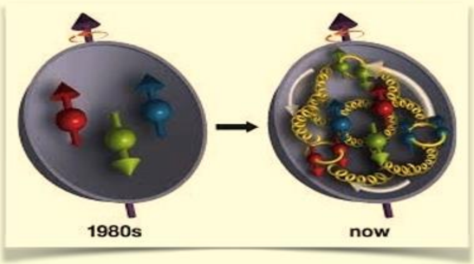
SLAC electron beam Generating optical vortex beam

Can we use SLAC-type facilities to generate other particle beams?

Nuclear physics with vortex beams

Nuclear Physics with vortex beams

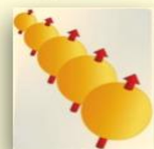
Polarized deep inelastic scattering (DIS): ~30% of the protons's spin is due to quarks spins



Quest for the remaining ~70% is a major enterprise in NP

- **quark OAM**
- **gluon spin**
- **gluon OAM**

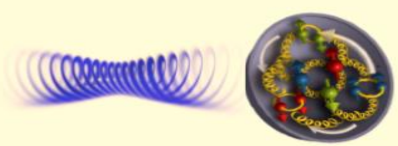
Significant effort underway: both theoretical **(GPDs, TMDs)** & experimental **(JLab, RHIC, CERN, & proposed EIC)**



Spin polarized beams & targets have been the only tools in use

It is high time we explore new tools such as OAM/vortex beams (electrons)

photon vortex beams in next talk (Y. Taira)



Leader & Lorcé: the formalism of spin+OAM for photons, applies to quarks and gluons too.

E. Leader and C. Lorcé, Phys. Rept. 541, 163 (2014).

Can we do deep inelastic scattering with vortex beams?

Courtesy D. Dutta, Sept 2016

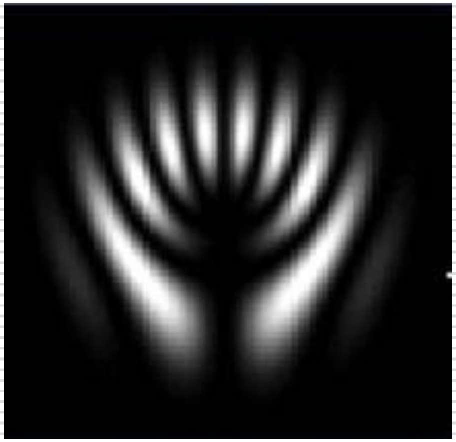
Production of atom/ion vortices.

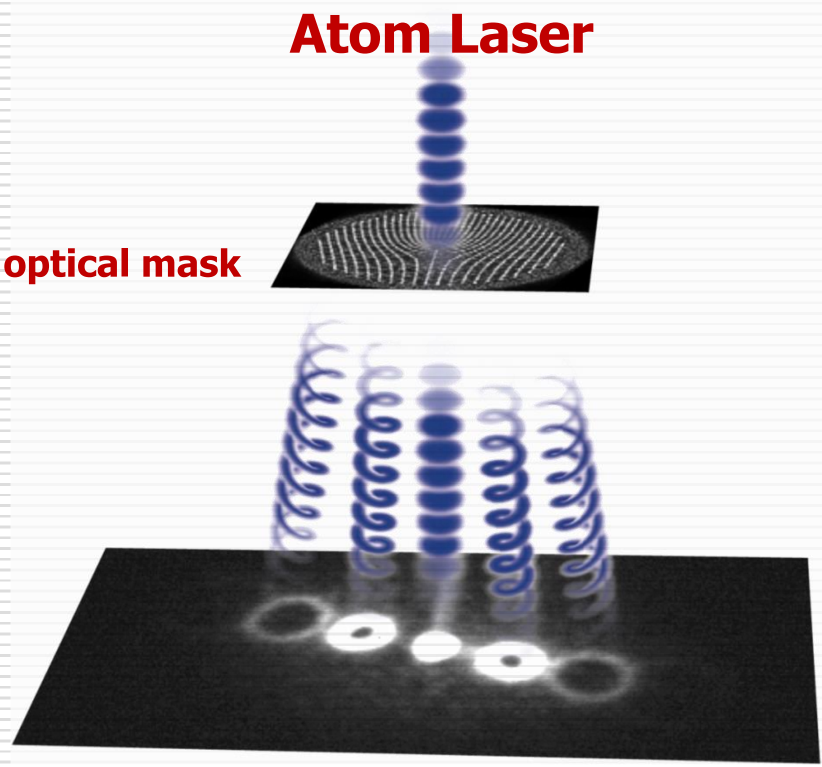
Typically for sodium cold atoms

$$E \approx 1\mu K \qquad \lambda_d \approx 1\mu m$$

Corresponding to speeds of order

$$v \approx \text{a few } mm\,\text{s}^{-1}$$

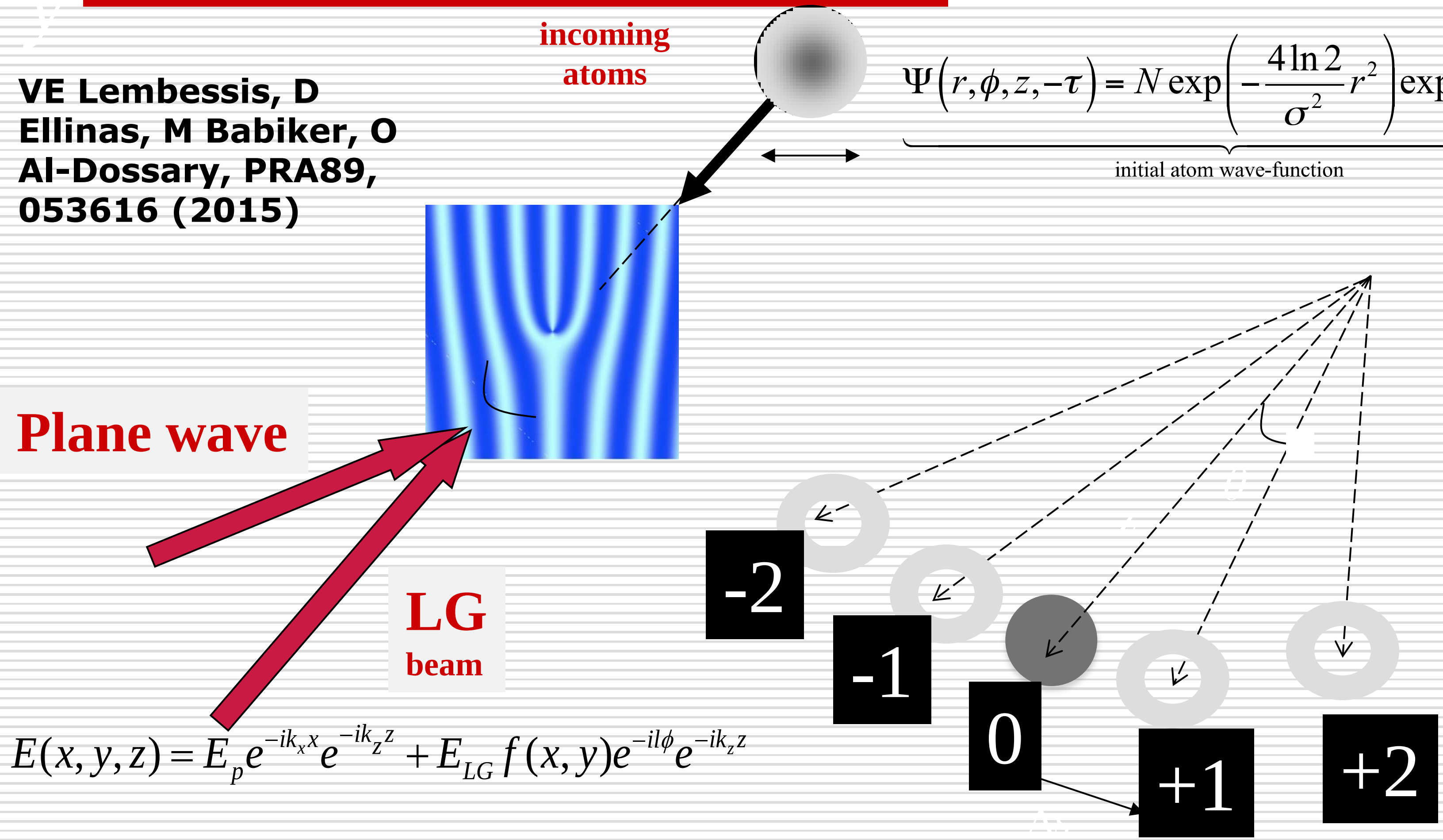




Atom vortex beams – optical mask

VE Lembessis, D Ellinas, M Babiker, O Al-Dossary, PRA89, 053616 (2015)

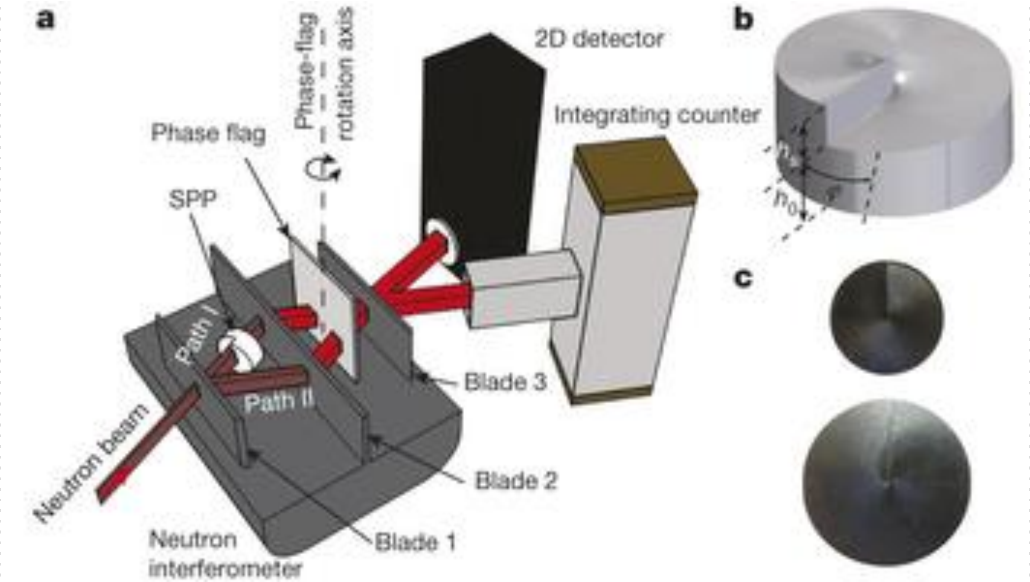
$$\underbrace{\Psi(r,\phi,z,-\tau)=N\exp\left(-\frac{4\ln 2}{\sigma^2}r^2\right)\exp\left(ik_z^{dB}z\right)}_{\text{initial atom wave-function}}$$



$$E(x,y,z)=E_p e^{-ik_x x}e^{-ik_z z}+E_{LG}f(x,y)e^{-il\phi}e^{-ik_z z}$$

Neutron Vortex beams

Clerk et al, Nature
525, 504-506 (2015)

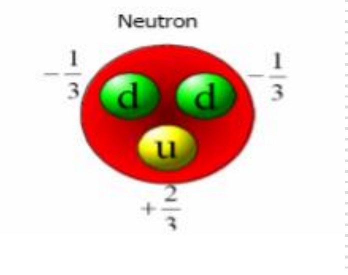


Neutrons do not normally have orbital angular momentum. But the demonstration that a beam of neutrons can acquire this property, 23 years after it was shown in photons, offers the promise of improved imaging technologies. R W Boyd: Nature (2015)

The phase plate for imparting extra quantum units of orbital angular momentum. Neutron waves fall on the face of this plate, made by milling a dowel of aluminum into a ramp-shaped spiral. The steeper the pitch of the milled phase plate, the more orbital angular momentum will be imparted to the neutron beam. Neutron interferometry

Neutrons

- Half-life of about 15 minutes (beta decay) $n^0 \rightarrow p^+ + e^- + \bar{\upsilon}_e$
- Neutral, so cannot be accelerated
- Typical release energies of order several MeV (e.g. reactor)
- Magnetic moment= **−1.913** μ_N .
- High penetration in most materials – imaging
- The structures of metal hydride complexes
  e.g. MgFeH have been assessed by neutron diffraction



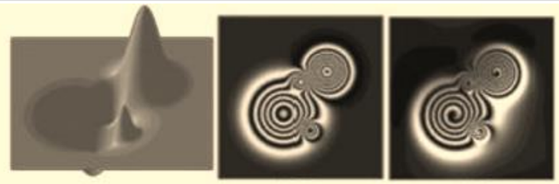
- Neutron analysis of materials – use in sensors
- Are these properties likely to be enhanced with neutron vortex beams?

Neutrinos

- Spin $\frac{1}{2}$; zero charge; tiny mass
- Interact weakly with other particles and fields
- Can be used to probe environments better than other forms of radiation (Earth's, Sun and other planets' cores)
- Inter-stellar exploration (neutrino telescopes)
- Communication: coherent neutrino message sent through 780 feet of rock: Stancil et al *Modern Physics Letters A*. 27 (2012). Future research may permit binary neutrino messages to be sent immense distances through even the densest materials, such as the Earth's core.
- Neutron vortex beams:first discussed by Hayrapetyan, Gotte and Dennis (NY ICOAM)
- What benefits can emerge from vortex neutrino beams?

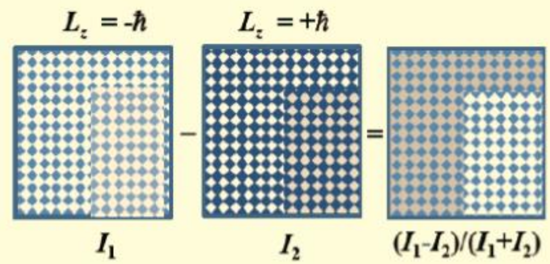
Applications (Electrons)

Higher resolution imaging
(spiral interferometry)



Fürhapter et al. Opt. Lett. 2005

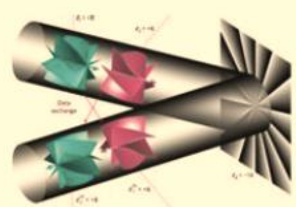
Novel microscopy
(STEM/EELS with vortex probe)



B. J. McMorran et al

atomic resolution map
of magnetic moments

Terabit communication
(OAM multiplexing)

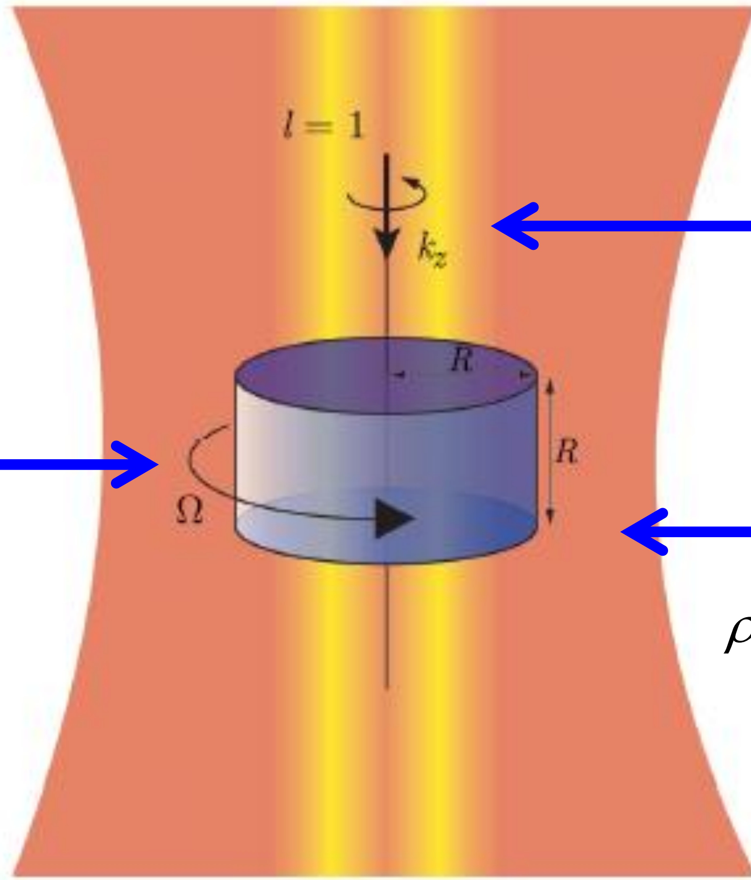


Wang et al. Nature Photon 2012

Nano fabrication

Opto-partico mechanics

SM Lloyd, M Babiker, J Yuan, PRA 88 031802 (2013)



Laser beam for levitation

Rotation due to particle vortex torque

Particle vortex beam

$$\rho(r) = m\Psi^* \Psi: \text{ mass density}$$

$$\text{Linear momentum} = \hbar k_z$$

$$\text{Angular momentum} = \hbar \ell$$

Conclusions

- Basic concept: quantum diffraction of particles – particles as large as Bucky-balls (C_{60})
- Particle vortex beam generation – electrons (being done: mostly mimicking optical vortex work).
- Neutrons: first experiments reported– further work needed
- Neutrinos – no experimental work yet; little theory
- Protons; work would be desirable
- Neutral atom and ion vortex beams: only theory so far
- Molecules: ditto
- Other elementary particles? Too early to say!
- Prospects of using particle vortex beams in communications; imaging; sensing; material analysis; astronomy, medicine

APS March Meeting 2016;Volume 61, Number 2; March 14–18, 2016; Baltimore, Maryland

Abstract: Y52.00013 : Particle beams carrying orbital angular momentum, charge, mass and spin

Authors: Teuntje Tijssen
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(Max Planck Institute for the Physics of Complex Systems, Dresden);Joerg Goette
(Max Planck Institute for the Physics of Complex Systems, Dresden);Mark Dennis
(H H Wills Physics Laboratory, University of Bristol)

Electron beams carrying vortices and angular momentum have been of much experimental and theoretical interest in recent years. In addition, optical vortex beams are a well-established field in optics and photonics. In both cases, the orbital angular momentum associated with the beam's axial vortex has effects on the overall spin of the beam, due to spin-orbit interactions. A simple model of these systems are Bessel beam solutions (of either the Dirac equation or Maxwell equations) with a nonzero azimuthal quantum number, which are found by separation in cylindrical coordinates. Here, we generalize this approach, considering the classical field theory of Bessel beams for particles which are either massive or massless, uncharged or charged and of a variety of different spins (0, 12, 1, …). We regard the spin and helicity states and different forms of spin-orbit terms that arise. Moreover, we analyse the induced electromagnetic field when the particles carry charge. Most importantly, this unified field theory approach leads to the prediction of effects for vortex beams of neutrons, mesons and neutrinos.

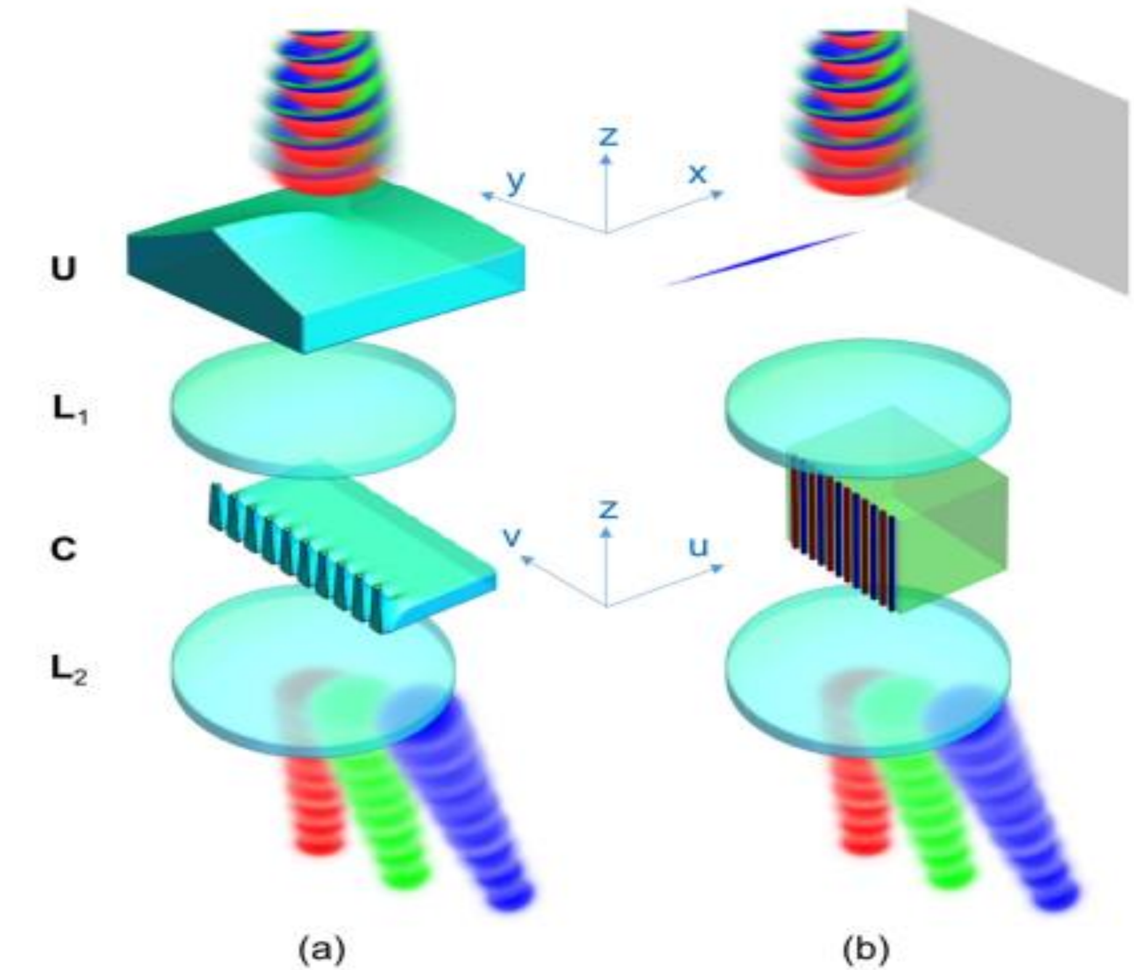


Figure 1. Schematic of the optical arrangement of OAM-sorting devices for (a) light and (b) electrons. Different OAM states are shown in different colors. Mixed OAM states are incident on the top of both systems, each of which consists of four elements. A phase unwrapper element U in the front focal plane of a lens L1 is followed by a phase corrector element C in the back focal place of L1. For electrons, the proposed element U is a charged needle or knife edge, and the corrector element C is an array of electrodes with alternating bias. Immediately after the corrector element C, different OAM components are separated in momentum space. At the bottom of each device, a Fourier-transforming lens L2 separates OAM components into different spots in real space at the output.

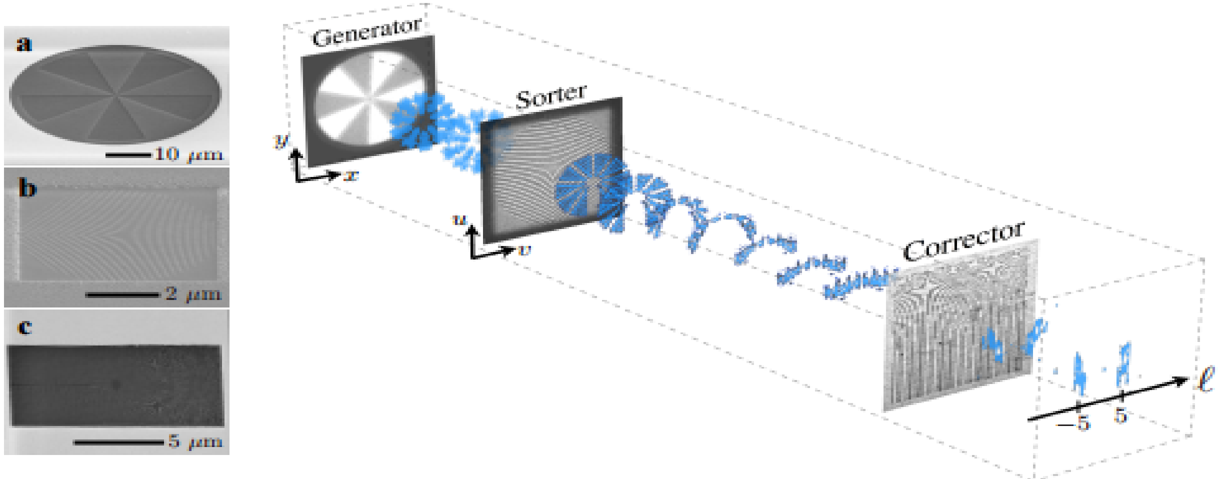


FIG. 1. **Schematics of the electron sorter depicting TEM images of the phase holograms.** These schematics also show an electron beam's experimental transverse intensity profile recorded at various planes in the sorting apparatus. A hologram in the sorter's generator plane, which corresponds to the electron microscope's condensor, produces an electron beam carrying OAM. In this particular case, the beam consists of a superposition of ±5 OAM states. The beam then goes through a hologram in the apparatus' sorter plane, positioned at the microscope's sample holder, which performs the required conformal mapping $(x, y) \mapsto (u, v)$. Once the beam is unwrapped, it passes through a hologram in the sorter's corrector plane corresponding to the microscope's SAD aperture. This hologram brings corrections to any phase defects to the beam in order to stabilize its propagation through the rest of the sorter. At the sorter's output, the original beam's OAM content is spatially resolved on a screen and captured by a CCD camera. Scanning electron microscopy (SEM) images of the depicted holograms, the ones in the generator, sorter, and corrector planes, are shown in **a**, **b**, and **c**, respectively.

Coherent Optical Vortices From Relativistic Electron Beams

Erik Hemsing, Michael Dunning, Dao Xiang, Agostino Marinelli, and Carsten Hast
SLAC National Accelerator Laboratory,

Nature Physics (2013) doi:10.1038/nphys2712.

- Recent advances in the production and control of high-brightness electron beams (e-beams) have enabled a new class of intense light sources based on the free electron laser (FEL) that can examine matter at ˚Angstrom length and femtosecond time scales. The free, or unbound, electrons act as the lasing medium, which provides unique opportunities to exquisitely control the spatial and temporal structure of the emitted light through precision manipulation of the electron distribution. We present an experimental demonstration of light with orbital angular momentum (OAM) generated from a relativistic e-beam rearranged into an optical scale helix by a laser. With this technique, we show that a Gaussian laser mode can be effectively up-converted to an OAM mode in an FEL using only the e-beam as a mode-convertor. Results confirm theoretical predictions, and pave the way for the production of coherent OAM light with unprecedented brightness down to hard x-ray wavelengths for wide ranging applications in modern light sources.

Holography with a neutron interferometer

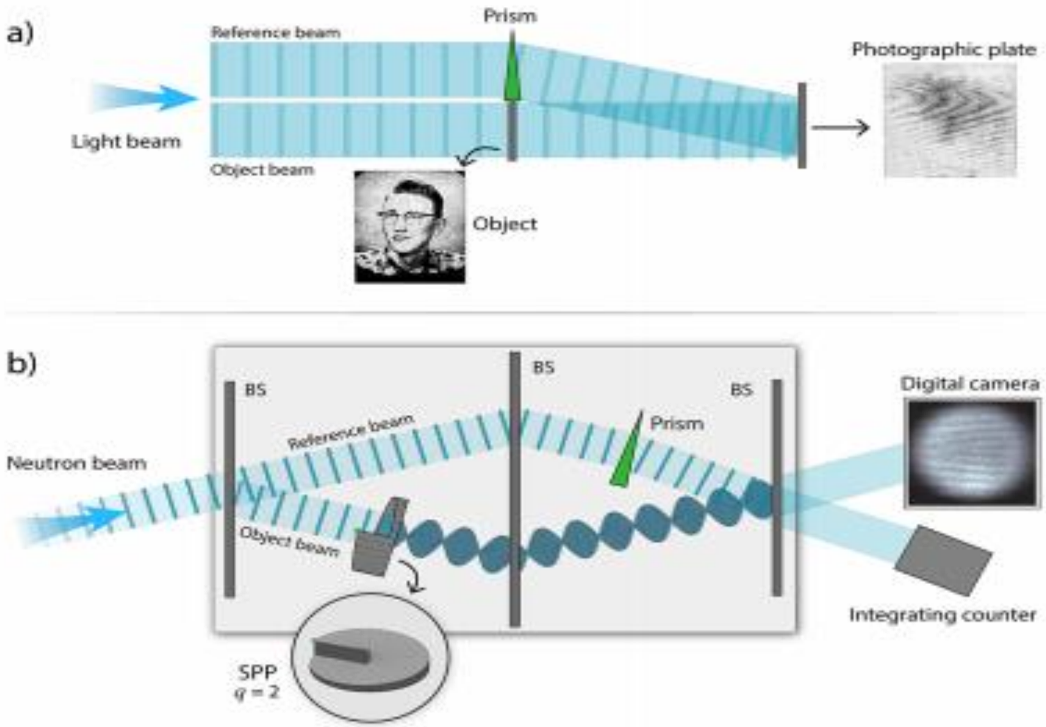


Fig. 1. (a) The off-axis method of optical holography of semitransparent objects introduced by Leith and Upatnieks. The object here is a continuous-tone transparency; its hologram was recorded on a photographic plate. Redrawn from figures of [8]: the object shown here is actually the holographic reconstruction of the original object, said to be a good facsimile of the original. (b) An artistic depiction of the neutron holography experiment. A neutron enters a single-crystal silicon Mach-Zehnder neutron interferometer (NI) and is separated into two paths by the left beamsplitter (BS). A spiral phase plate (SPP) with q = 2 is placed in the lower path, generating the object beam; a prism tilts the wavefront of the upper path to provide the reference beam. Object and reference beams are reflected at the central BS, and are coherently combined at the right BS. One of the output beams of the right BS is sent to an imaging detector, the other to an integrating counter that serves as an intensity monitor. Note that the experiment is an expectation valued measurement over many events, each of which involves only a single neutron. That is, there is one neutron at a time in the NI and the hologram is build up from an incoherent superposition of many events.

Abstract: We use a Mach-Zehnder interferometer to perform neutron holography of a spiral phase plate. The object beam passes through a spiral phase plate, acquiring the phase twist characteristic of orbital angular momentum states. The reference beam passes through a fused silica prism, acquiring a linear phase gradient. The resulting hologram is a fork dislocation image, which could be used to reconstruct neutron beams with various orbital angular momenta. This work paves the way for novel applications of neutron holography, diffraction and imaging.

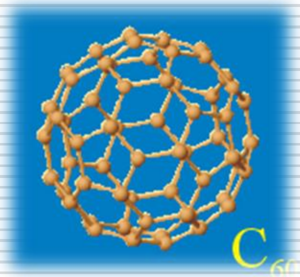
Sarenac et al, Optics Express 2016

Creation of vortex beams

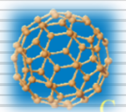
- Need a good source of particle beams
- Optical vortex beams: well studied; CGH and other methods , but SLAC experiments produced coherent OVs (including X-rays) from relativistic electron beams (Hemsing et al, 2013)
- Electron vortex beams: so far in electron microscopes also predominantly using CGH, but there are many other techniques
- Neutron vortex beams: phase plate method so far
- Neutral atoms vortex beams; CGH optical mask technique (theory: Lembessis et al)
- Neutrino vortex beams (theory: Hayrepetyan, Dennis, Gotte)
- Can we create a vortex beam of any particle kind?

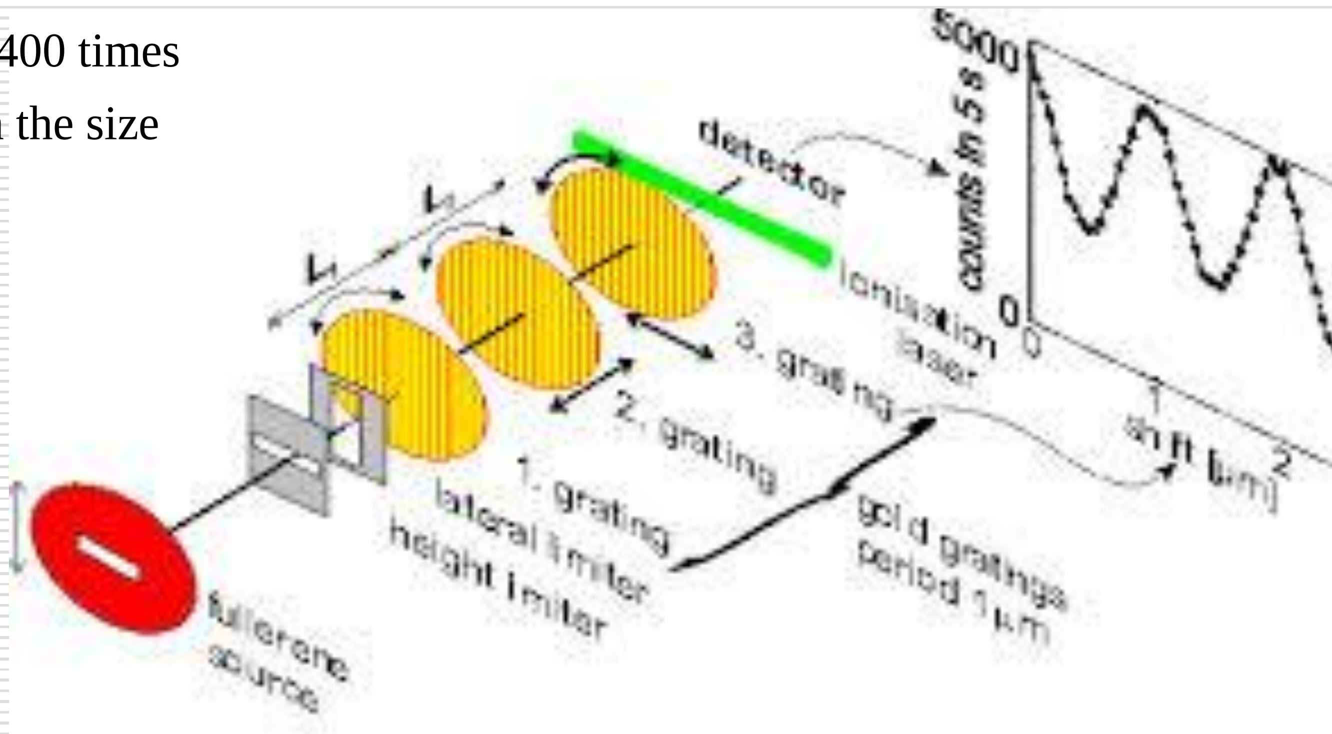
Interference of large molecules

The Vienna experiments with fullerene



$\lambda = 2.5$ pm 400 times smaller than the size of C_{60}



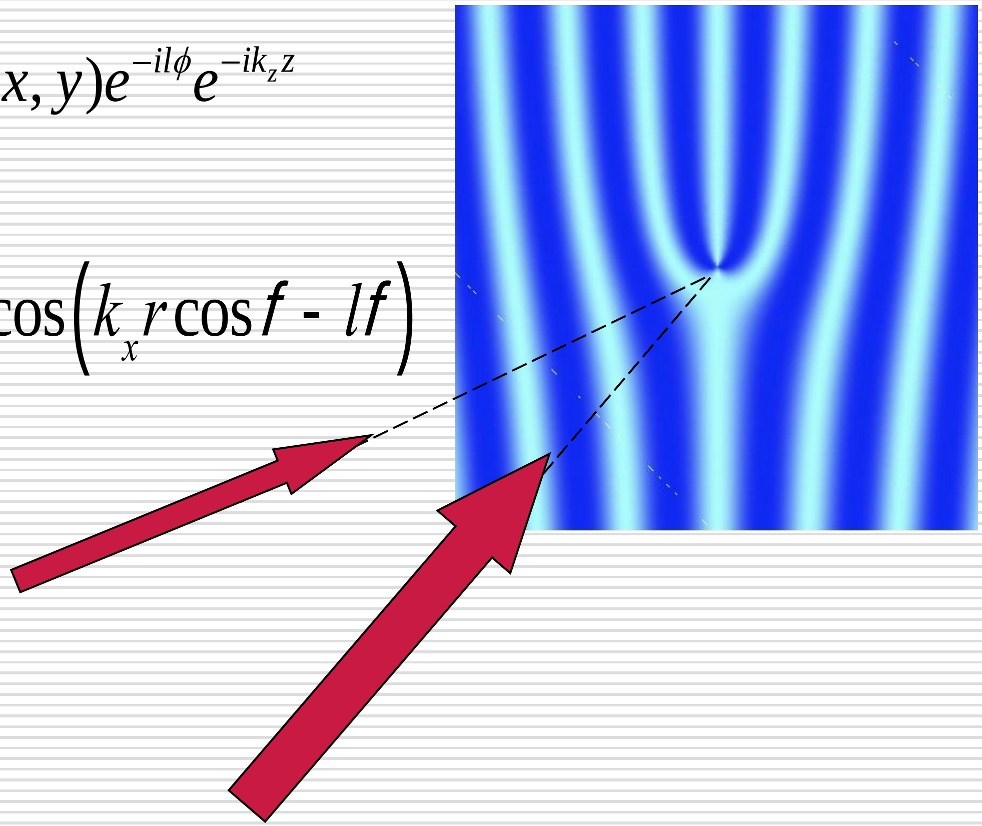


Talbot- Lau interferometer

Diffraction of atoms in fork-like light field

$$E(x,y,z) = E_p e^{-ik_x x} e^{-ik_z z} + E_{LG} f(x,y) e^{-il\phi} e^{-ik_z z}$$

$$I = I_p + I_{G-L} f^2(r) + 2\sqrt{I_p I_{G-L}}\, f(r) \cos\left(k_x r \cos f - l f\right)$$



Electron vortex beams: CGH method

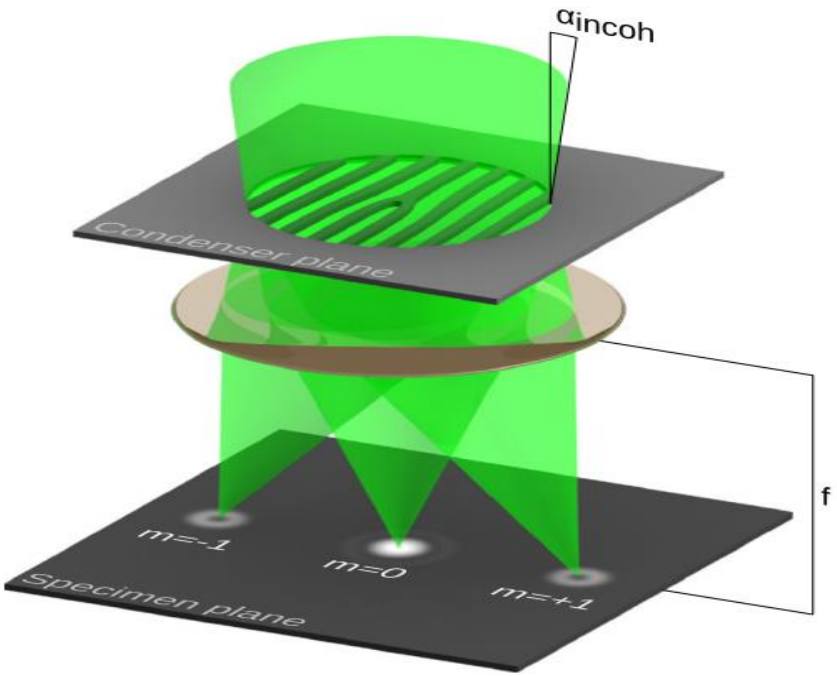


Fig. 1. Sketch of the experimental setup (not to scale). For simplicity, only the essential components are drawn. From the top, the electron beam hits the aperture with the screw dislocation (gray). There, it is diffracted into the central beam as well as side bands. These are focused by a lens (brown) on the object plane (black), where the typical donut shapes ($|m| \geq 1$) and Airy disk ($m = 0$) are well separated. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

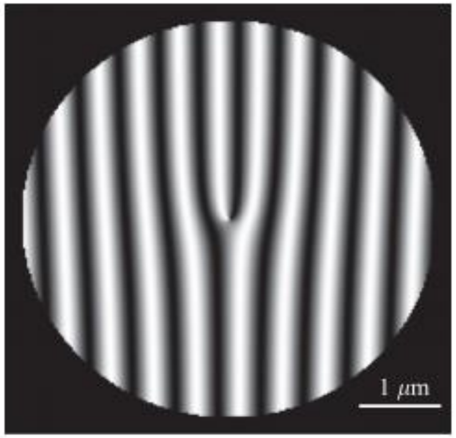


Fig. 4. Transmission function Eq. (21) for creating vortex beams. The aperture has 5 μm diameter. Scale in μm.

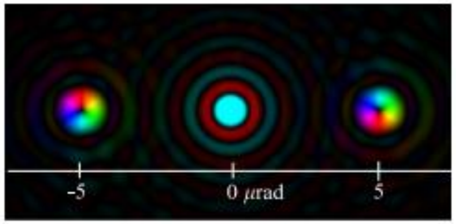


Fig. 5. Diffraction pattern of the mask in Fig. 4 obtained by Fourier transform, showing the central Airy disk and two focussed vortices. The Bragg angle at 200 kV is 2.5 μrad. The phase is coded as hue. The helical structure of the vortices is well visible (Color coding: Rainbow chart from 0 to 2π). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Schattschneider, Ultramic 115, 21-25 (2012).