Towards an open problem — polymer adsorption

A polymer near a sticky surface may undergo a transition

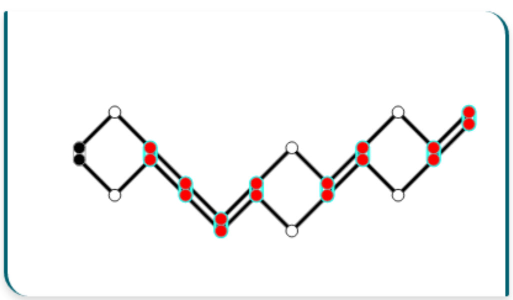
- Adsorption transition driven by short-range attractive force
- Relevant partition function is

$$Z_n(a) = \sum_{\varphi} a^{v(\varphi)}$$

where $v(\varphi)$ is number of visits to surface

Towards an open problem — polymer zipping

A pair of polymers may also undergo a zipping transition



- Zipping transition driven by short-range attractive force
- Relevant partition function is

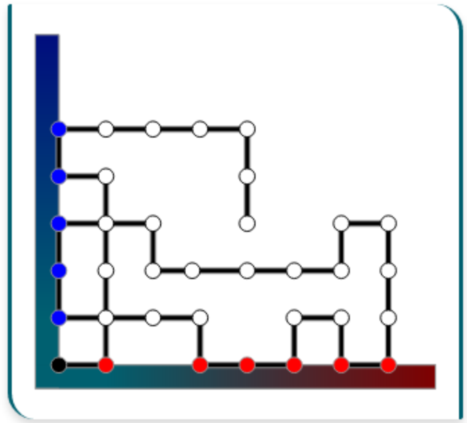
$$Z_n(c) = \sum_{\varphi} c^{m(\varphi)}$$

where $m(\varphi)$ is number of pairs of bound vertices
- Similarly for three or more polymers
— see Aleks' talk and Tabbara Owczarek & R. 2016

Quarter plane walks with interacting boundaries

We need to count walks in quarter plane with

- $a \cong$ visits to one wall
- $b \cong$ visits to the other wall



Uglification of coefficients

Return to $\mathcal{S} = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$ and double count carefully

$$f(r,s) + \frac{1}{a} f(r,0) + \frac{1}{b} f(0,s) + \left(\frac{1}{ab} - \frac{1}{a} - \frac{1}{b}\right) f(0,0)$$
$$= f(r,0) + f(0,s) - f(0,0) + \frac{1}{ab} + z(r + \bar{r} + s + \bar{s}) f(r,s)$$
$$- z\bar{s} f(r,0) - z\bar{r} f(0,s)$$

Clean up

$$f(r,s) \cdot K = \frac{1}{ab} + \left(1 - \frac{1}{a} - z\bar{s}\right) f(r,0) + \left(1 - \frac{1}{b} - z\bar{r}\right) f(0,s)$$
$$- \left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right) f(0,0)$$

Nature of solution changes

Step set unchanged, so same kernel

- kernel symmetries unchanged
- group unchanged
- system symmetry broken — $f(r,s) \neq f(s,r)$

But in this case half-orbit sums + careful coefficient extraction work

$$F(a,b) = \frac{1}{(a-1)(b-1)} + \frac{p_0(a,b;z)}{p_1(a,b;z)F(a,1) + p_2(a,b;z)F(1,b) + p_3(a,b;z)}$$

$$F(a,b) \equiv f(0,0) \text{ and } p_j = \text{polynomials}$$

Does not appear to remain D-finite.

On the other hand

The  solution has this form, as do:

-  solved general a,b — Tabbara Owczarek & R. 2014
-  solved $a=b$, but not $a \neq b$ — Tabbara Owczarek & R. 2016 (Aleks' talk)

On the other hand

-  remains D-finite and very similar
- Kreweras  remains algebraic but nastier
  - $a=b$ is algebraic Owczarek & R. (today minus 4 days)
  - $a \neq b$ appears to be algebraic based on series
  - broken symmetry means method of Bousquet-Mélou 2005 uglifies

Open problem 1'

Find a more direct combinatorial explanation of the form:

$$F(a,b) = \frac{1}{(a-1)(b-1)} + \frac{p_0(a,b;z)}{p_1(a,b;z)F(a,1) + p_2(a,b;z)F(1,b) + p_3(a,b;z)}$$

Stretch problem

Catalogue quarter plane walks with small steps and interacting boundaries