# A Fault in Time and Space: Spatial models for past and future Cascadia earthquakes

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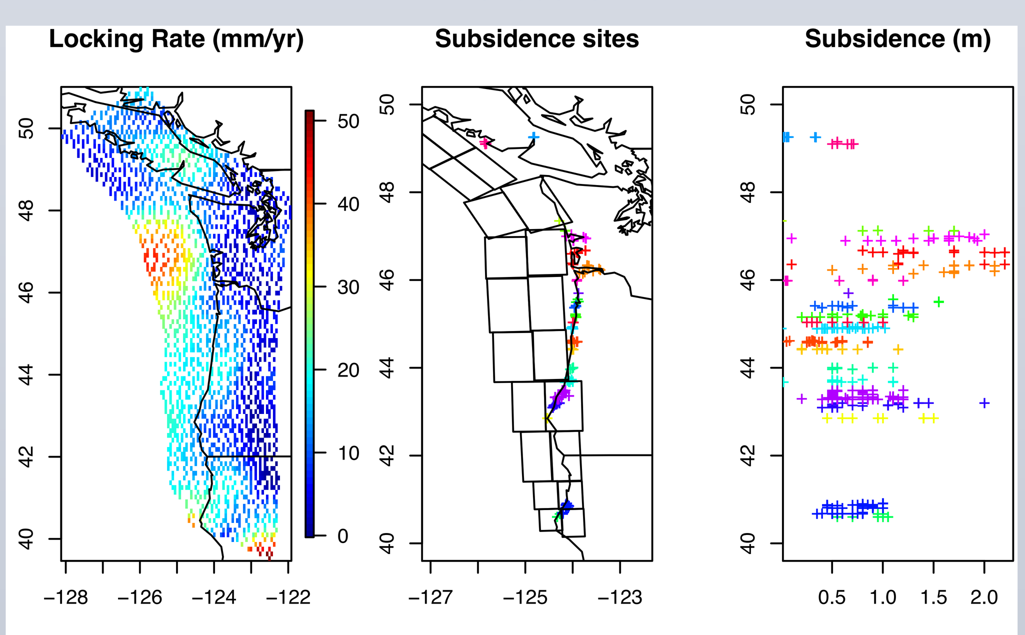
## Introduction

Substantial risk of an M9 earthquake on the Cascadia Subduction Zone (CSZ) exists, yet few fully likelihood-based spatial models have been developed for them. Many studies use just a handful of predetermined earthquakes to represent the full range of those possible. While Lévy Processes have been used to model slips due to their convenient stability properties [2], they are heavy-tailed to the point of having all moments infinite, which is unrealistic. Here we combine paleoseismic subsidence data [3] collected along the US and Canadian west coast with GPS-based fault locking rate estimates [1] over the CSZ megathrust to fit a fully stochastic spatial-statistical model for earthquake slips.

**Research aim: to better understand and predict variations in the spatial coseismic slip distributions of major Cascadia earthquakes.**

**Figure**: Possible subsidence evidence along the Duwamish river.

## Data




Subsidence data [3]:

How much the ground sunk along the coast from major EQs in the last 7,000 years

Locking rate data product [1]:

How tension is building up over the fault spatially. Based on GPS observations over last 30 years for idea of correlation and variability scales of EQ slip.

Uncertainty:

Rough estimates for both datasets. Subsidence SD inflation estimated from data.

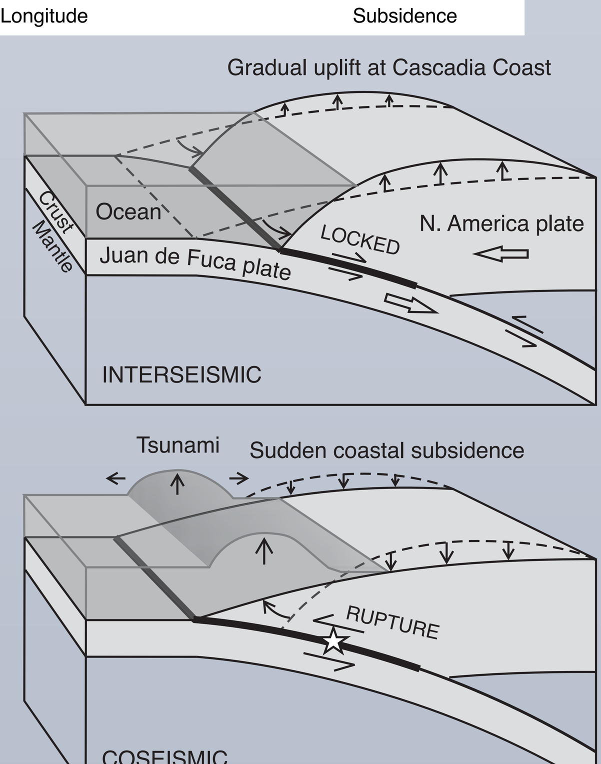
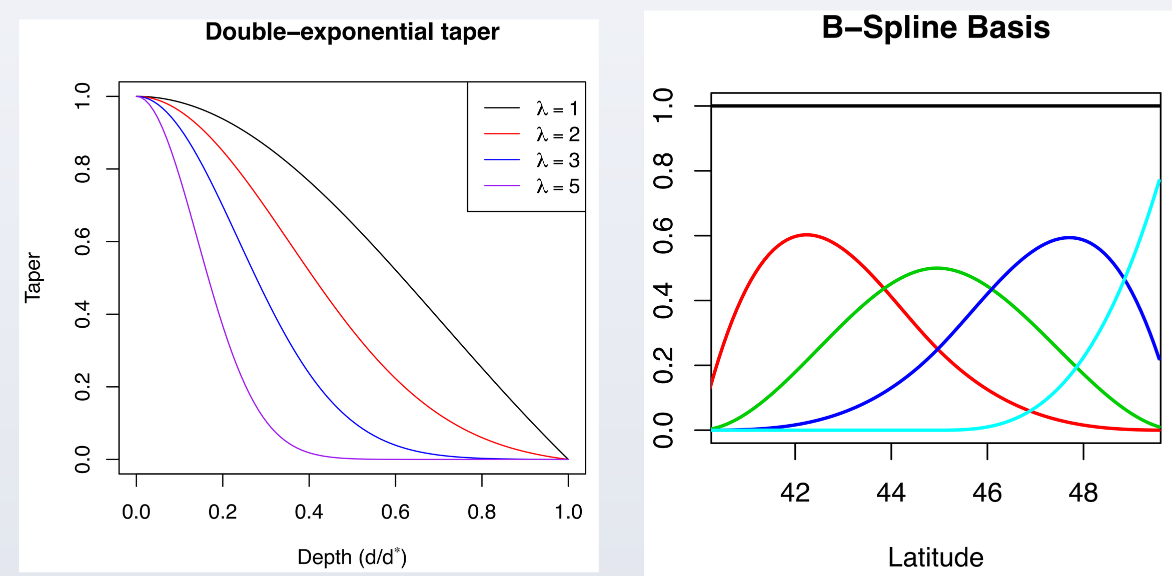



**Figure**: Image from [3]. Between earthquakes, the ground often rises as tension builds before the next quake. After the quake, the ground will subside back to its original state

## Model

### Slip Taper




$$t(d;\lambda) = 1 - \frac{1-\exp\{-(d/d^*)^2\lambda^2\}}{1-e^{-\lambda^2}}$$

$d$ : depth (m)

$d^*$ : max depth (m)

$\lambda$ : rate parameter

Taper function reduces earthquake slip as a function of depth along the fault.. The tapering rate is modelled as a function of latitude using a B-spline basis expansion

### Slip Distribution

$$\vec{X} = \gamma\vec{\zeta} + \vec{\xi} \quad \text{(Gaussian model)}$$

$$\log(\vec{X}) = \mu_\xi + \log(\vec{\zeta}) + \log(\vec{\xi}) \quad \text{(Lognormal model)}$$

$$\vec{Y} = GT\vec{\zeta} + \vec{\epsilon}$$

$\vec{X}$ : locking rate data

$\vec{Y}$ : subsidence data

$\gamma$ (or $\mu_\xi$) : scaling factor

$\vec{\zeta}$ : untapered slips

$\vec{\xi}$ : locking rate uncertainty

$G$ : Okada model giving subsidences from slips

$T$ : diagonal taper matrix

$\vec{\epsilon}$ : subsidence uncertainty

We model three difference marginal slip distributions: normal, positive (truncated) normal, and lognormal.

## Results

Normal — Positive Normal — Lognormal

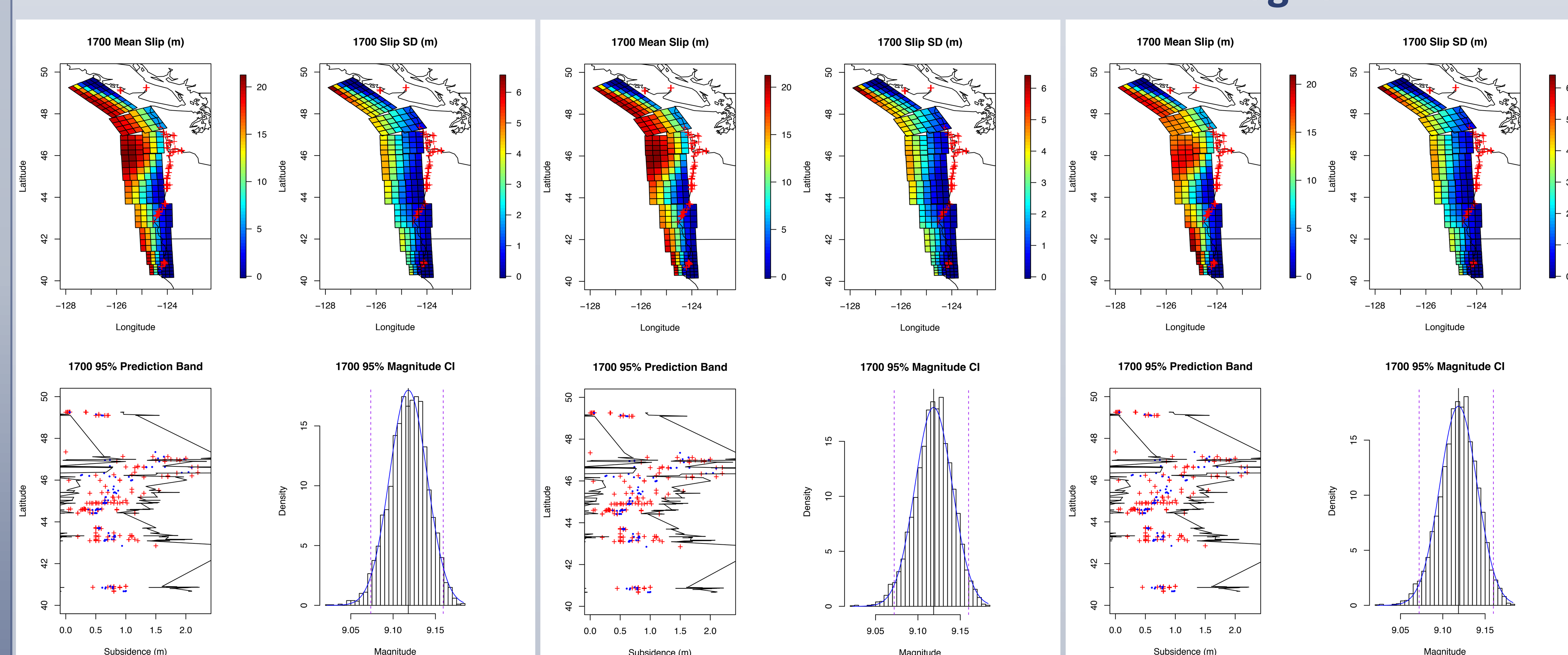



**Figure**: Expected earthquake slips (top-left), their standard deviations (top-right), subsidence data and predictions (bottom-left) and magnitude distribution (bottom-right) for normal, positive normal, and lognormal models, all predictions being for the 1700 event. Subsidence data is shown as the "+" symbol, and the 95% prediction bands for the subsidences is given as black lines in the bottom-left plot.

### Cross-Validation

| | | normal | positive normal | lognormal |
|---|---|---|---|---|
| Marginal | MSE | 0.47 | 0.33 | 0.74 |
| | bias | 0.34 | 0.17 | 0.52 |
| | variance | 0.16 | 0.10 | 0.27 |
| 1700 | MSE | 0.32 | 0.32 | 0.33 |
| | bias | 0.15 | 0.15 | 0.14 |
| | variance | 0.0061 | 0.0059 | 0.0089 |

**Table:** Cross-validation performed for subsidence data under the marginal distribution (what would be used to predict any future earthquake), and the predictive distribution for the 1700 earthquake event.

## Conclusions

While CV seems to imply the slip distributional assumptions do not impact 1700 event predictions, the assumptions make a much bigger difference for marginal distribution error, which is effectively the error for predicting future events where we have no subsidence observations to base predictions off of. The positive Gaussian model seems to perform the best in that setting, with uniformly less bias, variance, and MSE than the Gaussian model, which has lower bias, variance, and MSE than the lognormal model. The parameters and distributions used for the positive Gaussian model were the same as for the standard Gaussian model except simulations with negative values were thrown out, implying that the parameters of the positive Gaussian model could be further optimized to produce a better fit. This also implies that the fit of the model is highly dependent on distributional assumptions when making future predictions, while being more robust to distributional assumptions as the number of subsidence observations increases for historical earthquakes.

## Future Work

It would be possible to use a Gaussian-Log-Gaussian mixture model introduced in [4] or something similar to better account for varying skewness of the slip distribution throughout space. Additionally, it will be important to better account for nonstationarity in slip over the shallowest portions of the CSZ fault, since the taper primarily affects the medium depth portions of the fault. Aside from just creating better models for earthquake slip, it will also be important to use earthquake slip models for generating random tsunamis to account for the highly nonstationary and nonlinear tsunami inundation distributions of CSZ earthquakes along the west coast.

## References

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