Characterizing pseudolinear drawings in the plane

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Joint work with
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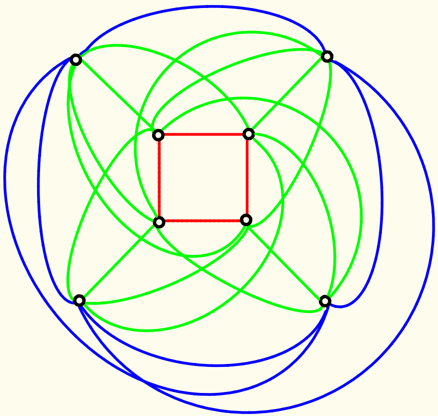
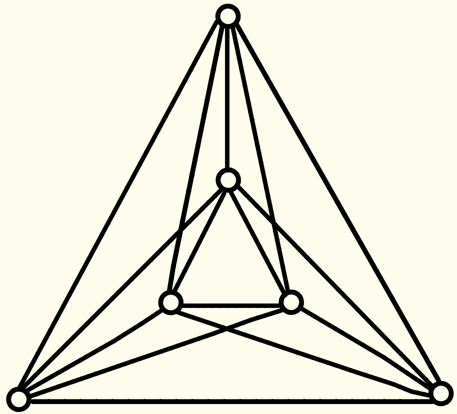
Harary-Hill Conjecture:

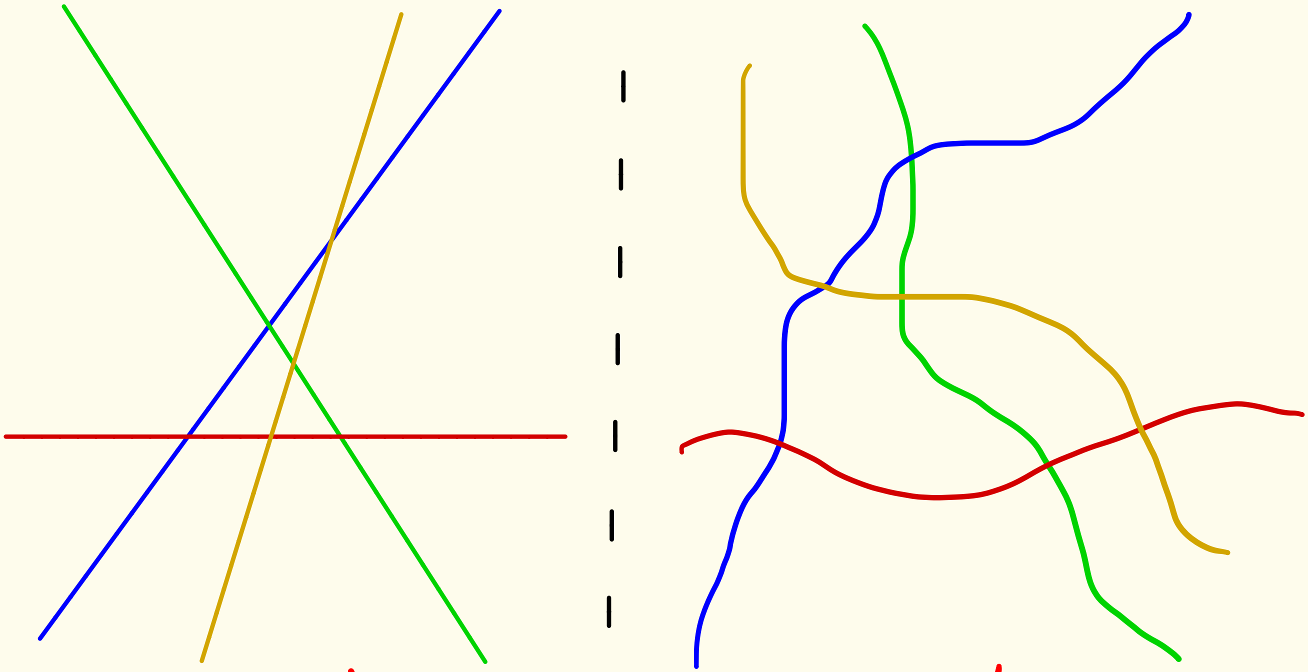
$$cr(K_n) = \frac{1}{4}\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-1}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\left\lfloor\frac{n-3}{2}\right\rfloor$$





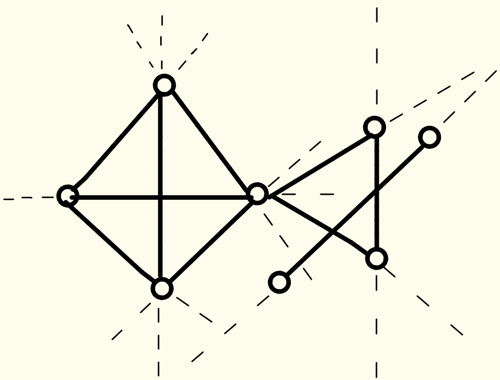
Arrangement of lines

Arrangement of pseudo lines

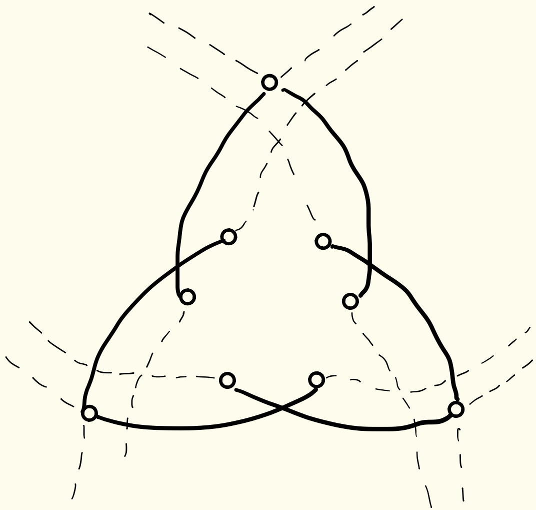


pairwise crossing

Rectilinear drawing



Pseudolinear drawing



Nice properties of pseudolinear drawings

1. Resemble in many aspects rectilinear drawings

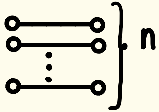
2. Proofs on pseudolinear drawings become more combinatorial.

3. We can decide whether a drawing is pseudolinear.

How to decide whether a drawing is pseudolinear?

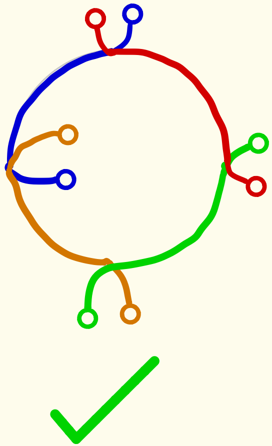
Example: Consider a drawing D of

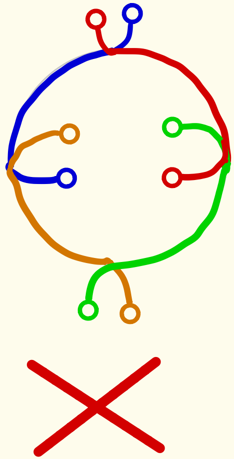
$$nK_2 =$$

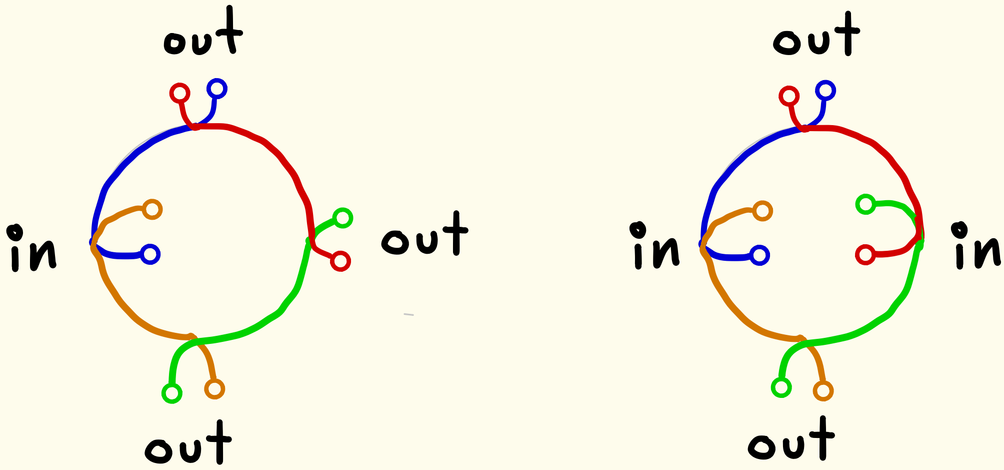


where the edges are "cyclically" arranged.

When is D pseudolinear?



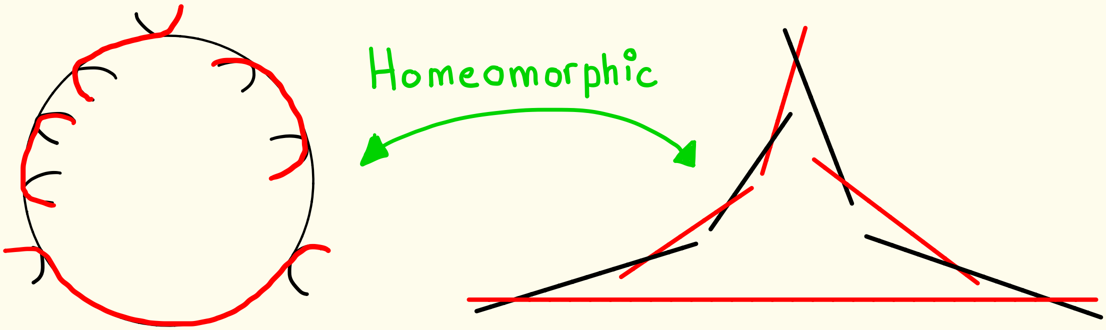




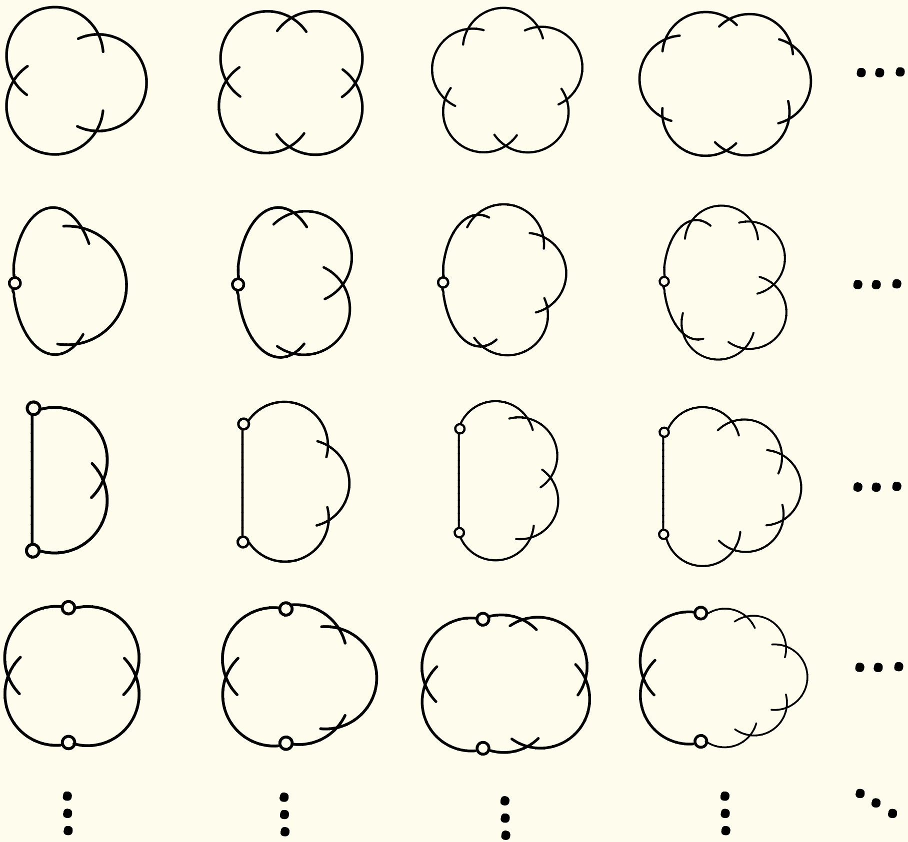
For which cyclic sequences $a_1, a_2, \ldots, a_n$ $(a_i \in \{in, out\})$

D is pseudolinear?

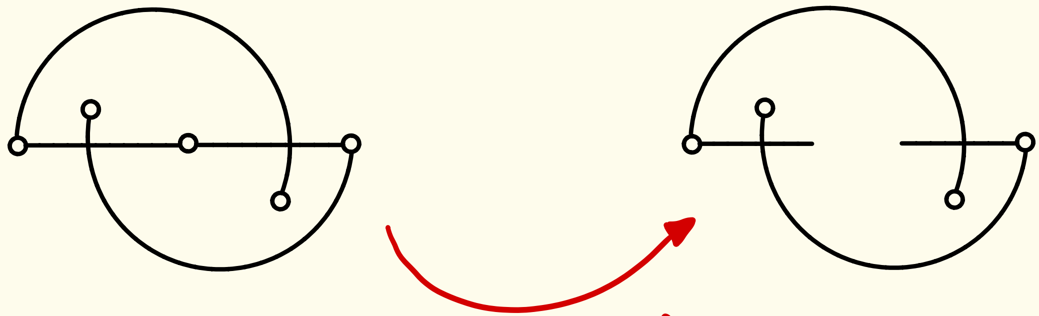
D is pseudolinear $\iff$ $a_1, \ldots, a_n$ has at least 3 outs.



So there are infinitely many (minimal) obstructions :



Are there other minimal obstructions?

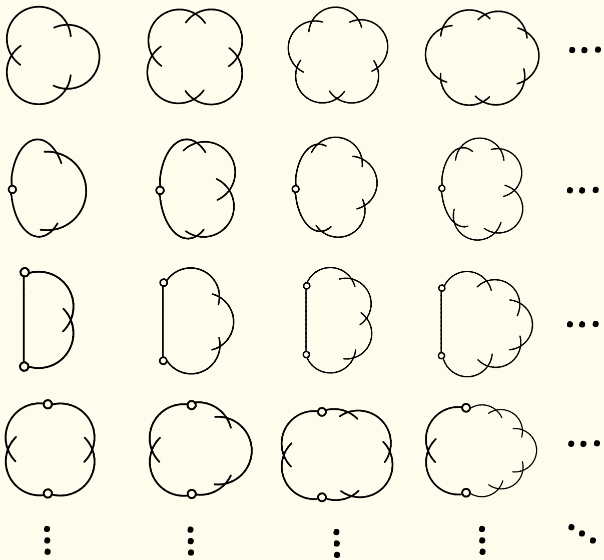


erase a bit from
the horizontal edges

Theorem (A., Bensmail, Richter, 17+)

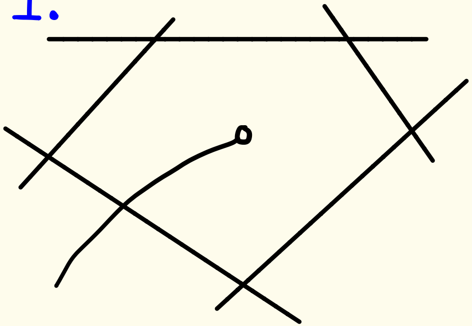
Let D be a drawing of a graph G. Then

D is not pseudolinear $\iff$ There are edges $e_1, e_2, \dots, e_n$ in $E(G)$, and subsegments $\sigma_1 \subseteq D[e_1], \dots, \sigma_n \subseteq D[e_n]$ that induce one of the drawings below

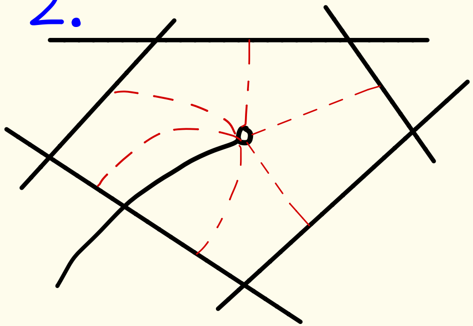


Key idea:

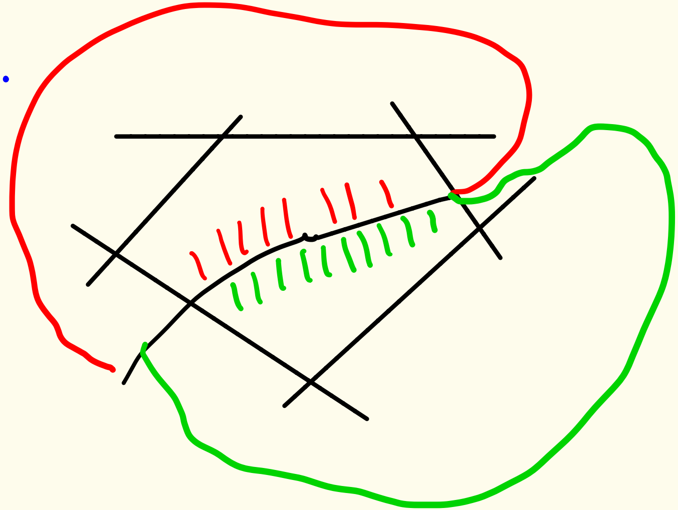
1.



2.

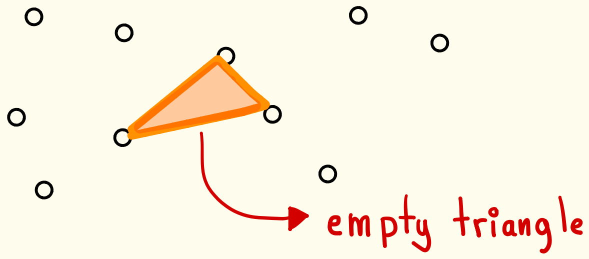


3.



Other interesting avenues ...

Generalizing results about configuration of points in the plane.

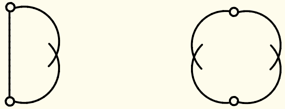


- Rectilinear drawings of K_n have at least $n^2 - O(n \log n)$ empty triangles (Bárány, Füredi, 87)
- We extend this result to pseudolinear drawings of K_n (A, McQuillan, Richter, Salazar, 17).

Question 1
Can we extend other geometric results to pseudolinear?

Other interesting avenues ...

In 88, Thomassen showed that a drawing where every edge is crossed at most once is homeomorphic to a rectilinear drawing $\Leftrightarrow$ it does not contain one of the following



- This result implies that "rectilinear" and "pseudolinear" are equivalent when every edge in a drawing is crossed at most once.

Question 2

If in a drawing every edge is crossed twice, is it true that if the drawing is pseudolinear then it is rectilinear?