Overview on $n-$gonal automorphism of Riemann surfaces

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González-Diez (1995)

If the automorphism group of a Riemann surface S, $\mathrm{Aut}(S)$, contains automorphism τ_1, τ_2 of same prime order and such that the quotient surface $S/ < \tau_i >$ $(i = 1, 2)$ are isomorphic to $\mathbb{P}^1$; then $< \tau_1 >$ and $< \tau_2 >$ are conjugate in $\mathrm{Aut}(S)$

Equivalently: For an arbitrary genus $g \geq 2$ there is a unique conjugacy $p-$gonal group in the full automorphism group of a $p-$gonal suface.

Remark 1: If the order is not prime there are Examples: [R.Hidalgo and G.González (1997), M.Carvacho (2013)]

Remark 2: If the genus of the quotient is not equal 0 is false. Examples [J.Cirre, A.Weaver, signature $(1; 2, 2, 2, 2)$]

Castelnuovo(1937)/Severi(1921)

$(p, \gamma)-$gonal group action on a surfaces of genus $g > 2p\gamma + (p-1)^2$ is unique and hece normal in the full automorphism group.

[p is prime number]

Farkas-Kra theorem

[Book: Riemann surface]

Theorem Let τ be a (n, γ) gonal group acting on a surface of genus g. Suppose

1. $g > n^2\gamma + (p-1)^2$ and,
2. there is a $\sigma \in \mathrm{Aut}(M)$ such that the number of fixed points is greater than $2n(\gamma + 1)$.

Then

- Each fixed point of τ is a fixed point of σ and $|\sigma| \leq |\tau| = n$
- If $|\sigma| = n$, then $\sigma \in \langle\tau\rangle$
- $\langle\tau\rangle$ is normal subgroup of $\mathrm{Aut}(M)$
- If $n = 2$, τ is in the center of $\mathrm{Aut}(M)$

Moduli space

g	Is J_0 unique?	Is J_1 unique?	Is J_2 unique?	$\mathcal{M}_g^0 \cap \mathcal{M}_g^1$	$\mathcal{M}_g^1 \cap \mathcal{M}_g^2$	$\mathcal{M}_g^0 \cap \mathcal{M}_g^2$
3	yes	?	?	?	?	?
4	yes	?	?	$\emptyset$	?	?
5	yes	?	?	$\emptyset$	?	?
6	yes	yes	?	$\emptyset$	?	?
7	yes	yes	?	$\emptyset$	?	?
8	yes	yes	?	$\emptyset$	$\emptyset$	?
9	yes	yes	?	$\emptyset$	$\emptyset$	?

Accola theorem

[Book:Topics in the Theorem of Riemann surfaces]

Let S be a compact Riemann surface of genus g. Suposse S admit a finite group of automorphism G_0 and $G_0 = \bigcup_{i=1}^{s} G_i$ where G_i is a subgroup of G_0 for each i. Let the order of G_i be p_i, let $S_i = S/G_i$and let n_i be the genus of S_i. Then

$$(s-1)g + p_0 n_0 = \sum_{i=1}^{s} p_i n_i$$

Questions

1. Is Gabino's theorem true when the order is not prime? Are there counterexample for each order not prime?
2. Are there counterexamples when the order is different of 2 and the genus of quotient greater than 0?
3. Does the inequality true if the order is not prime?
4. If you have a $(N, \gamma)-$gonal group acting on a surface of genus q, unique in the full automorphism group. Is it hold $g > 2N\gamma + (N-1)^2$?
5. Is the Accola's theorem equivalent to Farkas-Kra's theorem?
6. Goal: Give a characterization of normal subgroups of the Mapping class group.

On branch loci in Teichmller space.

Author: Harvey, W. J.

Trans. Amer. Math. Soc. 153 (1971), 387399.

Review: A point in the Teichmüller space T_g may be considered as an equivalence class of marked Fuchsian groups G. The corresponding Riemann surfaces S are of genus g. The Teichmüller modular group M_g acts discontinuously on T_g; the set of fixed points is called the branch locus. It is known that Δ_g is a countable union of complex submanifolds (not disjoint) of T_g , each one being stabilized by a cyclic subgroup of the modular group. Each submanifold can be associated with the Teichmüller space of another Fuchsian group. This new Teichmüller space uniformizes the quotient surfaces of the original surfaces identified under the subgroups of M_g which leave them fixed. All Fuchsian groups which arise in this way are classified, and the number of conjugacy classes of elements of prime order in the mapping class group of closed surfaces is computed.

On conjugacy classes in the Teichmüller modular group.

Author: Gilman, Jane

Michigan Math. J. 23 (1976), no. 1, 5363. 32G15

Review: Let $M(g, n)$ denote the mapping class group of a compact surface S having genus g and n punctures. By a theorem due to Nielsen and Fenchel, each element of $M(g, n)$ with finite order p can be deformed by a homotopy to a homeomorphism of order p on S, and this can be realised using uniformisation theory by a Fuchsian group , which represents $S = S/ < \alpha >$, and a homomorphism $\phi : \Gamma \rightarrow Z/pZ$ whose kernel is the defining subgroup of the branched covering S. Analysis of the ramification of $S \rightarrow S$ shows that a pair (S, α) determined, up to conjugacy by a homeomorphism, by the data consisting of cardinalities of the sets of standard generating elements of Γ is mapped by into the various elements of Z/pZ. The author obtains a generating function for the number $\lambda(p, g, n)$ of classes of (S, α) when p is prime by solving this combinatorial enumeration problem.

An example about normalizers in mapping class groups

Author: Jane Gilman

Journal: Proc. Amer. Math. Soc. 69 (1978), 115-118

https://doi.org/10.1090/S0002-9939-1978-0486493-0

Abstract: J. Birman has raised the question as to whether every element of the mapping class group (the Teichmüller modular group) is in the normalizer of some element of finite order. In this paper elements of the mapping class group which are not in the normalizer of any element of finite order are constructed.

Maximal groups of automorphisms of compact Riemann surfaces in various classes of finite groups.

Author: Gromadzki, Grzegorz

Rev. Real Acad. Cienc. Exact. Fs. Natur. Madrid 82 (1988), no. 2, 267-275.

Article: It is well known that a finite group can be represented as a group of automorphisms of a compact Riemann surface. For an integer $g \geq 2$ and a class F of finite groups, let $N(g, F)$ denote the order of the largest group in F that a compact Riemann surface of genus g admits as a group of automorphisms. Having a class F of finite groups it is natural to ask for a bound for $N(g, F)$ as well as for the characterizations of those g for which this bound is attained. For the classes of all finite, cyclic, abelian, nilpotent, $p-$groups (given p), soluble, metabelian, and finally for supersoluble groups, an upper bound for $N(g, F)$, as well as infinite series of g for which this bound is attained, were found in a series of papers by various authors. This paper is a survey of results concerning this problem published in recent years. We present some new proofs of known results, strengthen and generalize some of them as well as present new results not yet published.

The equisymmetric stratification of the moduli space and the Krull dimension of mapping class groups

Author Allen Broughton

Topology and its Applications Volume 37, Issue 2, November 1990, Pages 101-113

https://doi.org/10.1016/0166-8641(90)90055-7

Abstract: A stratification of the moduli space into strata consisting of the Riemann surfaces of the same symmetry type is discussed. The stratification is then employed to determine the Krull dimension of the mod p cohomology algebras of mapping class groups.

Loci of Curves Which are Prime Galois Coverings of $\mathbb{P}^1$

Authors: Gabino González-Díez

Proc. London Math. Soc. (3) 62 (1991), no. 3, 469489

Abstract: The author studies the moduli spaces parametrizing algebraic curves which are Galois coverings of $\mathbb{P}^1$ with prime order and with given ramification numbers. The main results proved in this paper are the existence of a parametrization of the Teichmüller spaces of the above class of curves in terms of theta functions and the application of this result to the construction, in Sections 5 and 6, of an explicit birational model of the corresponding moduli spaces and the study of the normality of these spaces. Results proved in this paper generalize some of the results proved by D. Mumford [Tata lectures on theta, II, Progr. Math., 43, Birkhäuser Boston, Boston, 1984; MR0742776] in the case of hyperelliptic curves.

Conformal Versus Topological Conjugacy of Automorphisms on Compact Riemann Surfaces

Bull. London Math. Soc. 29 (1997), no. 3, 280-284.

Authors Gabino González-Diez, Rubén A. Hidalgo.

Abstract: We produce a family of algebraic curves (closed Riemann surfaces) S admitting two cyclic groups $H1$ and $H2$ of conformal automorphisms,which are topologically (but not conformally) conjugate and such that S/Hi is the Riemann sphere . The relevance of this example is that it shows that the subvarieties of moduli space consisting of points parametrizing curves which occur as cyclic coverings (of a fixed topological type) of need not be normal.

On Cyclic Groups of Automorphisms of Riemann Surfaces

Authors: Emilio Bujalance, Marston Conder.

Journal of the London Mathematicas Society. Volume 59, Issue 2 April 1999. Pages 573-584.

DOI: 10.1112/S0024610799007115

Abstract: The question of extendability of the action of a cyclic group of automorphisms of a compact Riemann surface is considered. Particular attention is paid to those cases corresponding to Singerman's list of Fuchsian groups which are not finitely-maximal, and more generally to cases involving a Fuchsian triangle group. The results provide partial answers to the question of which cyclic groups are the full automorphism group of some Riemann surface of given genus $g > 1$.

Symmetries of real cyclic $p-$gonal Riemann surfaces

Authors: Antonio F. Costa, Milagros Izquierdo.

Pacific Journal of Mathematics. Vol. 213 (2004), No. 2, 231-243.

DOI: 10.2140/pjm.2004.213.231

Abstract: A closed Riemann surface X which can be realised as a p-sheeted covering of the Riemann sphere is called $p-$gonal, and such a covering is called a $p-$gonal morphism. A $p-$gonal Riemann surface is called real $p-$gonal if there is an anticonformal involution (symmetry) σ of X commuting with the $p-$gonal morphism. If the $p-$gonal morphism is a cyclic regular covering the Riemann surface is called real cyclic $p-$gonal, otherwise it is called real generic $p-$gonal. The species of the symmetry σ is the number of connected components of the fixed point set $Fix(\sigma)$ and the orientability of the Klein surface $X\sigma$. In this paper we find the species for the possible symmetries of real cyclic $p-$gonal Riemann surfaces by means of Fuchsian and NEC groups.

Multiple prime covers of the Riemann sphere

Authors: Aaron Wootton

Central European Journal of Mathematics. June 2005, Volume 3, Issue 2, pp 260-272.

https://link.springer.com/article/10.2478/2FBF02479202

Abstract: A compact Riemann surface X of genus $g \geq 2$ which admits a cyclic group of automorphisms C_q of prime order q such that X/C_q has genus 0 is called a cyclic $q-$gonal surface. If a $q-$gonal surface X is also $p-$gonal for some prime $p \neq q$, then X is called a multiple prime surface. In this paper, we classify all multiple prime surfaces. A consequence of this classification is a proof of the fact that a cyclic $q-$gonal surface can be cyclic $p-$gonal for at most one other prime p.

Conjugators of Fuchsian groups and quasiplatonic surfaces

Authors: Ernesto Girondo, Jürgen Wolfart

The Quarterly Journal of Mathematics, Volume56, Issue 4, 1 December 2005, Pages 525-540.

https://doi.org/10.1093/qmath/hah054

Abstract: Let G be a Fuchsian group containing two torsion free subgroups defining isomorphic Riemann surfaces. Then these surface subgroups K and $\alpha K \alpha^1$ are conjugate in $PSL(2, R)$, but in general the conjugating element α cannot be taken in G or a finite index Fuchsian extension of G. We will show that in the case of a normal inclusion in a triangle group G these can be chosen in some triangle group extending G. It turns out that the method leading to this result allows also to answer the question of how many different regular dessins of the same type can exist on a given quasiplatonic Riemann surface.

On symmetries of compact Riemann surfaces with cyclic groups of automorphisms

Author: E.Bujalance, J.Cirre, J.M.Gamboa, G.Gromadzki

Journal of Algebra. Volume 301, Issue 1, 1 July 2006, Pages 82-95.

https://doi.org/10.1016/j.jalgebra.2006.03.019G

Abstract: A Riemann surface X is said to be of type (n, m) if its full automorphism group $\mathrm{Aut}X$ is cyclic of order n and the quotient surface $X/AutX$ has genus m. In this paper we determine necessary and sufficient conditions on the integers n, m, g and γ, where n is odd, for the existence of a Riemann surface of genus g and type (n, m) admitting a symmetry with γ ovals.

Defining equations for cyclic prime covers of the Riemann sphere

Author: A. Wootton

Israel Journal of Mathematics. January 2007, Volume 157, Issue 1, pp 103-122.

Abstract: We determine a method to find explicit defining equations for each compact Riemann surface which admits a cyclic group of automorphisms C_p of prime order p such that the quotient space has genus 0.

The full automorphism group of a cyclic $p-$gonal surface

Author: A.Wootton

Journal of Algebra. Volume 312, Issue 1, 1 June 2007, Pages 377-396.

https://doi.org/10.1016/j.jalgebra.2007.01.018G

Abstract: If p is prime, a compact Riemann surface X of genus $g \geq 2$ is called cyclic $p-$gonal if it admits a cyclic group of automorphisms Cp of order p such that the quotient space X/C_p has genus 0. If in addition Cp is not normal in the full automorphism A, then we call X a non-normal $p-$gonal surface. In the following we classify all non-normal $p-$gonal surfaces.

Maximal order of automorphisms of trigonal Riemann surfaces

Author: Antonio F.Costa, Milagros Izquierdo.

Journal of Algebra Volume 323, Issue 1, 1 January 2010, Pages 27-31.

https://doi.org/10.1016/j.jalgebra.2009.09.041G

Abstract: In this paper we find the maximal order of an automorphism of a trigonal Riemann surface of genus g, $g \geq 5$. We find that this order is smaller for generic than for cyclic trigonal Riemann surfaces, showing that generic trigonal surfaces have less symmetry than cyclic trigonal surfaces. Finally we prove that the maximal order is attained for infinitely many genera in both the cyclic and the generic case.

The maximal order is $2g + 4$. The famliy of surfaces was given by Xin Lü

On cyclic $p-$gonal Riemann surfaces with several $p-$gonal mor..

Geometriae Dedicata August 2010, Volume 147, Issue 1, pp 139-147.

Authors: Antonio F. Costa, Milagros Izquierdo, Daniel Ying

Abstract: Let p be a prime number, $p > 2$. A closed Riemann surface which can be realized as a $p-$sheeted covering of the Riemann sphere is called $p-$gonal, and such a covering is called a $p-$gonal morphism. If the $p-$gonal morphism is a cyclic regular covering, the Riemann surface is called a cyclic $p-$gonal Riemann surface. Accola showed that if the genus is greater than $(p-1)^2$ the p-gonal morphism is unique. Using the characterization of $p-$gonality by means of Fuchsian groups we show that there exists a uniparametric family of cyclic $p-$gonal Riemann surfaces of genus $(p-1)^2$ which admit two $p-$gonal morphisms. In this work we show that these uniparametric families are connected spaces and that each of them is the Riemann sphere without three points. We study the Hurwitz space of pairs (X, f), where X is a Riemann surface in one of the above families and f is a $p-$gonal morphism, and we obtain that each of these Hurwitz spaces is a Riemann sphere without four points.

On the Number of $p-$gonal Coverings of Riemann Surfaces

Author: Grzegorz Gromadzki

Rocky Mountain J. Math. Volume 40, Number 4 (2010), 1221-1226.

Article: The classical Castelnuovo-Severi theorem implies that for $g > (p-1)^2$, a $p-$gonal automorphism group of a cyclic $p-$gonal Riemann surface X of genus g is unique. Here we deal with the case $g \leq (p-1)^2$. We find some bounds for the number of such coverings in terms of g and p and then we derive from them bounds depending only on p and even an absolute bound equal to 30. We also show that a Riemann surface of genus $g \geq 2$ having less than $3(g-1)$ automorphisms admits at most one $p-$gonal covering.

Stratifying the space of moduli.

Teichmüller theory and moduli problem, 597-618, Ramanujan Math. Soc. Lect. Notes Ser., 10, Ramanujan Math. Soc., Mysore, 2010.

Author: Weaver, Anthony

Review: The modular group Mod_g acts isometrically on T_g, and the Nielsen-Kerckhoff fixed point theorem is used to give an algebraic condition for a finite group H to be realized as a subgroup of Mod_g, and to identify the fixed point set $F(H) \subset T(\Gamma)$ with another Teichmller space $T(\Gamma^*)$ thus embedded in $T(\Gamma)$. Conformal conjugacy of Riemann surfaces admitting a fixed symmetry is distinguished from the notion of topological conjugacy, and described in terms of the relative Riemann space and the relative modular group. This finally leads to the construction of the stratification. The paper explains, "It is precisely the intersection of different strata which are of most interest: the intersection and nesting relationships between the strata echo those of the covering branch locus."

On gonality of Riemann surfaces

Authors: Grzegorz Gromadzki, Anthony Weaver,Aaron Wootton

Geometriae Dedicata December 2010, Volume 149, Issue 1, pp 1-14.

Abstract: A compact Riemann surface X is called a $(p, n)-$gonal surface if there exists a group of automorphisms C of X (called a $(p, n)-$gonal group) of prime order p such that the orbit space X/C has genus n. We derive some basic properties of $(p, n)-$gonal surfaces considered as generalizations of hyperelliptic surfaces and also examine certain properties which do not generalize. In particular, we find a condition which guarantees all $(p, n)-$gonal groups are conjugate in the full automorphism group of a $(p, n)-$gonal surface, and we find an upper bound for the size of the corresponding conjugacy class. Furthermore we give an upper bound for the number of conjugacy classes of $(p, n)-$gonal groups of a $(p, n)-$gonal surface in the general case. We finish by analyzing certain properties of quasiplatonic $(p, n)-$gonal surfaces. An open problem and two conjectures are formulated in the paper.

The gonality of Riemann surfaces under projections by normal coverings

Authors: E.Bujalance, J.J.Etayo, J.M.Gamboa, G.Gromadzki.

Journal of Pure and Applied Algebra. Volume 215, Issue 5, May 2011, Pages 983-988.

Abstract: A compact Riemann surface X of genus $g \geq 2$ which can be realized as a q-fold, normal covering of a compact Riemann surface of genus p is said to be $(q, p)-$gonal. In particular the notion of $(2, p)-$gonality coincides with $p-$hyperellipticity and $(q, 0)-$gonality coincides with ordinary q-gonality. Here we completely determine the relationship between the gonalities of X and Y for an $N-$fold normal covering $X \to Y$ between compact Riemann surfaces X and Y. As a consequence we obtain classical results due to Maclachlan (1971) [5] and Martens (1977) [6].

[5]C. Maclachlan Smooth coverings of hyperelliptic surfaces Quart. J. Math. Oxford, 22 (2) (1971), pp. 117-123

[6]H.H. Martens A remark on Abels Theorem and the mapping of linear series Comment. Math. Helv., 52 (1977), pp. 557-559

Curves Possessing No Thomae Formulae of BershadskyRadul Type

Letters in Mathematical Physics. November 2011, Volume 98, Issue 2, pp 193-205.

Authors: Gabino González-Diez, David Torres-Teigell.

Abstract: A Z_N-curve is one of the form $y^N = (x_1)^{m_1} \cdots (x\lambda_s)^{m_s}$. When $N = 2$ these curves are called hyperelliptic and for them Thomae proved his classical formulae linking the theta functions corresponding to their period matrices to the branching values $\lambda_1, \cdots, \lambda_s$. In his work on Fermionic fields on Z_N-curves with arbitrary N, Bershadsky and Radul discovered the existence of generalized Thomaes formulae for these curves which they wrote down explicitly in the case in which all rotation numbers m_i equal 1. This work was continued by several authors and new Thomaes type formulae for Z_N-curves with other rotation numbers m_i were found. In this article we prove that for some choices of the rotation numbers the corresponding Z_N-curves do not admit such generalized Thomaes formulae.

On isolated strata of p-gonal Riemann surfaces in the branch locus of moduli spaces.

Bartolini, Gabriel, Costa, Antonio F, Izquierdo, Milagros

Albanian J. Math. 6 (2012), no. 1, 11-19.

Abstract: This paper studies the topology of the branch locus B_g. A cyclic $p-$gonal Riemann surface X is a surface that admits a regular covering of degree p on the Riemann sphere. In this paper, the authors find isolated strata of any dimension, consisting of $p-$gonal surfaces.

As a consequence, the authors give an infinite family of genera for which B_g has an increasing number of isolated strata.

Nonequivalent families of group actions on Riemann surfaces

Author: Mariela Carvacho

Journal of Pure and Applied Algebra. Volume 217, Issue 12, December 2013, Pages 2345-2355.

https://doi.org/10.1016/j.jpaa.2013.03.011Get rights and content

Abstract:We produce for each natural number $n \geq 3$ two $1-$parameter families of Riemann surfaces admitting automorphism groups with two cyclic subgroups H_1 and H_2 of order 2^n, which are conjugate in the group of orientation-preserving homeomorphisms of the corresponding Riemann surfaces, but not conjugate in the group of conformal automorphisms.

This property implies that the subvariety $M_g(H_1)$ of the moduli space M_g consisting of the points representing the Riemann surfaces of genus g admitting a group of automorphisms topologically conjugate to H_1 (equivalently to H_2) is not a normal subvariety.

Normal coverings of hyperelliptic real Riemann surfaces

Authors: Cirre, Francisco-Javier, Hidalgo, Rubén A.

Riemann and Klein surfaces, automorphisms, symmetries and moduli spaces, 59-75, Contemp. Math., 629, Amer. Math. Soc., Providence, RI, 2014.

Review: The authors investigate normal coverings between hyperelliptic real Riemann surfaces. A real Riemann surface is, by definition, a pair (R, τ), where R is a compact Riemann surface and τ is an anticonformal involution of R onto itself. Let (S, η) be another real Riemann surface. A normal covering $\pi : (R, \tau) \to (S, \eta)$ means a normal, possibly branched, covering $\pi : R \to S$ with $\pi \circ \tau = \eta \circ \pi$ such that every element of the group$G$ of covering transformations commutes with τ . The authors study the case when (R, τ) and (S, η) are hyperelliptic real Riemann surfaces of genus greater than one, and give explicit algebraic equations for R, τ, S, η, π and formulae for a set of generators of G, complementing some results of E. Bujalance, F. J. Cirre and J. M. Gamboa [Conform. Geom. Dyn. 11 (2007), 107127; J. Pure Appl. Algebra 212 (2008), no. 9, 2011-2026].

Connecting $p-$gonal loci in the compactification of moduli space

Authors: Antonio F. Costa, Milagros Izquierdo, Hugo Parlier

Revista Matemática Complutense. May 2015, Volume 28, Issue 2, pp 469-486.

Abstract: Consider the moduli space Mg of Riemann surfaces of genus $g \geq 2$ and its DeligneMumford compactification $\overline{M}_g$. We are interested in the branch locus B_g for $g > 2$, i.e., the subset of Mg consisting of surfaces with automorphisms. It is well-known that the set of hyperelliptic surfaces (the hyperelliptic locus) is connected in Mg but the set of (cyclic) trigonal surfaces is not. By contrast, the set of (cyclic) trigonal surfaces is connected in $\overline{M}_g$. We exhibit an explicit nodal surface that lies in the completion of every equisymmetric set of $3-$gonal Riemann surfaces providing an alternative proof of a result of Achter and Pries (Math Ann 338:187206, 2007). For $p > 3$ the connectivity of the $p-$gonal loci becomes more involved. We show that for $p \geq 11$ prime and genus $g = p1$ there are one-dimensional strata of cyclic pp-gonal surfaces that are completely isolated in the completion $\overline{B}_g$ of the branch locus in $\overline{M}_g$.

Maximal group actions on compact oriented surfaces

Journal of Algebra. Volume 472, 15 February 2017, Pages 1-14.

Authors: Valerie Peterson, Jacob Russell, Aaron Wootton.

https://doi.org/10.1016/j.jalgebra.2016.10.004

Abstract: Suppose S is a compact oriented surface of genus $\sigma \geq 2$ and C_p is a group of orientation preserving automorphisms of S of prime order $p \geq 5$. We show that there is always a finite supergroup $G > C_p$ of orientation preserving automorphisms of S except when the genus of S/C_p is minimal (or equivalently, when the number of fixed points of C_p is maximal). Moreover, we exhibit an infinite sequence of genera within which any given action of C_p on S implies C_p is contained in some finite supergroup and demonstrate for genera outside of this sequence the existence of at least one C_p-action for which C_p is not contained in any such finite supergroup (for sufficiently large σ).

On periodic self-homeomorphisms of closed orientable surfaces determined by their orders

Collectanea Mathematica. September 2016, Volume 67, Issue 3, pp 415-429.

Authors: Bagiski, M. Carvacho, G. Gromadzki, R. Hidalgo.

Abstract: The fundamentals for the topological classification of periodic orientation-preserving self-homeomorphisms of a closed orientable topological surface X of genus $g \geq 2$ have been established, by Nielsen, in the thirties of the last century. Here we consider two concepts related to this classification; rigidity and weak rigidity. A cyclic action G of order N on X is said to be topologically rigid if any other cyclic action of order N on X is topologically conjugate to it. If this assertion holds for arbitrary other action but having, in addition, the same orbit genus and the same structure of singular orbits, then G is said to be weakly topologically rigid. We give a precise description of rigid and weakly rigid cyclic quasi-platonic actions which mean actions having three singular orbits and for which X/G is a sphere.

Counting cyclic quasiplatonic surfaces of prime power order

Preprint.

http://docs.wixstatic.com/ugd/4cead8_ca45abdca0344957a0ba32eafe00e2d6.pd

Author: Charles Camacho

Abstract: We give an explicit formula for the number of isomorphism classes of compact Riemann surfaces of genus $g \geq 2$ which admit a regular dessin with cyclic automorphism group of prime power order.