

Planting trees in random graphs (and finding them back)

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Joint work with

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Inference problems with planted structure

Objectives

- Detection (planted structure present or not)
- Recovery (estimation of planted structure)

Recurrent patterns / phases

(e.g. in community detection, planted clique detection, planted dense subgraph detection)

- Information-Theoretic impossibility
- Computational hardness though IT-feasible
- Computationally easy

A scenario from computer science

Attackers may be “plotting” or not

Plotting → communications among them

Goals:

-Infer from observed communication graph whether attack under way

-If attack detected, identify attackers

Model

Erdős-Rényi graph

Under attack: augmented by tree on nodes
chosen uniformly at random

Focus on simplest tree i.e. the line

Outline

- Phase diagram for lines
- Other trees

The argument for undetectability

Likelihood ratio between distributions without and with attack,

The argument for undetectability

Likelihood ratio between distributions without and with attack,

Bounding Markov chain

The argument for detectability,

- 1) Too many edges under
- 2) Counts of small connected components enable consistent estimation of

Arguments hold for any connected K -graph, not just lines and trees

The argument for reconstruction impossibility

Optimal overlap between estimated and planted
path:

Maximum a Posteriori, i.e. nodes on largest
number of K-paths

The argument for reconstruction impossibility

“Lures”: can construct confounding segment s.t.

More generally: construct T symmetric paths

Other trees: stars

- Stars hard to hide!

Threshold at

Detection easy: inspection of degrees

Other trees: D-regular

Conjectured phase diagram

Conclusions

Phase diagram for line detection & reconstruction

- No computationally hard phase
- Reconstruction impossible throughout, by presence of too many copies of planted structure

Planted subgraphs beyond cliques & dense subgraphs:

- Phase diagram for D -regular trees?
- What triggers hard phases?

Thanks!

