Ramsey Theory and Big Data

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Joint Work with M.Waddell (Columbia University)

Stories

1. Story 1: Motivations
2. Story 2: Goodman's Theorem
3. Story 3: How many bad colourings?

Beginning of the story

Calude and Longo, 2016: "The Deluge of Spurious Correlations in Big Data"

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M. Waddell (Columbia, PhD student in data science)

Hey, Mike, let's follow the lead of Calude-Longo.

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- The connections are largely unexplored.
- Impactful, applicable research.

Story 2: Goodman's Theorem

Spurious Correlations

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A **spurious correlation** is one that is a result of forced, geometric or combinatorial relations.

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Theorem (folklore?)

Every red/blue edge colouring of a K_6 contains <u>at least 2</u> monochromatic triangle.

Enter Goodman

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Every red/blue edge colouring of K_n must contain at least $\frac{1}{4}\frac{n-4}{n-1}$ fraction of monochromatic triangles.

Recall: $\binom{5}{3} = 10$, $\binom{6}{3} = 20$.

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Question 2

What is this measuring?

What about K_4?

Conjecture: Erdős

For large n, every red/blue edge coloured K_n, (asymptotically) at least $\frac{1}{32}$ many of the K_4 should be monochromatic.

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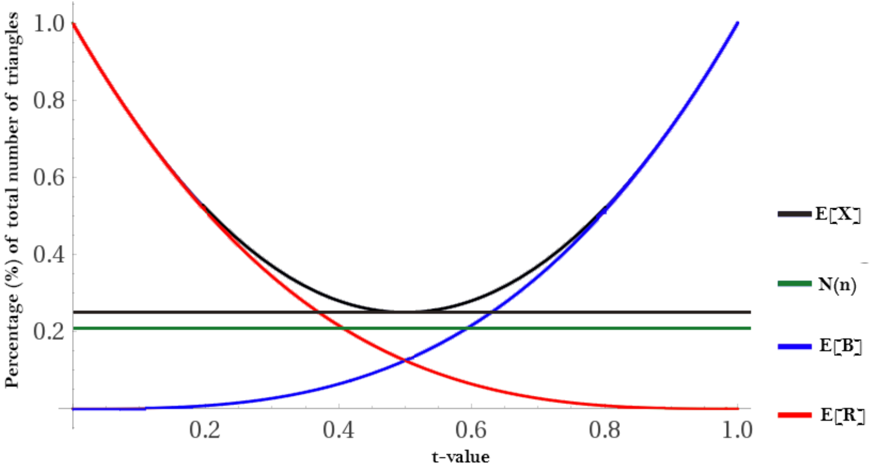
Thomason, 1989

There are red/blue edge colourings of (large) K_n with only $\frac{1}{33}$ many monochromatic triangles.

For K_m: there are colourings with $0.936 \cdot 2^{1-\binom{m}{2}}$ monochromatic K_m.

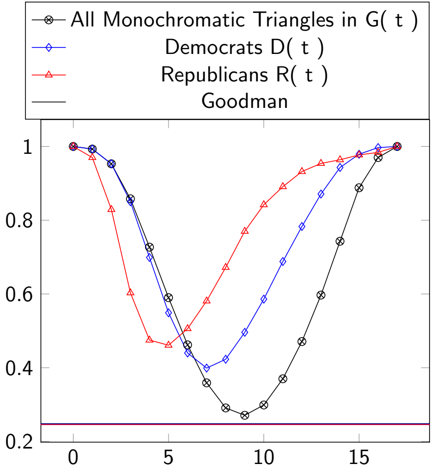
How can this be used?

This can be used to give a meaningful measure of randomness.



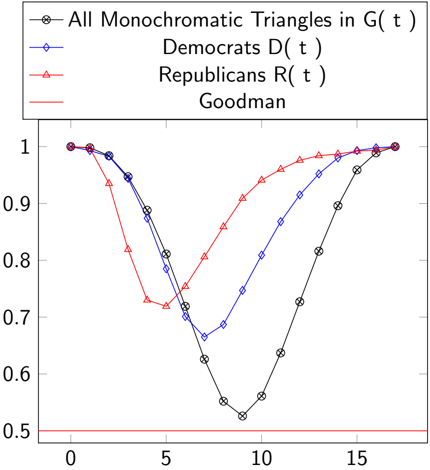
Real data set: 1984 US Congress voting

168 Republicans + 267 Democrats = 435 Voters. 16 votes, Hamming distance.



Advantages

1. It is computationally efficient to count triangles (but not K_4s).



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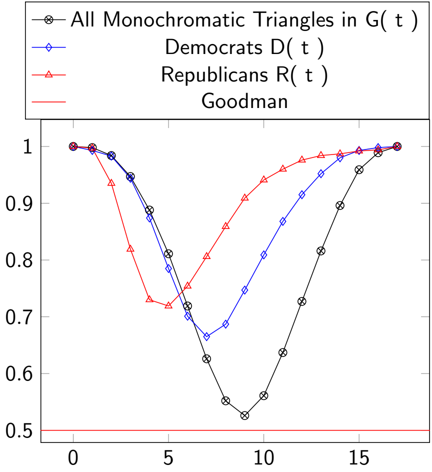
2. This picture is also a measure of how transitive the combined relations are.



Other "Goodman" theorems

1. Schur triples (2 colours). $\frac{1}{22}$. Story starts with Graham-Rödl-Ruciński 1996, "ends" with Robertson-Zeilberger 2003.
2. VdW (3 term, 2 colours). At least 25% of all 3-term such arithmetic progressions must be monochromatic. [Sjöland 2014, using Cameron-Cilleruelo-Serra 2007]
3. VdW (4 term, 2 colours). $\frac{7}{96} < \frac{1}{16}$. [Lu-Peng 2012, building off Wolf 2010]
4. See also work of Parillo-Robertson-Saracin 2008, Butler-Costello-Graham 2010.

Questions

Question 1

What are the other Goodman (quantitative) versions of Ramsey theorems?

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What are the other Goodman (quantitative) versions of Ramsey theorems?

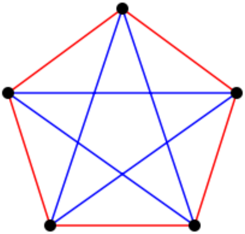
Question 2

What is a "physical" interpretation of monochromatic arithmetic progressions in a large data set?

Story 3: Bad colourings

Artist: M. Pawliuk (Age: 31).

$R(3,3) > 5$



Observation

There is an edge colouring of K_5 without a monochromatic K_3, but *most* edge colourings *do* have a monochromatic triangle.

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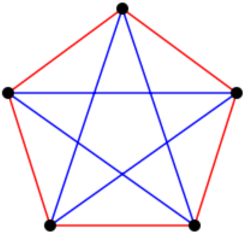

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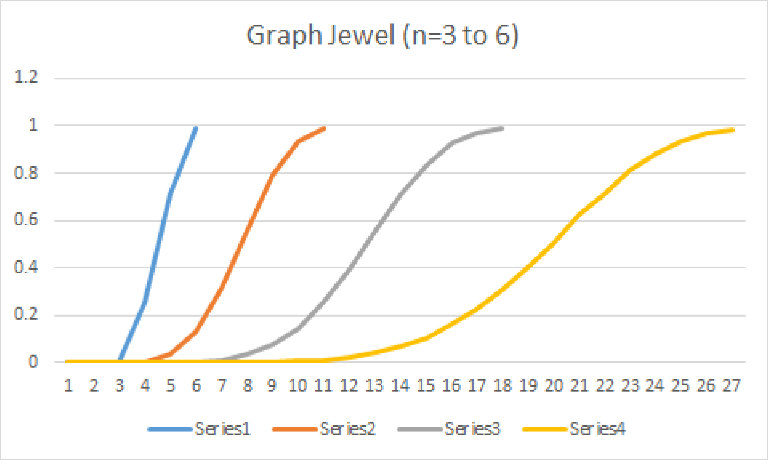
[

noframenumbering]Major Question

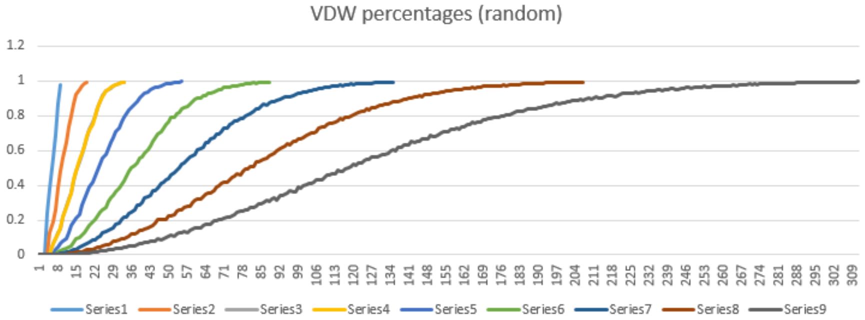
Given a Ramsey-style result, as the size n of the data set grows, what percentage of colourings have monochromatic witnesses?

Ramsey's Theorem



See Robertson-Cipolli-Dascălu 2017 for descriptions of these distributions.

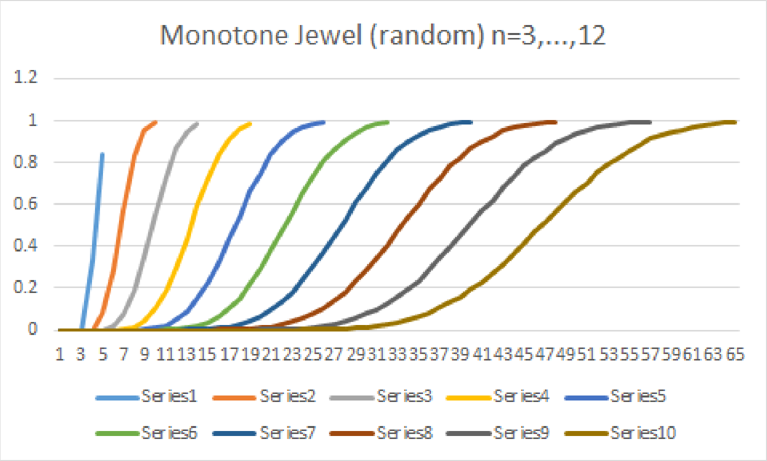
VdW. Arithmetic progressions of length n



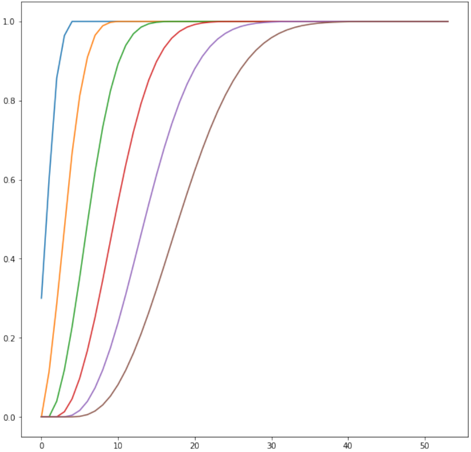
AP of lengths 3 to 10.

See Robertson-Cipolli-Dascălu 2017 for descriptions of these distributions.

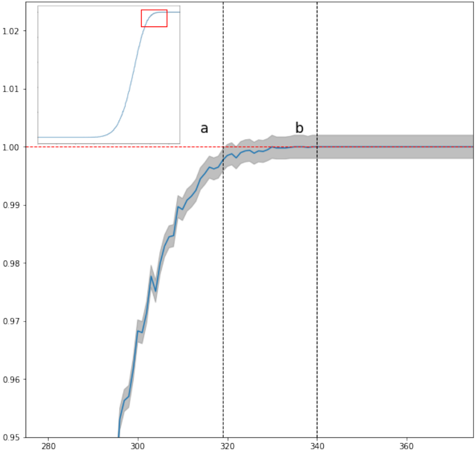
Erdős-Szekeres. Monotone subsequence of length n



Partitions of n objects into N boxes, with at least one box with N objects

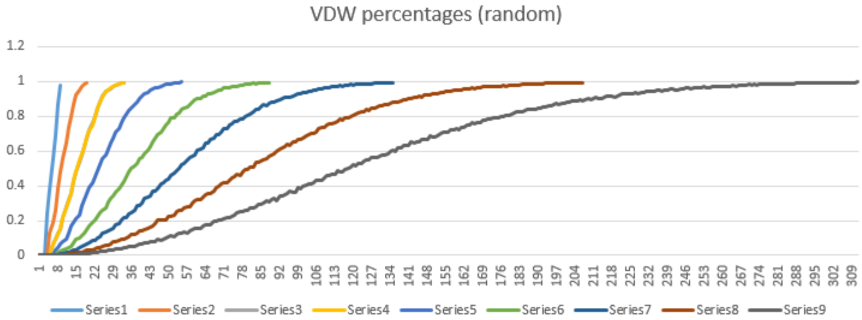


$N = 20$



The jaggedness is not noise! It is an essential feature of the graph.

Machine learning and classifiers



Using **one** of these distributions gives you an okay way to classify/partition graphs. Using **many** of these distributions gives you a better way to classify graphs.

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Call to action 1

Results in this area need to be made accessible to data scientists. We need to (if possible) include digestible results.

Call to action 2

Talk to a statistician and a data scientist.