
Banff Bonus Material:
F-theory & AdS/CFT
SSN

Dave's talk

F-theory = 10d f. gravity which at low energy is $SL_2\mathbb{Z}$-gauged IIB sugra coupled to 7-branes.

If $M^{10} = X \times Y$, $\boxed{Y = \text{Lorentzian } \mathbb{R}^{4,1}}$ $\Rightarrow X =$ Kähler and

$$\left.\begin{array}{l} f \in \Gamma\, \mathcal{O}(-4K_X) \\ g \in \Gamma\, \mathcal{O}(-6K_X) \end{array}\right\} \Rightarrow \tau \quad \& \quad \left.\begin{array}{c} E_\tau \\ \downarrow \\ X \end{array}\right\} Y : \underline{\underline{K_Y = 0}}$$

Recently more general vacua of F-theory were discussed:

$$\boxed{Y : AdS_{d+1} = \frac{SO(d,2)}{SO(d,1)}}$$

e.g. AdS_3 = 3-hyperbolic space.

Why is this interesting?

- $\boxed{\text{SCFTs}}$ (cf. Thursday) have a "holographic" dual description in terms of quantum gravity in AdS-spaces.

$\Rightarrow$ AdS / CFT

d-dim SCFTs: AdS_{d+1} dual gravity.

e.g. 4d $\mathcal{N}=4$ SYM $\longleftrightarrow$ $AdS_5 \times S^5$ solution in IIB sugra.
(τ constant).

gravity dual computes strong-coupling information for SCFT.

e.g. 6d SCFTs from Th $\rightsquigarrow$ AdS_7-duals in IIA & M.
(not IIB).

Question: Are there F-theoretic AdS_{d+1}-solutions i.e. IIB sugra solutions w/ τ varying &

$$M^{10} = Y \times AdS_{d+1} \;?$$

Key: Solutions use metric properties of spacetime.
What spaces Y are allowed?

Yes

- AdS_3 - solutions: dual to 2d SCFTs.

$\boxed{\text{SUSY} \Rightarrow \text{constraints on Killing spinors.}}$

Metric ansatz:

$$ds^2 = ds^2_{AdS_3} + ds^2_{M_7}$$

Flux:

$$F_5 = (1+*)\, Vol(AdS_3) \wedge F^{(2)}$$

$(G_3 \equiv 0 \quad \log)$.

$\Big\Downarrow$ SUSY
(0,2) in 2d.

$$ds^2_{M_7} = (dz+g)^2 + ds^2(M_6) \quad ; \; \tau \text{ varies holom. on } M_6.$$

S^1-fibration.   Kähler 3-fold.

auxiliary metric:

$$\boxed{\begin{gathered} ds^2_{M_8} = \frac{1}{\tau_2}\left((dx+\tau_1 dy)^2 + \tau_2^2 dy^2\right) + ds^2(M_6) \\ \Box_8 R_8 - \frac{1}{2} R_8^2 + R_{8\,ij} R_8^{ij} = 0 \end{gathered}}$$

Requiring $\mathcal{N}=(0,4)$:

$$M_6 = S^2 \times B \qquad\qquad M_4 \text{ Kähler sfc.}$$

and curvature condition $\Rightarrow$

$$\left.\begin{array}{ccc} E_\tau & \longrightarrow & Y_3 \\ & & \downarrow \\ & & B_2 \end{array}\right\} CY_3.$$

$SU(2)_R$

$\Rightarrow$ Complete solution is $\boxed{AdS_3 \times S^3/\mathbb{Z}_M \times CY_3^\tau}$

most general sol. w/ F_5.

[Couzens, Lawrie, Martelli, SSN, Wang].

Metric in IIB solution: $\boxed{\text{Metric on } B_2}$ is the one induced by CY metric on Y_3 $\rightsquigarrow$ $\boxed{\underline{\text{singularities}}}$

For τ constant: Reduces to well-known $AdS_3 \times S^3 \times CY_2$ solution.

- $\underline{(0,2) \text{ solutions}}$ [Couzens, Martelli, SSN].

$$AdS_3 \times \underset{\substack{\| \\ S^1 \to \mathbb{F}_0}}{Y^{p,q}} \times K3^\tau$$

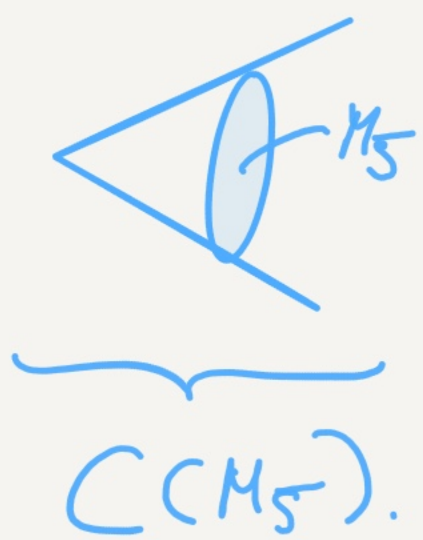
- AdS_5 solution w/ varying axio-dilaton

Most general duals to 4d $\mathcal{N}=1$ in IIB w/ varying τ

Recall: Constant τ: $AdS_5 \times M_5$ w/ $M_5 =$ Sasaki-Einstein.

$C(M_5)$ Kähler   Ricci $= \lambda g$

$M_5 \sim S^2 \times S^3$ topologically. $\boxed{C(M_5) = \text{CY cone}}$



F-theory:

Ansatz: $\boxed{AdS_5 \times M_5 \quad + F_5\text{-flux} \quad + \tau}$

Let again $C(M_5) = Y_3$.

$\Downarrow$ SUSY

$$\text{Ricci}(Y_3) = \frac{d\tau_1}{2\tau_2}\,,$$ Y_3 Kähler, but not CY; τ varies holomorph.

F-theoretic reformulation: $AdS_5 \times \boxed{\begin{array}{c} S^1 \to M_5 \quad E_\tau \\ \downarrow \\ B^2 \end{array}}$ M_7 w/ $C(M_7)$ an elliptic CY_4^τ.

In summary:

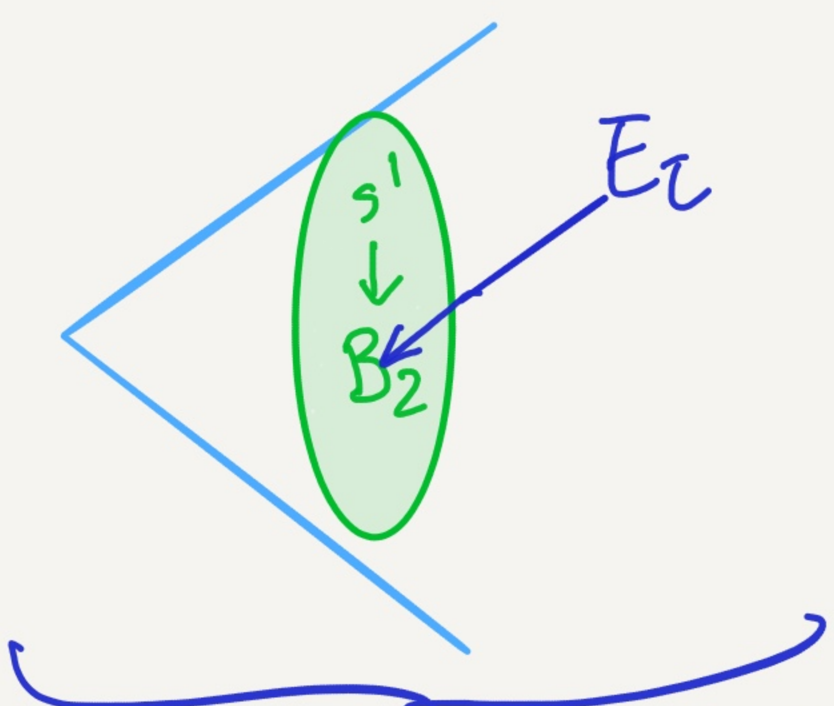
$$ds^2 = ds^2_{AdS_5} + \frac{1}{m^2}\left((d\psi+\varrho)^2 + ds^2(\overset{\text{Kähler surf.}}{\downarrow}B_2)\right)$$

w/ $\boxed{E_\tau \text{ varying over } B_2.}$

$$\Longleftrightarrow \quad ds^2 = \frac{1}{m^2}\left(r^2\, ds^2_{\mathbb{R}^{1,3}} + \frac{1}{r^2}\, ds^2(Y_3)\right)$$

w/ $$ds^2(Y_3) = dr^2 + r^2\Big(\underbrace{(d\psi+\varrho)^2 + ds^2(B_2)}_{M_5}\Big).$$

$$Y_3 = C(M_5).$$



X_F CY 4-fold

and $\boxed{Y_3 \text{ is Base of elliptic CY 4-fold } X_F.}$

Dual SCFTs

AdS_3 solutions: • D3-branes on $C \subseteq \left.\begin{array}{c} E_\tau \\ \downarrow \\ B \end{array}\right\} CY_3$

for $(0,4)$ [Lawrie, SSN, Weigand]

• $Y^{p,q}$ 4d $N=1$ theories on $C = \left.\begin{array}{c} E_\tau \\ \downarrow \\ \mathbb{P}^1 \end{array}\right\} K3$

for $(0,2)$. [Couzens, Martelli, SSN].

AdS_5 solutions: • D3-branes in K3-fibered CY_4 [CMS].

⇒ Summary: - There is more to F-theory than Minkowski space.
- Would be good to develop better tools to study singular metrics on Bases of F-theory elliptic fibrations ~ diff. geom.
- Full $\boxed{SL_2\mathbb{Z}}$ symmetry built into solutions.