A review of the action principle for hydrodynamics

Amos Yarom

Together with: K. Jensen, N. Pinzani, R. Marjieh

See also: Haehl, Loganayagam, Rangamani

together with Geracie, Narayan, Nizami, Ramirez

and: Crossley, Glorioso, Liu

together with Gao, Rajagopal

and earlier work by: Grozdanov, Polonyi

Schwinger-Keldysh

Given an action, S, we construct

$$Z = \int D\phi e^{\frac{i}{\hbar} S}$$

Schwinger-Keldysh

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$$Z[A] = \int D\phi e^{\frac{i}{\hbar} S[A]}$$

Schwinger-Keldysh

Recall that:

$$\langle 0 | \underbrace{J \ldots J}_{n} | 0 \rangle \sim \frac{\delta^n}{\delta A^n} \ln Z[A]$$

Schwinger-Keldysh

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$$\langle 0|\underbrace{J\ldots J}_{n}|0\rangle \sim \frac{\delta^n}{\delta A^n}\ln Z[A]$$

But also

$$\mathrm{Tr}\left(e^{-\beta H}\underbrace{J\ldots J}_{n}\right) \sim \frac{\delta^n}{\delta A^n}\ln Z_{SK}[A]$$

Schwinger-Keldysh

Recall that:

$$\langle 0|\mathcal{T}\left(\underbrace{J\ldots J}_{n}\right)|0\rangle = \frac{\delta^n}{\delta A^n} i \ln Z[A]$$

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Recall that:

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Our goal is to find S_{eff} .

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Symmetries:

Schwinger-Keldysh

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Symmetries:

- Doubled gauge/diff invariance.

$$Z_{SK}[A_1 + d\Lambda_1, A_2] = Z_{SK}[A_1, A_2 + d\Lambda_2] = Z_{SK}[A_1, A_2]$$

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$$Z_{SK}[A, A] = \mathrm{Tr}\left(U[A]e^{-\beta H}U^{\dagger}[A]\right) = 1$$

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- Reality & positivity

$$Z_{SK}[A_1, A_2]^* = \mathrm{Tr}\left(U^*[A_1^*]e^{-\beta H^*}U^T[A_2^*]\right)$$

$$= \mathrm{Tr}\left(\left(U^*[A_1^*]e^{-\beta H^*}U^T[A_2^*]\right)^T\right)$$

$$= Z_{SK}[A_2^*, A_1^*]$$

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- KMS (Kubo-Martin-Schwinger)

$$\mathrm{Tr}\left(e^{-\beta H}O_1(t_1)O_2(t_2)\right) = \mathrm{Tr}\left(e^{-\beta H}O_1(t_1)e^{\beta H}e^{-\beta H}O_2(t_2)\right) = \mathrm{Tr}\left(O_1(t_1 + i\beta)e^{-\beta H}O_2(t_2)\right)$$

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Schwinger-Keldysh

Degrees of freedom.

Schwinger-Keldysh

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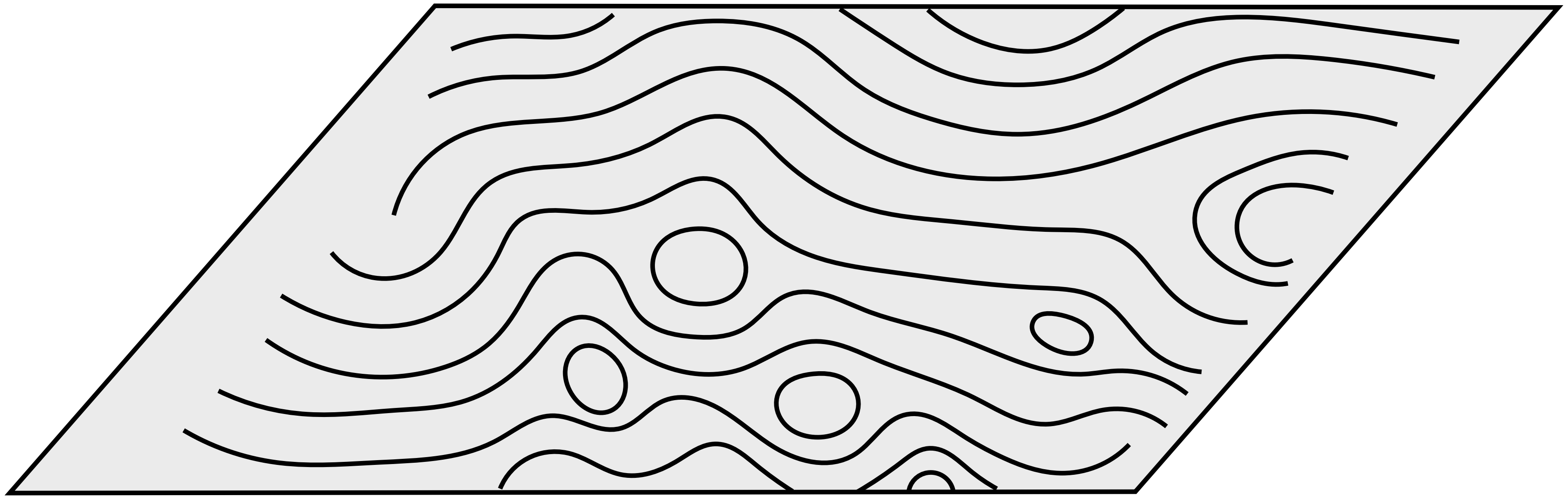
- Motivation 1 (fluid variables)



Schwinger-Keldysh

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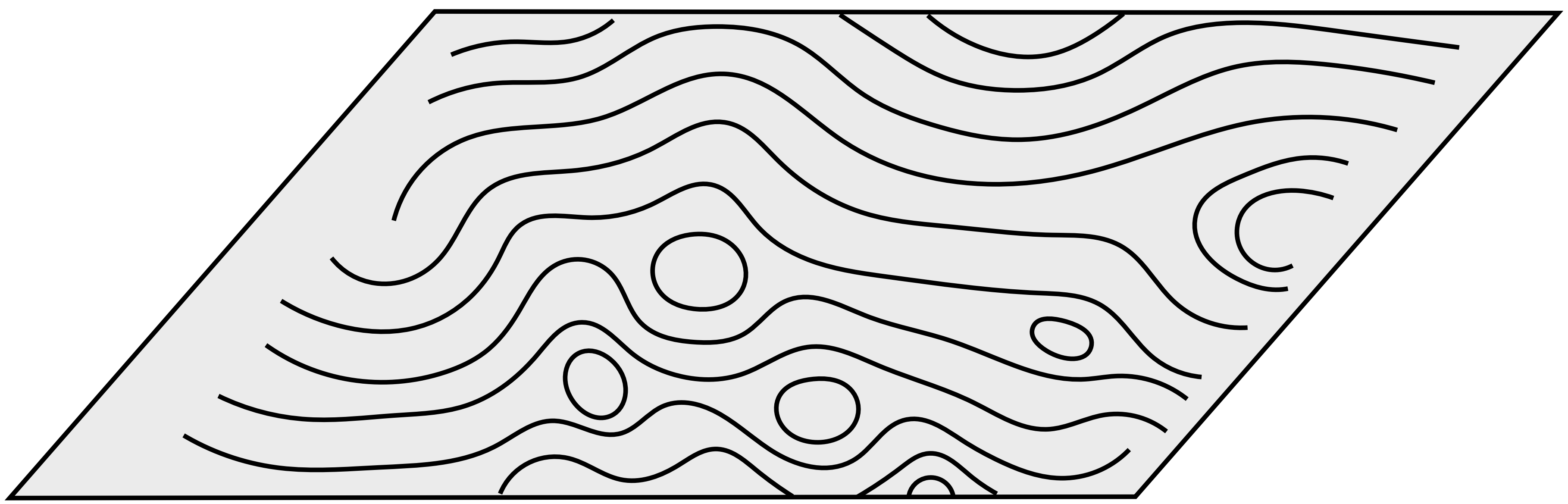
    Euler description of fluids:



Schwinger-Keldysh

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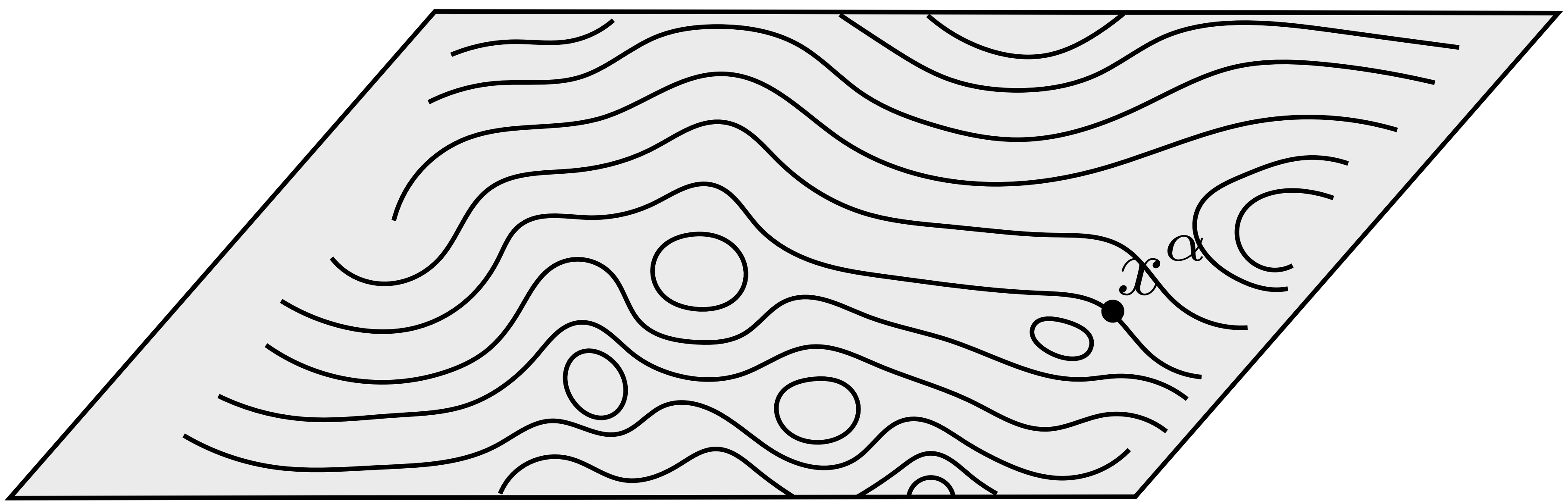

$u^\mu(x^\alpha)$ $T(x^\alpha)$

Schwinger-Keldysh

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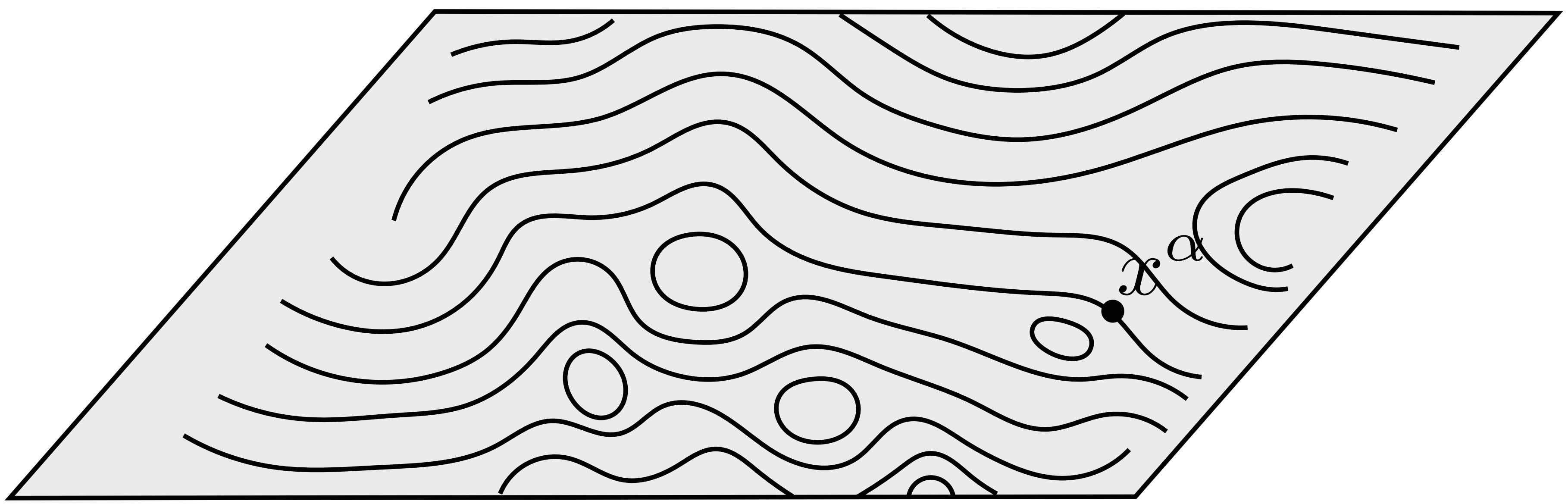

$X^\alpha(\sigma^0)$

Schwinger-Keldysh

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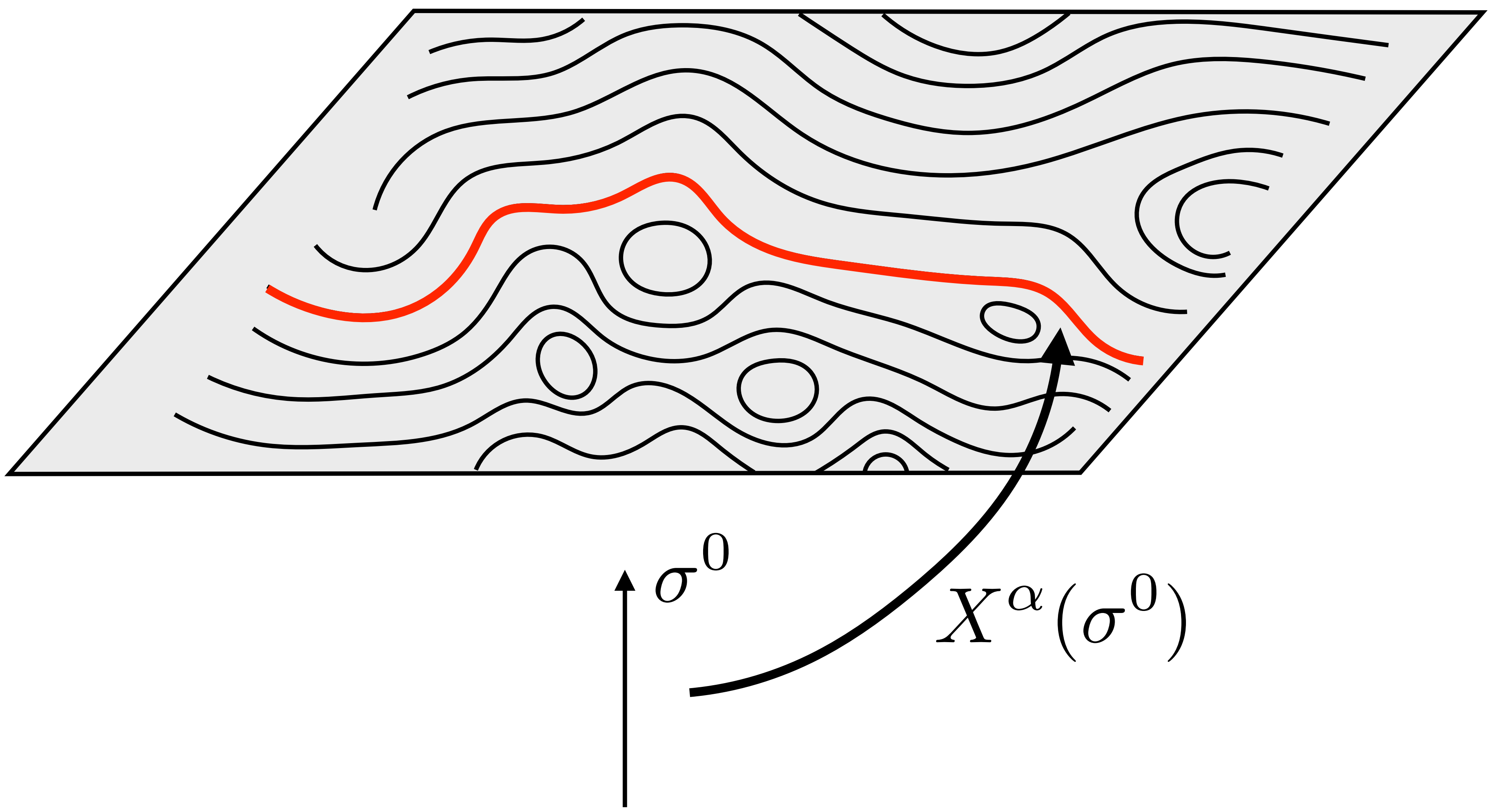

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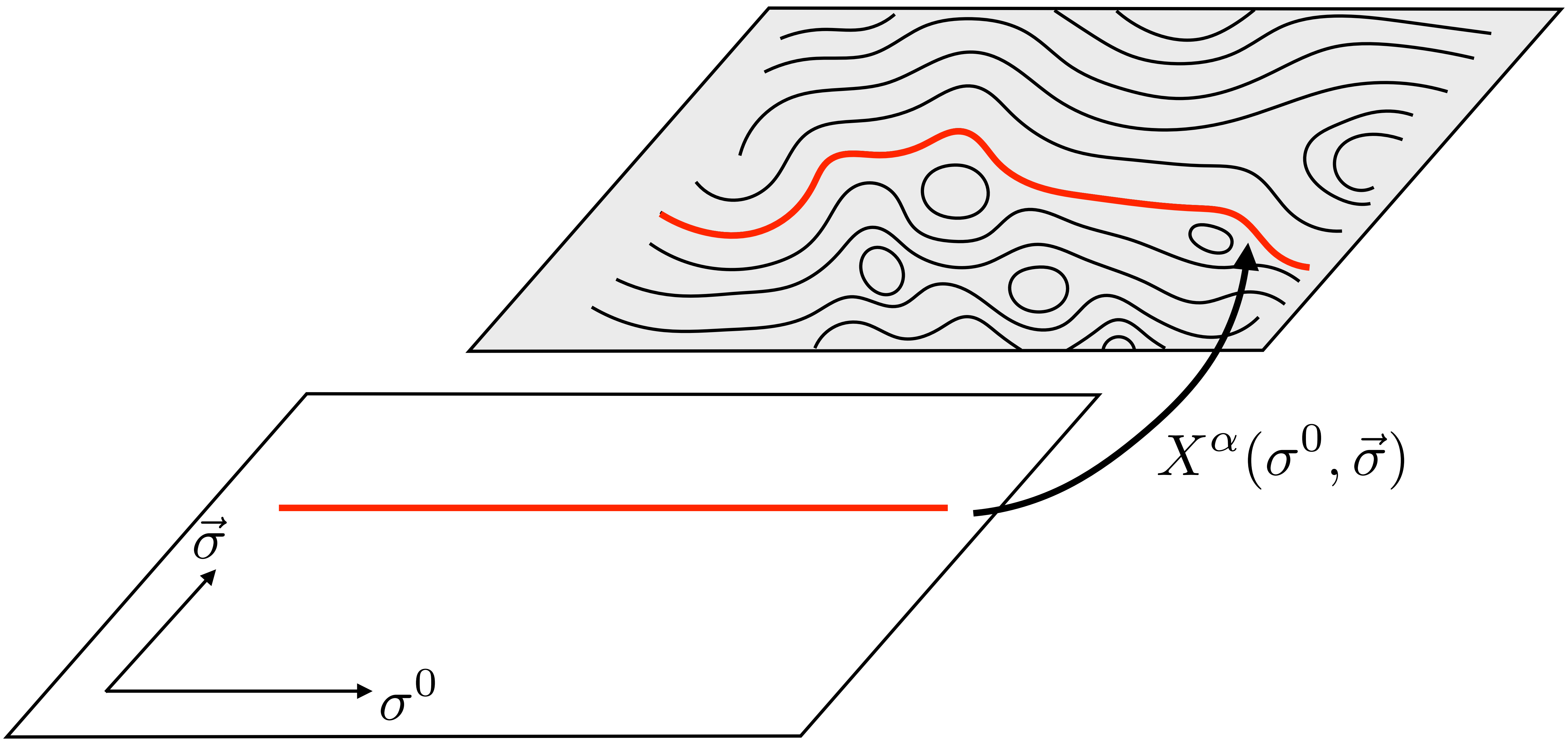

Schwinger-Keldysh

Degrees of freedom. $X^\alpha(\sigma^0, \vec{\sigma})$

- Motivation 1 (fluid variables)

Schwinger-Keldysh

Degrees of freedom. $X^{\alpha}(\sigma^0, \vec{\sigma})$

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- Motivation II (fluid equations of motion)

$$\partial_\mu T^{\mu\nu} = 0$$

Schwinger-Keldysh

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Schwinger-Keldysh

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$$\delta_X S = 0 \qquad \Rightarrow \nabla_\mu T^\mu{}_\nu = 0$$

(where $T^{\mu\nu} = \partial_i X^\mu \partial_j X^\nu T^{ij}$)

Schwinger-Keldysh

$$Z_{SK}[A_1, A_2] \underset{\frac{\mu}{\Lambda} \ll 1}{\to} \int D\xi_1 D\xi_2 e^{\frac{i}{\hbar} S_{eff}}$$

Our goal is to find S_{eff} .

Symmetries:

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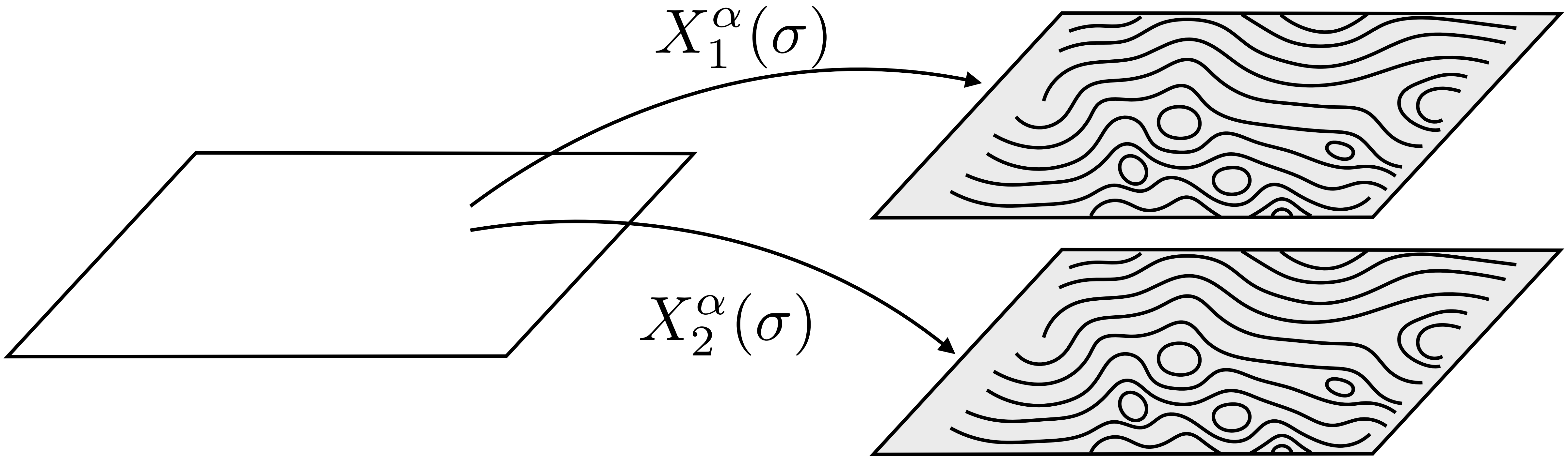
Schwinger-Keldysh



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End result:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}} \right)$$

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Recall that the KMS symmetry is a $\mathbb{Z}_2$ symmetry:

$$\begin{aligned} Z_{SK}[A_1, A_2] &= Z_{SK}[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)] \\ &= Z_{SK}[\eta^2_{A_1} A_1(t_1),\, \eta^2_{A_2} A_2(t_2 + i\beta - i\beta)] \end{aligned}$$

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Let us define the action of the $\mathbb{Z}_2$ symmetry on fields as K .

$\widetilde{\mathcal{L}}$ is the $\mathbb{Z}_2$ transform of $\mathcal{L}$: $K(\mathcal{L}) = \widetilde{\mathcal{L}}$

Schwinger-Keldysh

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- $Z_{SK}[A, A] = 1$

Schwinger-Keldysh

SK symmetry:

- $Z_{SK}[A, A] = 1$

If we change basis, we find

$$Z_{SK}\left[\frac{1}{2}\left(A_1 + A_2\right) = A, A_1 - A_2 = 0\right] = 1$$

Schwinger-Keldysh

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Thus:

$$\frac{\delta^n}{\delta(A_1 + A_2)^n} \ln Z_{SK}\bigg|_{A_1 - A_2 = 0} = 0$$

Schwinger-Keldysh

$$\frac{\delta^n}{\delta(A_1 + A_2)^n} \ln Z_{SK}\Big|_{A_1 - A_2 = 0} = 0$$

This is a topological symmetry. It is possible to construct topological theories in the following way:

Schwinger-Keldysh

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Schwinger-Keldysh

This is a topological symmetry. It is possible to construct topological theories in the following way:

1. A Grassmanian nilpotent operator Q
2. Physical operators (and the action) vanish under Q.
3. The energy momentum tensor is given by: $T^{\mu\nu} = \delta_Q V^{\mu\nu}$

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1. $\Phi = \phi + \theta\psi$

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where $\int d\theta = 0 \quad , \quad \int \theta d\theta = 1$

Schwinger-Keldysh

$$\frac{\delta^n}{\delta(A_1 + A_2)^n} \ln Z_{SK}\Big|_{A_1 - A_2 = 0} = 0$$

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For the Schwinger-Keldysh theory

1. $$\mathbb{X}_r = \frac{1}{2}(X_1 + X_2) + \theta X_{\bar{g}}$$
$$\mathbb{X}_a = X_g + \theta(X_1 - X_2)$$

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3. $S=\int d\theta d^d\sigma L(\mathbb{X}_r,\mathbb{X}_a)$

Schwinger-Keldysh

$$Z_{SK}[A_1, A_2] \underset{\frac{\mu}{\Lambda} \ll 1}{\longrightarrow} \int D\xi_1 D\xi_2 e^{\frac{i}{\hbar} S_{eff}}$$

Our goal is to find S_{eff} .

End result:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}}\right)$$

- $Z_{SK}[A, A] = 1$

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- $Z_{SK}[A, A] = 1$
- $Z_{SK}[A_1, A_2] = Z_{SK}[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)]$

We find that K and Q do not form a group. We add an extra nilpotent symmetry $\overline{Q}$.

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Recall:

$$\mathbb{X}_a = X_g + \theta(X_1 - X_2) \qquad \mathbb{X}_r = \frac{1}{2}(X_1 + X_2) + \theta X_{\bar{g}}$$

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We find that K and Q do not form a group. We add an extra nilpotent symmetry $\overline{Q}$.

Now:

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Recall $\rho = e^{-\beta H}$ or $\rho = e^{\beta^i P_i}$

and we define, e.g., $\pounds_\beta \phi = \beta^i \partial_i \phi$

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$$\mathcal{L} = \sqrt{-\mathbb{g}} \mathcal{L} \left(\mathbb{g}_{ij}, \, \beta^i \right)$$

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$$g_{1/2\,ij}(X) = \partial_i X^\mu \partial_j X^\nu g_{1/2\,\mu\nu}(X)$$

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*Infinitesimal coordinate transformations do not commute with Q. So far we have only retained all the symmetries in a limit where the sources are almost aligned and the temperature is large, or in a probe limit.

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In addition we impose

$$\mathrm{Im} S_{eff} \geq 0$$

due to

$$|Z_{SK}[A_1, A_2]|^2 \leq 1$$

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Example:

$$\mathcal{L} = \sqrt{-\mathbb{g}}P(-\beta^i \mathbb{g}_{ij} \beta^j)$$

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Leads to:

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Example:

$$\mathcal{L} = \sqrt{-\mathbb{g}}\left(P - \eta \mathbb{g}^{ik}\mathbb{g}^{jl} D_\theta \mathbb{g}_{ij} D_{\bar{\theta}} \mathbb{g}_{kl}\right)$$

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Leads to:

$$T^{ij} = \epsilon u^i u^j + (g^{ij} + u^i u^j) P - \eta \sigma^{ij}$$

Summary

$$Z_{SK}[A_1, A_2] \underset{\frac{\mu}{\Lambda} \ll 1}{\longrightarrow} \int D\xi_1 D\xi_2 e^{\frac{i}{\hbar} S_{eff}}$$

Our goal is to find S_{eff} .

Symmetries:

- $Z_{SK}[A_1 + d\Lambda_1, A_2] = Z_{SK}[A_1, A_2 + d\Lambda_2] = Z_{SK}[A_1, A_2]$
- $Z_{SK}[A, A] = 1$
- $Z_{SK}[A_1, A_2]^* = Z_{SK}[A_2^*, A_1^*] \qquad |Z_{SK}[A_1, A_2]|^2 \leq 1$
- $Z_{SK}[A_1, A_2] = Z_{SK}[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)]$

Degrees of freedom:

- $X_1^\alpha \qquad X_2^\alpha$

Summary

We found:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}} \right)$$

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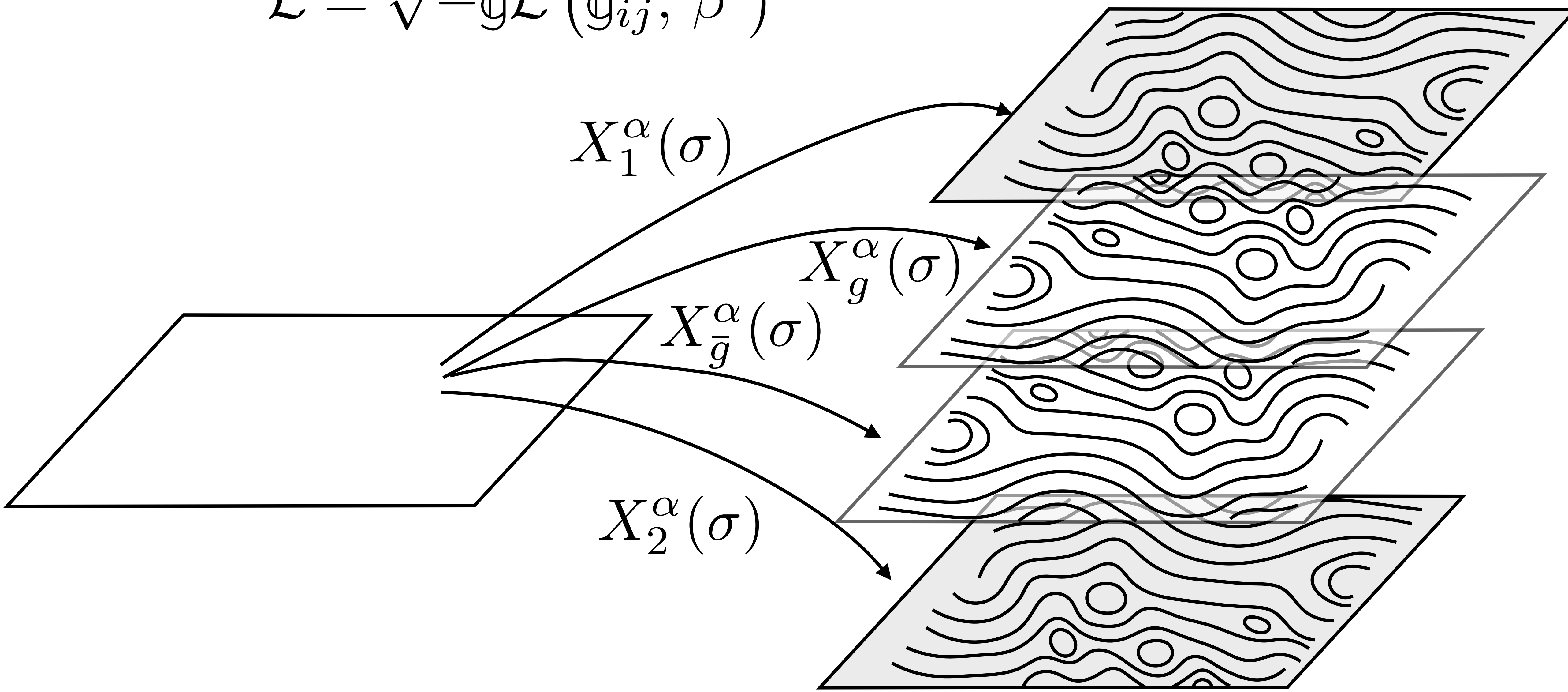

Outlook

- Generalizations
- Chaos
- Stochastic noise
- AdS/CFT
- Classification & constraints
- Hidden symmetries

Generalizations

- Generalizations to other fluids
  - Non relativistic fluids
  - Superfluids
  - Anomalies (Glorioso, Liu and Rajagopal 2017, Jensen, Marjieh, Pinzani-Fokeeva, AY, 2017)
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- Generalizations to more contours
  - Classification (Loganayagam, 2019)

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Chaos

Chaos can be characterised by

$$Tr\left(e^{-\beta H}\left[V(t), W(0)\right]^2\right) \sim e^{\lambda t}$$

where

$$\lambda \leq \lambda_{max} = 2\pi T$$

(Maldacena, Shenker, Stanford, 2019)

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(Maldacena, Shenker, Stanford, 2019)

It is possible to compute these 4-pt functions via Schwinger Keldysh theory?

(Blake, Lee, Liu, 2017, Blake, Davison, Grozdanov, Liu, 2018, Grozdanov 2019, Haehl, 2018)

Outlook

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Stochastic noise

The ‘a’ type fields in the action encode stochastic noise which, at the quadratic level is Gaussian-like

$$Z \sim \int e^{i \int i X_a^2 G(X_r) + \ldots d^d x} D X_a D X_r$$

$$\sim \int e^{- \int X_a^2 G(X_r) + \ldots d^d x} D X_a D X_r$$

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E.g., in (Chen-Lin, Delacretaz, Hartnoll, 2018) the authors looked at a theory of a single diffusion mode

$$\mathcal{L} = iT^2\kappa(\nabla\phi_a)^2 - \phi_a(\dot{\epsilon} - D\nabla^2\epsilon) + \nabla^2\phi_a\left(\frac{1}{2}\lambda\epsilon^2 + \frac{1}{3}\lambda'\epsilon^3\right) + icT^2\left(\nabla\phi_a\right)^2\left(\tilde{\lambda}\epsilon + \tilde{\lambda}'\epsilon^2\right) + \ldots$$

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This was preceded by (Kovtun, Moore, Romatschke, 2014)

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E.g., in (Chen-Lin, Delacretaz, Hartnoll, 2018) the authors looked at a theory of a single diffusion mode

$$\mathcal{L} = i T^2 \kappa (\nabla \phi_a)^2 - \phi_a (\dot{\epsilon} - D \nabla^2 \epsilon) + \nabla^2 \phi_a \left(\frac{1}{2} \lambda \epsilon^2 + \frac{1}{3} \lambda' \epsilon^3 \right) + i c T^2 (\nabla \phi_a)^2 \left(\tilde{\lambda} \epsilon + \tilde{\lambda}' \epsilon^2 \right) + \ldots$$

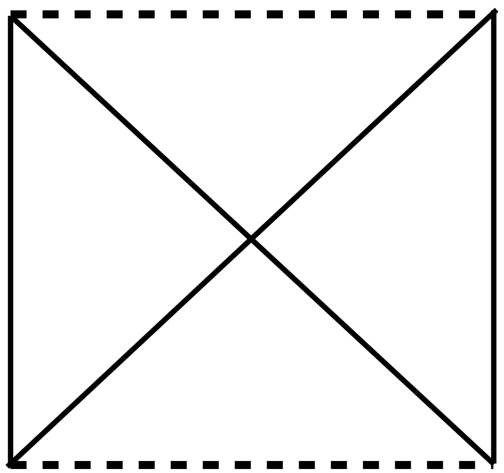
This was preceded by (Kovtun, Moore, Romatschke, 2014)

- Validity of hydro ?
- How do 3rd order terms contribute?
- What about noise associated with particular solutions?

Outlook

- Generalizations
- Chaos
- Stochastic noise
- AdS/CFT
- Classification & constraints
- Hidden symmetries

AdS/CFT



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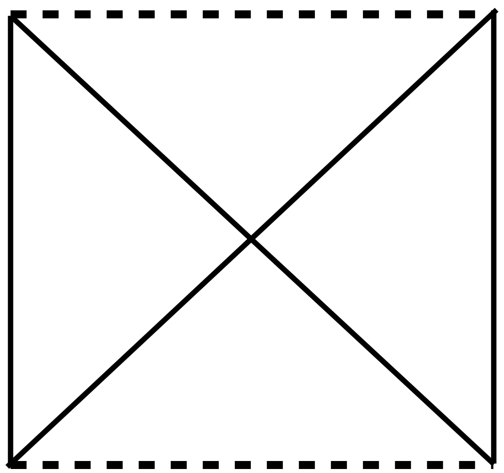

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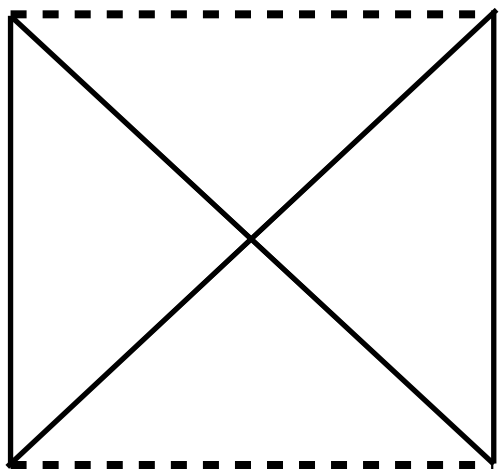

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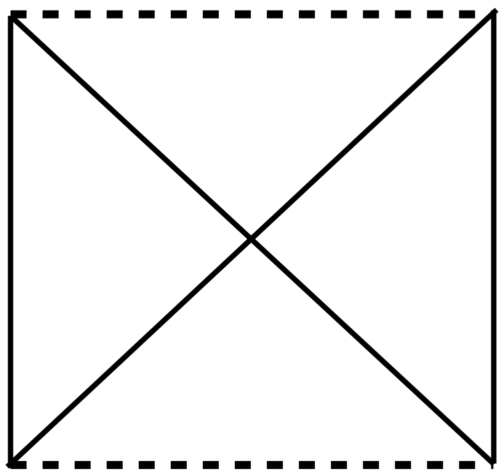

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Can one find a prescription which is independent of the background geometry?

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- Chaos
- Stochastic noise
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- Classification & constraints
- Hidden symmetries

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$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}}\right)$$

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$$\mathcal{L} = \sqrt{-\mathbb{g}}\mathcal{L}(\mathbb{g}_{ij},\, \beta^i)$$

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(Jensen, Marjieh, Pinzani-Fokeeva, AY, 2018, Haehl, Loganayagam, Rangamani, 2018)

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But positivity of the effective action implies:

$$\gamma^- = 0$$

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- Are there better examples?
- Is there a geometric interpretation in AdS/CFT?

Hidden symmetries

- Generalizations
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Hidden symmetries

The Navier Stokes equations are given by:

$$\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} + \vec{\nabla} p = \frac{1}{R} \nabla^2 \vec{v}$$

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From these it follows that

$$\partial_t E = -\frac{1}{R} \Omega$$

with

$$E = \frac{1}{2} \int v^2 d^d x \qquad \Omega = \frac{1}{2} \int \omega_{ij} \omega^{ij} d^d x$$

$$\omega_{ij} = \partial_i v_j - \partial_j v_i$$

Hidden symmetries

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This leads to Kolmogorov's theory where energy is dissipated at small scales.

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$$\sigma_{ij} = \partial_i v_j - \partial_j v_i$$

Taking a closer look:

$$\partial_t \Omega = \int \omega_{ji}\omega^{i}{}_{k}\sigma^{kj} d^d x - \frac{1}{R}\int \partial_k \omega_{ij}\partial^k \omega^{ij}$$

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So in 2 dimensions we have, for large R,

$$\partial_t E = 0 \qquad\qquad \partial_t \Omega = -\frac{1}{R}P$$

which leads to the inverse cascade picture.

Hidden symmetries

Is there an analog of enstrophy in relativistic flow?

Hidden symmetries

Is there an analog of enstrophy in relativistic flow?

For conformal, uncharged fluids,

$$J^\mu = \frac{\Omega_{\alpha\beta}\Omega^{\alpha\beta}}{T^2} u^\mu$$

with

$$\Omega_{\alpha\beta} = \partial_\alpha (T u_\beta) - \partial_\beta (T u_\alpha)$$

satisfies

$$\partial_\mu J^\mu = \mathcal{O}(\partial^4)$$

(Carrasco, Lehner, Myers, Reula, Singh, 2012)

Hidden symmetries

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In 2 spatial dimensions one finds that

$$\delta X^\mu = \frac{\Omega^2}{Ts^2} u^\mu - \frac{2}{sp'} P^{\mu\alpha} \left(2\nabla_\nu \Omega^\nu{}_\alpha + \frac{\Theta E_\alpha}{p'} + 2\Omega_{\nu\alpha} a^\nu + \frac{2}{s} \left(\frac{\partial s}{\partial T} \nabla_\nu T + \frac{\partial s}{\partial \mu} \nabla_\nu \mu \right) \Omega_\alpha{}^\nu \right)$$

$$\delta C = -\frac{\mu \Omega^2}{s^2 T}$$

with

$$P = p(Tf(\mu/T))$$

Hidden symmetries

More generally, we can generalise this to other equations of state by looking for symmetries of the effective action:

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with

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is a symmetry. The associated current is

$$J^\mu = \frac{\Omega^2}{s} u^\mu \qquad \Omega_{\alpha\beta} = \partial_\alpha \left(Tf(\mu/T) u_\beta \right) - \partial_\beta \left(Tf(\mu/T) u_\alpha \right)$$

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