Nonexistence results for elliptic problems in contractible domains

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NONLINEAR GEOMETRIC PDE'S
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Main problem - outline of the talk

$$(P) \qquad \boxed{-\Delta u = f(u) \ \text{ in } \Omega \qquad u = 0 \ \text{ on } \partial\Omega \qquad u \not\equiv 0}$$

$\Omega \subset\subset \mathbb{R}^n$, $n \geq 3$, $\quad f$ supercritical and regular

$\diamondsuit$ model case $f(u) = |u|^{p-2}u$, $\quad p > 2^* := \frac{2n}{n-2}$

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- known facts
- existence results in nearly star-shaped domains
- new nonexistence results
- extensions to the q-Laplace operator
- work in progress

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Well-known facts

- f has subcritical growth $\Longrightarrow$ a (positive) solution exists
- $f(u) = |u|^{p-2}u,\ p > 2^*,\ \Omega$ star-shaped $\Longrightarrow$ no solution: Pohozaev identity

$$\frac{1}{2}\int_{\partial\Omega} |Du|^2\, x\cdot\nu\, d\sigma = -\left(\frac{n-2}{2}\right)\int_\Omega |Du|^2 dx + \frac{n}{p}\int_\Omega |u|^p dx$$

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- If $f(u) = |u|^{p-2}u,\ \ p > 2,\ \ \Omega$ an annulus $\Longrightarrow$ infinitely many [Kazdan - Warner (1975)]

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Is the nontriviality of the topology of the domain sufficient or necessary for the existence of solutions?

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- Nonexistence results in contractible domains (solid-tori) for $n \geq 4$ and $p > \frac{2(n-1)}{n-3} =: 2^*_{n-1}$ [Passaseo (1993)]

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In the supercritical case the geometry of the domain affect the existence of solutions

[Dancer, Del Pino, Felmer, Guo, Micheletti, M., Musso, Pacard, Pistoia, Passaseo, Struwe, Wei, Yan, ...]

[Wei - Yan (2011)]: existence of infinitely many positive solutions in suitable contractible domains for $f(u) = |u|^{p-2}u$ with $p = 2^*_{n-k} := \frac{2(n-k)}{(n-k)-2}$

There exists solutions in nearly star-shaped domains?

Definition [M. - Passaseo (2002)]

$$\sigma(\Omega) = \sup_{x_0 \in \Omega} \inf \left\{ \frac{x - x_0}{|x - x_0|} \cdot \nu(x) \ : \ x \in \partial\Omega \right\}$$

$\nu(x)$ *is the outward normal to* $\partial\Omega$*.*

- Ω strictly star-shaped $\leftrightarrow$ $\sigma(\Omega) > 0$

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$\nu(x)$ *is the outward normal to* $\partial\Omega$.

- Ω strictly star-shaped $\leftrightarrow$ $\sigma(\Omega) > 0$
- "Ω nearly star-shaped $\leftrightsquigarrow$ $\sigma(\Omega)^- = \max\{0, -\sigma(\Omega)\}$ small"

In [Dancer - Zhang (2000)] a different definition of nearly star-shaped domains

- Theorem (2002) For every $\eta > 0$ there exists $\Omega_\eta \subset \mathbb{R}^n$ and $\varepsilon_\eta > 0$ such that $\sigma(\Omega)^- < \eta$ and problem

$$-\Delta u = u^{2^*-1+\varepsilon} \ \text{ in } \Omega_\eta \qquad u = 0 \ \text{ on } \partial\Omega_\eta$$

has multiple positive solutions for every $\varepsilon \in (0, \varepsilon_\eta)$.

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- Theorem (2006) For every $\eta > 0$ there exists $\Omega_\eta \subset \mathbb{R}^n$ and $p_\eta > 0$ such that $\sigma(\Omega)^- < \eta$ and problem

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What about nonexistence in domains far from star-shaped ones?

New nonexistence results

In our problem

$$(P) \qquad -\Delta u = f(u) \text{ in } \Omega \qquad u = 0 \text{ on } \partial\Omega \qquad u \not\equiv 0$$

we assume f a continuous function such that

$$(f) \qquad tf(t) \geq p \int_0^t f(\tau)d\tau \geq 0 \qquad \forall t \in \mathbb{R}$$

for a given $p > 2^*$

- ♢ p can be arbitrarily chosen near 2^*
- ♢ no symmetry assumption will be required for Ω

Notation: $F(t) = \int_0^t f(\tau)d\tau \quad \forall t \in \mathbb{R}$

.

Construction of the tubular domains:

- $\gamma \in \mathcal{C}^3([a,b],\mathbb{R}^n)$ injective and s.t. $\gamma' \neq 0$ in $[a,b]$
- $N_\varepsilon(t) = \{\xi \in \mathbb{R}^n \ : \ \xi \cdot \gamma'(t) = 0, \ |\xi| < \varepsilon\}$
- ε so small that $t_1 \neq t_2 \Longrightarrow$

$$[\gamma(t_1) + N_\varepsilon(t_1)] \cap [\gamma(t_2) + N_\varepsilon(t_2)] = \emptyset$$

$$\boxed{T_\varepsilon^\gamma := \bigcup_{t \in (a,b)} [\gamma(t) + N_\varepsilon(t)]}$$

Main result:

- Theorem (2019) If $f \in \mathcal{C}(\mathbb{R})$ satisfies

$$(f) \qquad tf(t) \geq p \int_0^t f(\tau)d\tau \geq 0 \qquad \forall t \in \mathbb{R}$$

with $p > 2^*$ then problem

$$-\Delta u = f(u) \text{ in } T_\varepsilon^\gamma \qquad u = 0 \text{ on } \partial T_\varepsilon^\gamma \qquad u \not\equiv 0$$

has no solution for ε small.

An integral identity:

- Lemma u a solution of (P), $V \in \mathcal{C}^1(\overline{\Omega}, \mathbb{R}^n) \implies$

$$\frac{1}{2}\int_{\partial\Omega} |Du|^2\, V \cdot \nu \, d\sigma =$$

$$\int_\Omega dV[Du] \cdot Du \, dx + \int_\Omega \operatorname{div} V \left(F(u) - \frac{1}{2}|Du|^2 \right) dx$$

here $dV[\eta] = \sum_{i=1}^n D_i V \, \eta_i, \forall \eta \in \mathbb{R}^n$.

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proof: apply Gauss-Green to $V \cdot Du \, Du$ and use (P)

♢ in Pohozaev $V(x) = x$

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The vector field:

$$V\big(\gamma(t)+\psi\big) = [1-\psi\cdot\gamma''(t)]\, t\,\gamma'(t) + \psi$$

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Properties:

(1) $V \cdot \nu > 0$ on $\partial \overline{T}_\varepsilon^\gamma$

(2) $\lim_{\varepsilon\to 0} \sup\{|1 - dV(x)[\eta]\cdot\eta| \;:\; x \in T_\varepsilon^\gamma,\ \eta \in \mathbb{R}^n,\ |\eta| = 1\} = 0$

(3) $\lim_{\varepsilon\to 0} \sup\{|n - \operatorname{div} V(x)| \;:\; x \in T_\varepsilon^\gamma\} = 0$

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u_ε a solution of (P) on T_ε^γ $\Longrightarrow$

$$\frac{1}{2}\int_{\partial T_\varepsilon^\gamma} |Du_\varepsilon|^2\, V\cdot\nu = \int_{T_\varepsilon^\gamma} dV[Du_\varepsilon]\cdot Du_\varepsilon + \int_{T_\varepsilon^\gamma} \operatorname{div} V\,\Big(F(u_\varepsilon) - \frac{1}{2}|Du_\varepsilon|^2\Big)$$

so

$$0 \le \Big(1-\frac{n}{2}+O(1)\Big)\int_{T_\varepsilon^\gamma} |Du_\varepsilon|^2 + (n+O(1))\int_{T_\varepsilon^\gamma} F(u_\varepsilon)$$

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End of the proof:

$$0 \leq \left(1 - \frac{n}{2} + O(1)\right) \int_{T_\varepsilon^\gamma} |Du_\varepsilon|^2 \, dx + (n + O(1)) \int_{T_\varepsilon^\gamma} F(u_\varepsilon) \, dx$$

$\Longrightarrow$ (by assumption (f))

$$0 \leq \left(1 - \frac{n}{2} + O(1)\right) \int_{T_\varepsilon^\gamma} |Du_\varepsilon|^2 \, dx + (n + O(1)) \frac{1}{p} \int_{T_\varepsilon^\gamma} u_\varepsilon f(u_\varepsilon) \, dx$$

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$\Longrightarrow$ (since u_ε solves (P))

$$0 \leq \left(1 - \frac{n}{2} + \frac{n}{p} + O(1)\right) \int_{T_\varepsilon^\gamma} |Du_\varepsilon|^2 \, dx$$

contrary to $1 - \frac{n}{2} + \frac{n}{p} < 0$ (i.e. $p > 2^*$), for small ε

$\square$

Tubular neighbourhood of closed circuit

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- Theorem Let $\gamma \in \mathcal{C}^2([a,b], \mathbb{R}^n)$ be a regular curve such that $\gamma(a) = \gamma(b)$ and $\gamma'(a) = \gamma'(b)$.
If $f \in \mathcal{C}(\mathbb{R})$ satisfies (f) with $\mathbf{p} > \mathbf{2}^*_{\mathbf{n-1}}$ then problem

$$-\Delta u = f(u) \text{ in } T^\gamma_\varepsilon \qquad u = 0 \text{ on } \partial T^\gamma_\varepsilon \qquad u \not\equiv 0$$

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Proof. The vector field:

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contrary to $1-\frac{(n-1)}{2}+\frac{(n-1)}{p}<0$ (i.e. $p>2^*_{n-1}$), for small ε $\square$

Higher dimensional tubular domains

Higher dimensional tubular domains - $\exists$ results

- $\gamma_k : \mathbb{R}^k \to \mathbb{R}^n$ the stereographic projection on a k-dim. sphere
- $\Gamma_k^r = \{\gamma_k(x) \ : \ |x| < r\}$
- $T_{\bar{\varepsilon}}(\Gamma_k^r)$ an $\bar{\varepsilon}$-normal tubular neighbourhood of Γ_k^r

Here: $2 \leq k \leq n-1$ and $2^*_{n-k+1} = \begin{cases} \frac{2(n-k+1)}{n-k-1} & \text{if } k < n-1 \\ \infty & \text{if } k = n-1 \end{cases}$

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(c) Fixed $r > 1$, there exists $\bar{p} < 2^*_{n-k+1}$ such that (P) has solution in $T_{\bar{\varepsilon}}(\Gamma_k^r)$, $\forall p \in (\bar{p}, 2^*_{n-k+1})$.

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$\diamondsuit$ $k \geq n-2$ or $k < n-2$ and $p < 2^*_{n-k}$ $\Rightarrow$ if Γ_k is a k-dimensional sphere then (P) has solution

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Nonexistence results for the q-laplacian

(-) $\gamma \in \mathcal{C}^3([a,b],\mathbb{R}^2)$ injective and s.t. $\gamma' \neq 0$ in $[a,b]$

(-) T_ε^γ the ε-neighbourhood of $\gamma([a,b])$

- Theorem $q \in (1,2)$. If $f \in \mathcal{C}(\mathbb{R})$ satisfies (f) with $p > q^* = \frac{2q}{2-q}$ then problem

$$(q) \qquad -\operatorname{div}(|Du|^{q-2}Du) = f(u) \text{ in } T_\varepsilon^\gamma \quad u = 0 \text{ on } \partial T_\varepsilon^\gamma \quad u \not\equiv 0$$

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◇ Similar results in $\mathbb{R}^n$ with $n \geq 3$

◇ Nonexistence results on contractible neighbourhood of graphs in $\mathbb{R}^2$

◇ *Conjecture*: nonexistence of solutions in contractible domains in $\mathbb{R}^2$

◇ … …

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Thanks for Your Attention