Long Transients in Ecology: Theory and Observations

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*Long Transients & Ecological Forecasting*

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Plan of the talk

- Introduction: what are long transients?
- Basic mechanisms generating long transients (nonspatial systems)
- Relation to tipping points
- A (brief) look at spatial systems
- Conclusions
What is it all about

*Transient*: lasting for only a short time; temporary

(Cambridge English Dictionary)

Typically, transients are associated with the effect of the initial conditions and disappear relatively fast.

Long-term dynamics are usually associated with the system’s attractors.

“Long transient” is apparently an oxymoron??

However...
Examples of long transients in population models

Dynamics of a nonspatial, time-discrete, single-species model:

(from Schreiber, 2003)
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Time-continuous single-species model with time-delay:

(from Morozov et al., 2016)
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Space-time-continuous, 3-species model (plankton dynamics):

(from Petrovskii et al., 2017)
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Empirical examples are abundant too

Flour beetle data (lab) (from Cushing et al., 1998)

Forage fishes (field) (from Frank et al., 2011)
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Flour beetle data (lab)  
(from Cushing et al., 1998)

Forage fishes (field)  
(from Frank et al., 2011)
In all above examples, a regime shift occurs

A well-known theory of regime shifts relates it to a tipping point: a bifurcation (e.g. saddle-node) due to a slow change in some system’s parameter (environmental conditions) (e.g. Scheffer et al. 2009, 2012; Kuehn 2011; Dakos et al., 2012, 2014)

Interestingly, in all above examples, parameters (environmental conditions) are constant!

How that can be possible?
Overview of the baseline mechanisms

“Crawl-by”: transients induced by a saddle

Here $A$ is the ‘small’ vicinity of the saddle, $B$ the range of appropriate initial conditions.
Consider a generic population dynamics model:

\[
\frac{du_k(t)}{dt} = f_k(u), \quad k = 1, \ldots, n,
\]

where \( u = (u_1, \ldots, u_n) \) are the population densities, \( t \) is time.

Linearized system in the vicinity of a steady state \( \bar{u} \):

\[
\frac{dx_k(t)}{dt} = a_{k1}x_1 + \ldots a_{kn}x_n, \quad k = 1, \ldots, n,
\]

where \( x_k(t) = u_k(t) - \bar{u}_k \).

Solution is a linear combination of exponents \( e^{\lambda_i t} \). Let \( \lambda_1 \) be the eigenvalue with the largest real part, \( \text{Re}\lambda_1 > 0 \). The time spent in the vicinity of the (unstable) steady state is estimated as

\[
\tau \propto \frac{1}{\text{Re}\lambda_1}.
\]
Nonlinear effects can substantially increase the range of appropriate initial conditions:

A is the 'small' vicinity, B the range of appropriate initial conditions, S is a separatrix
Example: Rosenzweig–MacArthur model

\[
\frac{du(t)}{dt} = \alpha u \left(1 - \frac{u}{K}\right) - \frac{\gamma uv}{u + h}, \quad \frac{dv(t)}{dt} = \frac{\nu \gamma uv}{u + h} - mv
\]
This will result in **recurrent** long transients:

The system stays in the vicinity of (1,0)

The system stays in the vicinity of (0,0)
Generalization 1

A modified prey-predator system can have a saddle point in the interior of the domain (not at the origin), so that the decay to low density is not a necessary property.

Example: strong Allee effect for prey, quadratic mortality for predator.

(Sen & Banerjee 2015)
Saddle-induced transients in a higher-dimensional systems

A case of more complex dynamics: connected saddles:

(Ashwin & Timme, 2005)
Ghost attractors

Consider a generic two-species system:

\[
\begin{align*}
\frac{du}{dt} &= F(u, v; p), \\
\frac{dv}{dt} &= G(u, v; p)
\end{align*}
\]

Two-species nonlinear competition model (Hastings et al. 2018)
Ghost attractors

A change in the parameter value can bring the system beyond the saddle-node bifurcation:

However, the local bifurcation does not change the global structure of the phase flow: the system slows down in the vicinity of the pre-bifurcation steady state location.
Ghost attractors

The long transient dynamics occur:

![Graph showing long term transient and true asymptotics]

The transient’s duration depends on the closeness to the bifurcation:

\[ \tau \propto |\rho - \rho_c|^{-0.5}. \]
Ghost attractors

A similar mechanism applies to more complicated dynamics, e.g. periodic solutions (limit cycles) and chaos.

Example: long-term chaotic transient (chaotic ghost) in a resource-consumer-predator system (Hastings and Powell 1991; McCann and Yodzis 1994)

Pre-bifurcation: chaotic attractor coexists with a stable limit cycle

Post-bifurcation: the two basins merge, chaotic attractor disappears

Chaotic transients can be particularly long: $\tau \propto \exp\left(k|p - p_c|^{-\gamma}\right) \quad (k, \gamma > 0)$

(Grebogi et al. 1983, 1985)
Ghost attractors

Example of the time-series generated by a chaotic ghost:

(Petrovskii et al., 2017)
Consider
\[ \frac{du(t)}{dt} = f(u, v, \epsilon), \quad \frac{dv(t)}{dt} = \epsilon g(u, v, \epsilon), \quad \epsilon \ll 1. \quad (1) \]

Introducing a rescaled time \( \tau = \epsilon t \), it turns into
\[ \epsilon \frac{du(\tau)}{d\tau} = f(u, v, \epsilon), \quad \frac{dv(\tau)}{d\tau} = g(u, v, \epsilon). \quad (2) \]

In the limit \( \epsilon \to 0 \), system (1) turns into
\[ \frac{du(t)}{dt} = f(u, v, 0), \quad \frac{dv(t)}{dt} = 0, \]
and system (2) turns into
\[ 0 = f(u, v, 0), \quad \frac{dv(\tau)}{d\tau} = g(u, v, 0), \]
Slow-fast systems

Example 1: periodical dynamics in a prey-predator system ($\epsilon = 0.01$)
Slow-fast systems

Example 2: aperiodical dynamics in a two-species competition system

Black (dashed) curve for $\epsilon = 1$, red curve for $\epsilon = 0.002$
Relation between long transients and tipping points
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Regime shift as a system’s response when parameter change is “not too slow” – no LTs

Parameter change very slow or with limited variation: regime shift after LT ghost dynamics

Long transients

- Saddles
- Ghost attractors due to reset of initial conditions
- Slow-fast dynamics
- LTs created by noise
- LTs due to time-delay
Long transients in higher dimensional systems

- Effect of time-delay is known to generate long transients but the scaling law is unknown.

- Effect of noise - broad and variable. For non-chaotic systems (saddles and ghosts), tends to decrease the transient’s life-time but would not normally destroy it. Can create the transient dynamics (e.g. in bistable systems):

  ![Graph of transient dynamics](image)

  For chaotic transients, noise can increase as well as decrease the transient’s life-time (Grebogi et al. 1983; Do and Lai 2004, 2005).

- Spatial systems: new types of transients (e.g. related to population waves propagation).
A brief look at the spatial systems

What are the new phenomena brought in by explicit space?

- Pattern formation
- Synchronization / desynchronization & onset of spatiotemporal chaos
- Travelling waves
A brief look at the spatial systems

Consider the space-continuous, time-discrete single-species system:

\[ u(x, t + 1) = \int_{0}^{L} g(x - y)F(u(x, t))dx, \quad F(u) = ue^{r(1-u)}. \]

For distributed random initial conditions, the system’s dynamics exhibit a chaotic saddle:

(Hastings and Higgins, 1994)
A brief look at the spatial systems

The above system exhibits long transients in terms of the spatially average values.

Knowledge of the spatial population distribution can provide a different angle on long transients.

Example: “wave of chaos” in a space-time-continuous prey-predator system:

Spread of the chaotic phase over the system can take a very long time, $\tau \propto \frac{L}{c}$. 

(Petrovskii and Malchow, 2001)
A brief look at the spatial systems

For compact initial conditions, the system’s dynamics usually consists of a succession of population waves.

Example: space-time-continuous (diffusion-reaction) prey-predator system, invasion of predator; dynamical stabilization in the wake of the invasion front.

(Petrovskii and Malchow, 2000)
A brief look at the spatial systems

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Conclusions

- Long transients do occur
- The life-time of long transients can be arbitrary long (cf. scaling laws)
- We have identified a few basic mechanisms for the long transients to occur
- Long transients provide an alternative scenario of regime shifts
References


Thanks for listening