Evaporation of a thin sessile droplet in a shallow well

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BIRS Workshop on Modelling of Thin Liquid Films – Asymptotic Approach vs. Gradient Dynamics, 28th April to 3rd May 2019

Talk Outline

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- Assumptions and model

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- Solution after touchdown
- Comparison with experiments
- Extension to different well shapes
- Conclusions and further work

Motivation

- Droplet evaporation occurs commonly in a vast range of circumstances in nature, industry and biology

neelywang.com

comsosmagazine.com

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- Crucial process in technological applications - inkjet printing, coating, spray cooling, etc.

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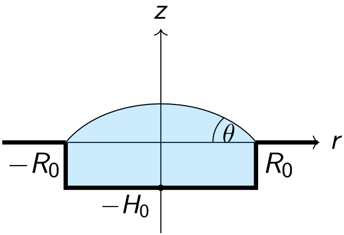
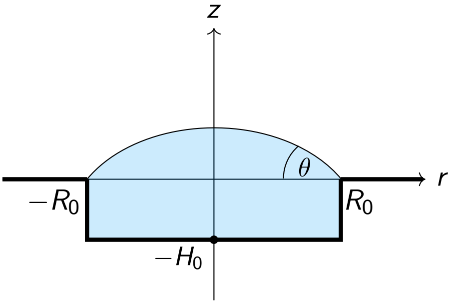
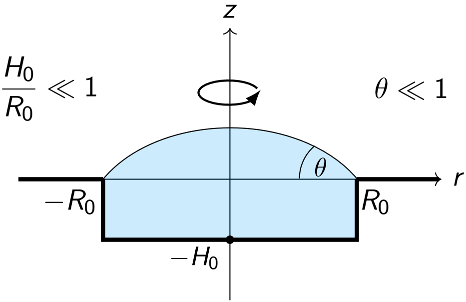
- The aim of the project is to understand droplet evaporation on textured substrates
- In this talk we look at the evolution and lifetime of a drying droplet in a shallow well



The Model - Assumptions





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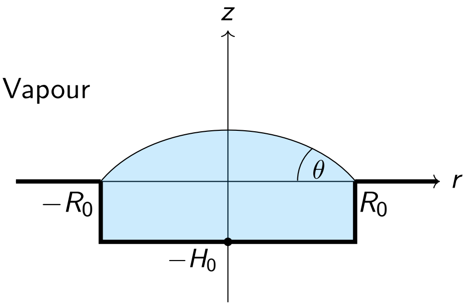

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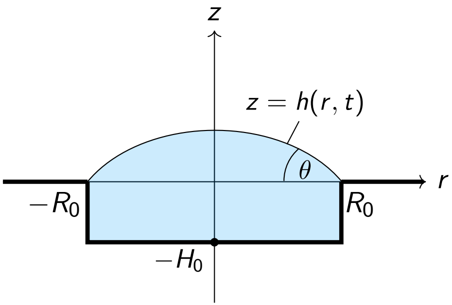

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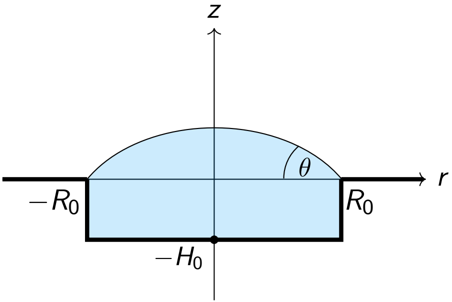

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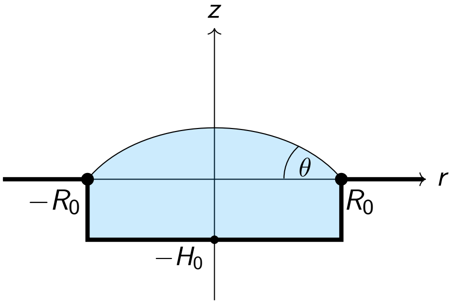

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- Diffusion-limited evaporation under ambient conditions
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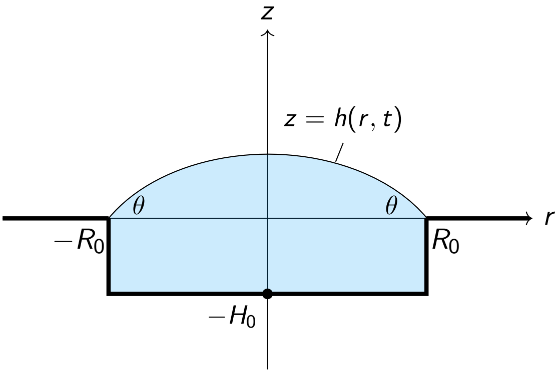
- Free surface evolves quasi-statically
- Contact line remains pinned at the lip of the well throughout the entire evaporation

The Model - Prior to Touchdown



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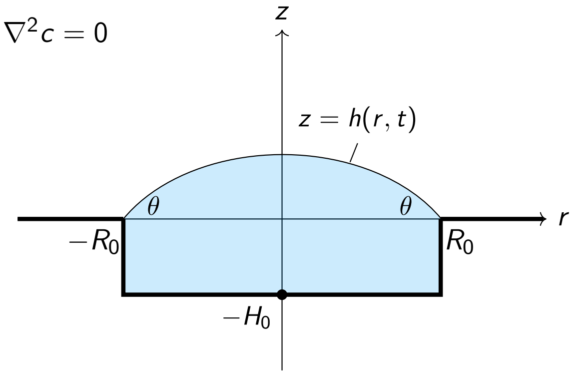

- c is the vapour concentration in the air

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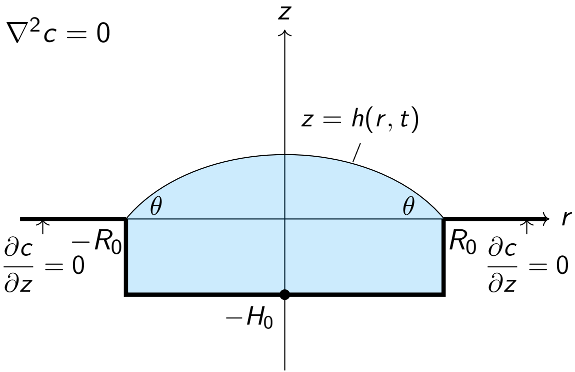

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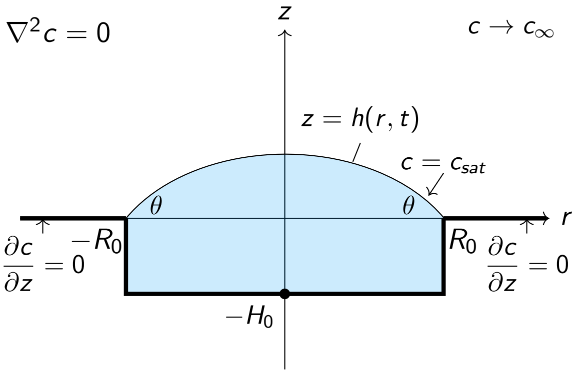

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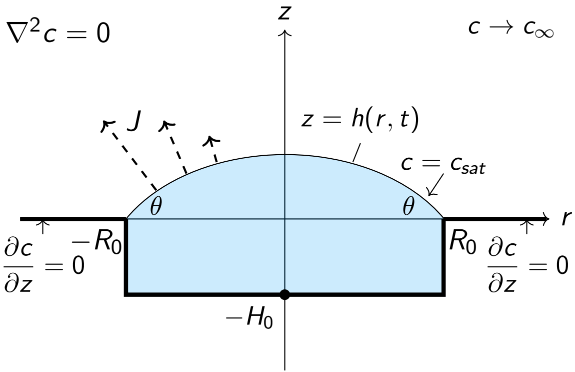

- c is the vapour concentration in the air
- J is the local evaporative flux

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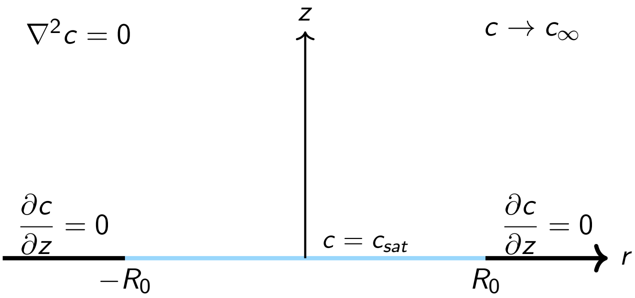

Figure: View of the droplet on the scale of the atmosphere

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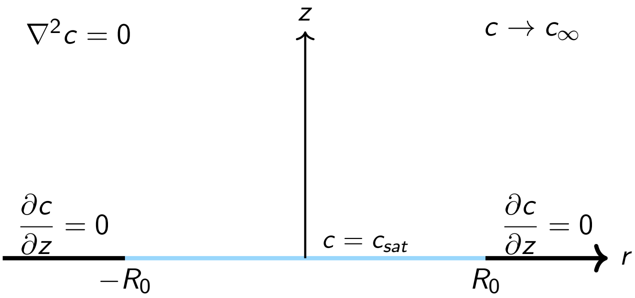

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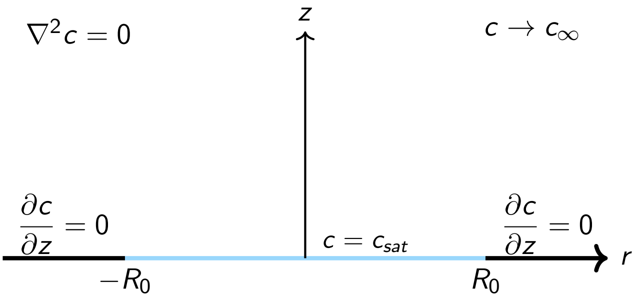

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Volume: $V(t) = V_{well} + V_{drop} = \pi H_0 R_0^2 + \frac{\pi R_0^3 \theta}{4}$

The Model - Prior to Touchdown

- Mathematical model for a thin droplet evaporating in a shallow well prior to touchdown

$$\nabla^2 c = 0 \qquad \text{for the half space } z > 0$$

$$c = c_{sat} \qquad \text{on } z = 0 \text{ for } r \leq R_0$$

$$c \to c_\infty \qquad \text{as } |\mathbf{r}| \to \infty$$

$$\frac{\partial c}{\partial z} = 0 \qquad \text{on } z = 0 \text{ for } r > R_0$$

$$J = -D\frac{\partial c}{\partial z} \qquad \text{on } z = 0 \text{ for } r \leq R_0$$

$$\rho\frac{dV}{dt} = -2\pi\int_0^{R_0} J\, r\, dr$$

The Vapour Problem - Prior to Touchdown

- We rescale so that the problem for the concentration c of vapour is

$$\nabla^2 c = 0 \qquad \text{for the half space } z > 0$$

$$c = 1 \qquad \text{on } z = 0 \text{ for } r \leq 1$$

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- where

$$h = h_m(1 - r^2), \quad h_m = \frac{\theta}{2}$$

$$V = \pi H_0 + \frac{\pi\theta}{4}$$

Solution for Concentration - Prior to Touchdown

- The exact solution for c is well known and may be written as

$$c = \frac{2}{\pi}\sin^{-1}\left(\frac{2}{[(1+r)^2+z^2]^{1/2}+[(1-r)^2+z^2]^{1/2}}\right)$$

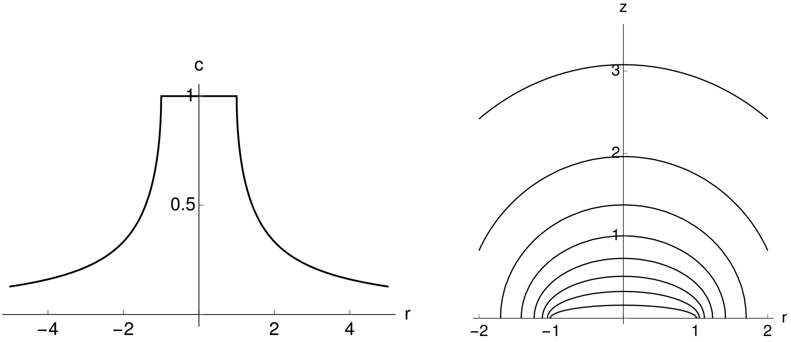

Figure: Plot of the concentration on $z = 0$ and a contour plot of c.

Solution for the Flux - Prior to Touchdown

- The evaporative flux from the free surface J is

$$J = -\frac{\partial c}{\partial z}\bigg|_{z=0} = \frac{2}{\pi(1 - r^2)^{1/2}} \qquad \text{for } r < 1$$

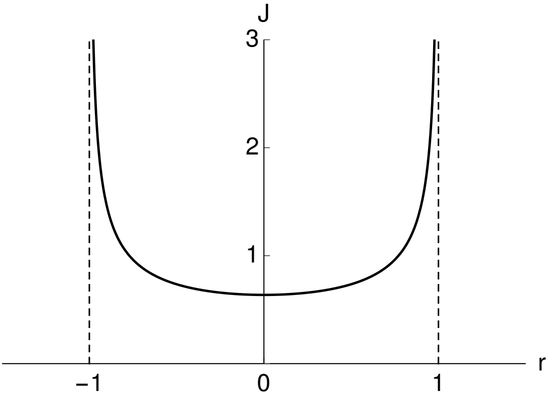

Figure: Solution for the evaporative flux J prior to touchdown

Evolution - Prior to Touchdown

- $\dfrac{dV}{dt}$ is given by

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- The evolution of the droplet prior to touchdown is therefore given by

$$V = V_0 - 4t, \qquad V_0 = V(0) = \pi H_0 + \frac{\pi}{4}$$
$$\theta = \theta_0 - \frac{16}{\pi}t, \qquad \theta_0 = \theta(0) = 1$$

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- we now find t_{flat} and $t_{\text{touchdown}}$ to be

$$t_{\text{flat}} = \frac{\pi}{16} \approx 0.1963$$

$$t_{\text{touchdown}} = \frac{\pi(1 + 2H_0)}{16}$$

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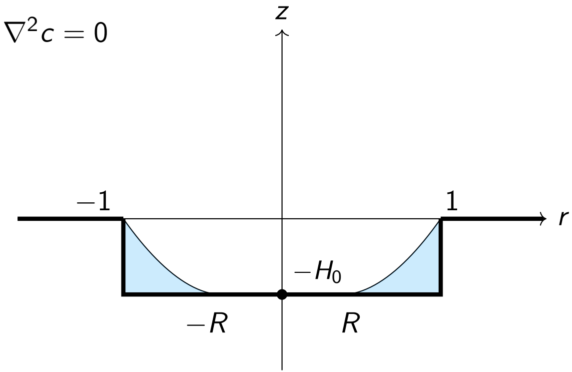


Figure: Sketch of the annular droplet at some instant after touchdown

The Droplet - After Touchdown

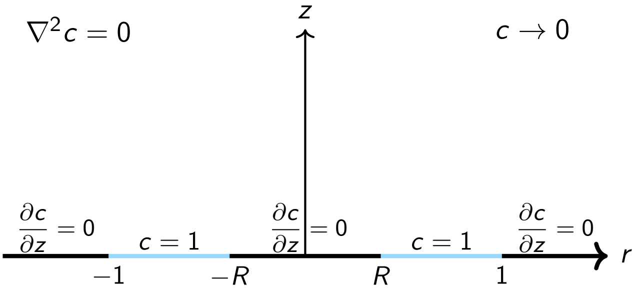
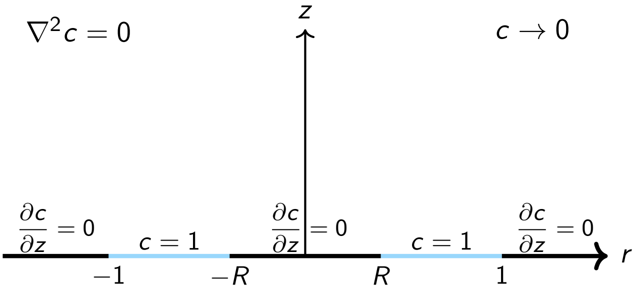


Figure: View of the droplet on the scale of the atmosphere after touchdown

The Droplet - After Touchdown



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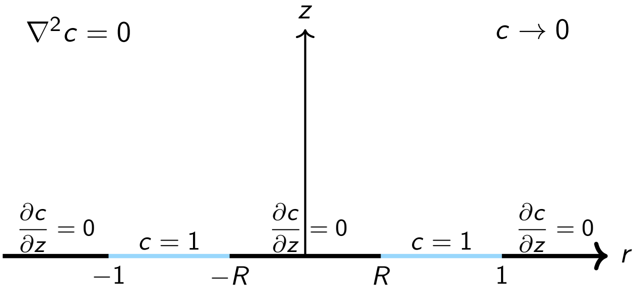

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- No simple closed form solution is available for c
- Our main concern is finding the total flux F

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  - We also used the finite-element package COMSOL to find the solution for c and hence J and F
  - The two approaches were found to be in very good agreement for F

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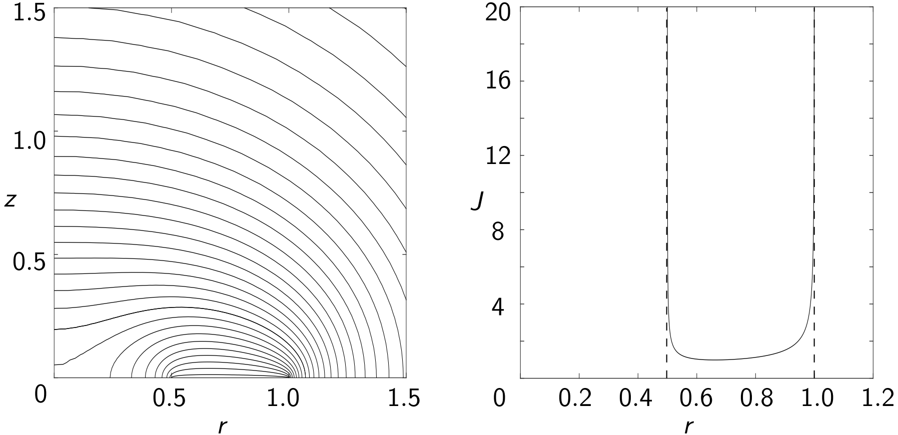

Figure: Contour plot of the concentration c and a plot of the evaporative flux J for an annular droplet in $R \leq r \leq 1$ in the case $R = 1/2$

Solution - After Touchdown

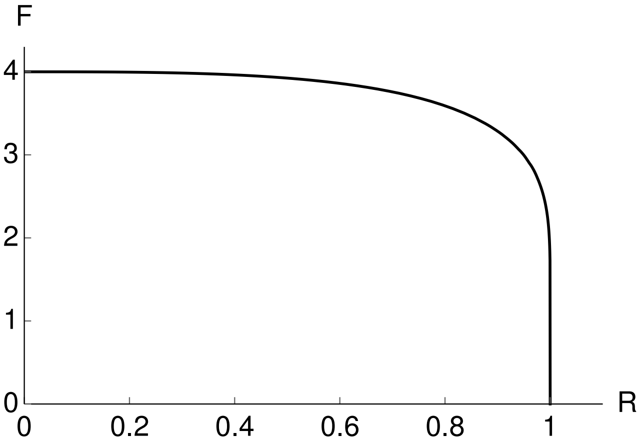

Figure: The numerical solution for the total evaporative flux F, obtained via COMSOL

Lifetime

$$V = \frac{\pi H_0(1 - R^4 + 4R^2 \log R)}{2(1 - R^2 + 2R^2 \log R)} = \pi H_0 f(R), \qquad \frac{dV}{dt} = -F(R)$$

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- Then using the fact that $R(t_{\text{lifetime}}) = 1$ we obtain

$$t_{\text{lifetime}} = t_{\text{touchdown}} + \pi\alpha H_0 = \frac{\pi}{16}\left[1 + 2(1 + 8\alpha)H_0\right],$$

$$\text{where } \alpha = -\int_0^1 \frac{f'(R)}{F(R)}\, dR \approx 0.1369$$

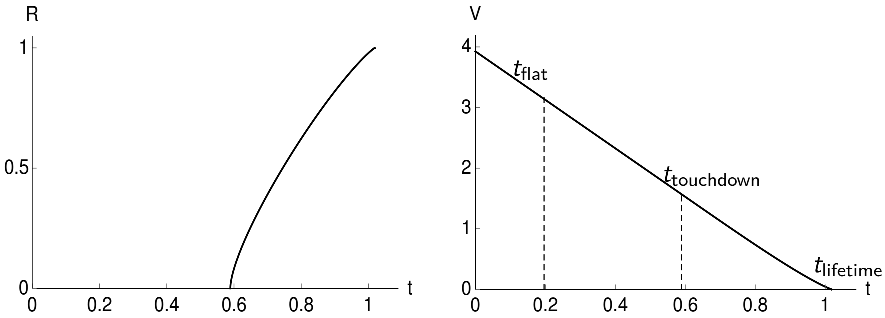

Figure: Evolution of the moving inner contact radius R and the volume V for $t = 0 \ldots t_{\text{lifetime}}$ in the case $H_0 = 1$

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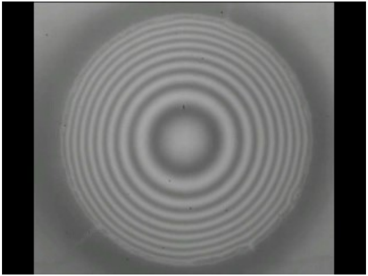

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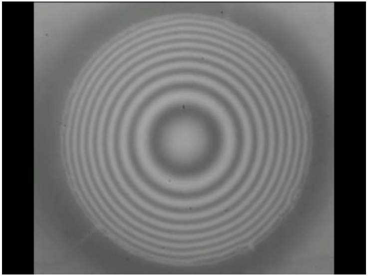
- Behaviour of the height profile, R, and V were measured



Comparison with experiments

- The following parameter values were used in the comparison of the theory with experiments

$$D = 6.899 \times 10^{-6}\ \mathrm{m}^2\ \mathrm{s}^{-1}, \quad \rho = 1.085 \times 10^{3}\ \mathrm{kg}\ \mathrm{m}^{-3},$$

$$c_{\mathrm{sat}} = \begin{cases} 2.330 \times 10^{-3}\ \mathrm{kg}\ \mathrm{m}^{-3} & \text{Book Value 1} \\ 2.252 \times 10^{-3}\ \mathrm{kg}\ \mathrm{m}^{-3} & \text{Book Value 2} \end{cases}$$

$$c_{\infty} = 0$$

Comparison with Experiments - Height Profile

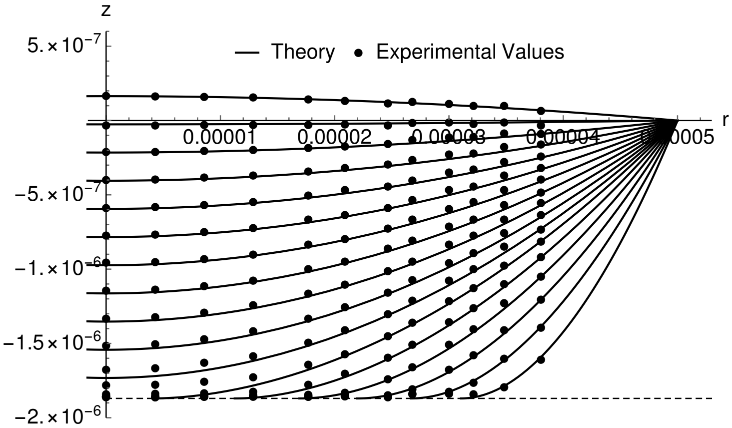


Figure: Comparison of the height profile predicted by the theory with the measured experimental values of a methyl benzoate droplet in a well of radius 50 μm at times $t = 0, 0.26 \ldots 4.16$ s

Comparison with Experiments - R

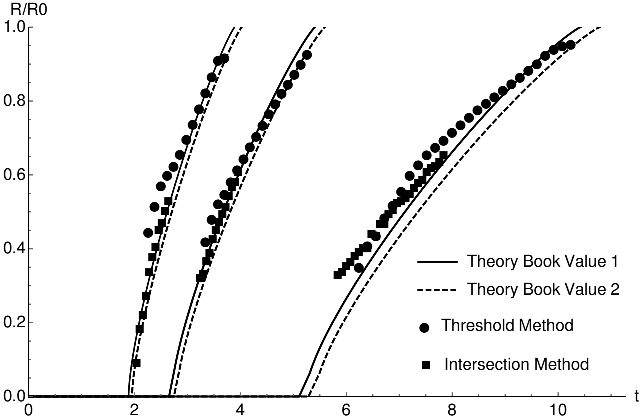


Figure: Comparison of the evolution of the moving inner contact radius R predicted by the theory with the measured experimental values for droplets of methyl benzoate in wells of radii 30, 50 and 75 μm

Comparison with Experiments - V

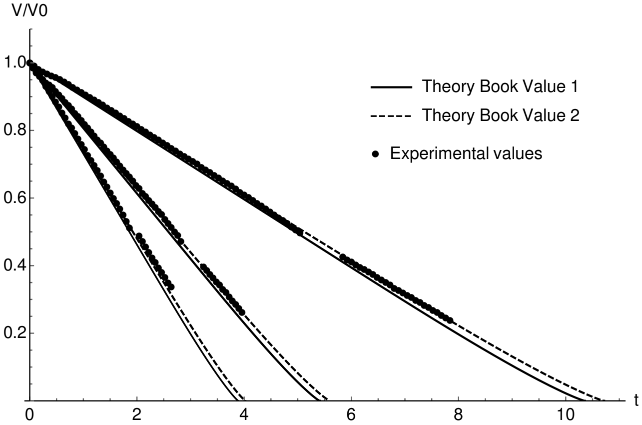


Figure: Comparison of the evolution of the volume V predicted by the theory with the measured experimental values for droplets of methyl benzoate in wells of radii 30, 50 and 75 μm

Comparison with Experiments - Critical Times

Well Dimensions (μm)	Critical Times	Experiments	Theory BV 2	% diff.
$R_0 = 30,\ H_0 = 2.38$	t_{flat}	0.07 s	0.06 s	-14%
	$t_{\text{touchdown}}$	1.90 s	1.95 s	$+3\%$
	t_{lifetime}	3.98 s	4.03 s	$+1\%$
$R_0 = 50,\ H_0 = 1.87$	t_{flat}	0.23 s	0.23 s	$\pm 0\%$
	$t_{\text{touchdown}}$	2.88 s	2.79 s	-3%
	t_{lifetime}	5.44 s	5.60 s	$+3\%$
$R_0 = 75,\ H_0 = 2.39$	t_{flat}	0.46 s	0.49 s	$+6\%$
	$t_{\text{touchdown}}$	5.05 s	5.40 s	$+7\%$
	t_{lifetime}	10.40 s	10.79 s	$+4\%$

Table: Comparison of experimental results for the critical times with theory

Different Shapes of Well

- We have extended this approach to wells with height profile

$$z = H(r) = -H_0\left[1 - \left(\frac{r}{R_0}\right)^n\right]$$

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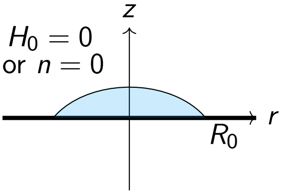

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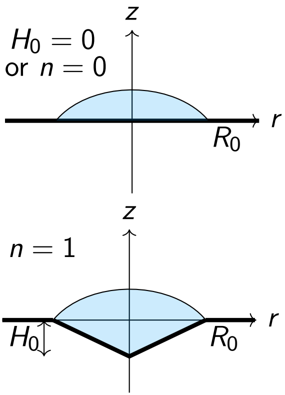

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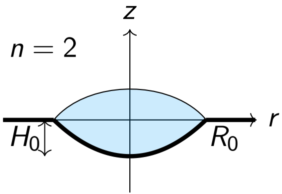

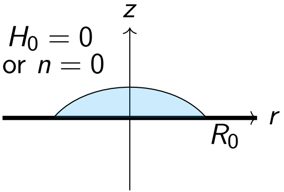

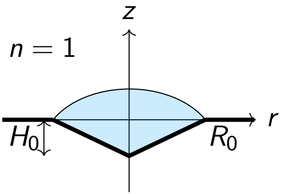

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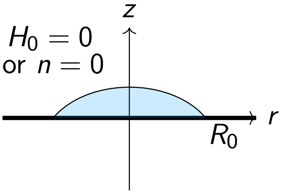

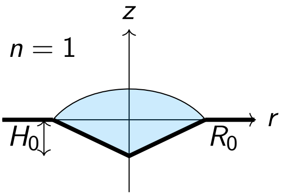

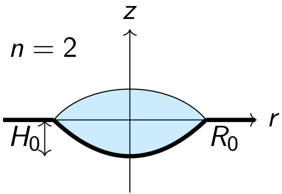

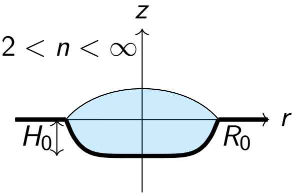

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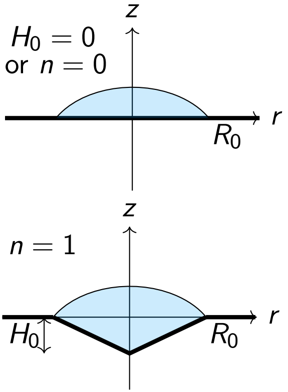

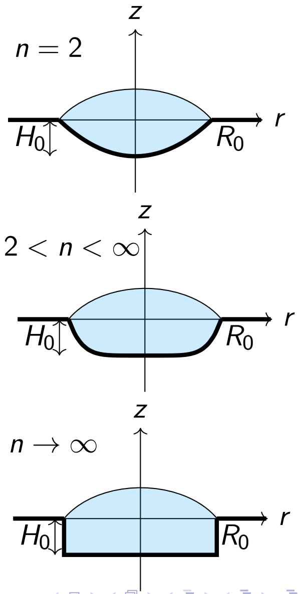

Conclusions and Future Work

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- We used the COMSOL Multiphysics package and Chebyshev-Gauss quadrature to obtain the evaporative flux J and the total flux F after touchdown
- We found that the lifetime of the droplet is linear in H_0
- We found good agreement with experimental data for the height profile, R, V and the critical times

Future Work

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Acknowledgements

Thank You for Listening!

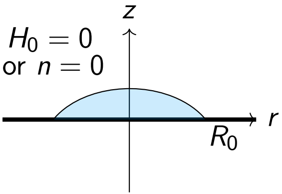
Hannah-May D'Ambrosio
Brian Duffy
Teresa Colosimo
Colin Bain
Dan Walker

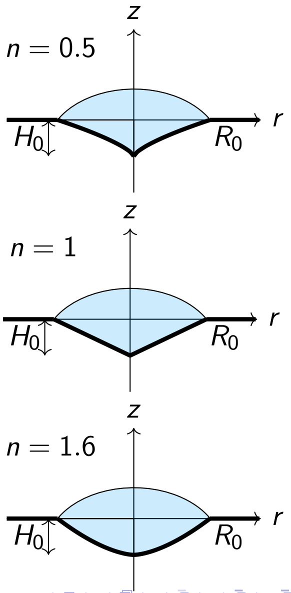
Different Shapes of Wells

- For $0 < n < 2$

$$t_{\text{touchdown}} = \frac{\pi(1 + nH_0)}{16}$$

$$t_{\text{lifetime}} = \frac{\pi(1 + n + 3nH_0)}{16(1 + n)}$$





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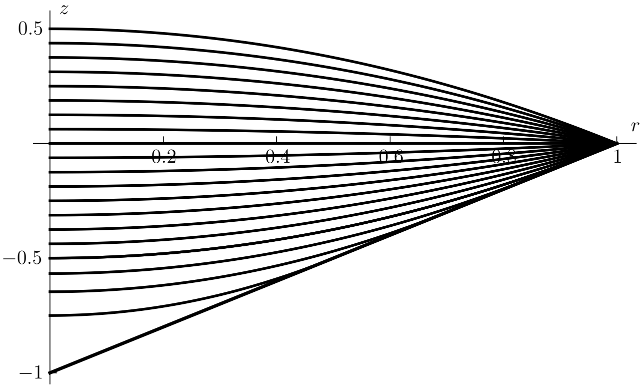
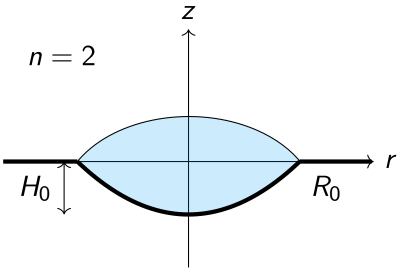


Figure: Evolution of the height profile of a droplet in a conical well from the theory where n=1.

Different Shapes of Wells

- For $n = 2$

$$t_{\text{touchdown}} = t_{\text{lifetime}} = \frac{\pi(1+2H_0)}{16}$$



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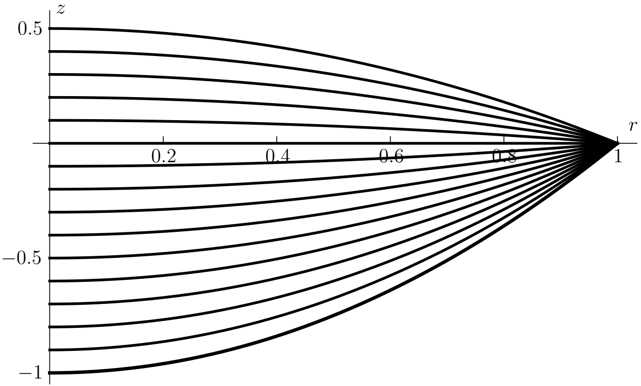


Figure: Evolution of the height profile of a droplet in a parabolic well from the theory where n=2.

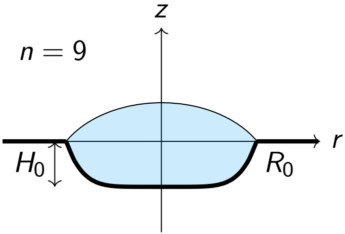
Different Shapes of Wells

- For $2 < n < \infty$

$$t_{\text{touchdown}} = \frac{\pi(1 + 2H_0)}{16}$$

$$t_{\text{lifetime}} = \frac{\pi}{16}[1 + 2(1 + 8\alpha)H_0]$$

$$\alpha = -\int_0^1 \frac{f'(R)}{F(R)} dR (> 0)$$



Different Shapes of Wells

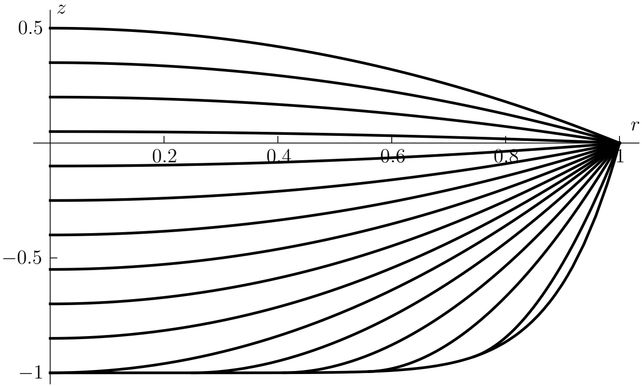


Figure: Evolution of the height profile in an axisymmetric well from the theory where n=9.

Different Shapes of Wells - Evolution of α

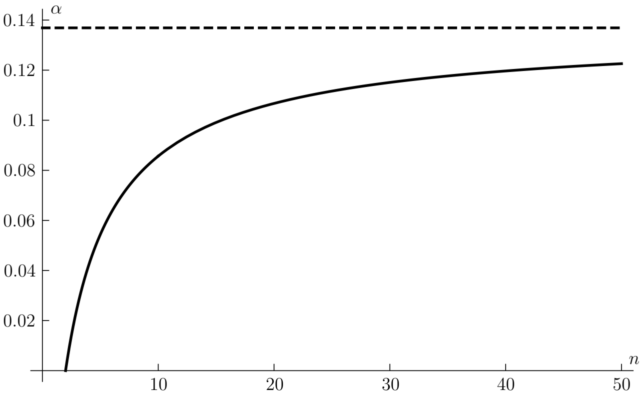


Figure: Evolution of parameter α for varying n.

Experimental Analysis

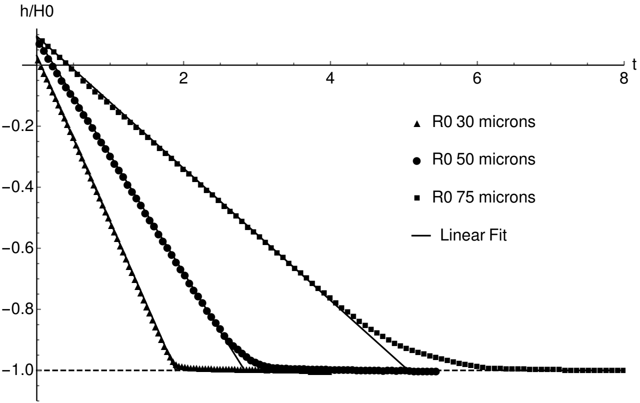
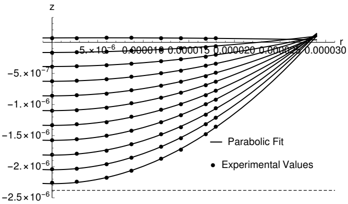
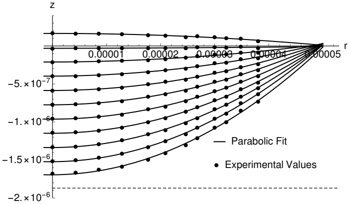


Figure: Experimental values for the height in the middle of the droplet h_m for three wells of radius 30, 50 and 75 μm.

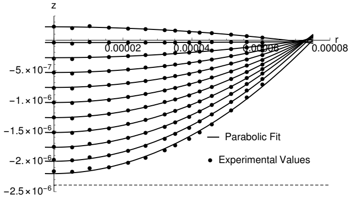
Experimental Analysis



(a) $R_0 = 30\ \mu$m



(b) $R_0 = 50\ \mu$m



(c) $R_0 = 75\ \mu$m

Figure: Parabolic fits of the experimental values for the height profile of a droplet in wells of radius 30, 50 and 75 μm before touchdown at time intervals of (a) $t = 0,\ 0.18 \ldots 1.80$ s, (b) $t = 0,\ 0.26 \ldots 2.60$ s and (c) $t = 0,\ 0.48 \ldots 4.80$ s.

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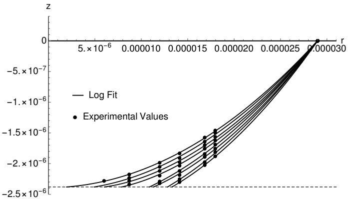

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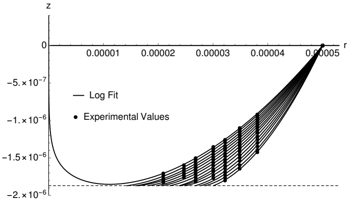

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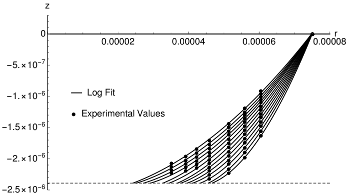

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Figure: Modified fits of the experimental values for the height profile of a droplet in wells of radius 30, 50 and 75 μm after touchdown at time intervals of (a) $t = 1.92, 1.98 \ldots 2.52$ s, (b) $t = 3.12, 3.18 \ldots 3.78$ s and (c) $t = 5.68, 5.84 \ldots 7.52$ s