# Multi-fluid representation of convection

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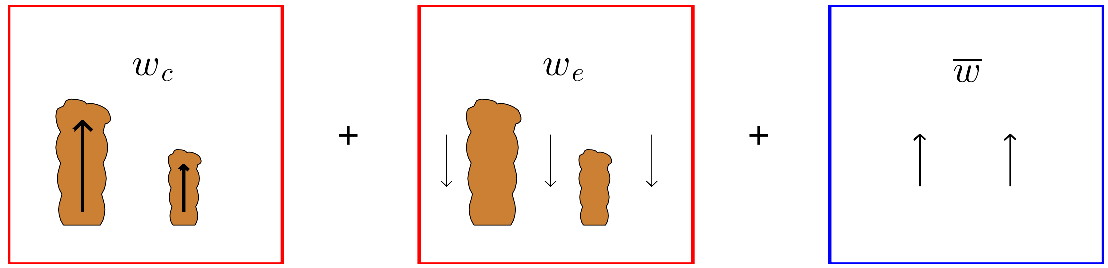
## **Motivation:** Some limitations of typical convection schemes

- Causality:

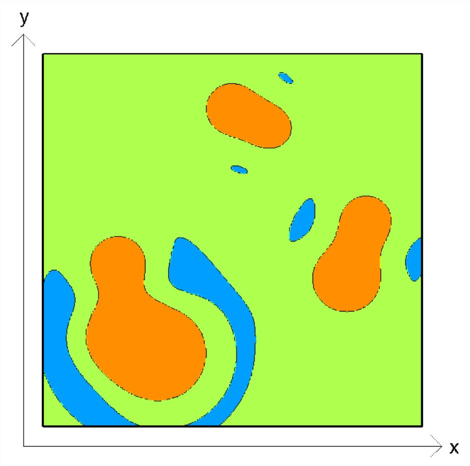
[ Physics ] [ Dynamics ]

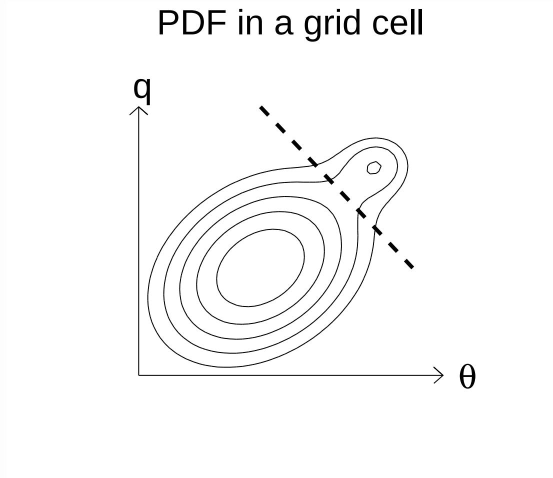



- Instantaneous: no memory, intermittency
- Local: no communication with or propagation to neighbouring columns
- Grey zone
- Consistency of dynamical and thermodynamic approximations (e.g. shallow atmosphere, hydrostatic, $q$ dependence of specific heat capacities)
- Not expressed as PDEs

# Idea!

Divide fluid into “convecting” and “non-convecting” components (perhaps more...)

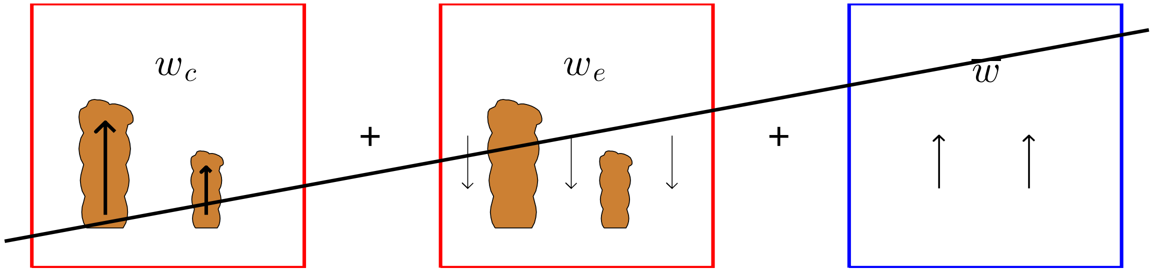


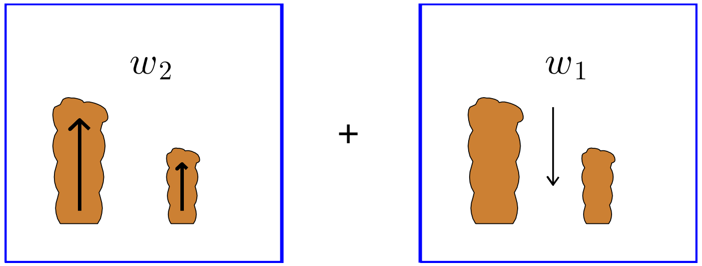

and obtain coupled governing equations for each component

Allow the dynamical core to handle the dynamics of both components

[ Physics ] [ Dynamics ]




[ Dynamics ]



## Governing equations

$$\frac{\partial}{\partial t}(\sigma_i \rho_i) + \nabla \cdot (\sigma_i \rho_i \mathbf{u}_i) = \sum_{j \neq i} (\mathcal{M}_{ij} - \mathcal{M}_{ji})$$

where $\mathcal{M}_{ij}$ is the rate at which mass is relabelled from $j$ to $i$,

$$\frac{D_i q_i}{Dt} = \frac{1}{\sigma_i \rho_i} \left[ \sum_{j \neq i} \{ \mathcal{M}_{ij} (\hat{q}_{ij} - q_i) - \mathcal{M}_{ji} (\hat{q}_{ji} - q_i) \} - \nabla . \mathbf{F}^{q_i}_{\mathrm{SF}} \right]$$

where $\hat{q}_{ij}$ is a representative $q$ for the fluid relabelled from $j$ to $i$,

$$\frac{D_i \mathbf{u}_i}{Dt} + \frac{1}{\rho_i} \nabla \overline{p} + \nabla \Phi = \frac{1}{\sigma_i \rho_i} \{ \mathcal{M}_{ij} (\hat{\mathbf{u}}_{ij} - \mathbf{u}_i) - \mathcal{M}_{ji} (\hat{\mathbf{u}}_{ji} - \mathbf{u}_i) - \mathcal{P}_i - \nabla \cdot \mathsf{F}^{\mathbf{u}_i}_{\mathrm{SF}} \}$$

# Parameterized terms:

Certain terms must still be parameterized:

**Subfilter-scale fluxes:** $\mathbf{F}^{q_i}_{\mathrm{SF}}$, $\mathbf{F}^{\mathbf{u}_i}_{\mathrm{SF}}$, etc.

**Effects of** $p \neq \overline{p}$: $\mathcal{P}_i$

**Relabelling terms, i.e. entrainment and detrainment:** $\mathcal{M}_{ij}$

Existing schemes should provide a useful starting point.

## Comments

• If we neglect certain terms, the multi-fluid equations reduce to the equations for a mass flux scheme.

• The multi-fluid equations have some nice mathematical properties.

• We can write down a multi-fluid version of the Mellor-Yamada high-order turbulence closure hierarchy.

# Further Reading

## Derivation

Thuburn et al., J. Atmos. Sci., **75**, 965–981 (2018).

## Conservation and normal mode properties

Thuburn and Vallis, QJRMS, **144**, 1555–1571 (2018).

## Application to single-column dry CBL

Thuburn et al., QJRMS, doi:10.1002/qj.3510

## Numerical solution

Thuburn et al., QJRMS, doi:10.1002/qj.3510

Weller and McIntyre, QJRMS, doi:10.1002/qj.3490

## Extended EDMF (essentially the same idea)

Tan et al., JAMES, **10**, 770–800.

# SPARE SLIDES

# Outline of derivation

Compressible Euler equations (omitting rotation and sources)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad \frac{D\eta}{Dt} = 0 \qquad \frac{Dq}{Dt} = 0$$

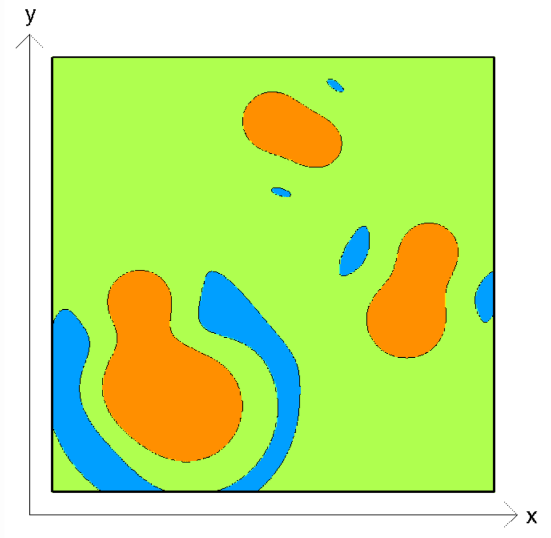
$$\rho \frac{D\mathbf{u}}{Dt} + \nabla p + \rho \nabla \Phi = 0 \qquad p = P(\rho, \eta, q)$$

plus a filter

$$\overline{X}(\mathbf{x}) = \int_D G(\mathbf{x} - \mathbf{x}', \Delta) X(\mathbf{x}')\, d\mathbf{x}'$$

...plus some Lagrangian labels $I_i$ equal to $0$ or $1$:

$$\frac{DI_i}{Dt} = 0.$$




Applying the filter to

$$\frac{\partial}{\partial t}(I_i \rho) + \nabla \cdot (I_i \rho \mathbf{u}) = 0 \qquad \text{gives} \qquad \frac{\partial}{\partial t}(\sigma_i \rho_i) + \nabla \cdot (\sigma_i \rho_i \mathbf{u}_i) = 0.$$

where
$\sigma_i = \overline{I_i}$ is the volume fraction of the $i^{\text{th}}$ fluid,
$\rho_i = \overline{I_i \rho}/\sigma_i$ is the average density of the $i^{\text{th}}$ fluid,
$\mathbf{u}_i = \overline{I_i \rho \mathbf{u}}/(\sigma_i \rho_i)$ is the density-weighted average velocity of the $i^{\text{th}}$ fluid.

Following a similar procedure for the $q$ equation gives

$$\frac{D_i q_i}{Dt} = -\frac{1}{\sigma_i \rho_i} \nabla . \mathbf{F}_{\mathrm{SF}}^{q_i}$$

where

$$q_i = \overline{I_i \rho q} / \sigma_i \rho_i, \qquad \frac{D_i}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{u}_i \cdot \nabla,$$

and $\mathbf{F}_{\mathrm{SF}}^{q_i}$ is the subfilter-scale flux of $q$ by the $i^{\mathrm{th}}$ fluid.

Similarly for $\eta$.

For the momentum equation we want to end up with a single pressure field (inter-fluid acoustic adjustment)

so write

$$\begin{aligned} \overline{I_i \nabla p} &= \sigma_i \nabla \overline{p} + \left( \overline{I_i \nabla p} - \sigma_i \nabla \overline{p} \right) \\ &= \sigma_i \nabla \overline{p} \qquad\quad + \mathcal{P}_i \end{aligned}$$

So the momentum equation for the $i^{\text{th}}$ fluid is

$$\frac{D_i \mathbf{u}_i}{Dt} + \frac{1}{\rho_i} \nabla \overline{p} + \nabla \Phi = - \frac{1}{\sigma_i \rho_i} \left\{ \nabla \cdot \mathsf{F}_{\mathrm{SF}}^{\mathbf{u}_i} + \mathcal{P}_i \right\},$$

where $\mathsf{F}_{\mathrm{SF}}^{\mathbf{u}_i}$ is the subfilter-scale momentum flux tensor.

## Some mathematical properties

- Conservation of mass, entropy, momentum, energy
- Variational formulation
- Normal modes
- Variant in which all fluids components have the same horizontal velocity

$$\left( \sum_i \sigma_i \rho_i \frac{D_i}{Dt} \right) \mathbf{v} + \nabla_H \overline{p} + \left( \sum_i \sigma_i \rho_i \right) \nabla_H \Phi = 0.$$