# Topology, Locality, and Passivity in Nonreciprocal Electromagnetics

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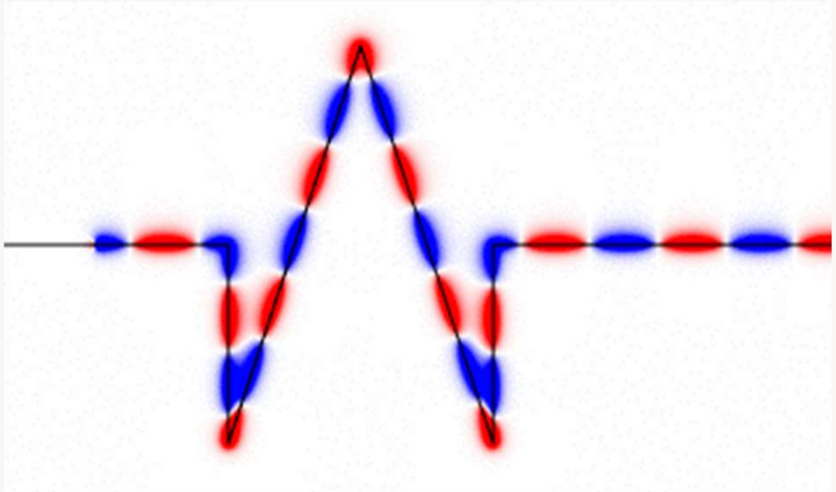

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# Introduction

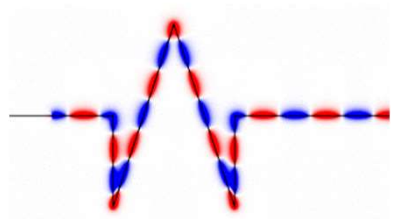
## Nonreciprocal electromagnetic materials and structures

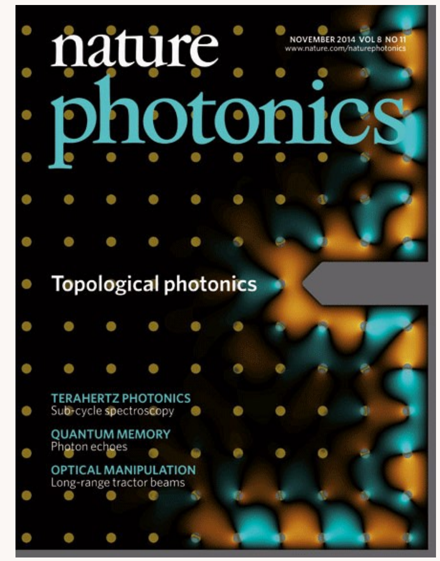
**Nonreciprocal materials**

**"Strongly nonreciprocal" platforms**
supporting unidirectional wave

**Nonreciprocal (Chern-type) topological insulators**

topologically-protected **surface-wave** propagation






L. Lu, J. D. Joannopoulos, and M. Soljačić, Nat. Photonics **8**, 821 (2014).

PHYSICAL REVIEW B **67**, 165210 (2003)

**Electromagnetic unidirectionality in magnetic photonic crystals**


A. Figotin and I. Vitebskiy
*Department of Mathematics, University of California at Irvine, California 92697*


(Received 22 August 2002; revised manuscript received 22 November 2002; published 28 April 2003)

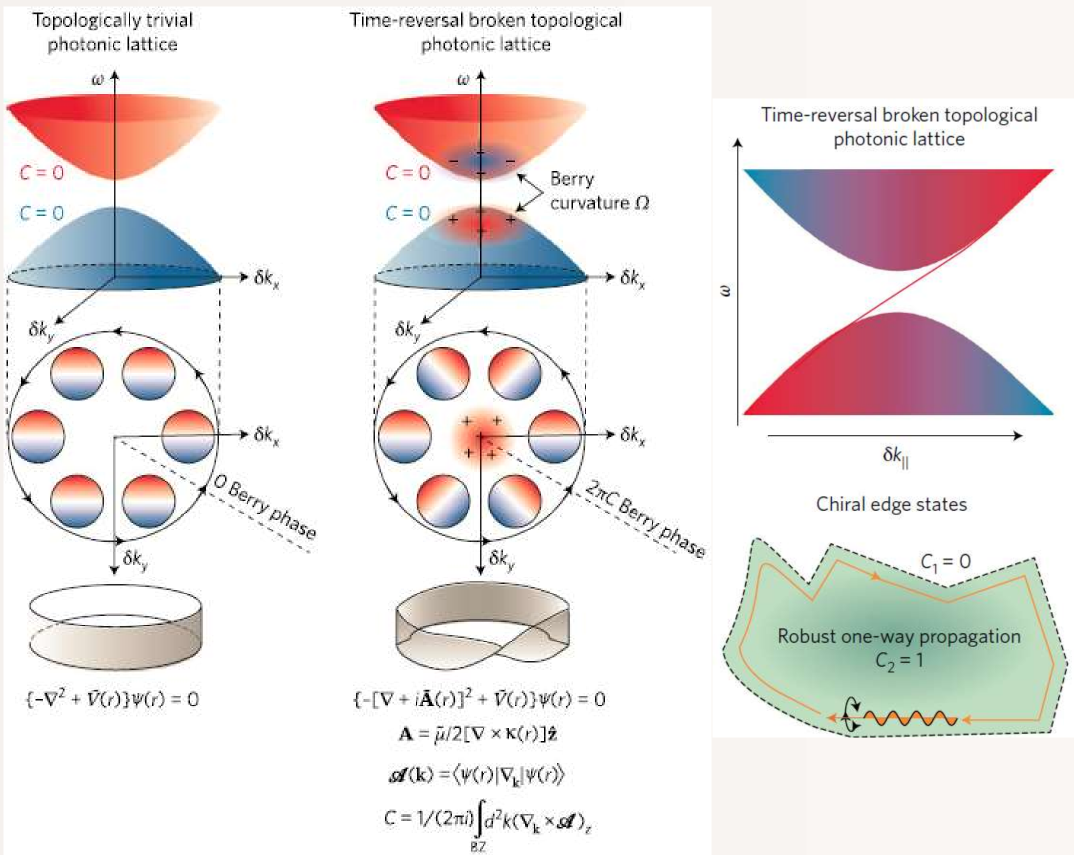

"The electric boy". In Cosmos 2.0: A spacetime odyssey (2014).

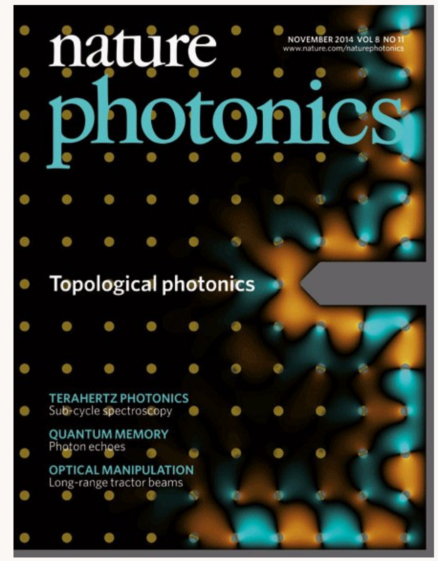
# Topological Transport in Electronic and Photonic Systems

nature photonics — FOCUS | REVIEW ARTICLE
https://doi.org/10.1038/s41566-017-0048-5

**Two-dimensional topological photonics**

Alexander B. Khanikaev[1,2]* and Gennady Shvets[3]*



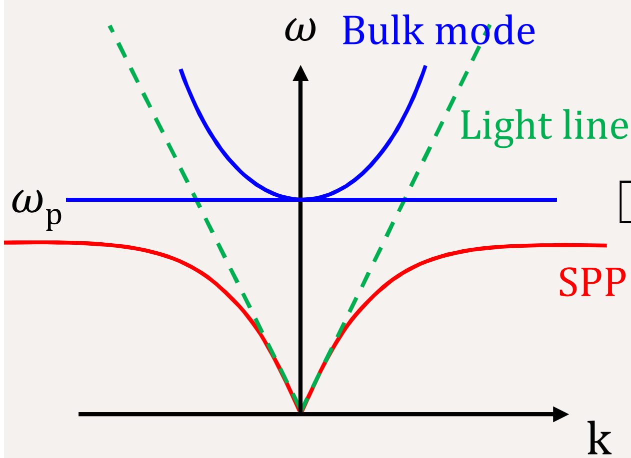



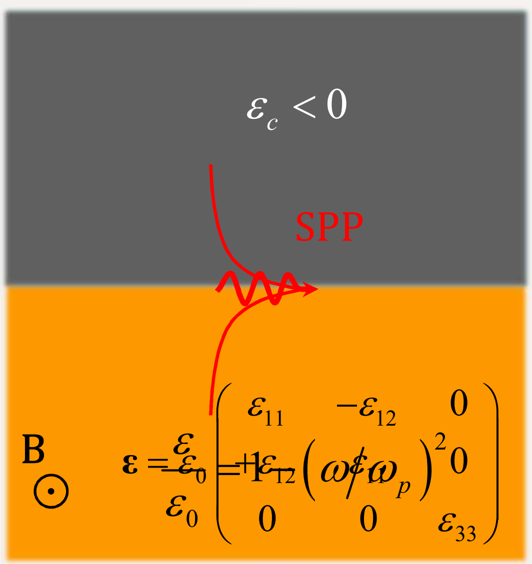

L. Lu, J. D. Joannopoulos, and M. Soljačić, Nat. Photonics **8**, 821 (2014).

# Bulk and Surface Modes of a Nonreciprocal Plasmonic Material

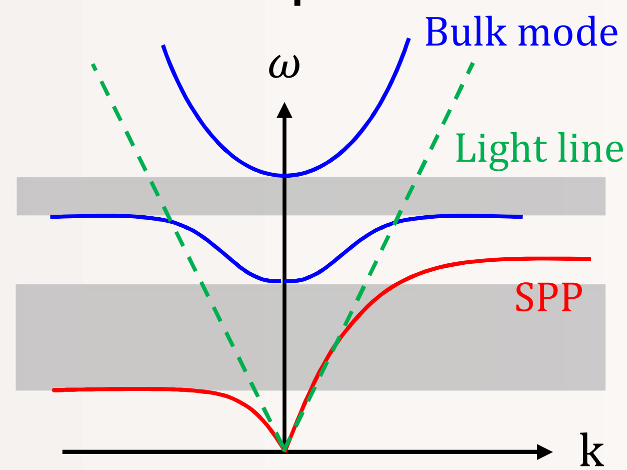
Consider a simple magnetized plasma (gyrotropic nonreciprocal material).

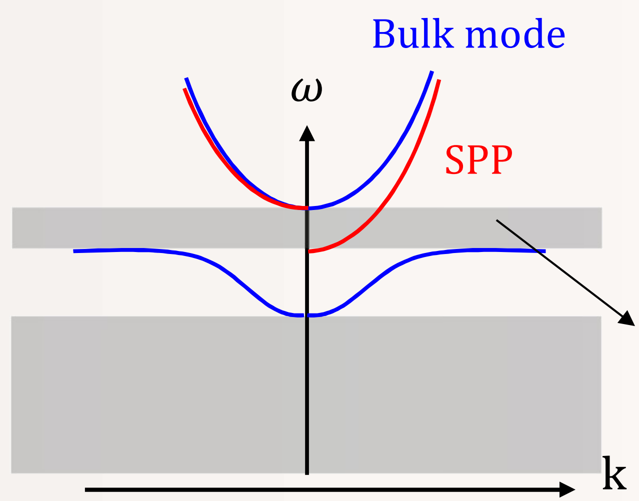
**Unbiased plasma**




**Biased plasma**




Propagation in the plane orthogonal to the bias:

$$k^2 = \frac{\epsilon_{11}^2 + \epsilon_{12}^2}{\epsilon_{11}} \left(\frac{\omega_n}{c}\right)^2, \quad \text{TM mode}$$

$$k^2 = \epsilon_{33} \left(\frac{\omega_n}{c}\right)^2, \quad \text{TE mode}$$

(1) Interface with a transparent medium (e.g., vacuum or a dielectric)

⇨ **Surface magneto-plasmons**

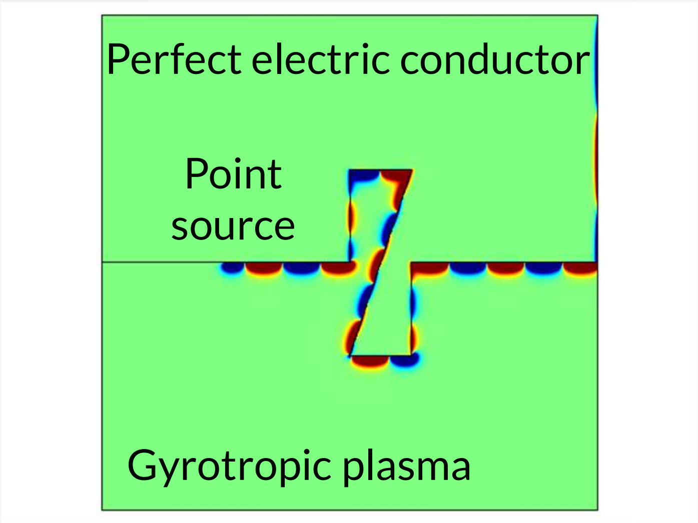
(2) Interface with an opaque medium (e.g., perfect electric conductor)

**Topological invariant : gap Chern number (=1)**

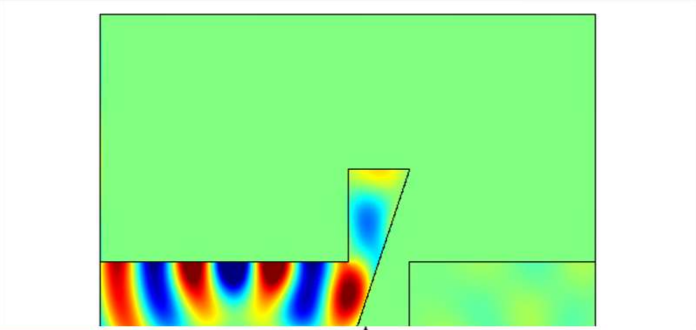
⇨ **Topological surface plasmon-polaritons**

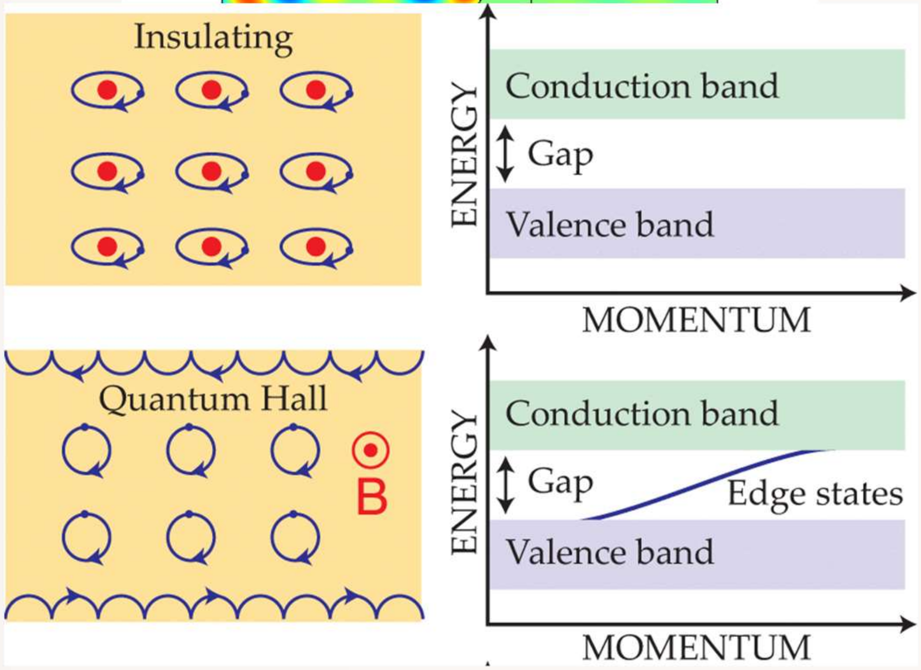
# Topologically-protected unidirectional surface mode

Biased, frequency inside
the bulk-mode bandgap




No magnetic bias



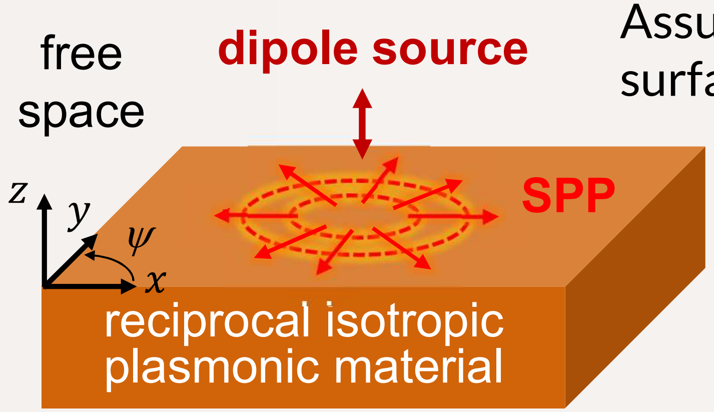



**Example of a *continuous* topological photonic insulator**

**(analogue of Quantum Hall Insulator in electronic systems)**

Can we realize a mode that is not only unidirectional but also diffractionless?

# Excitation and Propagation of Surface Plasmon-Polaritons (SPPs)




Assuming a single surface mode and a subwavelength dipole-surface distance, the **in-plane SPP pattern** is given by:

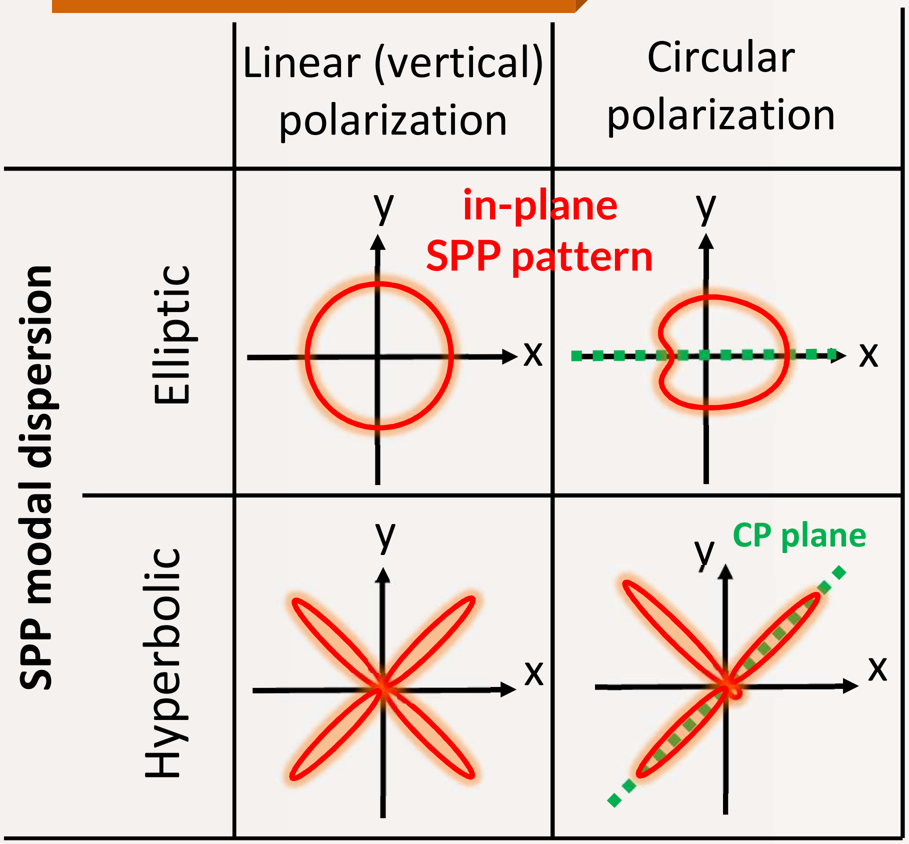
$$U(\psi) \approx \frac{\omega^2}{16\pi} \frac{1}{|\nabla_k \omega(\mathbf{k})|} \frac{1}{C(\mathbf{k})} |\mathbf{p}^* \cdot \mathbf{E}_\mathbf{k}(z_0)|^2$$

modal wavenumber $\mathbf{k}(\psi)$ modal field $\mathbf{E}_\mathbf{k}$

Inverse of **group velocity**

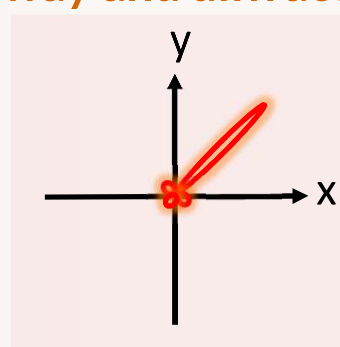
**Curvature (shape)** of iso-frequency contour of modal dispersion surface

**Source-mode coupling**



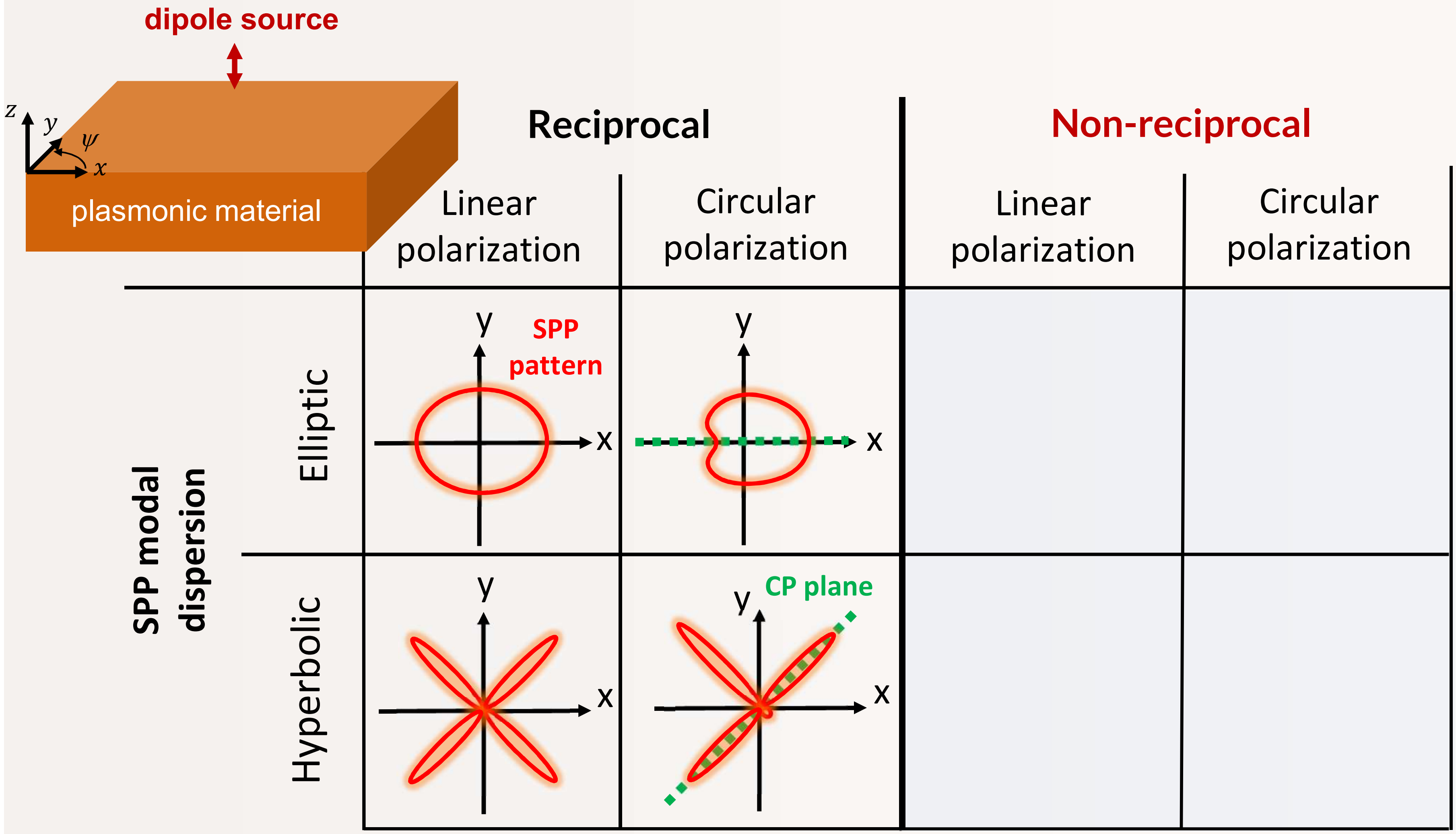

SAH Gangaraj, GW Hanson, MG Silveirinha, K Shastri, M Antezza, and F. Monticone, Physical Review B 99 (24), 245414

**Can we realize a delta-function-like SPP pattern (one-way and diffractionless beam)?**

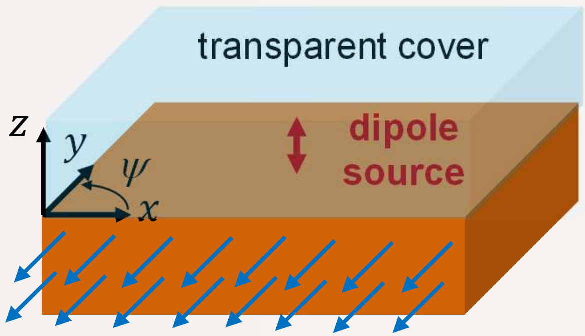


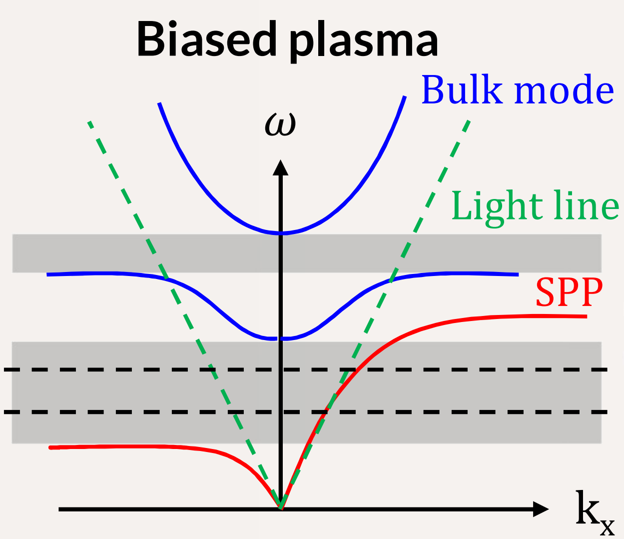

(without breaking the transverse homogeneity of the surface)

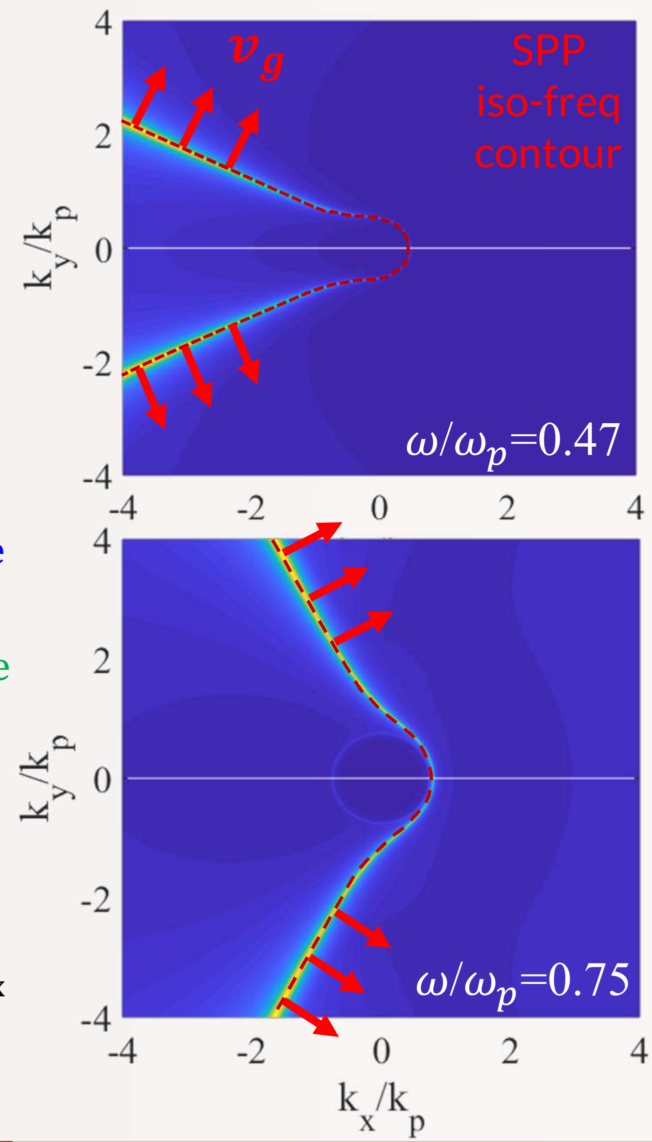
# Excitation and Propagation of Surface Plasmon-Polaritons (SPPs)

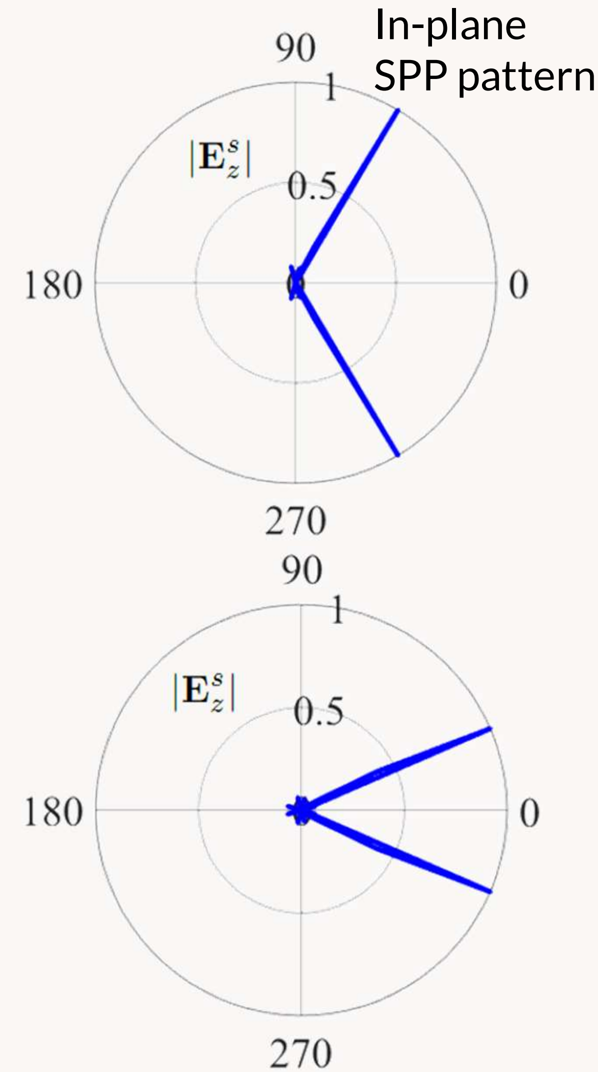


# Semi-Hyperbolic Nonreciprocal Surface Plasmon-Polaritons

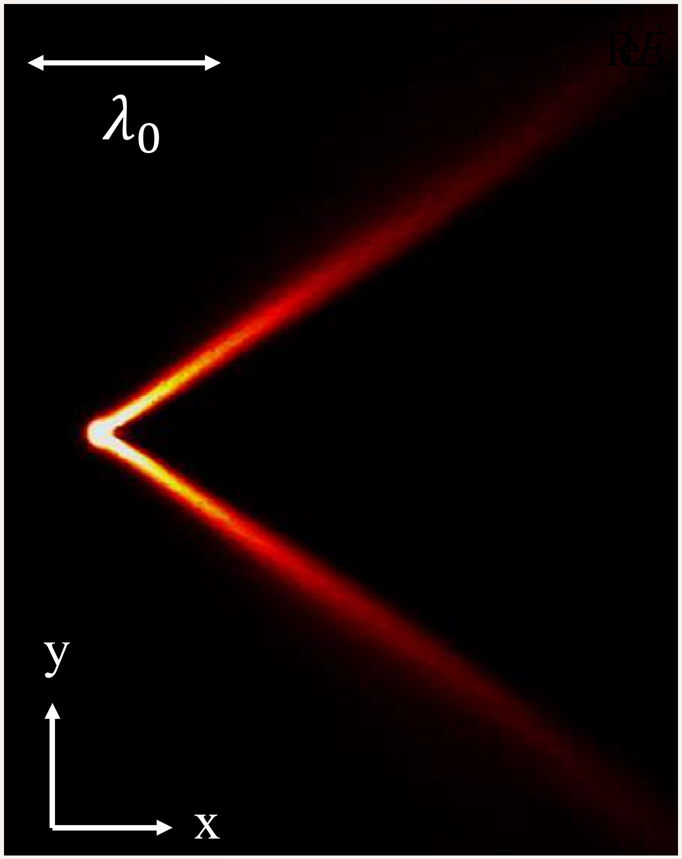


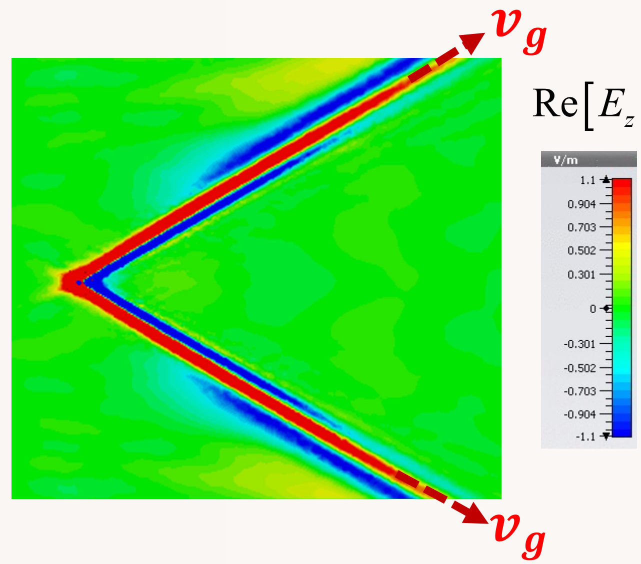



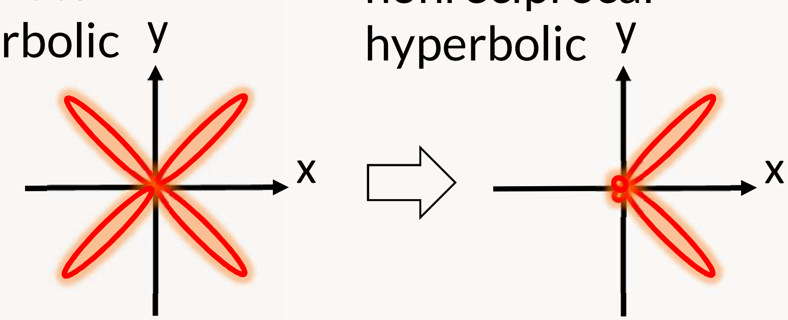
# Semi-Hyperbolic Nonreciprocal Surface Plasmon-Polaritons




reciprocal hyperbolic

nonreciprocal hyperbolic

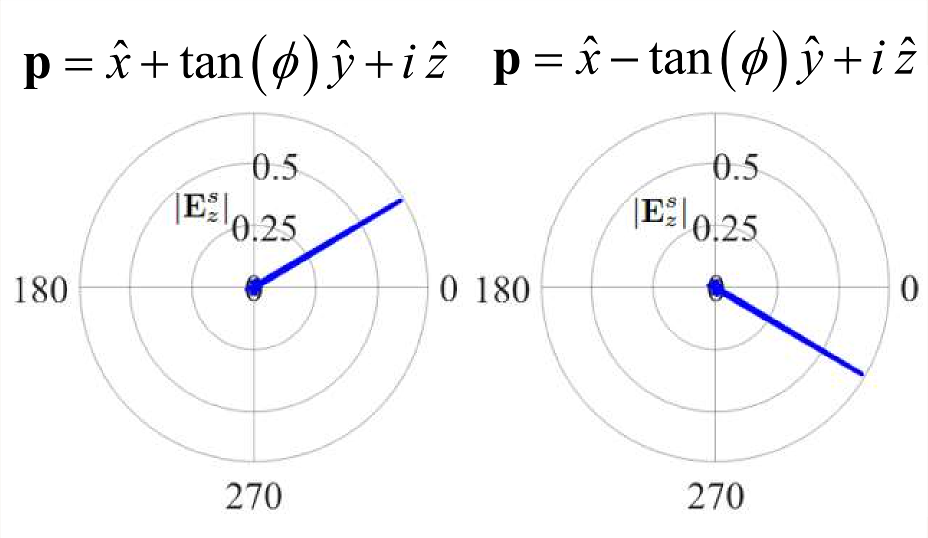


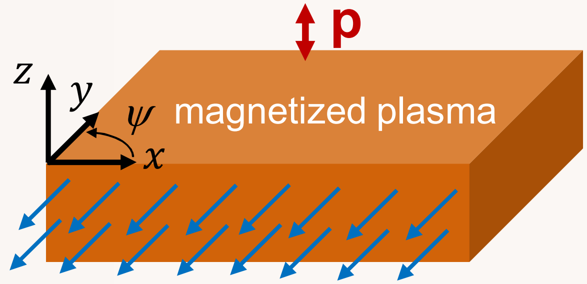

**2 diffractionless hyperbolic beams instead of the usual 4**

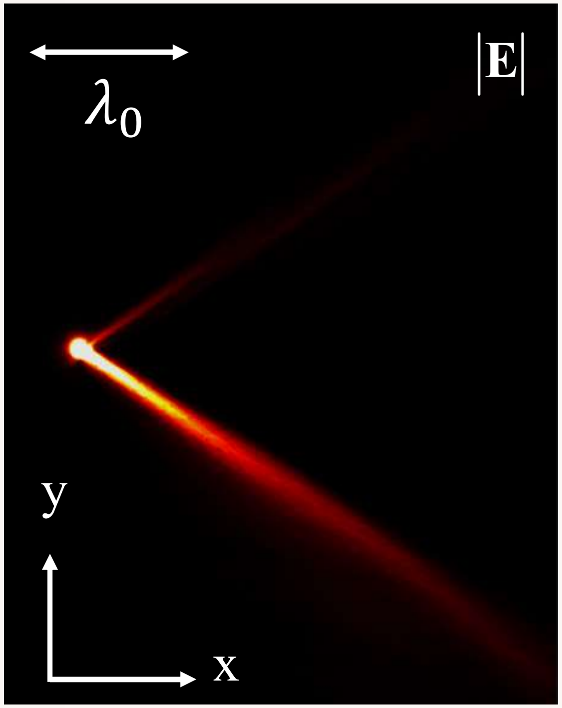
(backward propagation is prohibited)

# Unidirectional and diffractionless surface plasmon-polaritons

We can use the **source polarization** to selectively excite **one** of the two beams!

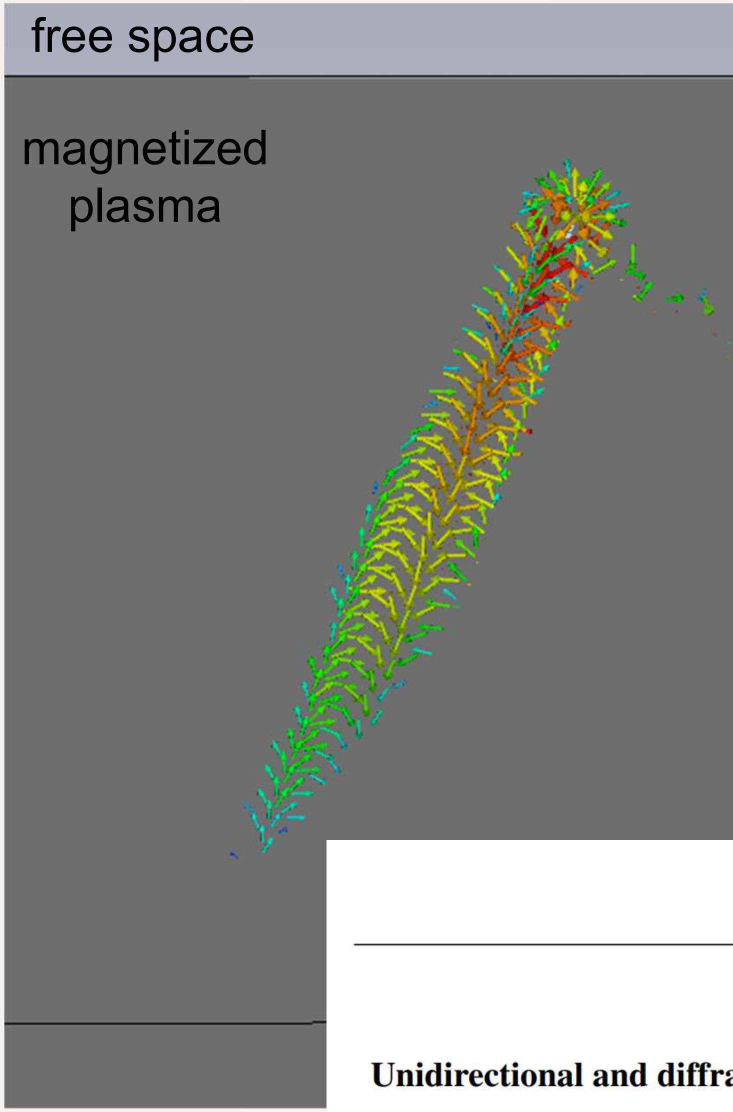


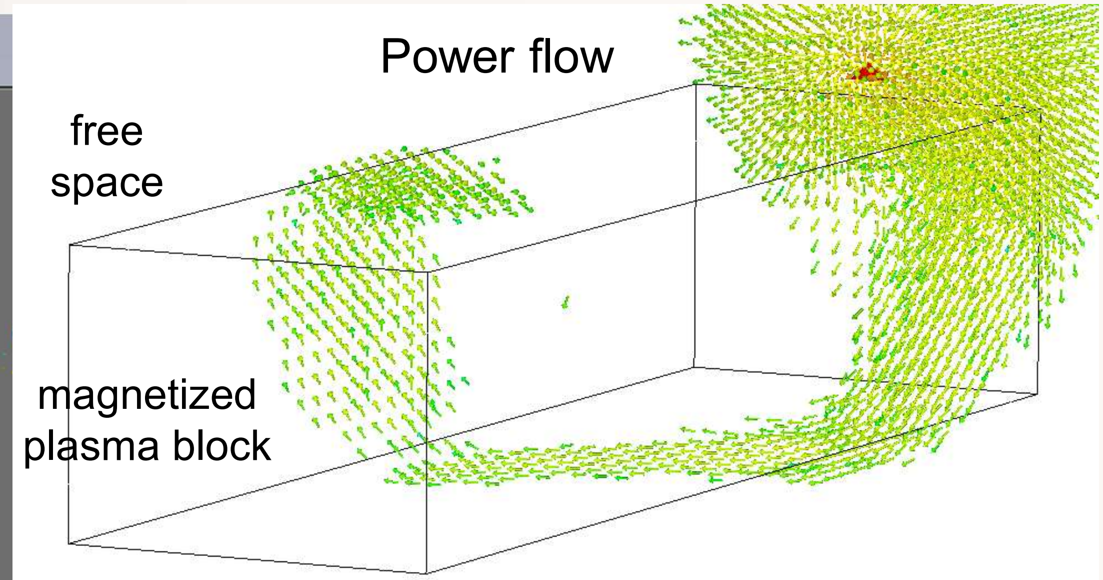




**Delta-function-like surface-wave pattern (one-way and diffractionless)!**

# Unidirectional and diffractionless surface plasmon-polaritons



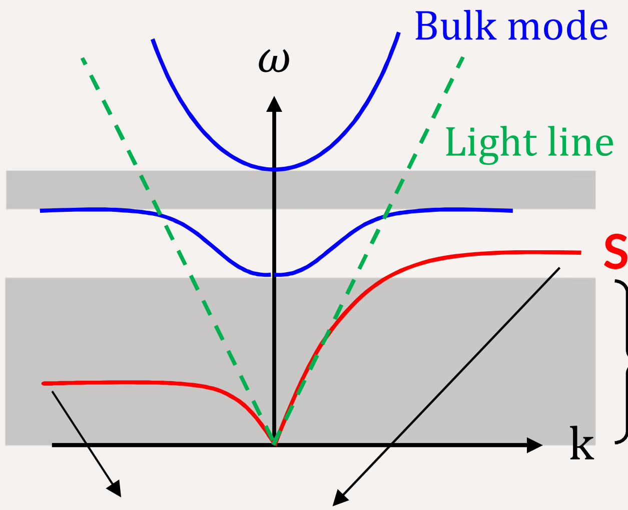

PHYSICAL REVIEW B **99**, 245414 (2019)

**Unidirectional and diffractionless surface plasmon polaritons on three-dimensional nonreciprocal plasmonic platforms**

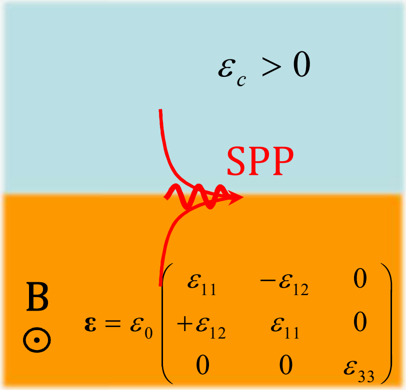
S. Ali Hassani Gangaraj,[1] George W. Hanson,[2] Mário G. Silveirinha,[3] Kunal Shastri,[1] Mauro Antezza,[4,5]
and Francesco Monticone[1,*]

## Are these modes truly unidirectional ?

# Unidirectionality of Surface Magneto-Plasmons




Interface with a **transparent medium** (e.g., vacuum or a dielectric)




**No topological invariant associated with this gap!**
(gap Chern number = 0)

This dispersion seem to suggest that, at a finite frequency ω, the material response persists for arbitrarily large wavevectors k.

However, in a realistic material, a field with very fast spatial (or frequency) variation cannot polarize the microscopic constituents of the medium.

⇨ The material response is **expected to vanish** when $k \to \infty$

The issue stems from assuming a **local material model** (local Drude) independent of k !

# Effect of Nonlocality (Spatial Dispersion) on Magneto-Plasmons

Intuitively, the nonlocal response is mainly due to the movement of electrons during an optical cycle (due to **convection** and **diffusion**, which act to avoid electron localization).

A. J. Bennett, Phys. Rev. B 1, Jan. 1970.

S. Raza, et al., Journal of Physics: Condensed Matter 27, 183204 (2015).

⇨ In general, this determines a **high spatial-frequency cutoff for the material response** (in particular, for longitudinal modes)

**Drift-diffusion model** for lossless electron gas ⇨

Transverse and longitudinal permittivity:

$$\frac{\varepsilon_T}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\omega^2} \qquad \frac{\varepsilon_L}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\left(\omega^2 - v^2 k^2\right)} \rightarrow 1$$

for $k \rightarrow \infty$

**Nonlocal hydrodynamic model** (no diffusion) for a magnetized plasma:

$$\beta^2 \nabla(\nabla \cdot \mathbf{J}) + \omega(\omega + i\gamma)\mathbf{J} = -i\omega(\omega_p^2 \epsilon_\infty \epsilon_0 \mathbf{E} - \frac{e}{m^*} \mathbf{J} \times \mathbf{B}_0)$$

$\beta$ is the nonlocal parameter proportional to the Fermi velocity.

$$\frac{\epsilon\,(\omega,\mathbf{k})}{\varepsilon_0} = \mathrm{I}_{2\times 2} - \omega_p^2 \begin{bmatrix} \dfrac{\omega^2}{-\beta^2\omega^2 k^2 + \omega^2(\omega^2 - \omega_c^2)} & \dfrac{-i\omega\omega_c}{-\beta^2\omega^2 k^2 + \omega^2(\omega^2 - \omega_c^2)} \\ \dfrac{+i\omega\omega_c}{-\beta^2\omega^2 k^2 + \omega^2(\omega^2 - \omega_c^2)} & \dfrac{-\beta^2 k^2 + \omega^2}{-\beta^2\omega^2 k^2 + \omega^2(\omega^2 - \omega_c^2)} \end{bmatrix}$$

$$\mathbf{k} = k\,\hat{x}$$

⇨

$$\frac{\epsilon\,(\omega,\mathbf{k})}{\varepsilon_0} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 - \dfrac{\omega_p^2}{\omega^2} \end{bmatrix}$$

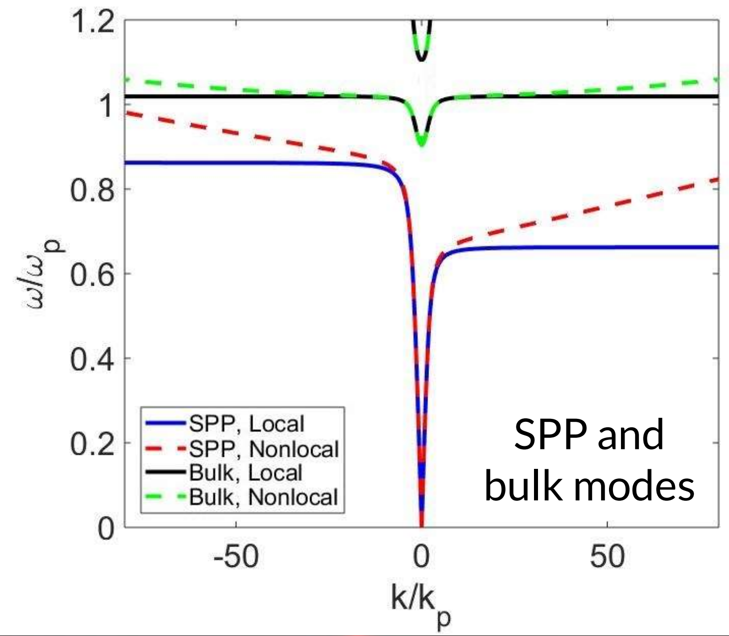
for $k \rightarrow \infty$

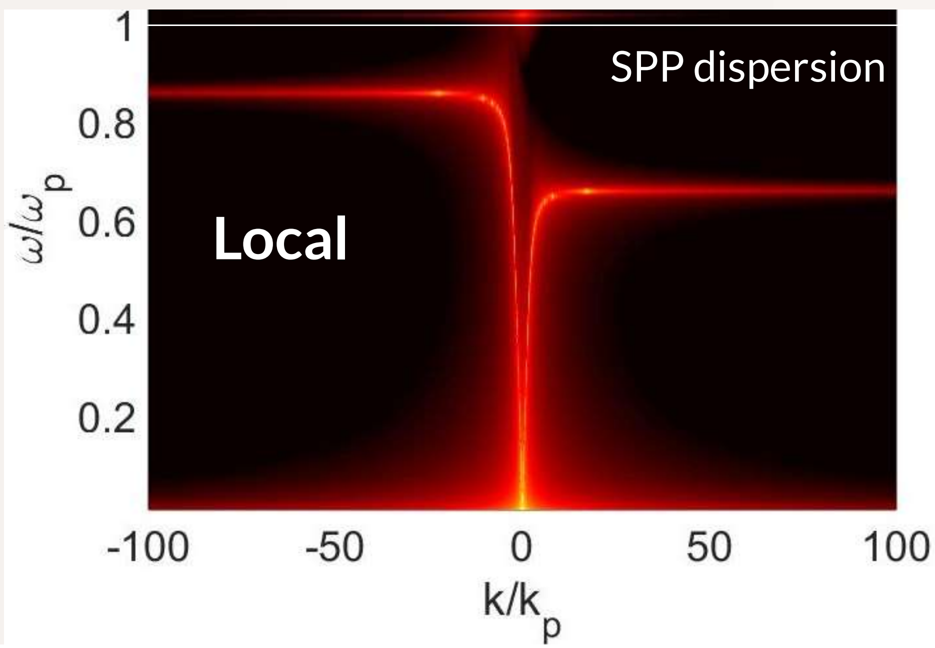
# Effect of Nonlocality (Spatial Dispersion) on Magneto-Plasmons

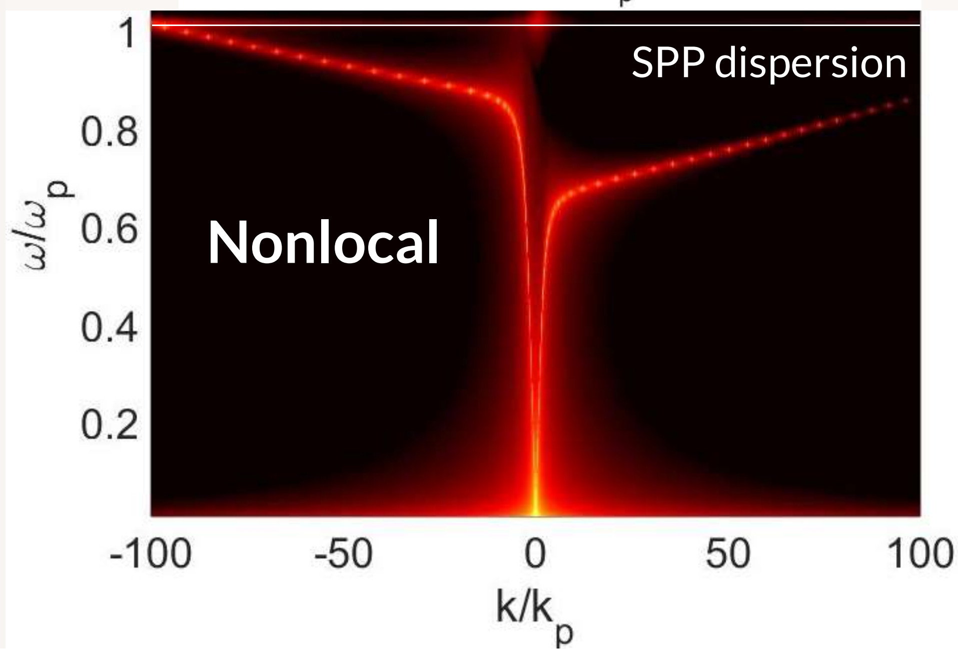
$$\beta^2\nabla(\nabla\cdot\mathbf{J})+\omega(\omega+i\gamma)\mathbf{J}=-i\omega(\omega_p^2\epsilon_\infty\epsilon_0\mathbf{E}-\frac{e}{m^*}\mathbf{J}\times\mathbf{B}_0)$$

One additional boundary conditions: $\hat{y}\cdot\mathbf{J}=0$

S. Buddhiraju, Y. Shi, A. Song, C. Wojcik, M. Minkov, I.A.D. Williamson, A. Dutt, and **Shanhui Fan**, arXiv:1809.05100v1, 2018.

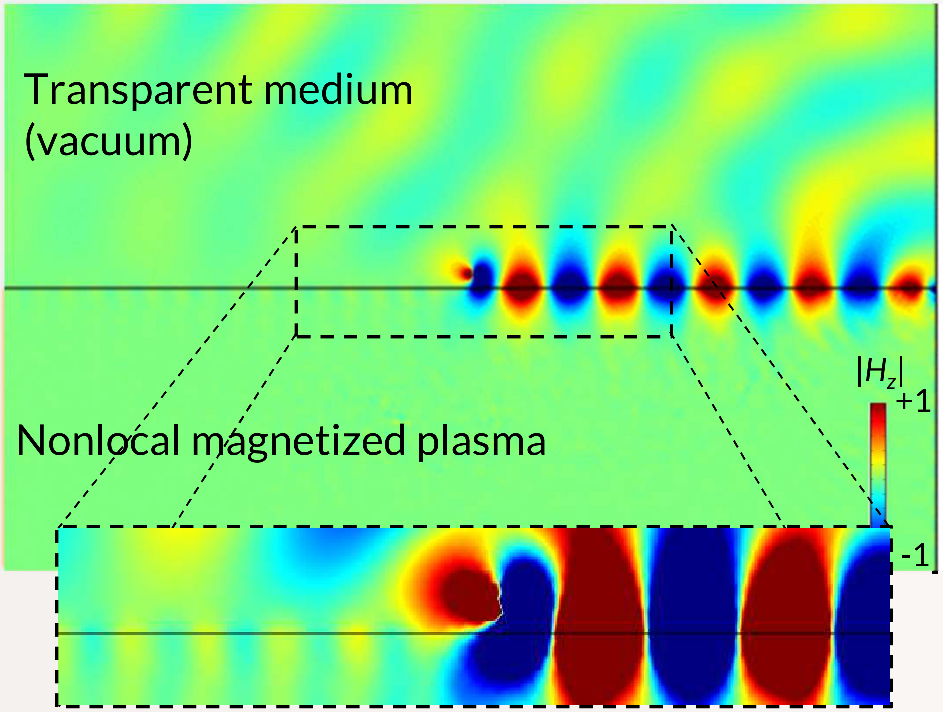


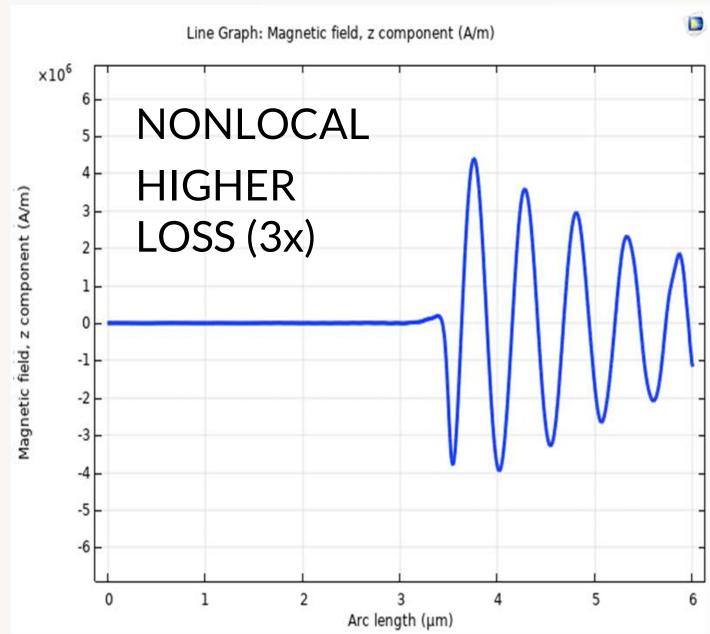



# Effect of Nonlocality (Spatial Dispersion) on Magneto-Plasmons

Implemented the hydrodynamic equation in COMSOL as a weak-form PDE with an additional boundary condition




**PEC** (perfect electric conductor) termination



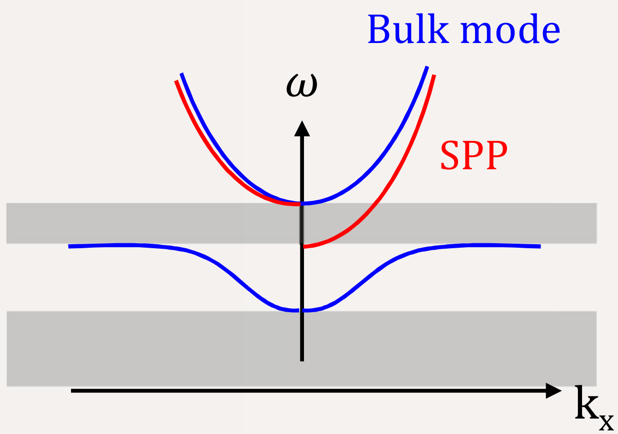

**A backward mode can be excited**

The energy "escapes" the termination via losses, radiation, AND a backward mode

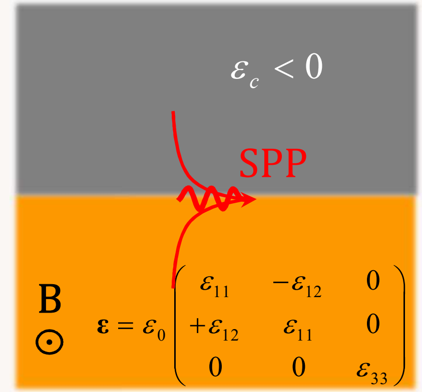
**Surface magneto-plasmons are not fundamentally unidirectional, but they may be in practice, depending on the material, loss, configuration, etc..**

# Topological Surface Plasmon-Polaritons

## Another surface-wave propagation regime:




Interface with an **opaque medium**
(e.g., perfect electric conductor)




Non-trivial bulk-mode bandgap (opened by breaking T-symmetry)

**Topological invariant : gap Chern number (=1)**

⇨ **Topological surface plasmon-polaritons**

## Impact of nonlocality?

Actually, a nonlocal material response (high spatial frequency cutoff) is **necessary** to justify the topological properties of **continuum plasmonic media!**

(otherwise the pseudo-Hamiltonian of the medium is not "sufficiently well behaved" for infinite wavenumber, and the Chern number may not be an integer)


PHYSICAL REVIEW B **92**, 125153 (2015)


**Chern invariants for continuous media**


Mário G. Silveirinha*
*University of Coimbra, Department of Electrical Engineering – Instituto de Telecomunicações, Portugal*



(Received 1 August 2015; revised manuscript received 2 September 2015; published 30 September 2015)



Here, we formally develop theoretical methods to topologically classify a wide class of bianisotropic continuous media. It is shown that for continuous media, the underlying wave vector space may be regarded as the Riemann sphere. We derive sufficient conditions that ensure that the pseudo-Hamiltonian that describes the electrodynamics of the continuous material is well behaved so that the Chern numbers are integers. Our theory brings the powerful ideas of topological photonics to a wide range of electromagnetic waveguides and platforms with no intrinsic periodicity and sheds light over the emergence of edge states at the interfaces between topologically inequivalent continuous media.



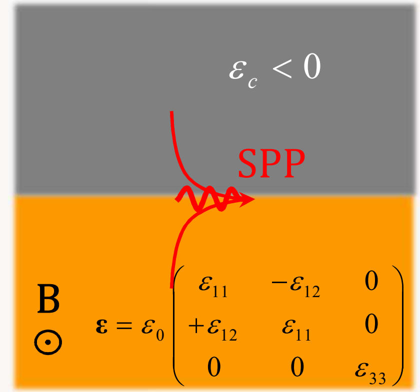
DOI: 10.1103/PhysRevB.92.125153 PACS number(s): 42.70.Qs, 03.65.Vf, 78.67.Pt, 78.20.Ls

# Topological Surface Plasmon-Polaritons

**For both media at the interface:**

$$\beta_1^{\,2}\nabla(\nabla\cdot\mathbf{J}_1)+\omega(\omega+i\gamma_1)\mathbf{J}_1=-i\omega\left(\omega_{p,1}^2\epsilon_{\infty,1}\epsilon_0\mathbf{E}-\frac{e}{m_1^*}\mathbf{J}_1\times\mathbf{B}_0\right)$$

$$\beta_2^{\,2}\nabla(\nabla\cdot\mathbf{J}_2)+\omega(\omega+i\gamma_2)\mathbf{J}_1=-i\omega\left(\omega_{p,2}^2\epsilon_{\infty,2}\epsilon_0\mathbf{E}-\frac{e}{m_2^*}\mathbf{J}_2\times\mathbf{B}_0\right)$$




**Two additional boundary conditions:**

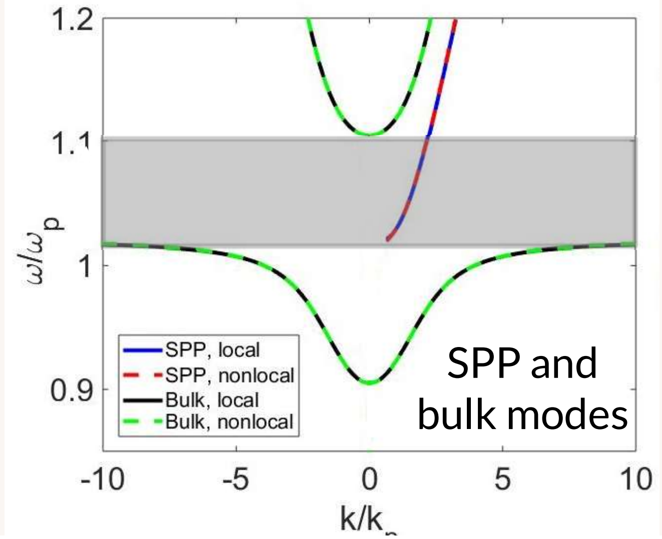
- **Continuous normal velocity of the electrons across the interface** (acoustic condition in hydrodynamics → it ensures plasma continuity).
- **Continuous mechanical pressure from the electrons across the boundary.**

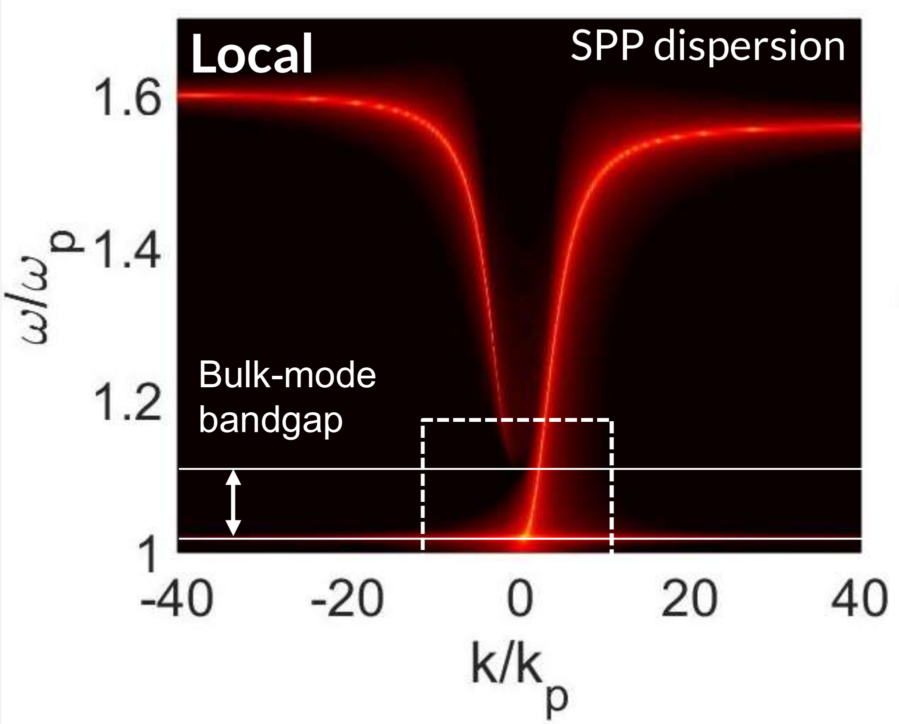
A. D. Boardman and R. Ruppin, "The boundary conditions between spatially dispersive media," Surf. Sci. **112**, 153 (1981).

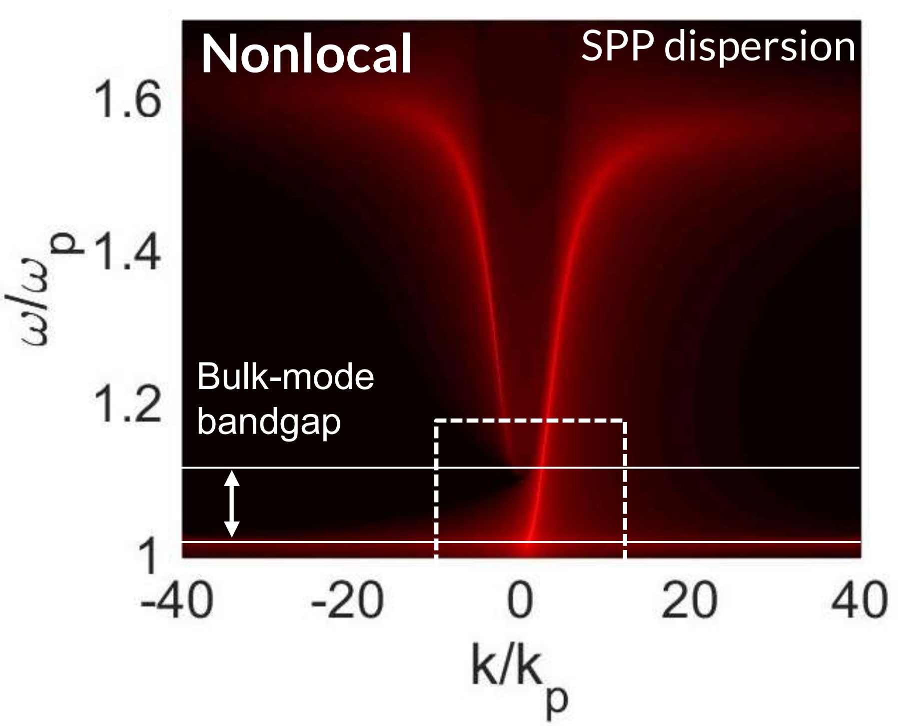
# Topological Surface Plasmon-Polaritons

The topological surface mode, which usually exists for **low values of wavenumber**, is unaffected by nonlocality (high spatial-frequency cutoff)

S.A.H. Gangaraj, and F. Monticone, "Do truly unidirectional surface plasmon-polaritons exist?" *Optica* **6** (9), 1158-1165

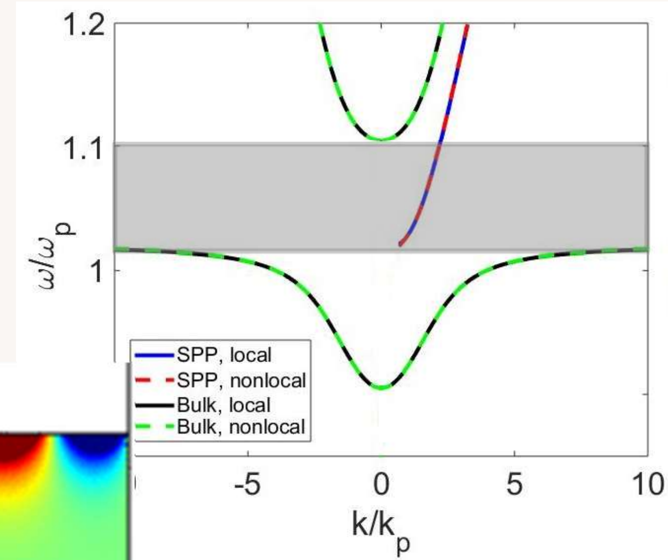


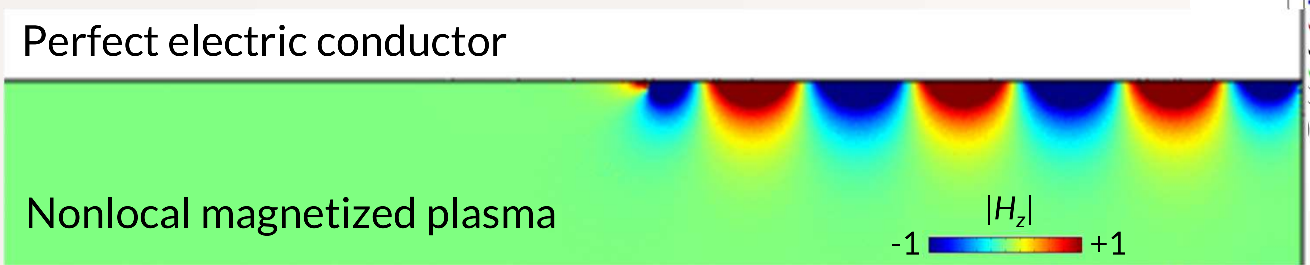



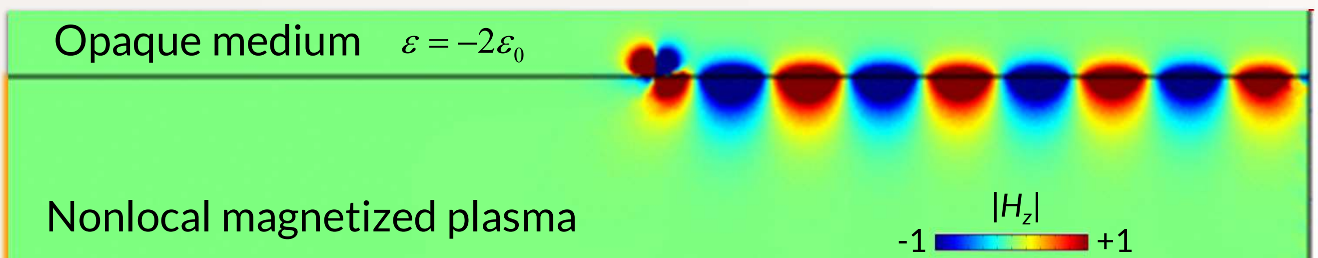



# Terminated Unidirectional Structure

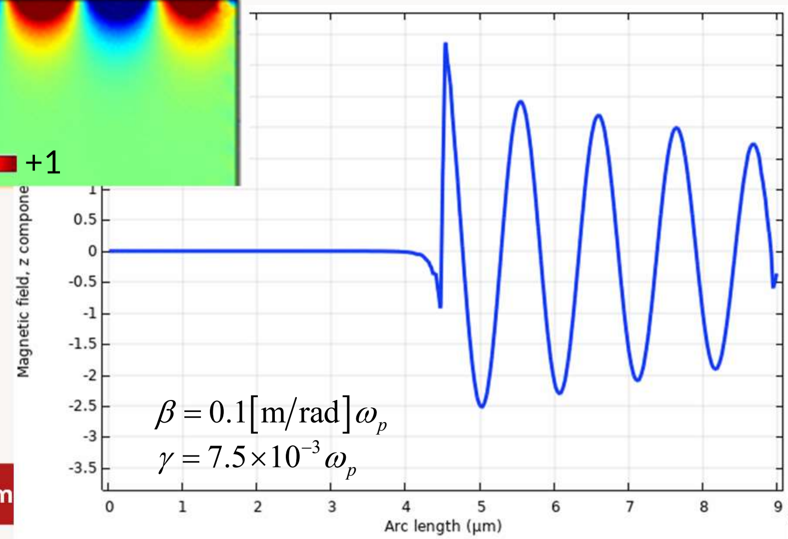
The topological surface mode, which usually exists for **low values of wavenumber**, is unaffected by nonlocality (high spatial-frequency cutoff)




**PMC** (perfect magnetic conductor) termination



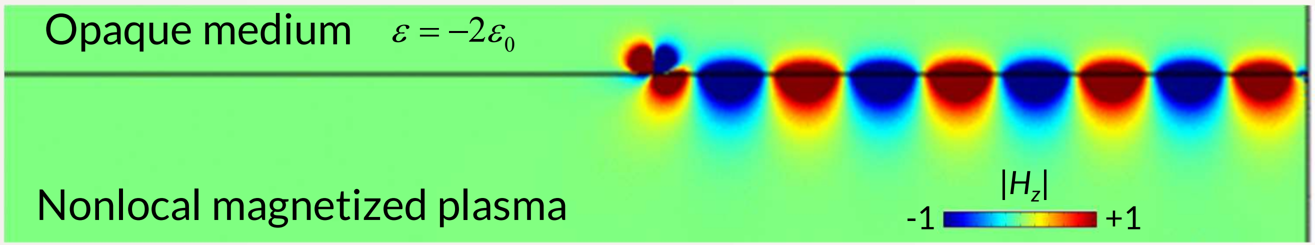

**No backward mode**

The energy can "escape" the termination only via absorption losses.

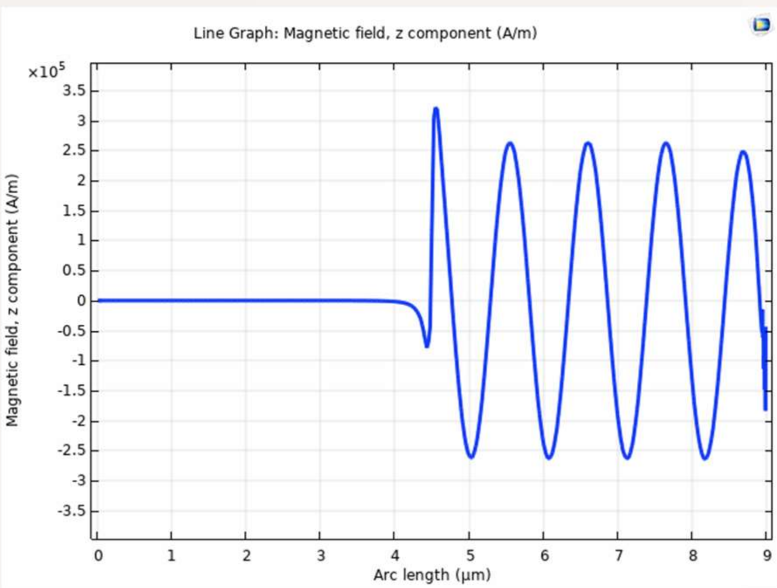
# Terminated Lossless Unidirectional Structure

$\gamma = 0$

$$\beta^2\nabla(\nabla\cdot\mathbf{J})+\omega(\omega+i\gamma)\mathbf{J} = -i\omega\left(\omega_p^2\epsilon_\infty\epsilon_0\mathbf{E} - \frac{e}{m^*}\mathbf{J}\times\mathbf{B}_0\right)$$




**PMC** (perfect magnetic conductor) termination




**No backward mode even in the lossless nonlocal case**

**Where does the energy go?**

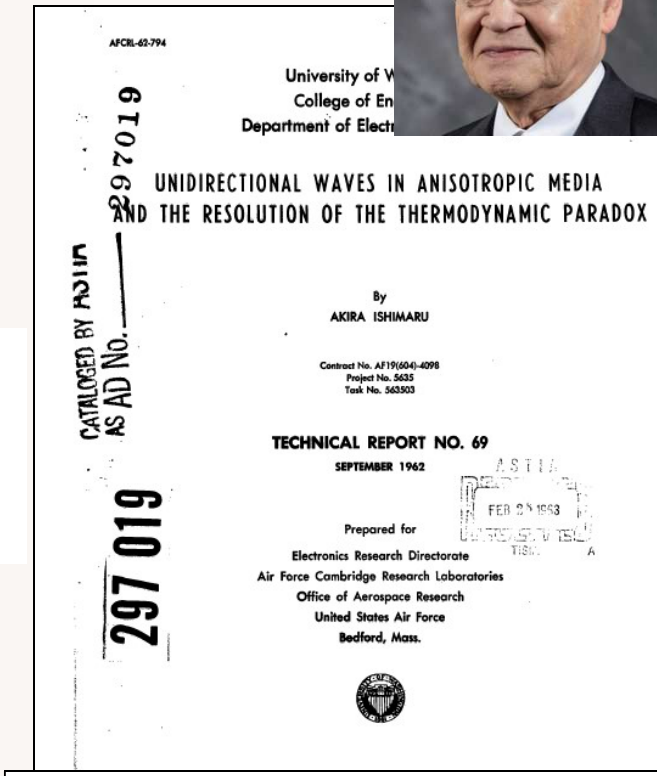
**Thermodynamic paradox in the ideal lossless scenario?**

# Thermodynamic Paradox for Topological SPPs?

## No!

This is an **ill-posed boundary-value problem** because the solution is not unique, and does not "depend continually on the data" (for loss = 0 and loss → 0 , we get different solutions)

It is shown that Maxwell's equations with completely lossless medium which leads to thermodynamic paradox is in fact "Improperly-posed problem," which does not correspond to physical reality.



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UNIDIRECTIONAL WAVES IN ANISOTROPIC MEDIA AND THE RESOLUTION OF THE THERMODYNAMIC PARADOX

By

AKIRA ISHIMARU

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Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Mass.

$$\int_V \lim_{\sigma\to 0} j\omega (E\cdot D^* - H^*\cdot B)\, dv = \text{Not defined}$$

$$\lim_{\sigma\to 0}\int_V j\omega (E\cdot D^* - H^*\cdot B)\, dv = \frac{v_1\phi_0}{2\omega\epsilon_T}\ \text{real} \rightarrow$$

real non-zero power dissipation

Power is dissipated in a "**wedge mode**" (λ **shrinks to zero**)

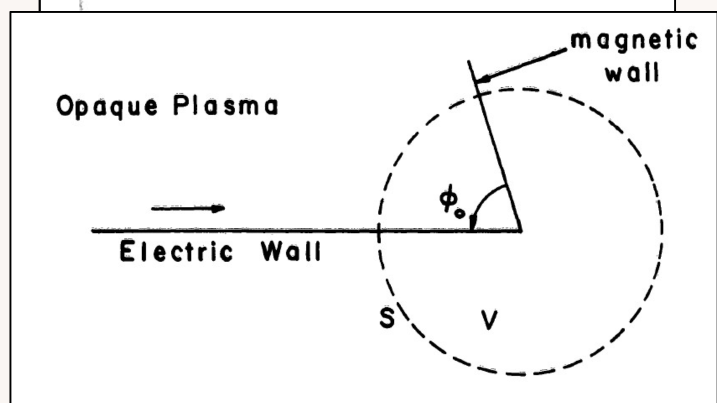
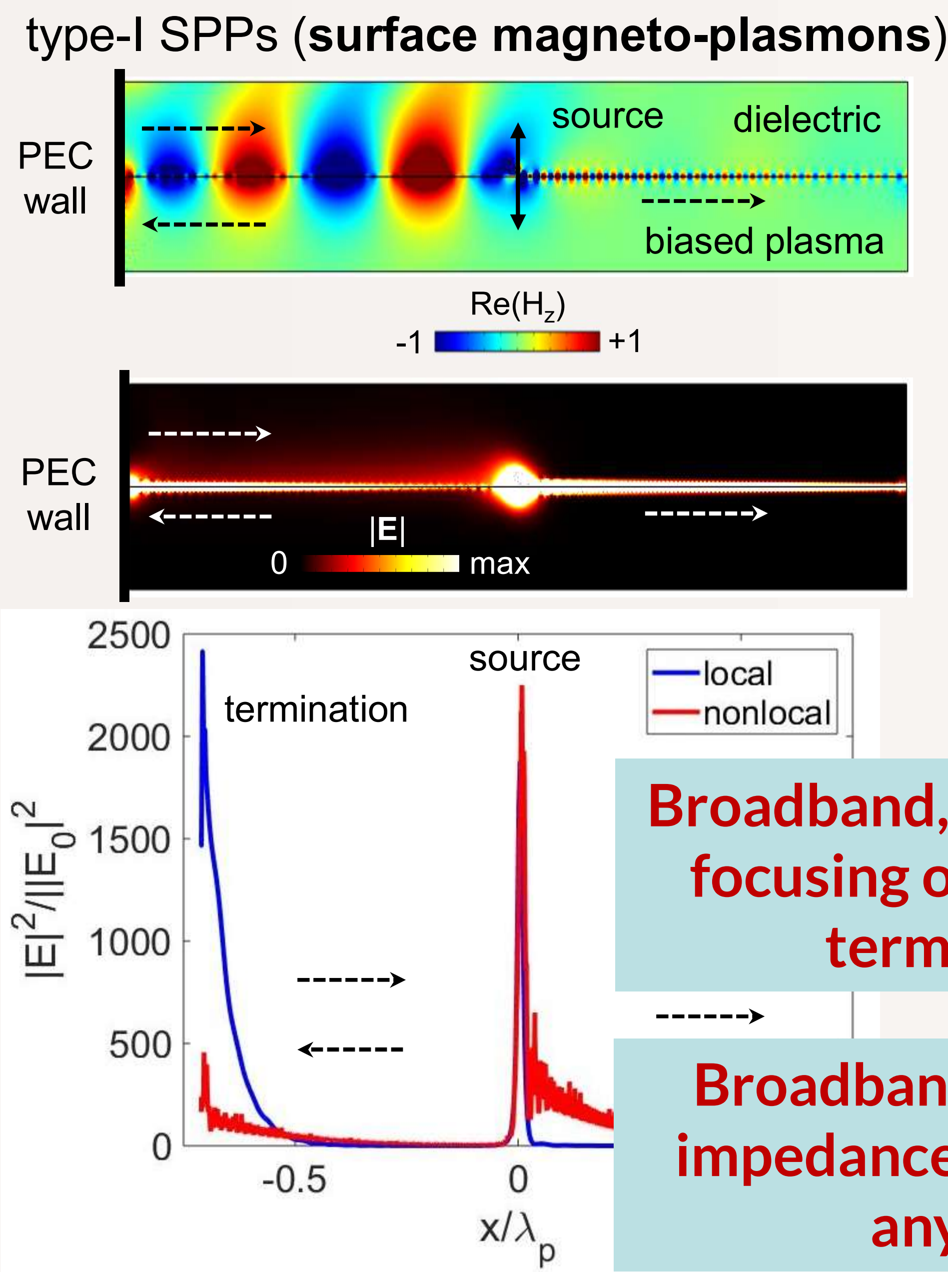
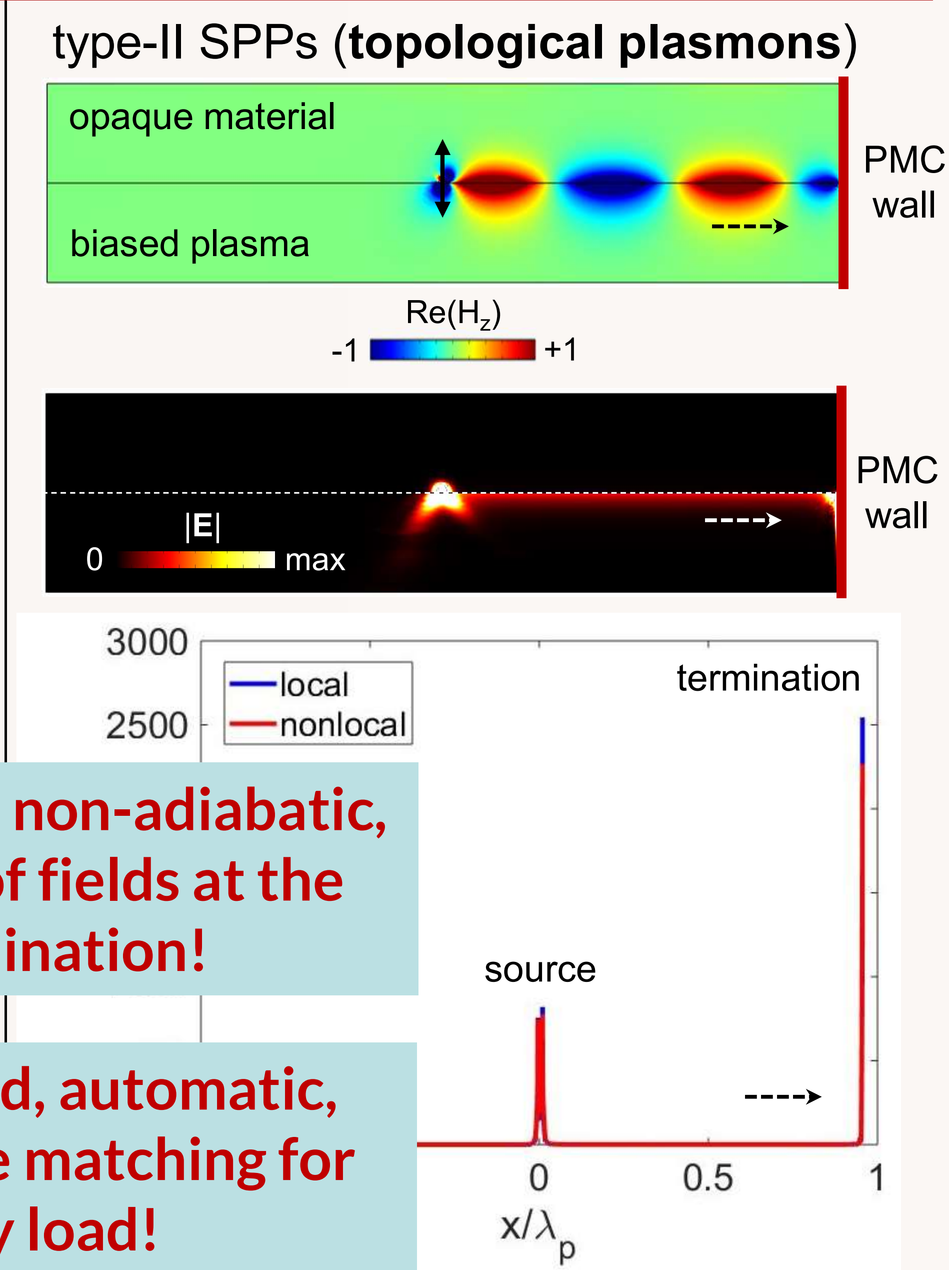



Figure 3 Unidirectional wave along an electric wall terminated by a magnetic wall.


S. A. Mann, D. L. Sounas, and A. Alu, "Nonreciprocal cavities and the time–bandwidth limit," Optica **6**, 104-110 (2019).

# Intensity Enhancement in Terminated One-Way Channels




**Broadband, non-adiabatic, focusing of fields at the termination!**

**Broadband, automatic, impedance matching for any load!**

**“Broadband, automatic, impedance matching for any load!”**

**Do nonreciprocity or unidirectionality break any conventional limit on broadband impedance matching for passive systems?**



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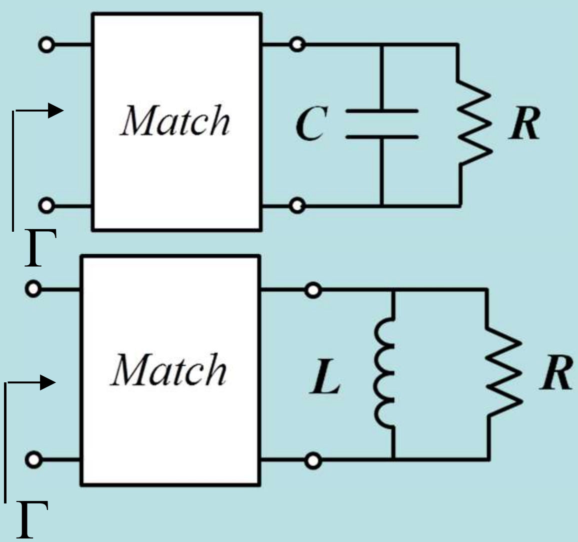
# Bounds on Impedance Matching in Nonreciprocal Systems

Microwave circuit theorists developed physical bounds on broadband impedance matching long time ago:




$$\int_0^\infty \ln\frac{1}{|\Gamma|}\,\mathrm{d}\omega \le \frac{\pi}{RC}$$

$$\int_0^\infty \omega^{-2}\ln\frac{1}{|\Gamma|}\,\mathrm{d}\omega \le \frac{\pi L}{R}$$

**Assumptions:**

(1) LTI system

**(2) Passivity and causality**

**(3) Reactive** matching network

**Bandwidth limitations on $\Gamma$ uniquely determined by the load itself.**

Herglotz-Nevanlinna function $h(\omega) = -i\ln\left(|\Gamma(\omega)|\right)$

$$\frac{2}{\pi}\int_0^\infty \mathrm{Im}\left[h(\omega)\right]\mathrm{d}\omega \le H$$

$$h(\omega) \sim -H/\omega \quad \text{for } \omega \to \infty$$

$$\frac{2}{\pi}\int_0^\infty \omega^{-2}\,\mathrm{Im}\left[h(\omega)\right]\mathrm{d}\omega \le L$$

$$h(\omega) \sim L\omega \quad \text{for } \omega \to 0$$

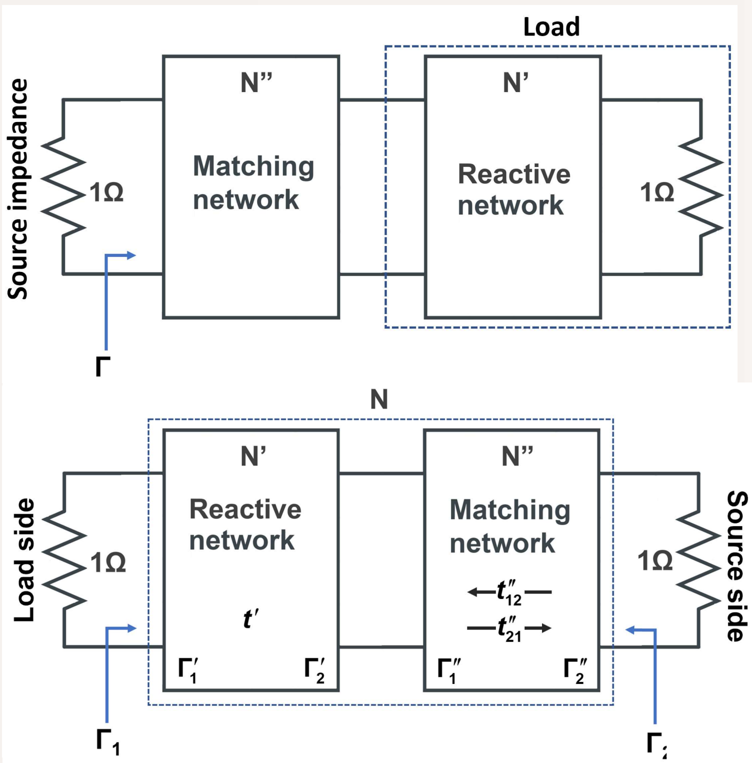
Bode-Fano inequalities = bounds on integral identities for Herglotz functions, which depend on the **low-frequency and high-frequency asymptotic expansions of *h***

# Validity of Bode-Fano limit for non-reciprocal matching

## Is the Bode-Fano limit valid for a non-reciprocal matching network?

**Check Fano's derivation :** Fano assumed a lossless reciprocal matching network. But, the steps of his derivation **don't change when the lossless matching network is non-reciprocal**!

R.M. Fano, J. Franklin Inst. **249**, 57 (1950).




$$S' = \begin{pmatrix} \Gamma'_1 & t' \\ t' & \Gamma'_2 \end{pmatrix} \quad S'' = \begin{pmatrix} \Gamma''_1 & t''_{12} \\ t''_{21} & \Gamma''_2 \end{pmatrix} \quad S = \begin{pmatrix} \Gamma_1 & t_{12} \\ t_{21} & \Gamma \end{pmatrix}$$

Network N is lossless ⇒ S is unitary

Unitary matrix $\begin{pmatrix} a & b \\ -e^{j\varphi} b^* & e^{j\varphi} a^* \end{pmatrix}, \quad |a|^2 + |b|^2 = 1$

**Lossless nonreciprocal matching…**

→ Nonreciprocal transmission manifests only in terms of an asymmetric phase difference…

→ $|\Gamma|$ is unaffected by this type of nonreciprocity

**Bode-Fano limit on |Γ| is unaffected!**

**One-way system used for matching**

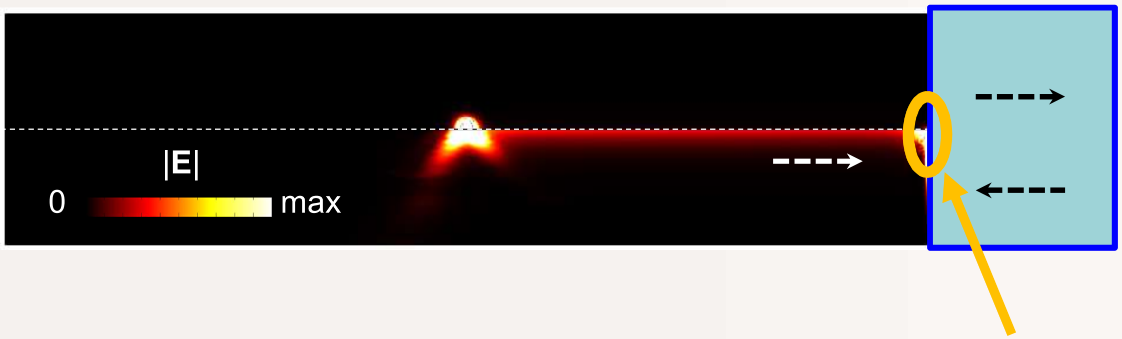
→ Necessary lossy!

→ **Bode-Fano theory does not apply** (as for any other matching technique based on absorption)

# Validity of Bode-Fano limit for non-reciprocal matching

One-way topological system with vanishing loss

Reactive load (e.g., cavity)




Dissipation at this corner (wedge) **even in the limit of vanishing losses**

$$\int_V \lim_{\sigma\to 0} j\omega\,(E\cdot D^* - H^*\cdot B)\,dv = \text{Not defined}$$

$$\lim_{\sigma\to 0}\int_V j\omega(E\cdot D^* - H^*\cdot B)\,dv = \frac{v_1\phi_0}{2\omega\varepsilon_T}\ \text{real}$$

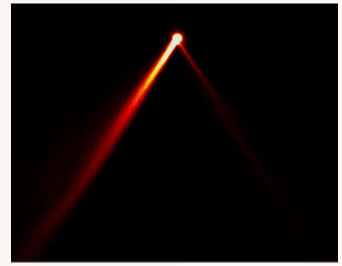
→ real non-zero power dissipation in the limit of vanishing losses

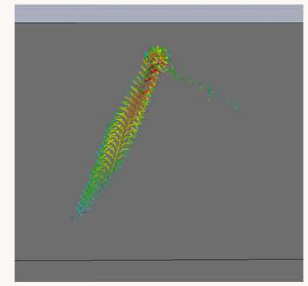
**Bode-Fano limit does not apply because this is a form of absorption-based impedance matching (even for vanishing material absorption)**

# Conclusion

## Role of Topology, Locality, and Passivity in Nonreciprocal Electromagnetics

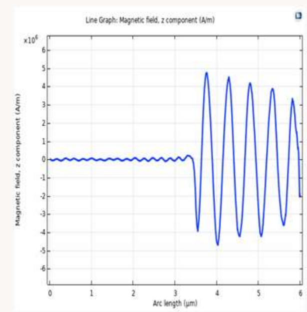
**Unidirectional AND diffractionless** surface waves on nonreciprocal substrates



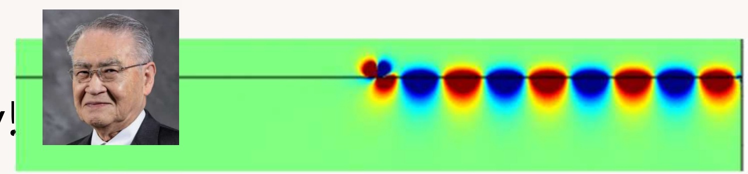


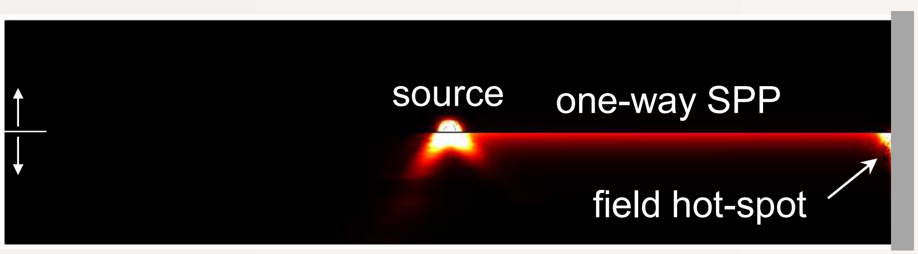
**Impact of nonlocality (spatial dispersion)** on unidirectional and topological surface waves on a continuous medium

$$\beta^2 \nabla(\nabla \cdot \mathbf{J}_p) + \omega(\omega + i\gamma)\mathbf{J}_i = -i\omega(\omega_p^2 \epsilon_\infty \epsilon_0 \mathbf{E} - \frac{e}{m^*}\mathbf{J}_i \times \mathbf{B}_0)$$



**No thermodynamic paradoxes, nor breaking of impedance matching bounds** due to unidirectionality!






Giant field enhancement in **terminated one-way channels** to boost weak light-matter interactions, for example, nonlinear effects!

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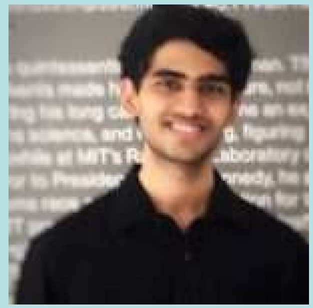

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# Do truly unidirectional surface plasmon-polaritons exist?

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# Unidirectional and diffractionless surface plasmon polaritons on three-dimensional nonreciprocal plasmonic platforms

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