Friction at the base of glaciers: insights from modeling at various scales

Olivier Gagliardini, Julien Brondex, Fabien Gillet-Chaulet

Mathematical Modelling in Glaciology BIRS workshop
Banff, January 2020

UNIVERSITÉ Grenoble Alpes · cnrs · IGE · SAUSSURE · elmer ICE

Two scales – two parts

1/ micro-meso scale: friction law formulation

2/ macro scale: grounding line sensitivity to the choice of the friction law

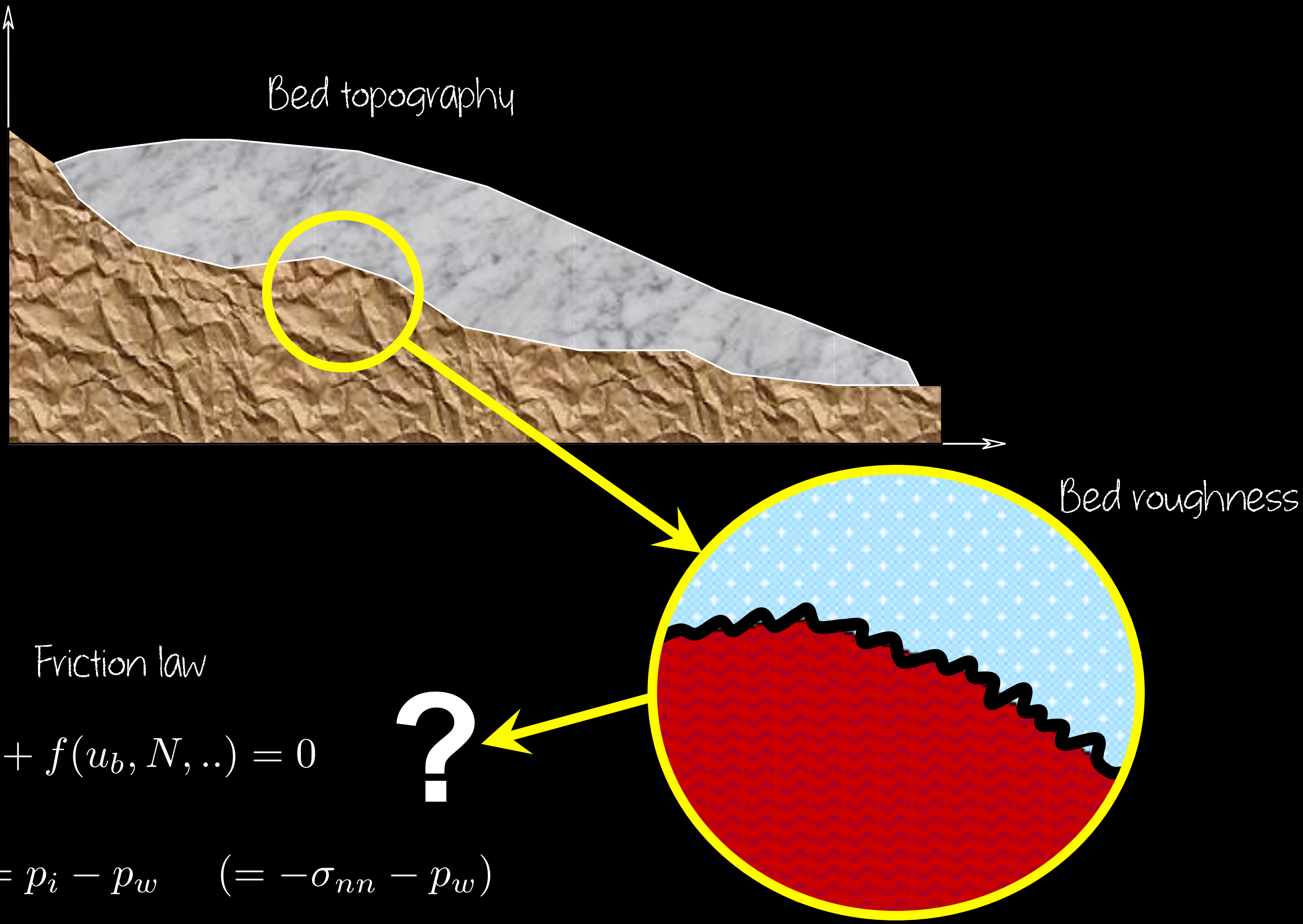
A unique modelling tool: Elmer/Ice

Open Source Finite Element Software for Ice Sheet, Glaciers and Ice Flow Modelling

http://elmerice.elmerfem.org/

Concept of friction law in Glaciology and role of water

Bed topography / roughness



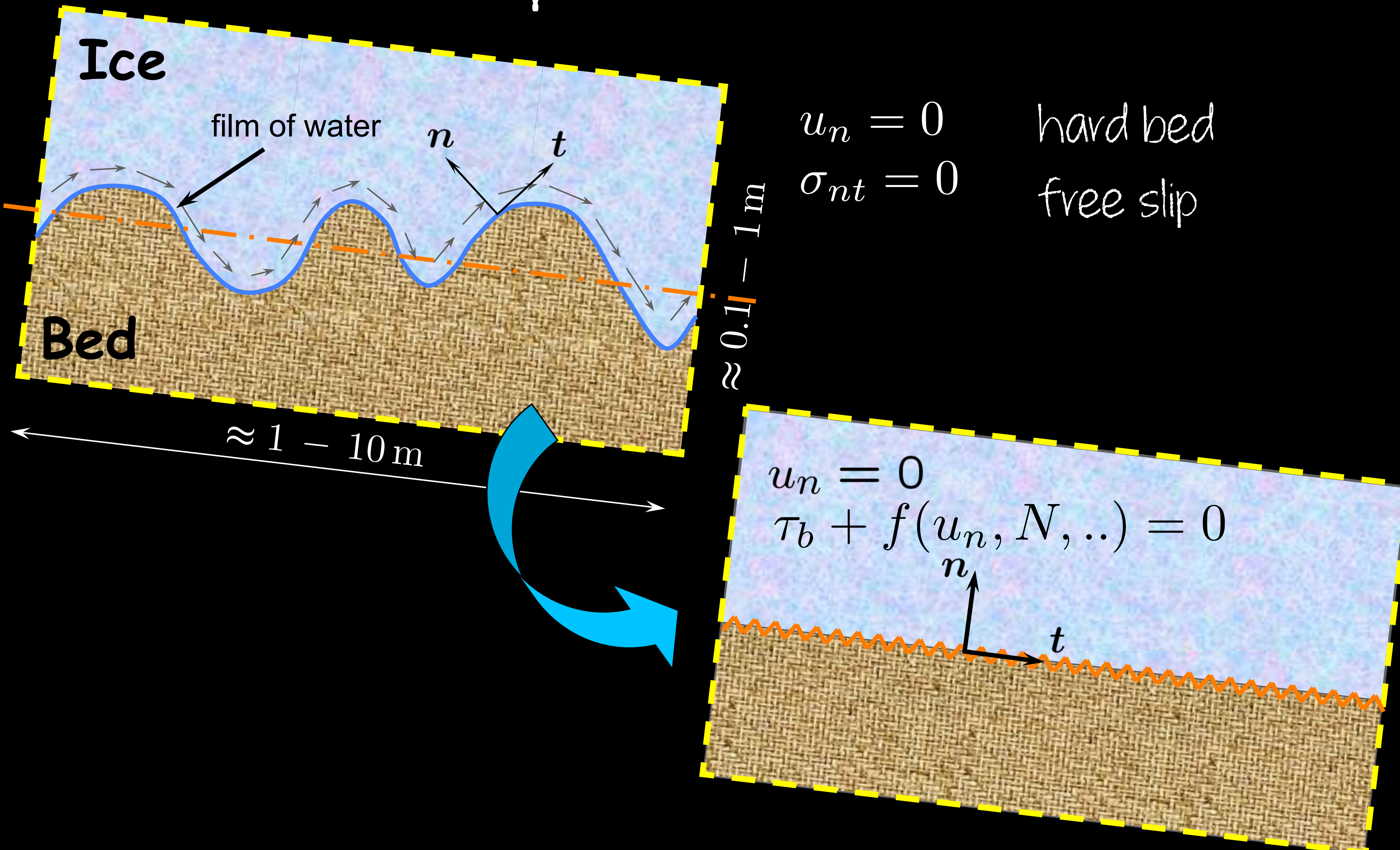
Friction law

$$\tau_b + f(u_b, N, ..) = 0$$

?

$$N = p_i - p_w \quad (= -\sigma_{nn} - p_w)$$

Concept of friction law



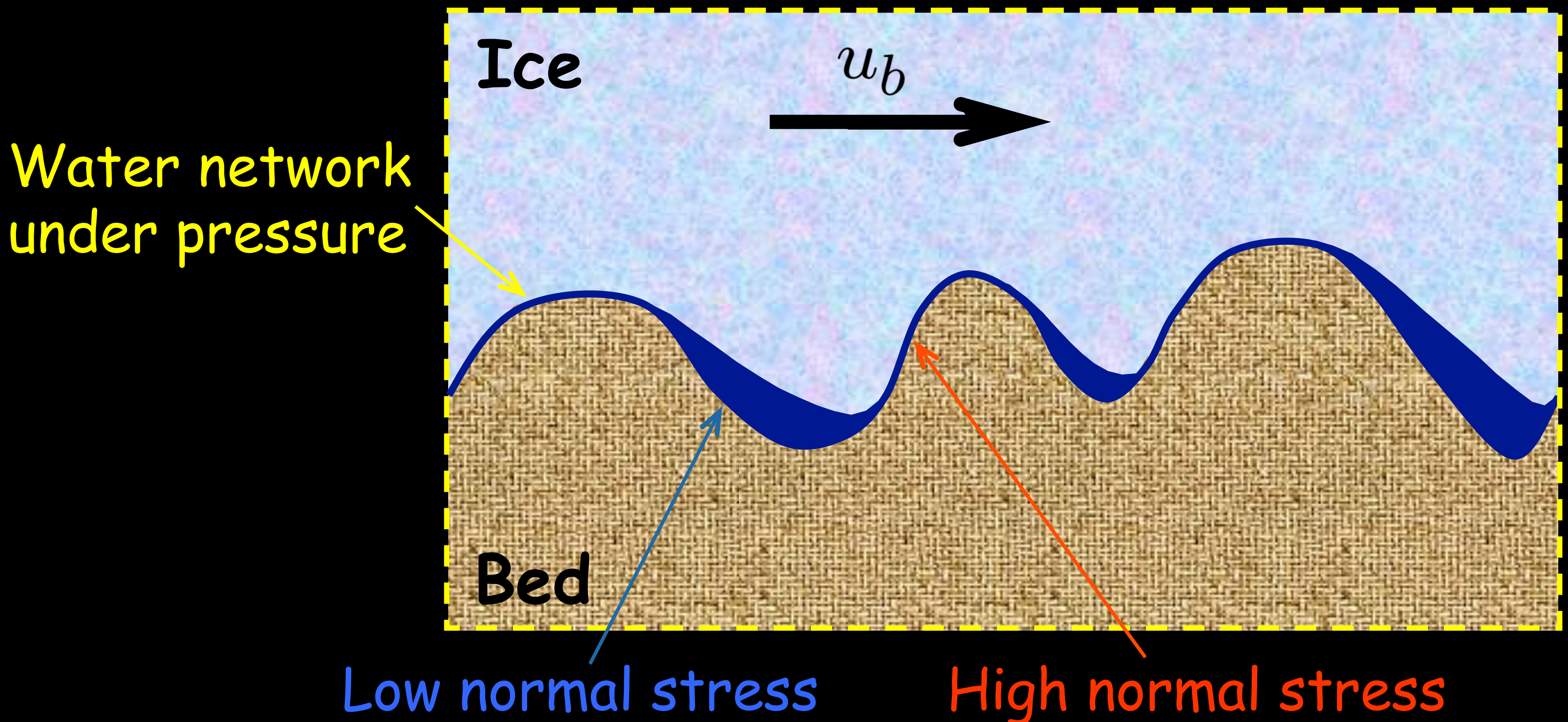
$u_n = 0$ hard bed

$\sigma_{nt} = 0$ free slip

[Weertman (1957), Fowler (1981), Gudmundsson (1997)]

Effect of water

If water pressure and/or velocity increase



➡ What is the form of the friction law ?

$$\tau_b - f(u_b, N, ...) = 0$$

A bit of history...

Journal of Glaciology, Vol. 56, No. 200, 2010

Weertman, Lliboutry and the development of sliding theory

A.C. FOWLER

read more: https://saussure.osug.fr/-History-

Johannes 'Hans' Weertman (1925-2018)

1957, JoG

ON THE SLIDING OF GLACIERS

By J. WEERTMAN

(Naval Research Laboratory, Washington, D.C.)

ABSTRACT. A model is proposed to explain the sliding of any glacier whose bottom surface is at the pressure melting point. Two mechanisms are considered. One is pressure melting and the other is creep rate enhancement through stress concentrations. Neither of the mechanisms operating alone is sufficient to explain sliding. If both mechanisms operate together appreciable sliding can occur.

RÉSUMÉ. On propose un modèle pour expliquer le glissement d'un glacier dont le fond se maintient au point de fusion. On considère deux mécanismes : le fusion de pression et l'augmentation de la vitesse de déformation causée par les concentrations de tension. Ni l'un ni l'autre en agissant seul ne suffit à expliquer le glissement. Mais ensemble ils occasionneraient un glissement assez important.

2 mechanisms:

- pressure melting/refreezing
- creep rate enhancement

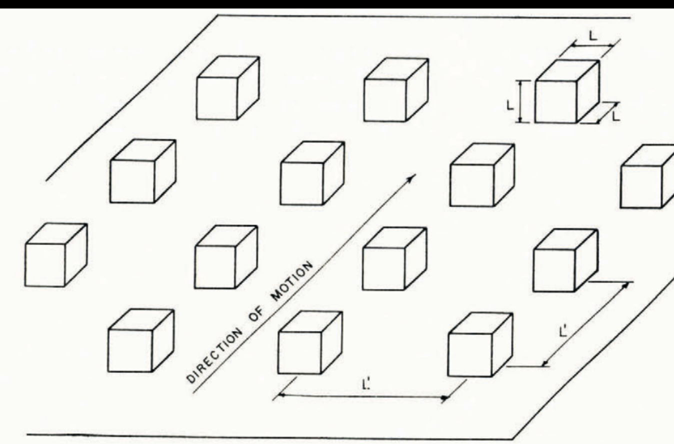


Fig. 1. Idealized glacier bed

read more: https://saussure.osug.fr/-History-

Louis Lliboutry (1922-2007)

1958, CRAS (in French)

GLACIOLOGIE. — *Contribution à la théorie du frottement du glacier sur son lit.* Note (*) M. **Louis Lliboutry**, présentée par M. Léon Moret.

L'introduction d'un troisième mécanisme de franchissement des protubérances permet d'expliquer l'indépendance du frottement dynamique vis-à-vis de la vitesse, la valeur plus élevée du frottement statique (et donc le mouvement par saccades), l'accélération du glacier aux époques chaudes, et enfin l'usure caractéristique qui conduit à des roches moutonnées.

3rd mechanisms:

opening of **cavities** on the lee side of bed bumps

read more: https://saussure.osug.fr/-History-

Lliboutry will publish more than 20 papers on that specific subject!

This one in 1968 :

Journal of Glaciology, Vol. 7, No. 49, 1968

GENERAL THEORY OF SUBGLACIAL CAVITATION AND SLIDING OF TEMPERATE GLACIERS

By L. LLIBOUTRY
(Laboratoire de Glaciologie du C.N.R.S., Grenoble, Isère, France)

followed by one from Weertman in 1972:

REVIEWS OF GEOPHYSICS AND SPACE PHYSICS, VOL. 10, NO. 1, PP. 287–333, FEBRUARY 1972

General Theory of Water Flow at the Base of a Glacier or Ice Sheet

J. WEERTMAN
Scott Polar Research Institute
Cambridge, England CB2 1ER

read more: https://saussure.osug.fr/-History-

10 march 2005, SHF, Grenoble

S.H.F. Section de Glacologie-Nivologie. Réunion du 10 Mars 2005

Glissement et hydraulique sous-glaciaires

Louis LLIBOUTRY

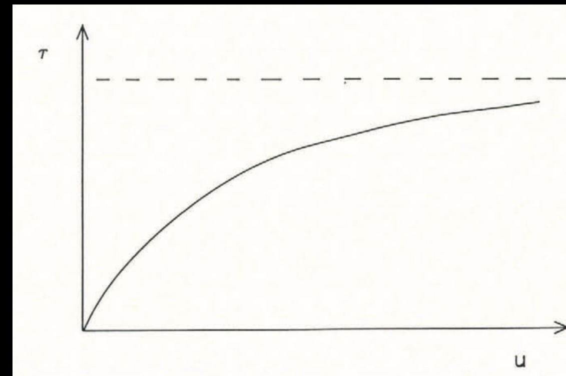
3, Av. de la Foy, Corenc, 38700 La Tronche

Llibourty : "*..., on en est resté aux théories grossières, incomplètes et erronées émises dans les années 60 par Weertman, moi-même, Nye, Röthlisberger, et Budd, qui n'ont plus qu'un intérêt historique aujourd'hui.*"

He presented a new theory (in 10 minutes) which should be published in 3 papers that he was about to submit to Journal of Glaciology

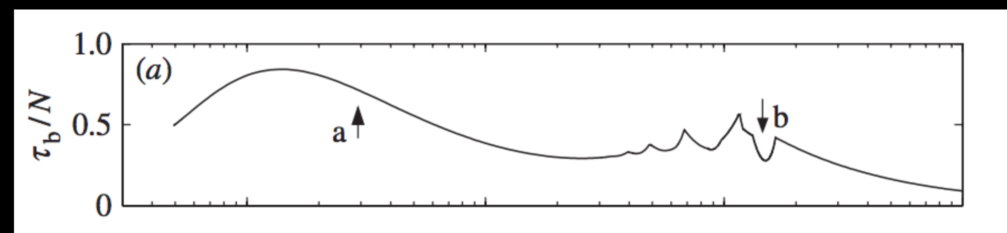
These papers will never be submitted... and the manuscripts are lost...

Fowler, 1981, 1987



Schoof 2005

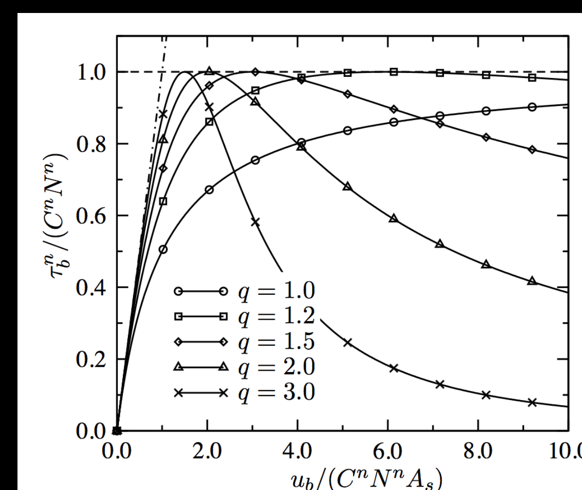
$$\frac{\tau_\mathrm{b}}{N} = C\left(\frac{\Lambda}{\Lambda + \Lambda_0}\right)^{1/n}, \quad \Lambda = \frac{u_\mathrm{b}}{N^n}$$



Gagliardini et al., 2007

$$\frac{\tau_b}{CN} = \left(\frac{\chi}{1 + \alpha\chi^q}\right)^{1/n}$$

$$\chi = \frac{u_b}{A_s C^n N^n} \; ; \; \alpha = \frac{(q-1)^{(q-1)}}{q^q}$$



All these formulations are based on strong assumptions:

- **two-dimensional** bed geometry see Helanow et al., GRL 2019

- **pure sliding** at the interface

wait for Roldan Blasco et al. , in preparation

- **steady** water pressure This talk!

and

- have **not yet been validated** against real data

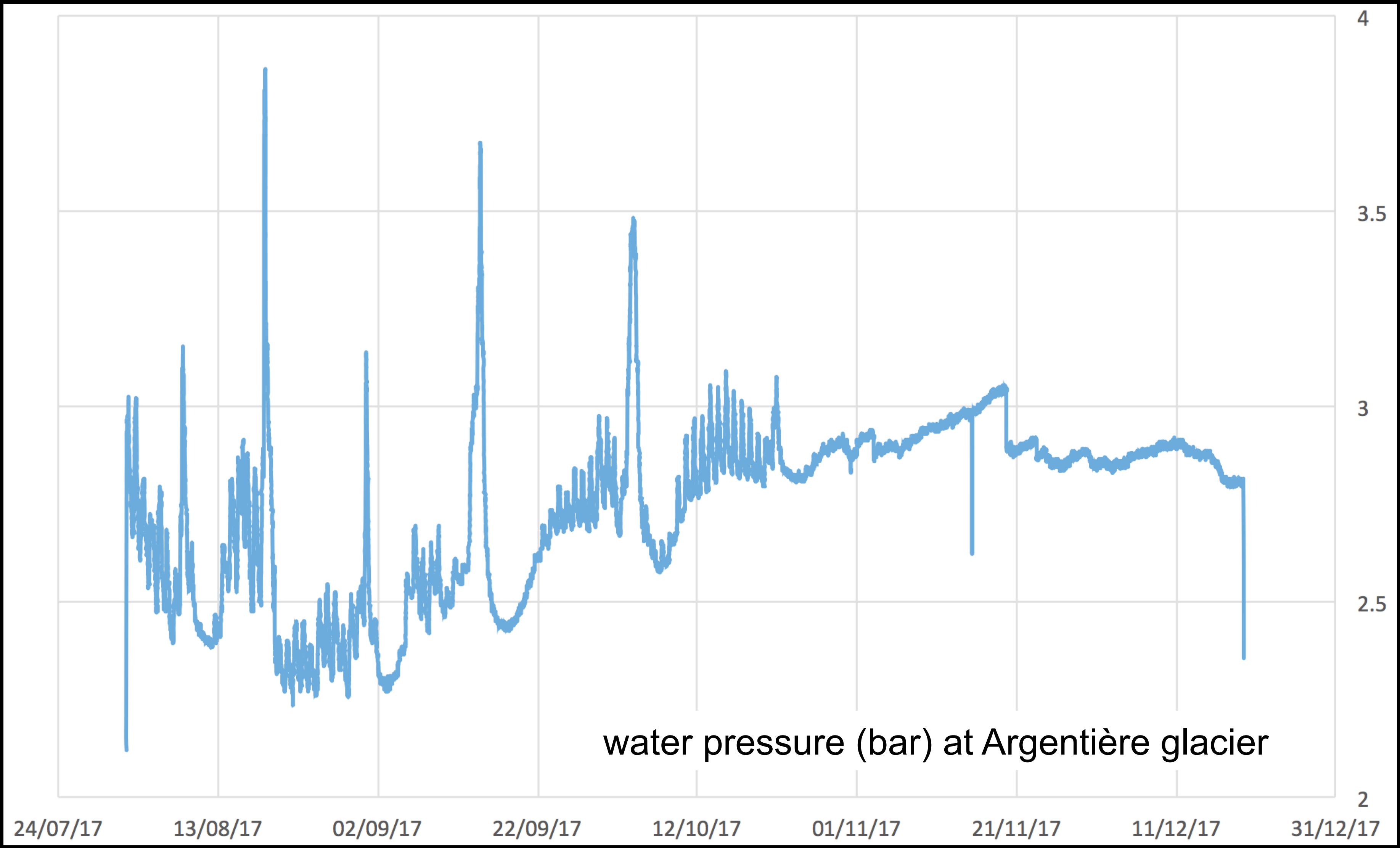
wait for Gimbert et al. , submitted

- their **link with basal hydrology** is still unclear

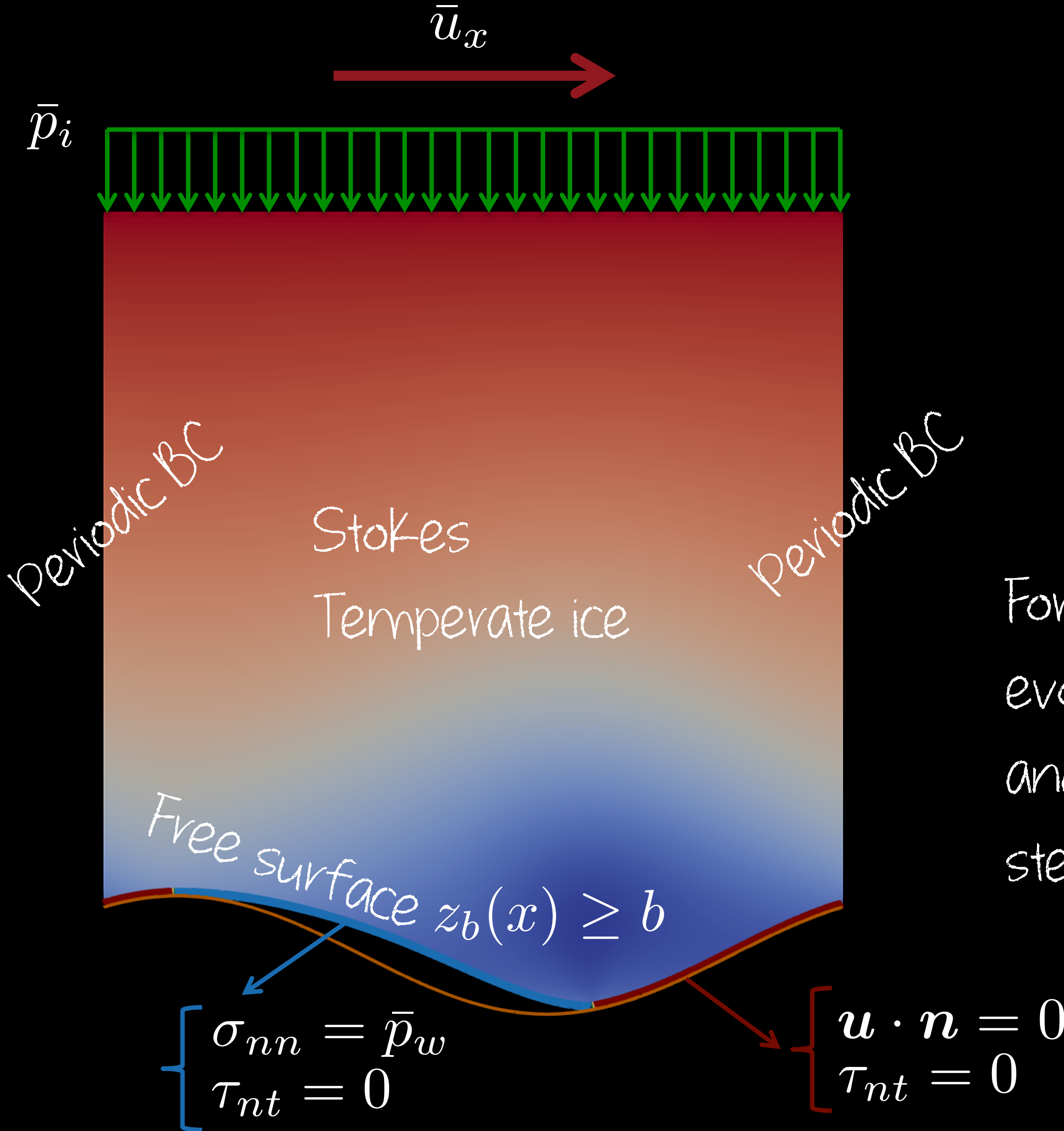
wait for Gilbert et al. in preparation

How these laws are modified if water pressure at the base is **unsteady**?

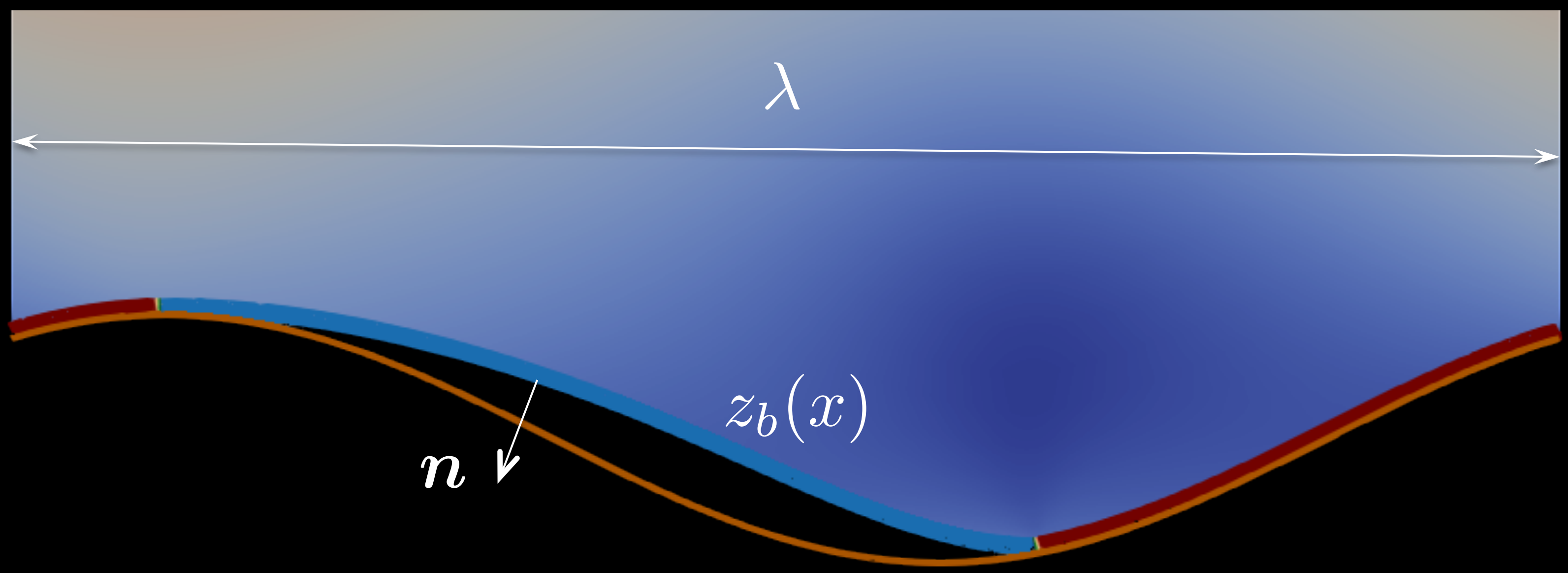
and water pressure is not steady...



@Piard 2018



For given $\bar{p}_i$ and $\bar{u}_x$ evolve $\bar{p}_w / \bar{p}_i$ and look for steady state



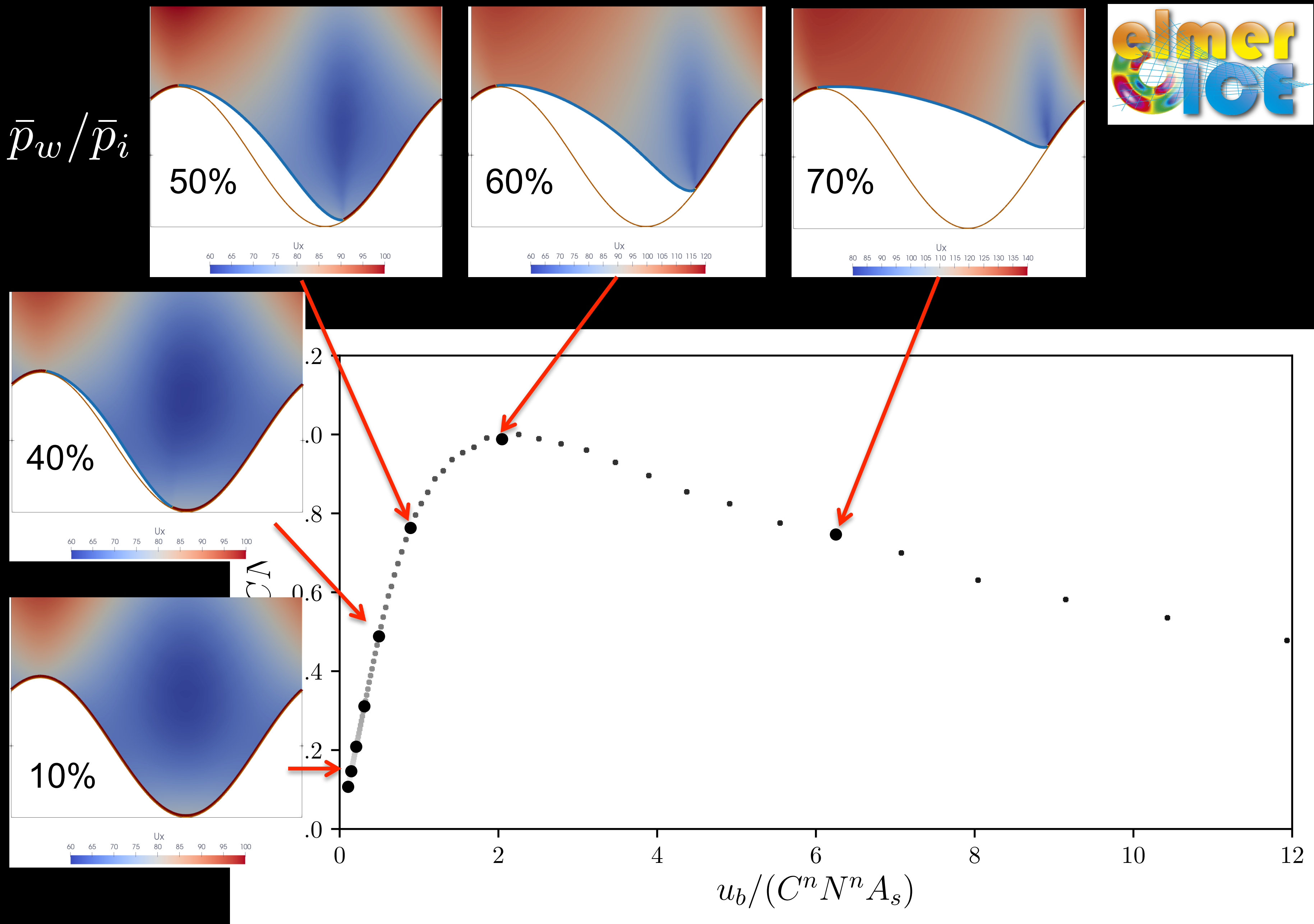
$$\tau_b = \frac{1}{\lambda}\int_0^{\lambda} \sigma_{nn} n_x ds = \frac{1}{\lambda}\int_0^{\lambda} \sigma_{nn}\frac{\partial z_b}{\partial x}dx$$

$$u_b = \frac{1}{\lambda}\int_0^{\lambda} u dx$$

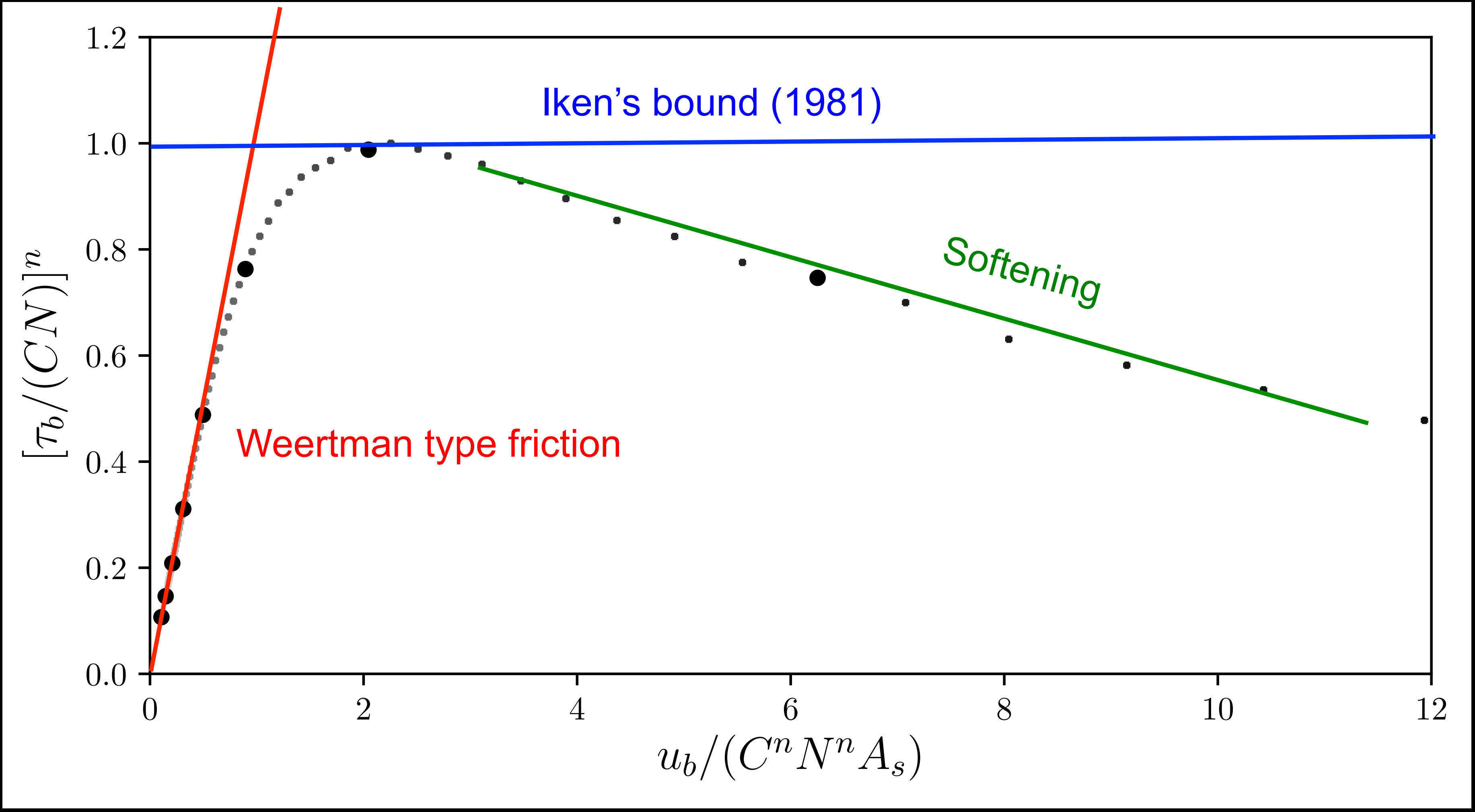
$$\rightarrow \tau_b = f(u_b, N)$$

$$N = p_i - \bar{p}_w$$

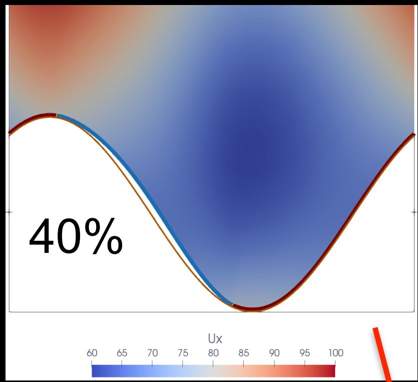
$$p_i = \frac{1}{\lambda}\int_0^{\lambda} \sigma_{nn} n_y ds = -\frac{1}{\lambda}\int_0^{\lambda} \sigma_{nn} dx \approx \bar{p}_i$$



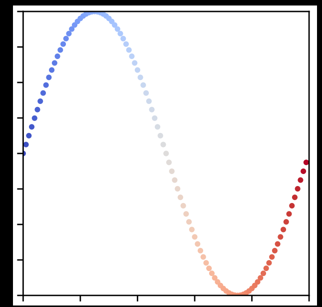
$\bar{p}_w/\bar{p}_i$
50%
60%
70%
40%
10%
Ux
60 65 70 75 80 85 90 95 100
60 65 70 75 80 85 90 95 100 105 110 115 120
80 85 90 95 100 105 110 115 120 125 130 135 140
.2
.0
.8
0.6
.4
.2
.0
0 2 4 6 8 10 12
$u_b/(C^n N^n A_s)$



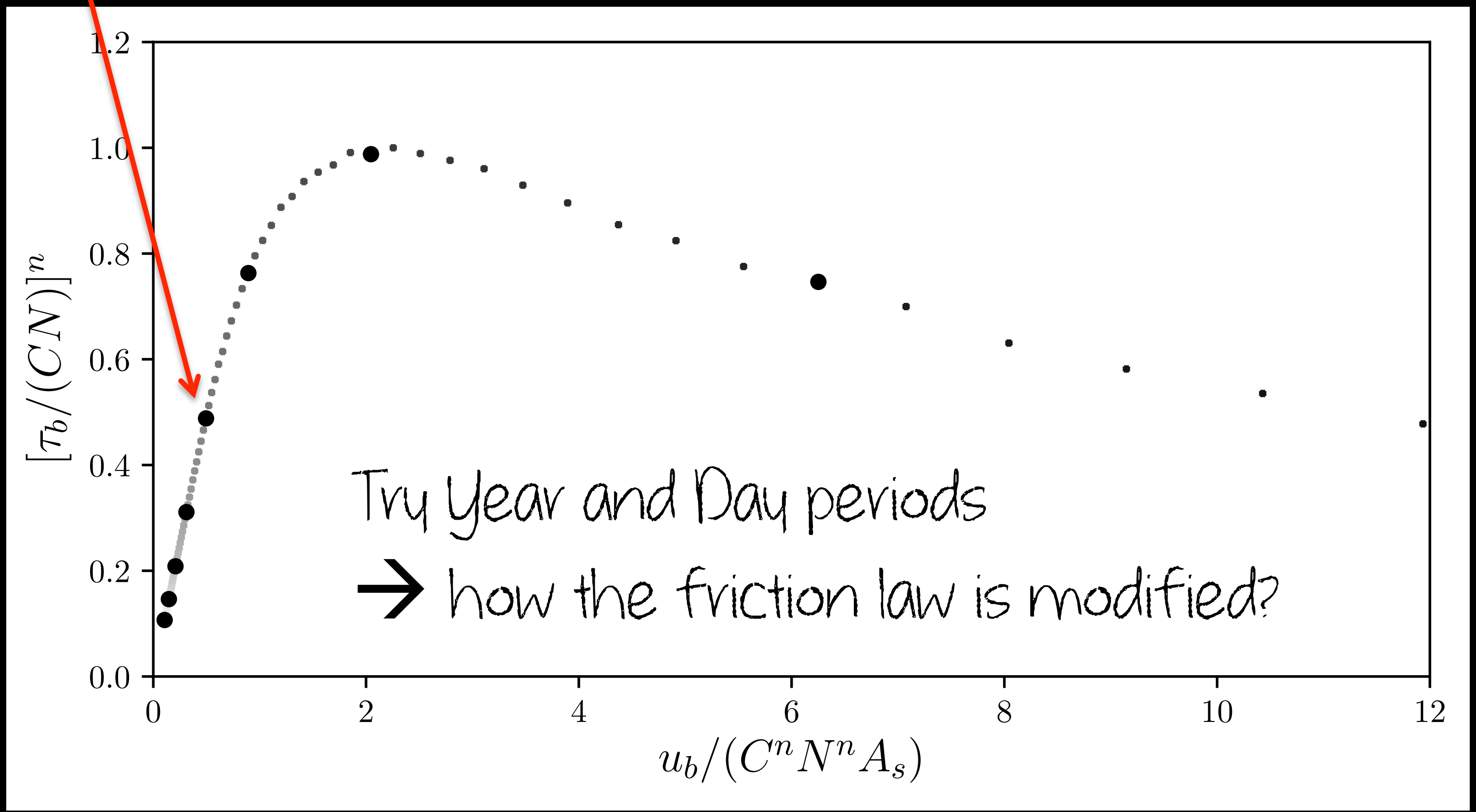
Iken's bound (1981)
Softening
Weertman type friction
$[\tau_b/(CN)]^n$
$u_b/(C^n N^n A_s)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
2
4
6
8
10
12



$$p_w(t) = \bar{p}_w + \Delta p_w \sin(\omega t)$$

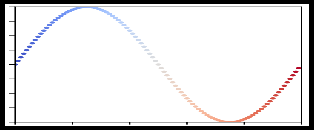


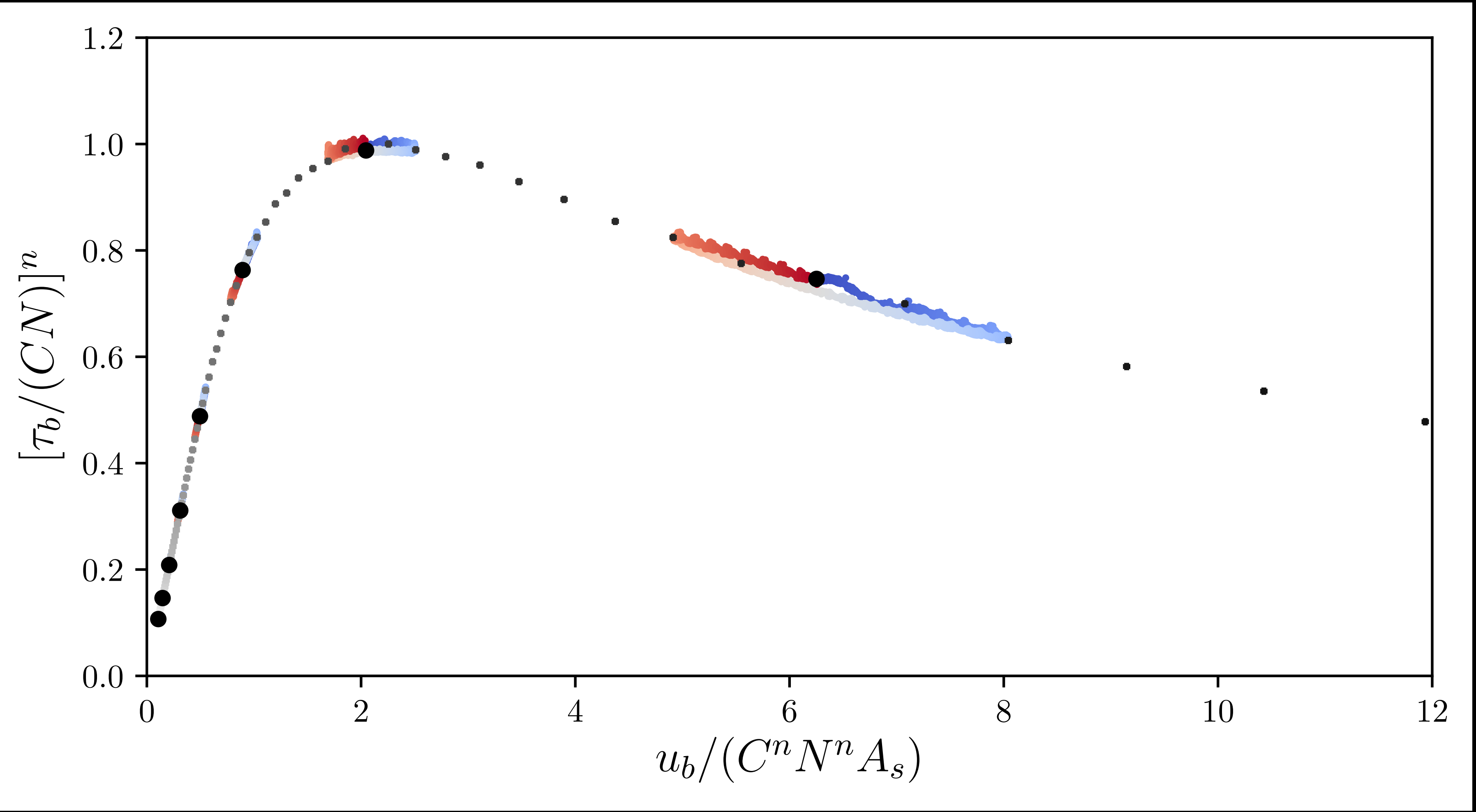
$$\Delta p_w / \bar{p}_i = 2\%$$



One **year** period

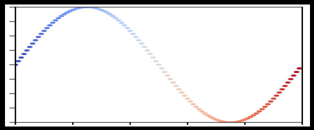
$\Delta p_w / \bar{p}_i = 2\%$

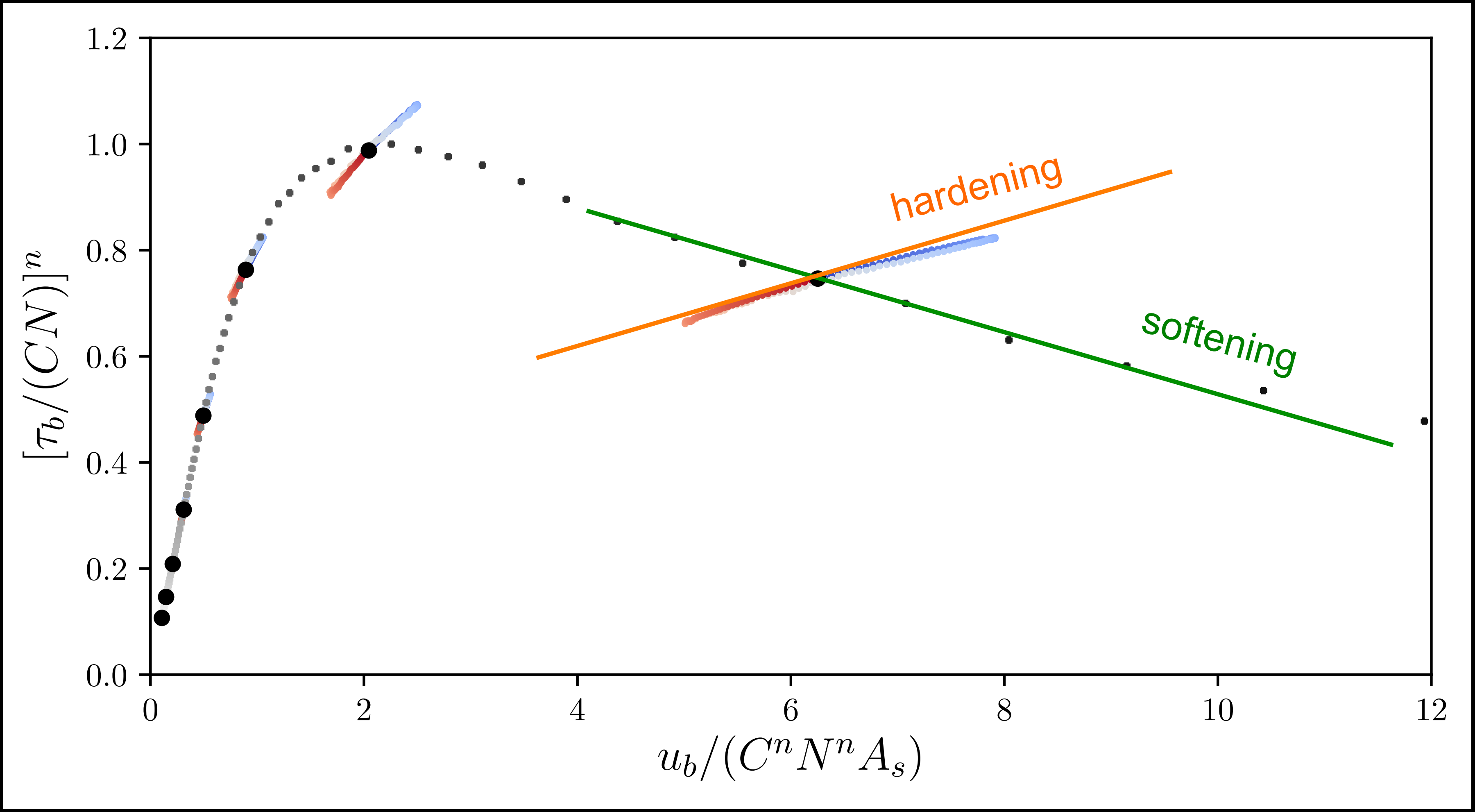




One **day** period

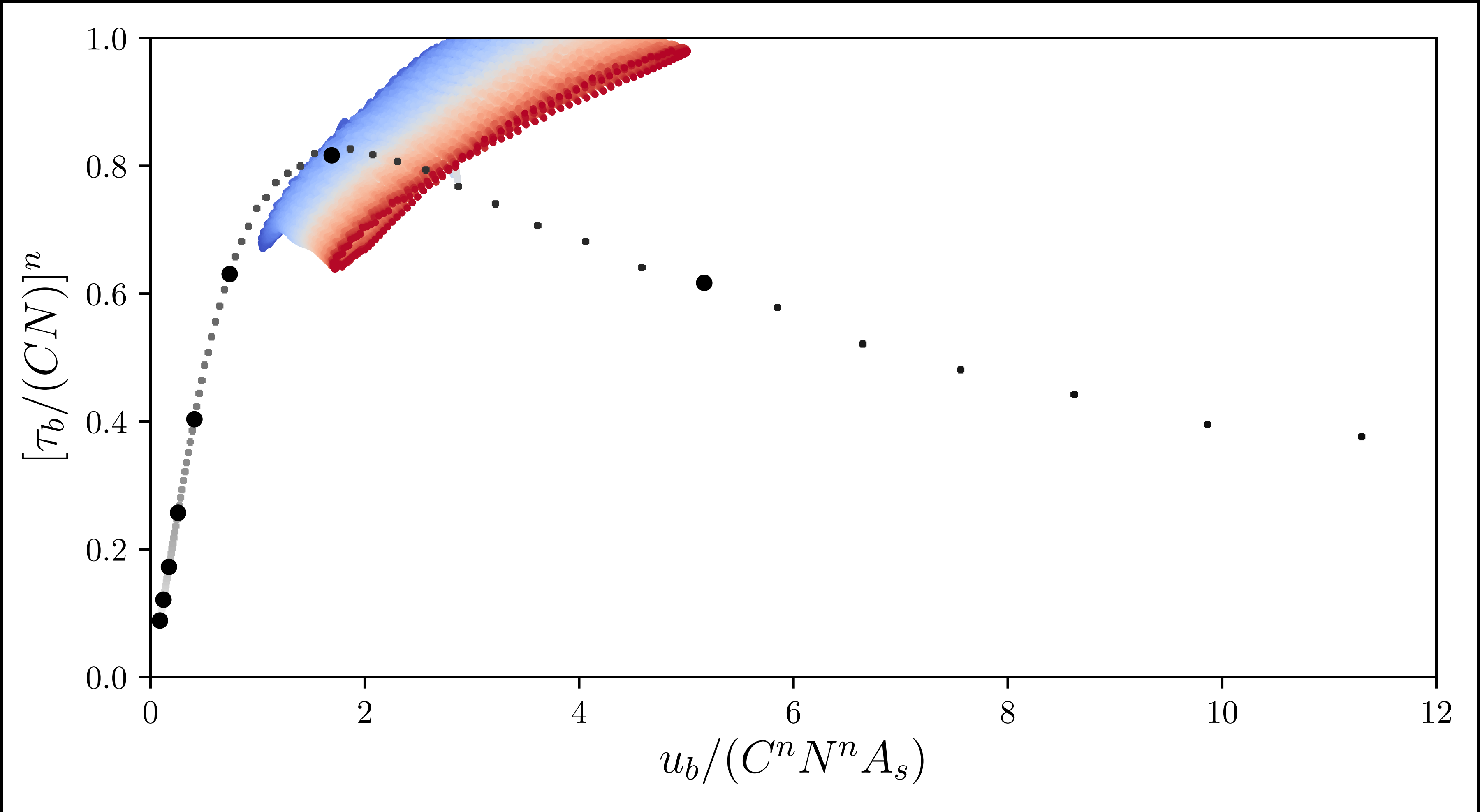
$$\Delta p_w / \bar{p}_i = 2\%$$



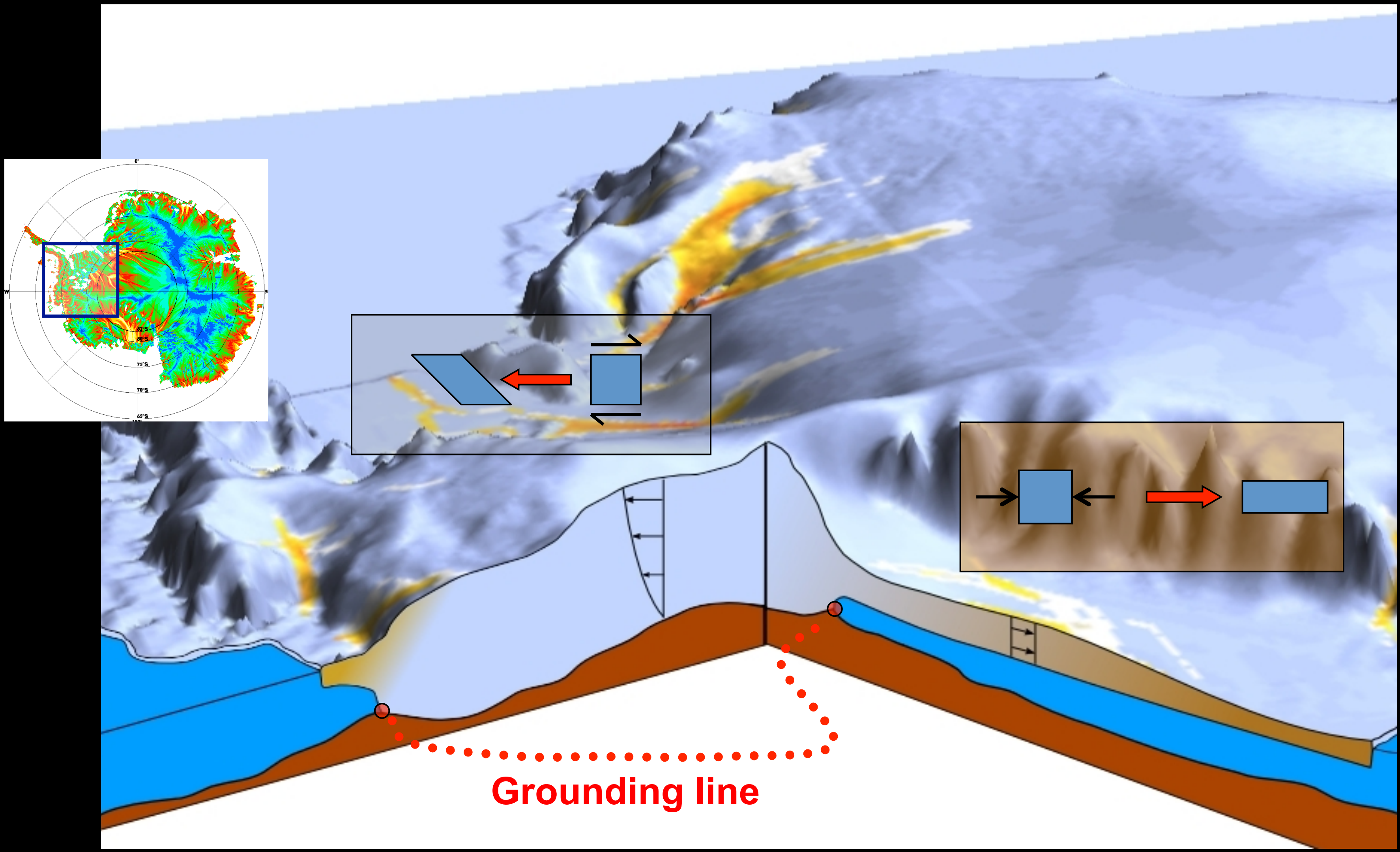


Year + day period

$\Delta p_w / \bar{p}_i = 5\%$



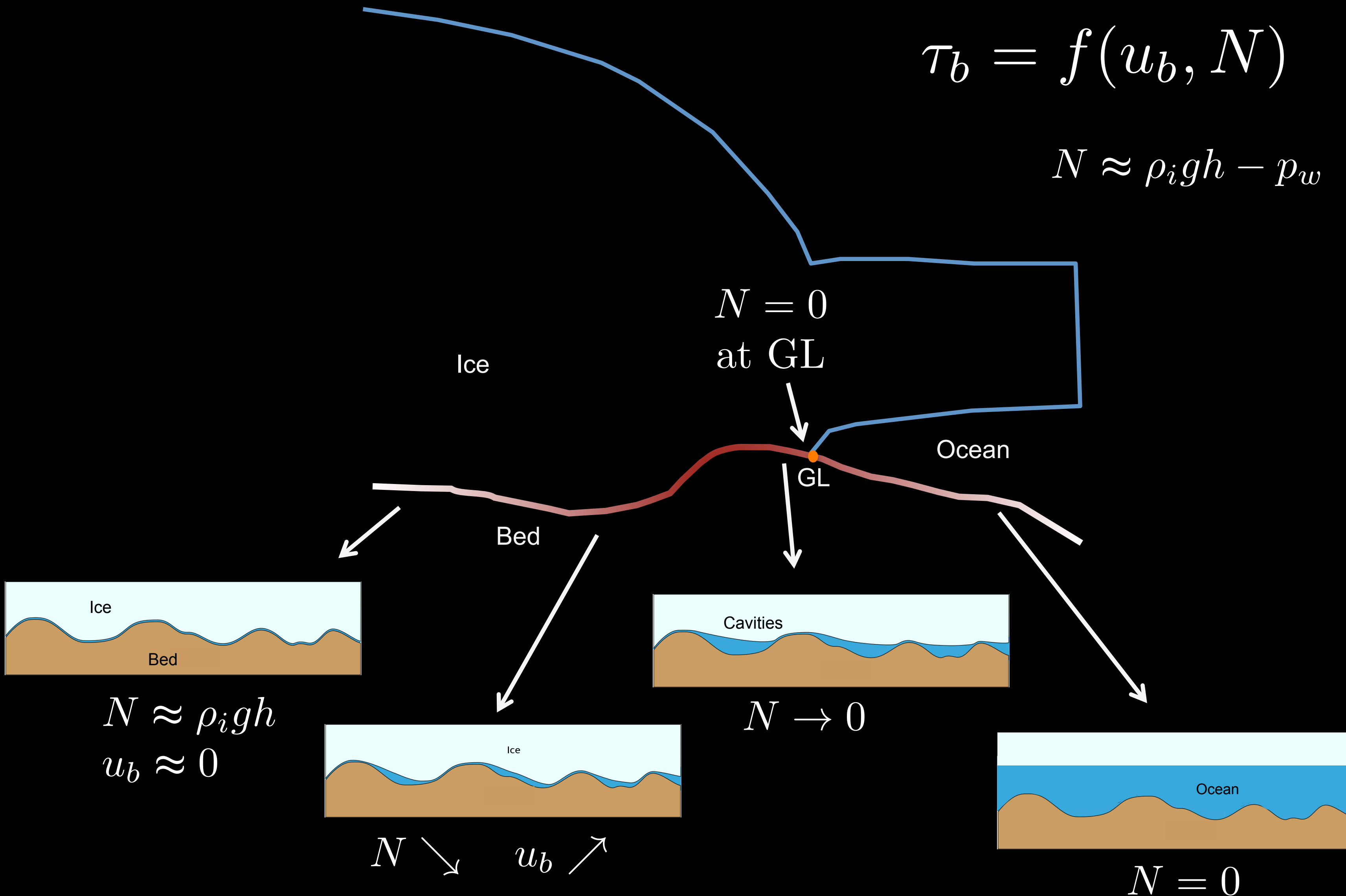
Grounding line



Types of basal conditions

$$\tau_b = f(u_b, N)$$

$$N \approx \rho_i g h - p_w$$



Which friction laws?

Weertman

$$\tau_b = C_{\mathrm{W}} u_b^m$$

Coulomb

$$\tau_b = f_{\mathrm{C}} N$$

Budd

$$\tau_b = C_B u_b^m N$$

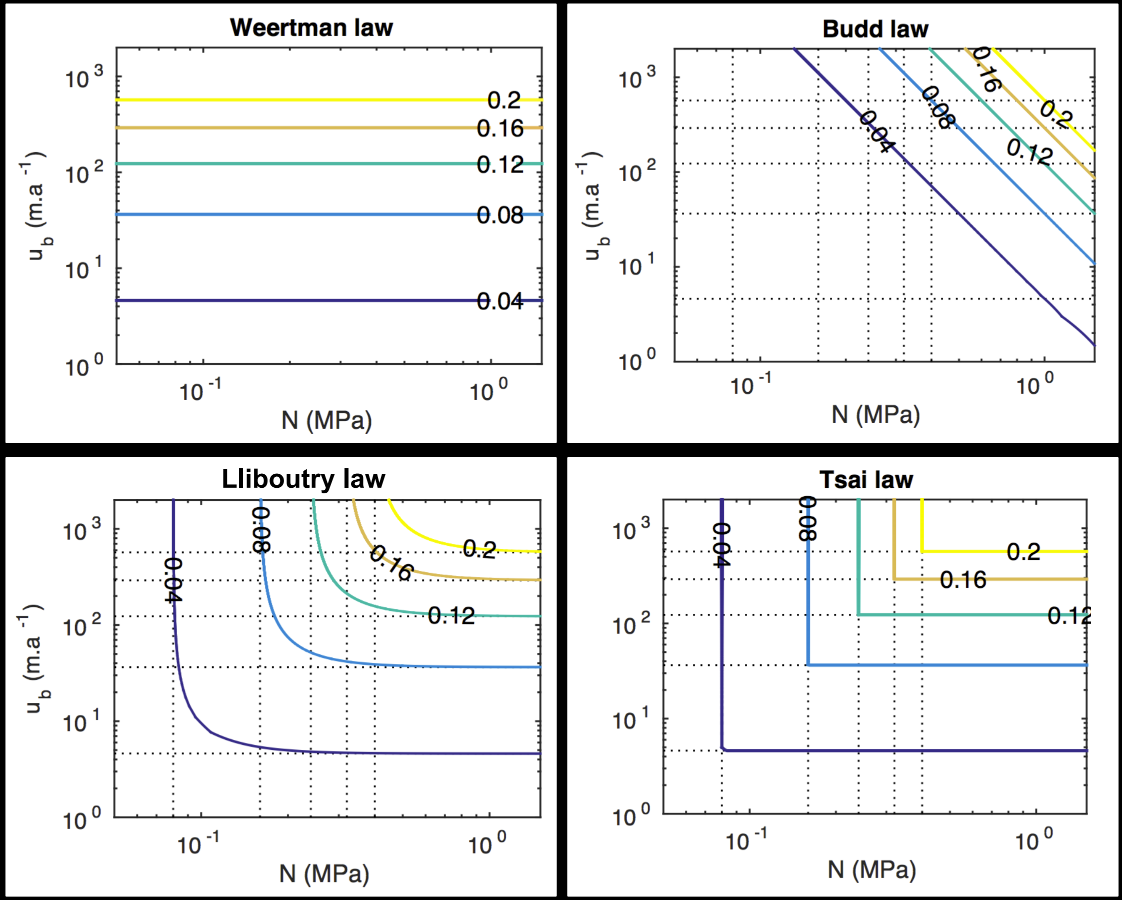
Lliboutry

$$\tau_b = \frac{C_{\mathrm{S}} u_b^m}{\left(1 + \left(\frac{C_{\mathrm{S}}}{C_{\max} N}\right)^{1/m} u_b\right)^m}$$

Tsai

$$\tau_b = \min\left[C_{\mathrm{W}} u_b^m, f_{\mathrm{C}} N\right]$$

Isovalues of τ_b [MPa]

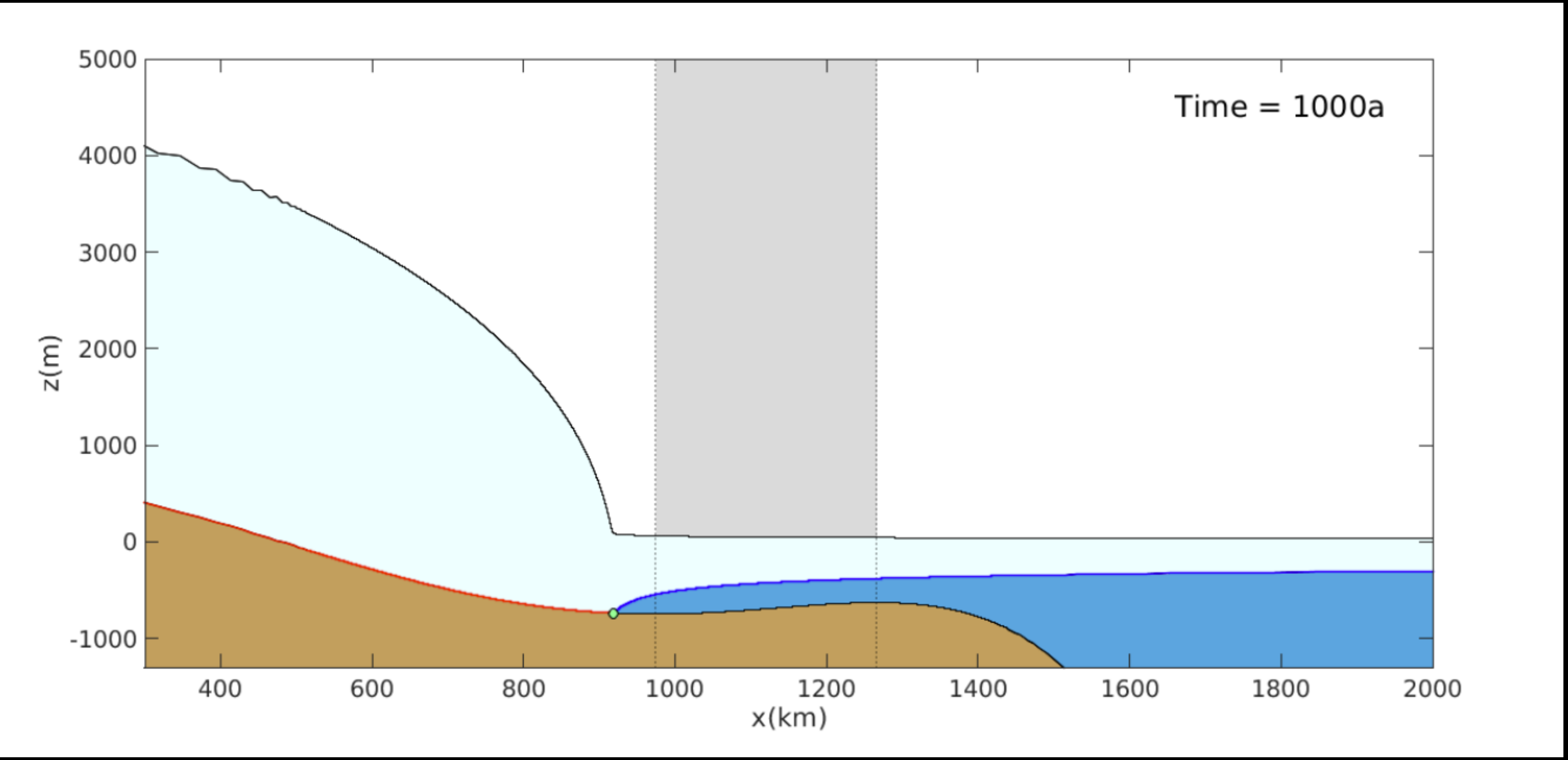


$$m = 1/3, \quad C_{\mathrm{W}} = C_{\mathrm{B}} = C_{\mathrm{S}}, \quad C_{\max} = f_{\mathrm{C}}$$

[Weertman 1957; Budd et al., 1984; Schoof 2005; Gagliardini et al., 2007; Tsai et al., 2015]

[Brondex et al., 2017]

2D exp – MISMIP setup



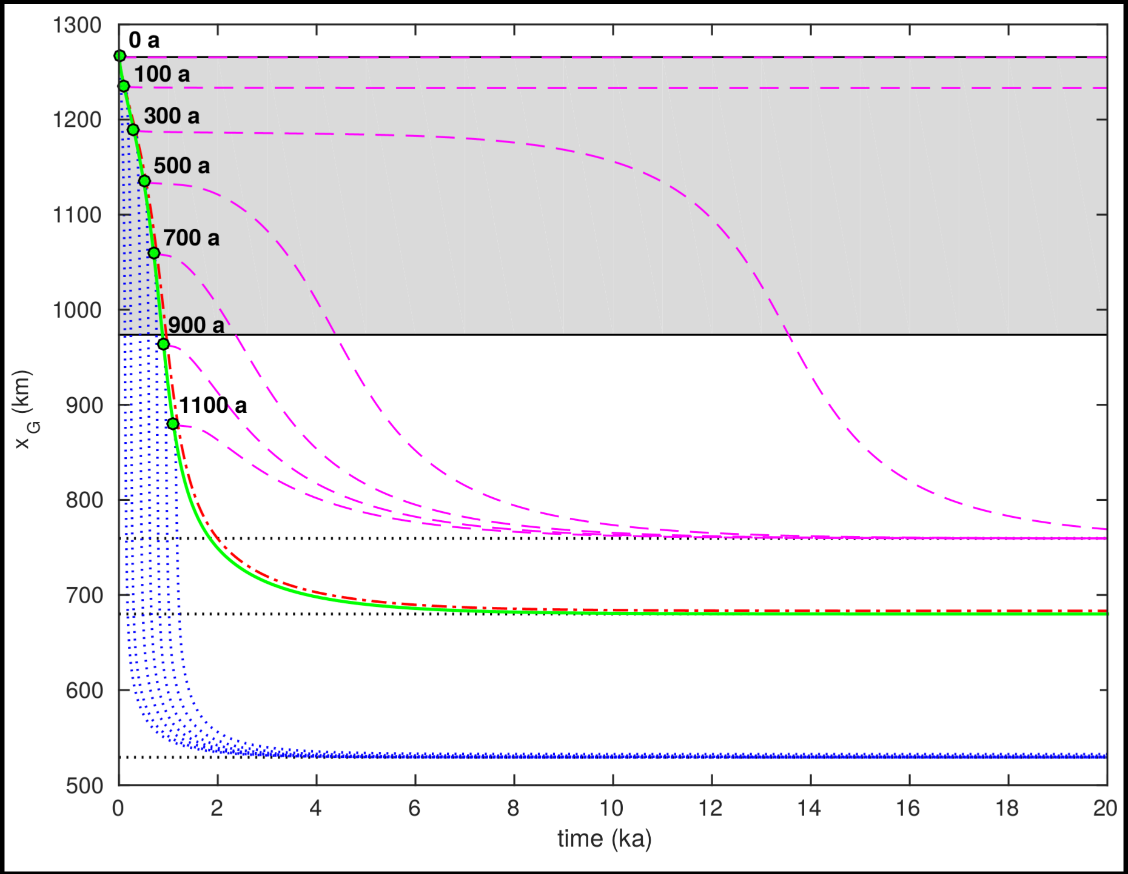
[Brondex et al., 2017]

Assume $N = \rho_i g h - \rho_w g (Z_{sl} - Z_b)$

Use Lliboutry solution as "observations"

Compare Weertman, Budd, Tsai to Lliboutry

2D exp - GL retreat speed



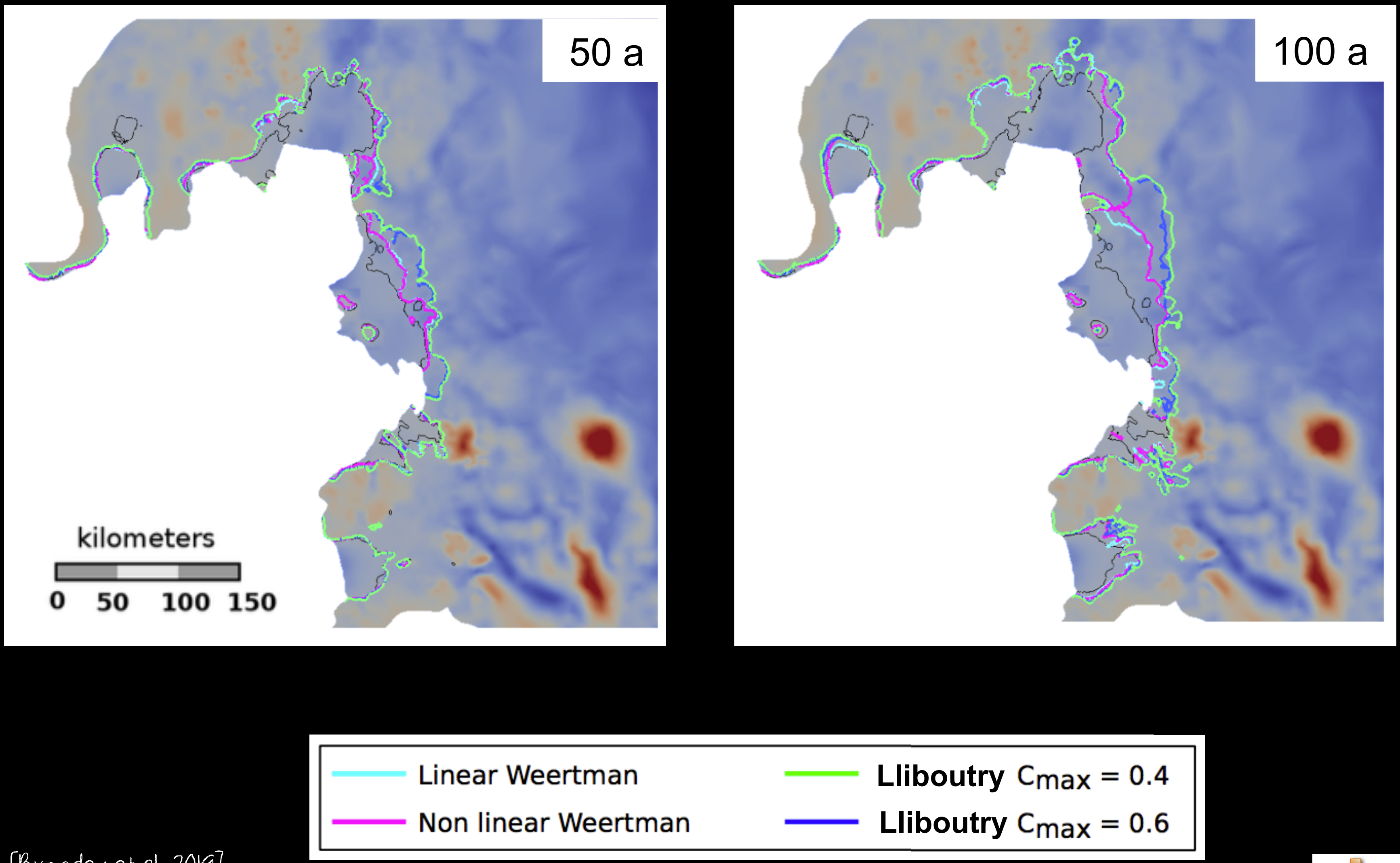
Weertman ~ 100 m/a

Lliboutry / Tsai ~ 500 m/a

Budd ~ 9000 m/a

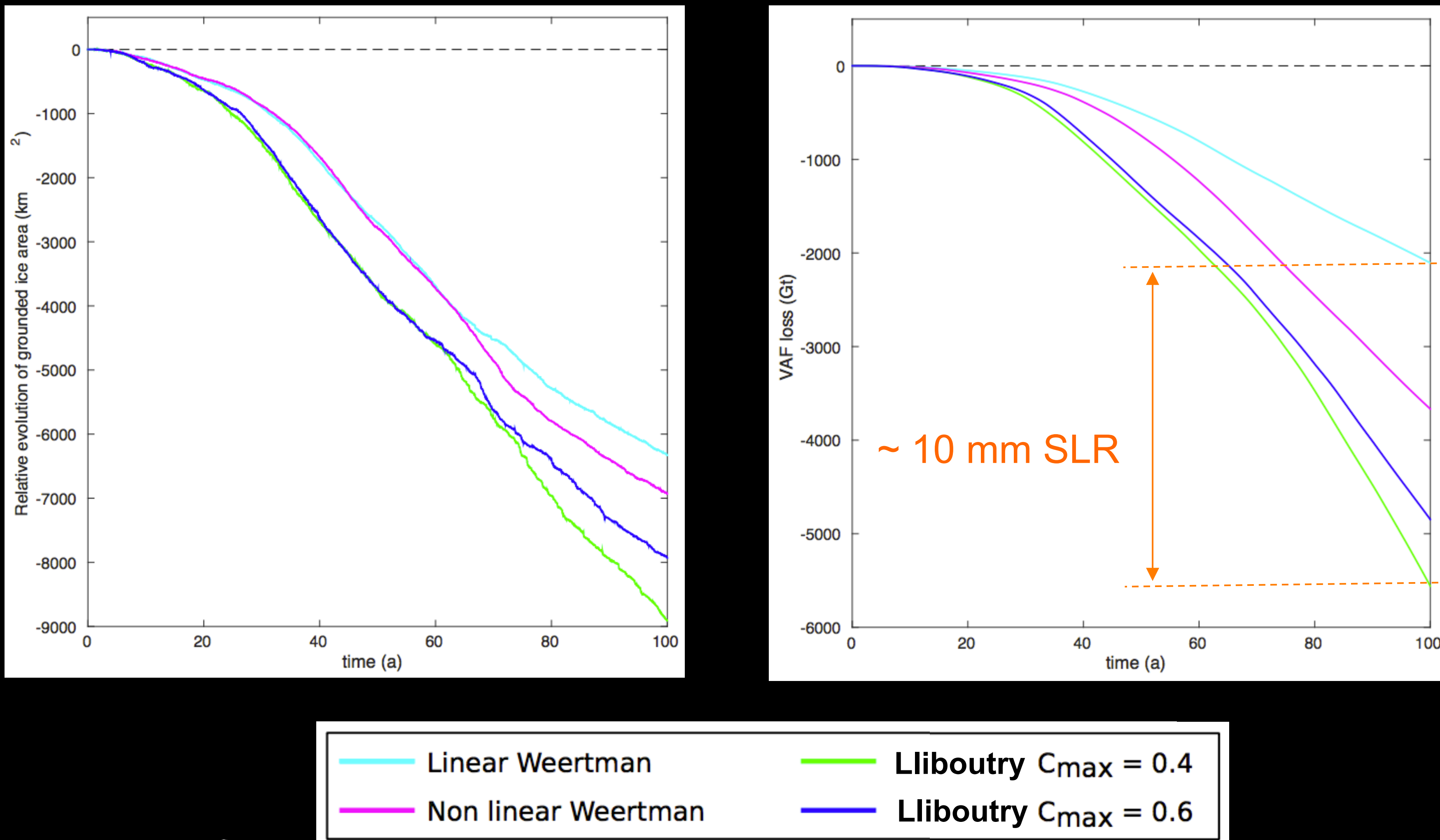
[Brondex et al., 2017]

Admundsen



[Brondex et al., 2019]

Admundsen - Results



[Brondex et al., 2019]

Conclusions

- The ice dynamics strongly depend on the choice of the form of the friction law...

- and we are not yet sure what is the appropriate form for such friction law...

- more work to be done... on the path of Lliboutry and others

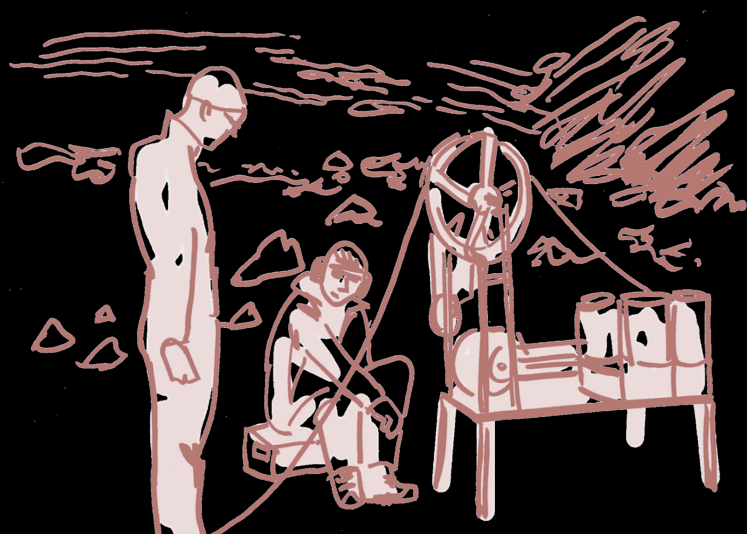
ANR SAUSSURE (2019-2023)

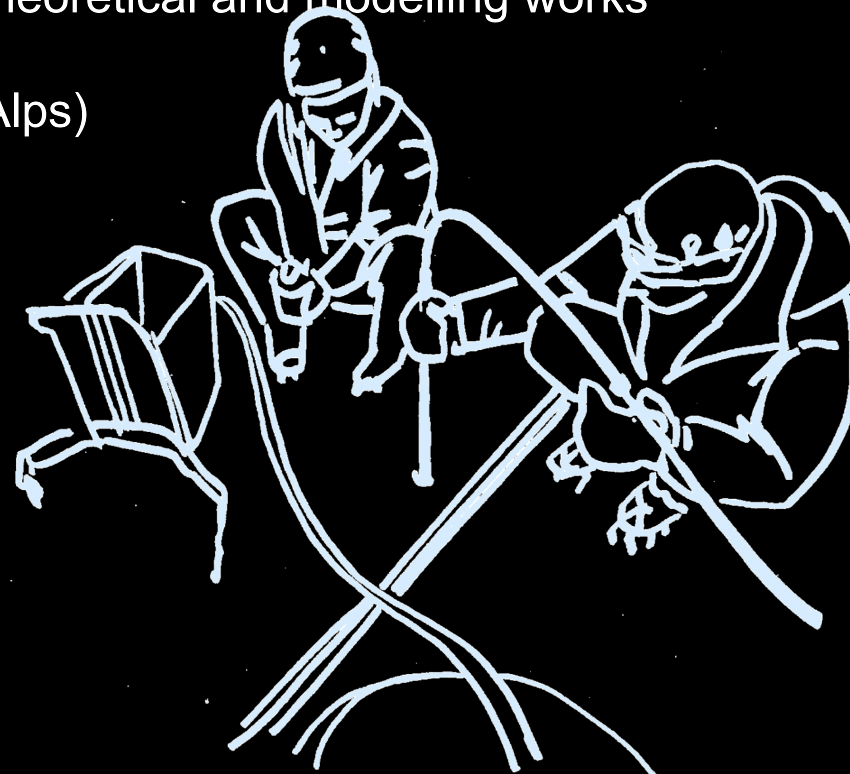
Objectives : evaluate, improve and validate various friction laws in a natural, geophysical scale configuration

Methods : Observation combined with theoretical and modelling works

Site study : Argentière glacier (French Alps)

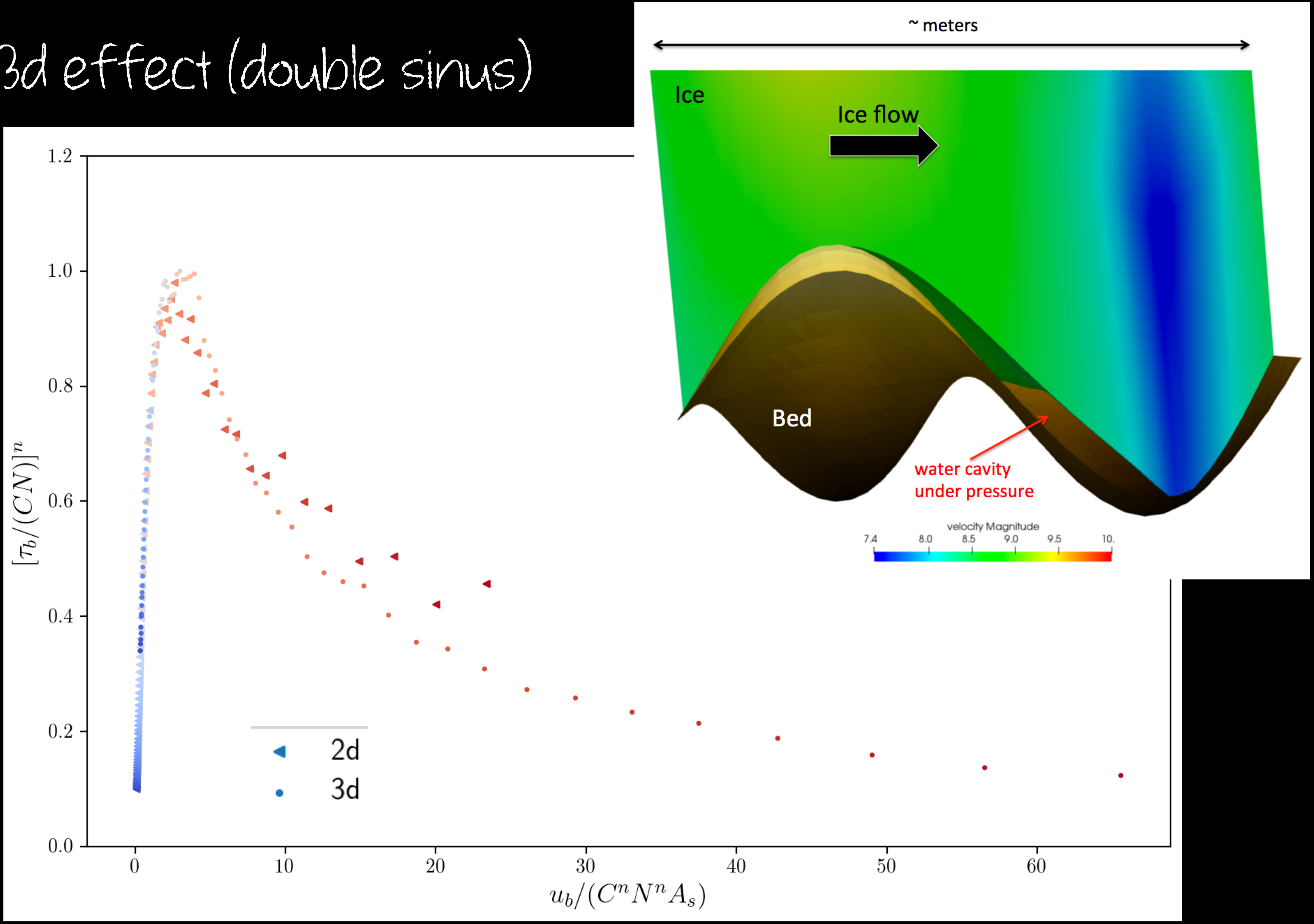
Involved lab : IGE, IRSTEA, IsTerre



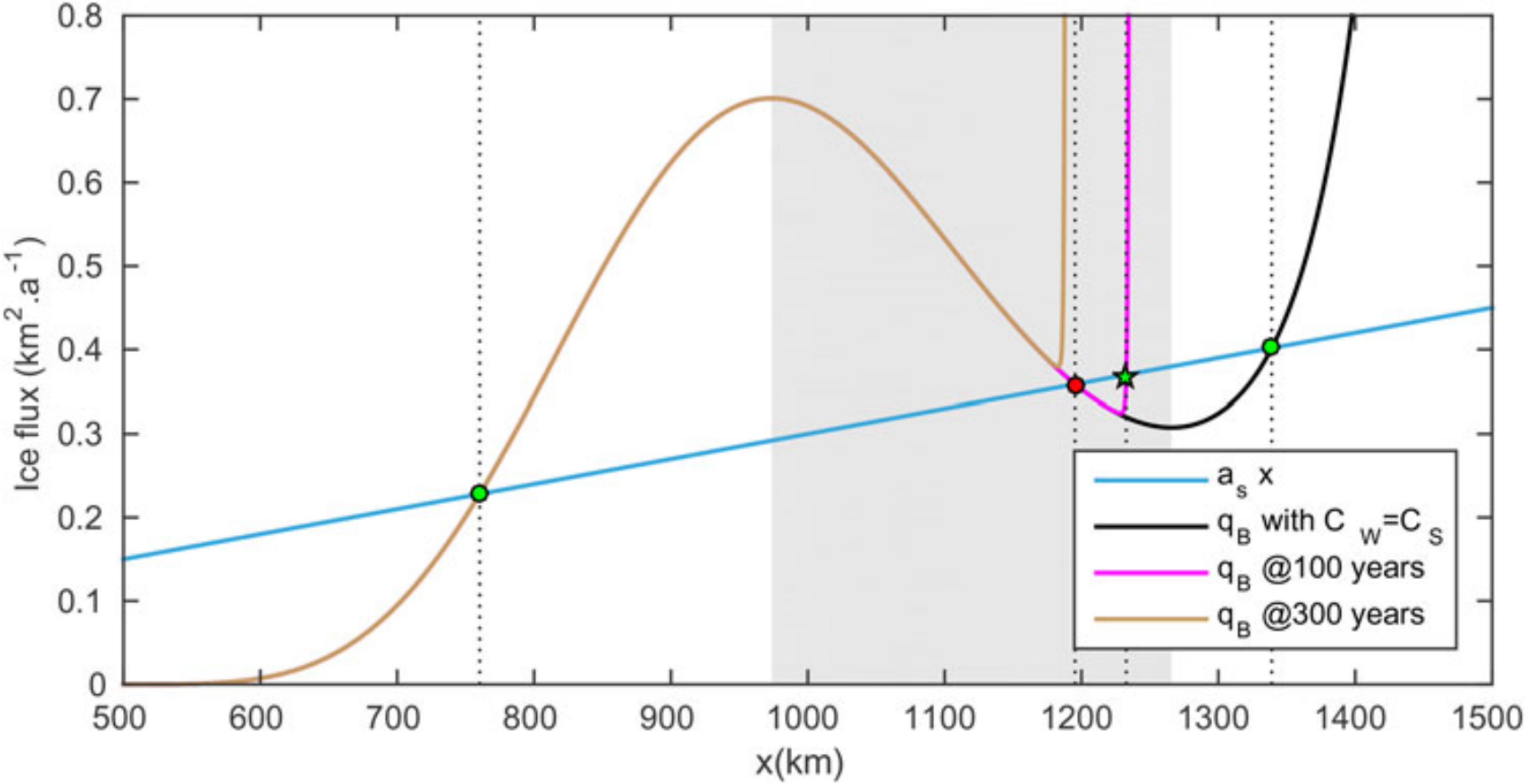


www.saussure.osug.fr

3d effect (double sinus)



C value lower ; peak reached for lower water pressure



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Ice flux (km^2.a^-1)
500
600
700
800
900
1000
1100
1200
1300
1400
1500
x(km)
a_s x
q_B with C_W=C_S
q_B @100 years
q_B @300 years