Using Extended Source Inversion to solve an Acoustic Transmission Inverse Problem, Extensions to Microseismic Source Estimation

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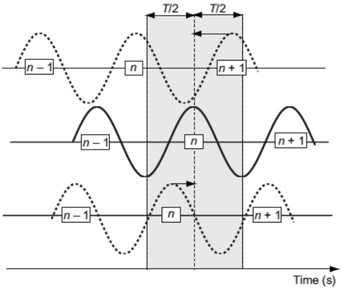
Outline

- Simple Problem Setup and Motivation
- The problem with least-squares inversion (full waveform inversion): cycle-skipping!
- Introduction of Source Extended Objective Function (ESI)
- Why ESI helps
- Discrepancy Algorithm and Numerical Examples
- Extensions: Microseismic Source Estimation
- Conclusions

Motivation Behind Extended Objective Functions

- Full Waveform Inversion (FWI) is now well-established as a useful tool for estimating parameters in the earth.
- Unfortunately, the FWI objective function is not convex. FWI stagnates at geologically uninformative earth models (local minima).



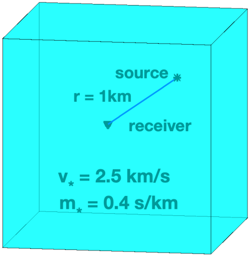
Schematic of cycle-skipping artifacts in FWI. Solid black line is seismogram of period T. Upper dashed line is seismogram with a time delay greater than $T/2$. Bottom example, has time delay less than $T/2$.[1]

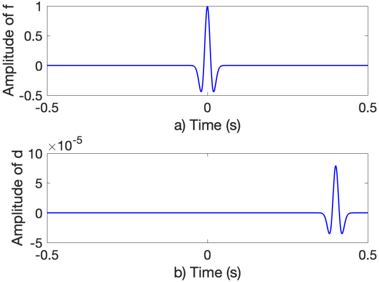
[1] Virieux, J., and S. Operto, 2009, "An overview of full-waveform inversion in exploration geophysics", *Geophysics*, **74**, WCC1–WCC26.

Motivation Behind Extended Objective Functions

- Extended inversion is one of the many ideas that have been advanced to overcome cycle-skipping. We will focus on "source extension".
- "Extended" signifies that additional degrees of freedom are provided to the modeling process.
- These extended degrees of freedom should be suppressed in the eventual solution since they are not physical.
- In the case of a very simple model problem, all computations can be done analytically. Results can be theoretically justified.
- Simple problem illustrates the same cycle skipping issues one encounters in FWI for more realistic problems.

Simple Experimental Setup





Left: single-trace experimental setup. Right top: the source wavelet (a 20 Hz Ricker). Right bottom: data.

Acoustic Wave Equation

- Assume small amplitude constant-density, acoustic wave propagation in $3D$.
- An isotropic point source and receiver.

$$(m^2 \frac{\partial^2}{\partial t^2} - \nabla^2) p(x,t) = w(t)\delta(x - x_s)$$
$$p(x,t) = 0, \quad t < 0$$

- The pressure trace recorded at the receiver position is given by:

$$p(x_r, t) = \frac{1}{4\pi r} w(t - mr) = F[m]w(t)$$

$F[m] =$ operator of convolution with acoustic $3D$ Green's function
$w(t) =$ time dependence of the point source ("wavelet")
$r =$ distance between the source and receiver
$m =$ slowness (reciprocal v)

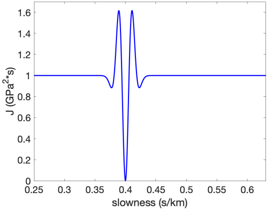
Inverse Problem: Given ϵ, $\lambda > 0$, find the slowness m and wavelet w so that:

- $w(t) = 0$ if $|t| > \lambda$
- $\|F[m]w - d\| \leq \epsilon \|d\|$

Definition

The *basic FWI objective* function e of slowness m and wavelet w is

$$e[m, w; d] = \frac{1}{2} \frac{\|F[m]w - d\|^2}{\|d\|^2} \quad (1)$$



- There are entire intervals of local minimizers far from the global minimizer m_*.
- Initial guess for slowness m must be within $2\lambda/r$ of the global minimizer m_*, or we fail to solve the inverse problem.[2] **"cycle-skipping"!!**

[2] Symes, W. W., 2021. "Solution of an acoustic transmission inverse problem by extended inversion: theory", arXiv:2110.15494.

Source Extended FWI

- Add degrees of freedom to F to avoid local minima.
- By including the source wavelet as one of the modeling parameters and dropping the support constraint on w, we extend space of possible solutions.

Definition

The Extended Source Inversion ("ESI") objective function J_α is defined by

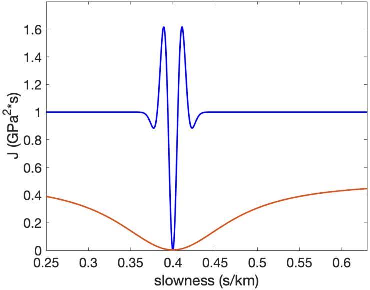
$$J_\alpha[m, w; d] \quad = \quad \frac{1}{2}(\|F[m]w - d\|^2 + \alpha^2\|Aw\|^2)/\|d\|^2. \tag{2}$$

- A is an annihilator. We choose A to penalize energy away from $t = 0$:

$$Aw(t) = tw(t) \tag{3}$$

Variable Projection Method

- ESI objective hard to minimize for both *m* and *w* simultaneously.
- Use Variable Projection Method[3] with inner minimization over *w* then an outer minimization over *m*.
- In this case, wavelet solution given analytically by the normal equations.



The FWI (blue curve) and ESI (red curve) objective functions versus slowness for data from a 40 Hz Ricker source.

[3] Golub, G., and V. Pereyra, 2003, "Separable nonlinear least squares: the variable projection method and its applications", *Inverse Problems*, **19**, R1-R26.

Extended FWI

We can show[4] that

$$J^{\alpha}_{ESI}[m;d] = \frac{1}{2(4\pi r)^2}\int [1-(1+(4\pi r)^2\alpha^2(t+(m_*-m)r)^2)^{-1}]|w(t)|^2 dt,$$

$$\nabla J^{\alpha}_{ESI}[m] = -r\alpha^2 \int_{-(m-m_*)r-\lambda}^{-(m-m_*)r+\lambda} \frac{t(w(t+(m-m_*)r)^2)}{(1+(4\pi r)^2\alpha^2 t^2)}dt.$$

Since $w(t)=0$ if $|t|>\lambda$, we can see that

- if $m > m_* + \lambda/r$, then $\nabla J^{\alpha}_{ESI}[m] > 0$, and
- if $m < m_* - \lambda/r$, then $\nabla J^{\alpha}_{ESI}[m] < 0$.

That it, J^{α}_{ESI} has no local minima further than $O(\lambda)$ from the global minimum.

[4] Symes, W. W., Chen, H., and Minkoff, S. E., "Full waveform inversion by source extension: why it works," *Proceedings of the 90th Annual International Meeting of the Society of Exploration Geophysicists*, pp. 765-769, 2020.

Extended FWI Theory[5]

Result

Suppose that $d = F[m_*]w_* + n$ *with* target slowness $m_* > 0$, target wavelet $w_*(t) = 0$ *for* $|t| > \lambda$, *noise trace* n, *and* $\alpha > 0$. *Define the* noise-to-signal ratio η *by* $\eta = \|n\|/\|d_*\|$. *If* $\eta < \frac{\sqrt{5}-1}{2}$, *then any stationary point* m *of* $\tilde{J}_\alpha[\cdot; d]$ *satisfies*

$$|m - m_*| \leq (1 + f(\eta))\frac{\lambda}{r}, \tag{4}$$

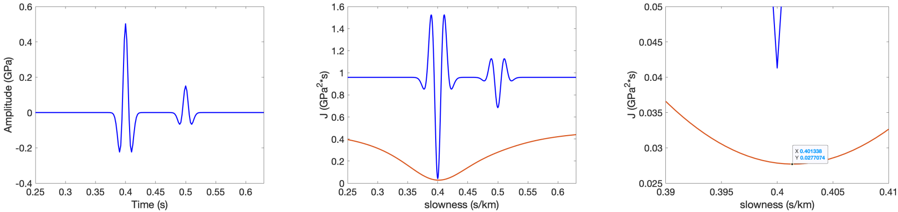
where $f(\eta) = \frac{2\eta(1+\eta)}{1-\eta(1+\eta)} = 2\eta + O(\eta^2)$.

- Special case: data is noise-free, then error between any stationary point of the reduced ESI objective and the target slowness is at most the maximum lag λ of the target wavelet divided by the source-receiver offset r.

[5] Symes, W. W., 2021. "Solution of an acoustic transmission inverse problem by extended inversion: theory", arXiv:2110.15494.

Example with 30% Coherent Noise[6]

- noise-to-signal ratio is $\eta = 0.3$.
- $\lambda = 0.025$
- $|m - m_*| \leq \left(1 + \frac{2\eta(1+\eta)}{1-\eta(1+\eta)}\right) \frac{\lambda}{r} \approx 0.057$.
- Estimated error $|m - m_*| \approx 0.01338 < 0.057$ (upper bound on error).

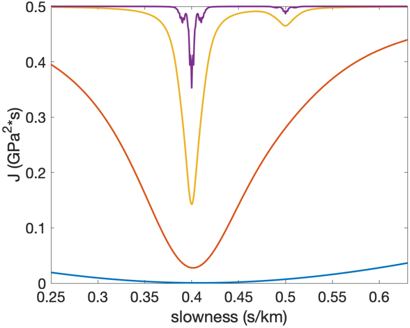


Left: data with noise. Middle: reduced FWI and ESI objective functions versus slowness. Right: Zoom in of middle figure.

[6] Symes, W. W., Chen, H., and Minkoff, S. E., "Solution of an Acoustic Transmission Inverse Problem by Extended Inversion," submitted 2021.

How do you choose α?

- Results above were for fixed values of the penalty parameter α.
- α has a big impact on the rate of convergence of the algorithm.
- If α can increase dynamically during run we see improved performance of the algorithm.



ESI objective functions plotted with blue curve: $\alpha = 0.1$, red: $\alpha = 1.0$, yellow: $\alpha = 10.0$, purple: $\alpha = 100.0$.

Discrepancy Algorithm[7]:

Given data $d \in D$ and a range of minimum and maximum allowable errors $0 < e_- < e_+$, find the slowness m and the scalar α so that

(i) *m is a stationary point of the reduced objective function $\tilde{J}_\alpha[\cdot; d]$, and*

(ii) $e_- < e[m, w_\alpha[m; d]; d] < e_+$.

Start with arbitrary m, $\alpha = 0$, gradient tolerance δ,

Then alternate:

1. first fix m, update α so that e so that $e_- \leq e \leq e_+$
2. then fix α, update m so that $|\nabla \tilde{J}_\alpha| < \delta$ (use local descent method)
3. repeat until $e_- \leq e \leq e_+$ AND $|\nabla \tilde{J}_\alpha| < \delta$

For the experiment

- noise-to-signal ratio of 30%, corresponding to $e \approx 0.045$.
- choose $[e_-, e_+] = [0.027, 0.11]$.
- $\delta = 0.01$

[7] L. Fu and W. W. Symes, 2017, "A discrepancy-based penalty method for extended waveform inversion", *Geophysics*, **82**, no. 5, R287-R298.

Example of Discrepancy Algorithm[8]

iteration:	α	g	e
1	0.284184	0.371103	0.003140
2	0.568368	0.311447	0.022460
3	1.136737	0.204342	0.102216

Table: α updates for initial $m = 0.343$. Initial $\alpha = 0$.

i:	g	e	m	$\tilde{J}_\alpha$	$\nabla \tilde{J}_\alpha$
1	0.035018	0.403247	0.622695	0.448496	0.463686
2	0.089906	0.140974	0.478014	0.257147	2.614541
3	0.011959	0.017344	0.405674	0.032797	0.803269
4	0.028659	0.025577	0.381536	0.062608	-3.049986
5	0.009643	0.018478	0.400642	0.030938	-0.070100
6	0.010393	0.017888	0.403158	0.031317	0.370812
7	0.009914	0.018178	0.401900	0.030989	0.151012
8	0.009752	0.018327	0.401271	0.030929	0.040569
9	0.009691	0.018402	0.400956	0.030925	-0.014743
10	0.009720	0.018364	0.401114	0.030924	0.012919
11	0.009705	0.018383	0.401035	0.030924	-0.000911

Table: updates of m after first update of $\alpha = 1.136737$.

[8] Symes, W. W., Chen, H., and Minkoff, S. E., "Solution of an Acoustic Transmission Inverse Problem by Extended Inversion," submitted 2021.

Example of Discrepancy Algorithm

iteration:	α	g	e	m
1	2.273473	0.009705	0.033737	0.401035

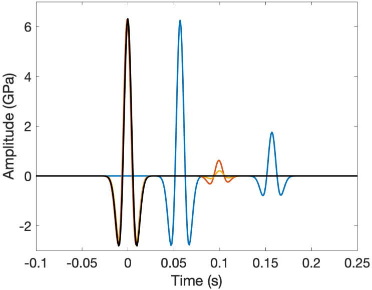
Table: second update of α.

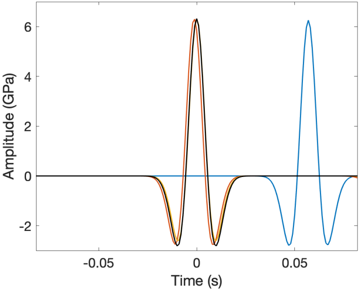
i:	g	e	m	$\tilde{J}_\alpha$	$\nabla \tilde{J}_\alpha$
1	0.002303	0.475832	0.637763	0.487735	0.114887
2	0.011948	0.336897	0.485548	0.398651	0.700990
3	0.007396	0.037167	0.409441	0.075396	5.288562
4	0.020844	0.111823	0.371387	0.219561	-7.541345
5	0.007280	0.040128	0.390414	0.077754	-5.521535
6	0.002986	0.033854	0.399927	0.049290	-0.128092
7	0.004197	0.034122	0.404684	0.055816	2.827748
8	0.003301	0.033728	0.402306	0.050789	1.382890
9	0.003068	0.033732	0.401116	0.049590	0.631507
10	0.003008	0.033779	0.400522	0.049327	0.252196
11	0.002992	0.033813	0.400225	0.049280	0.062106
12	0.002988	0.033833	0.400076	0.049278	-0.032988
13	0.002990	0.033823	0.400150	0.049278	0.014561
14	0.002989	0.033828	0.400113	0.049278	-0.009213

Table: final update of m.

Discrepancy Algorithm Example: Wavelets

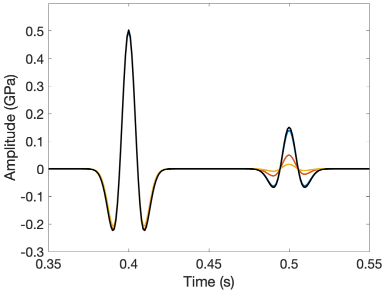
Note: to solve the inverse problem which includes the source having support $\subset [-\lambda, \lambda]$, we have to truncate the wavelet as the final step.

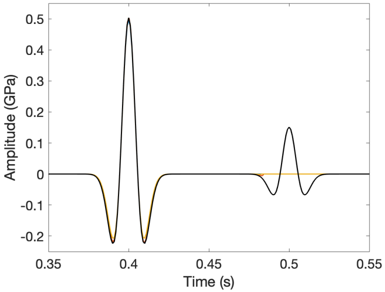




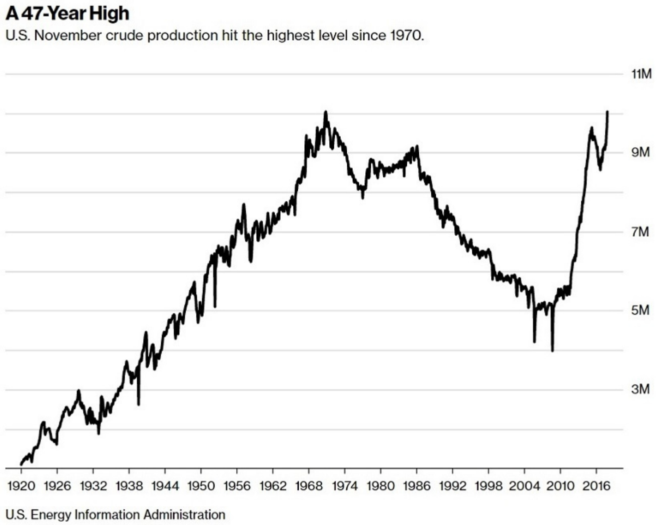
Left: Estimated wavelets. Right: truncated estimated wavelets. Blue curve is the initial wavelet. Red curve is the estimated wavelet after the first update of m. Yellow is the estimated wavelet after the final m update. Black curve is the target.

Discrepancy Algorithm Example: Predicted Data

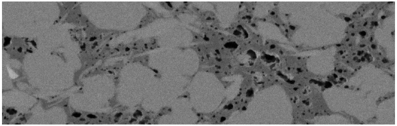




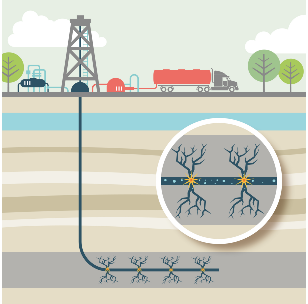
Left: Predicted Data. Right: Predicted data from truncated wavelets. Blue curve is the initial data. Red curve is the estimated data after the first update of *m*. Yellow is the estimated data after the final *m* update. Black curve is the true data.


A 47-Year High
U.S. November crude production hit the highest level since 1970.
11M
9M
7M
5M
3M
1920 1926 1932 1938 1944 1950 1956 1962 1968 1974 1980 1986 1992 1998 2004 2010 2016
U.S. Energy Information Administration

Why is Hydraulic Fracturing Done?



(a) Scan of Shale



(b) Experiment

- Fracking is used to extract oil and gas from materials with low permeability such as shale.
- High pressure liquid is injected into the well to create fracture openings that allow oil and gas to flow more freely.
- Buildup of pressure and stress may result in a microseismic event (small earthquake).
- Distribution of microseismic events gives an indication of the extent of flow paths.

Microseismic Event Magnitudes[9]

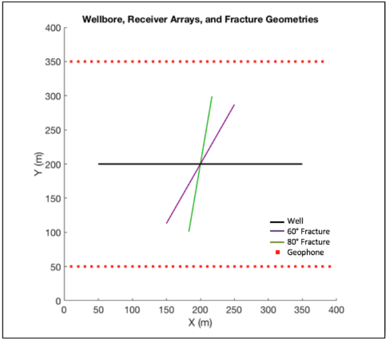
Table 2. Energy release for different microseismic magnitudes with examples of equivalent weight drops and kinetic energies.

Magnitude	Seismic moment (Nm)	Energy (Joules)	Potential energy 1 M weight drop	Kinetic energy Projectile
0	1,000,000,000	63,000	6,300 kg (minivan)	
				Rifle
−1	32,000,000	2,000	200 kg (bbl of oil)	
				Pistol
−2	1,000,000	63	6 kg (jug of milk)	
				Air rifle
−3	32,000	2	200 g (can of pop)	
				Champagne cork
−4	1,000	0.06	6 g (coin)	

Microseismicity is often below magnitude zero.
Events of magnitude 3 or greater are felt at the surface (1000's of times larger than recorded microseismic events).

[9] S. Maxwell, "Microseismic Imaging of Hydraulic Fracturing: Improved Engineering of Unconventional Shale Reservoirs," *2014 SEG Distinguished Instructor Short Course.*

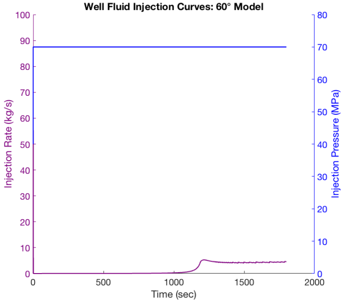
Setting Up the Numerical Experiment (Synthesizing a Microseismic Event on a Computer)

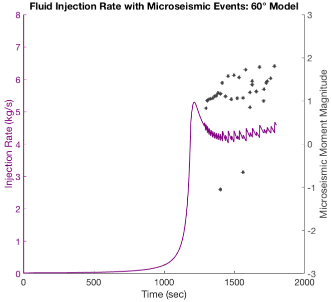


1. Synthesize microseismic events produced by hydraulic fracturing.
2. Code[10] models hydraulic injection of water into fractures.
3. Two natural fractures cross an open wellbore.
4. Two 1D receiver arrays record the emitted energy.

[10] M. W. McClure, and R. N. Horne, 2011, "Investigation of injection-induced seismicity using a coupled fluid flow and rate/state friction model," *Geophysics*, 76, WC181–WC198.

Output from Flow and Deformation Code





- Fluid is injected into the fracture for 1800 s.
- The pressure in the fracture is low compared to the forces acting on the fracture so not much happens till about 1100 s.
- Define microseismic events to have started when the velocity along the fault exceeds a specified value. [11]

[11] M. D. McChesney, S. E. Minkoff, and G. A. McMechan, "Rate and state flow and deformation simulation of microseismicity with elastic emission wavefield synthesis," *Proceedings of the 86th Annual International Meeting of the Society of Exploration Geophysicists,* (Dallas, TX.), pp. 5055-5059, 2016.

Elastic FWI: Governing Equations

Assuming an isotropic medium, the 3D velocity-stress equations for particle velocity $v_i(\vec{x},t)$ and stress tensor components $\sigma_{ij}(\vec{x},t)$ $(i,j=1,2,3)$ are

$$\frac{\partial v_i(\vec{x},t)}{\partial t} - b(\vec{x})\frac{\partial \sigma_{ij}(\vec{x},t)}{\partial x_j} = b(\vec{x})\left[f_i(\vec{x},t) + \frac{\partial m^a_{ij}(\vec{x},t)}{\partial x_j}\right],$$

$$\frac{\partial \sigma ij(\vec{x},t)}{\partial t} - \lambda(\vec{x})\frac{\partial v_k(\vec{x},t)}{\partial x_k}\delta_{ij} - \mu(\vec{x})\left[\frac{\partial v_i(\vec{x},t)}{\partial x_j} + \frac{\partial v_j(\vec{x},t)}{\partial x_i}\right] = \frac{\partial m^s_{ij}(\vec{x},t)}{\partial t}$$

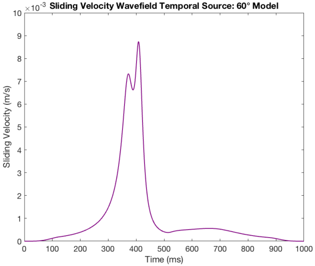
where $b = 1/\rho$ is mass buoyancy, and λ and μ are Lamé parameters.

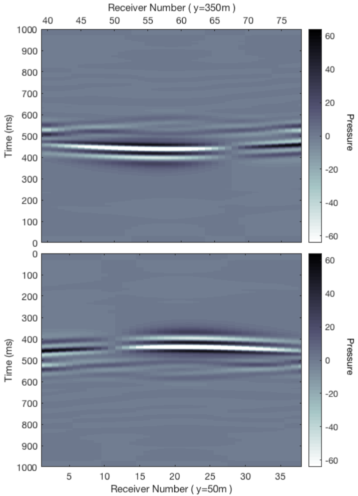
The source can be written in separable form as a moment density source:

$$m_{ij}(\vec{x},t) = -Mw(t)d_{ij}\delta(\vec{x}-\vec{x_s}).$$

Here M a moment amplitude and d_{ij} is a second-rank tensor giving the orientation of the applied moment. $m^s_{ij}(\vec{x},t)$ is the symmetric and $m^a_{ij}(\vec{x},t)$ the anti-symmetric part of the moment tensor.

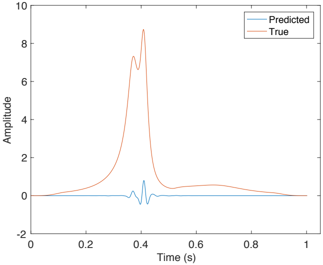
- Spatial Source:
  $(x, y, z) = (199, 198, 100)$ m
- Dip $= 90°$, Strike $= 60°$, Rake $= 0°$.
- Seismic Moment $= 3.94^{10}$ N-m
- Wavelet from flow and deformation simulation sliding velocity evolution.



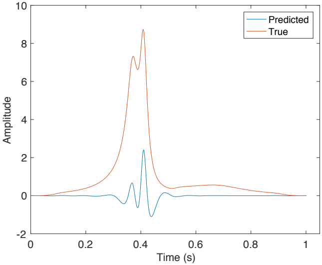


[12] McChesney, M. D., Minkoff, S. E., and McMechan, G. A., "Investigation and Analysis of Seismic Wavefield Response from Full Hydraulic Fracturing Flow and Geomechanics Modeling," submitted 2021.

Inverting for Realistic Microseismic Source

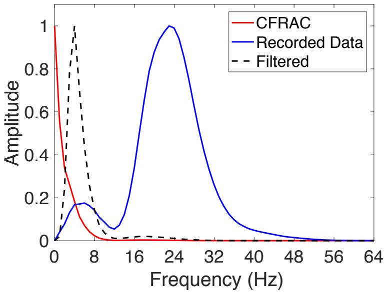


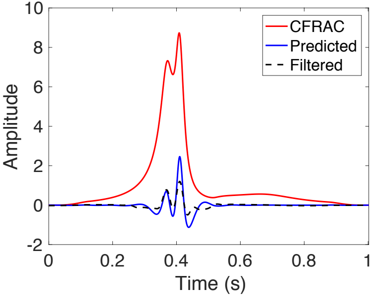
(f) Iteration 1



(g) Iteration 50

What Went Wrong?





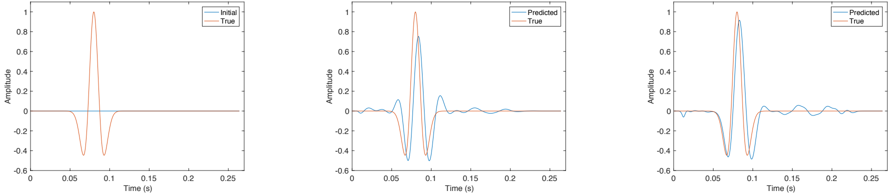
Data does not have the energy it needs to completely recover the source.
Does the best it can.[13]

[13] J. Kaderli, M. D. McChesney, and S. E. Minkoff, "A Self-Adjoint Velocity-Stress Full Waveform Inversion Approach to Microseismic Source Estimation," *Geophysics*, **83**, pp. 1–15, 2018.

Inverting for a Ricker wavelet $w(t)$ (Incorrect Earth Model)[14]

- Spatial components of source are known.
- All other parameters same as previous experiment except wrong earth model used.

	P-wave velocity (m/s)		S-wave velocity (m/s)	
Depth (m)	True	Perturbed	True	Perturbed
0 - 48	1200	1000	600	500
48 - 96	1500	1650	1000	1100
96 - 144	2500	2600	1500	1650
144 - 192	3000	2900	2000	1800
192 - 240	3500	3300	2250	2100



True and initial wavelets — Iteration 1 — Iteration 15

[14] J. Kaderli, M. D. McChesney, and S. E. Minkoff, "A Self-Adjoint Velocity-Stress Full Waveform Inversion Approach to Microseismic Source Estimation," *Geophysics*, **83**, pp. 1–15, 2018.

Conclusions

- Even this very simple single-trace transmission problem exhibits cycle skipping so FWI can fail without a good enough initial guess.
- By extending the problem to include inverting for the wavelet without support constraint as well as the sound velocity, we may bypass local minima.
- The ESI objective function can be efficiently solved using the Discrepancy Algorithm which maintains the data misfit within a reasonable range while also increasing the penalty parameter.
- ESI avoids cycle-skipping, allowing us to solve the inverse problem using standard local optimization.
- Stationary points of the ESI objective function lie near the global minimizer of the FWI objective function with an error bounded by a multiple of the wavelet support and noise level in the data.
- Goal is to apply method to more realistic microseismic event estimation problem when earth model is not known (real world situation).