Convective scale data assimilation perspectives

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MIDS
Mathematical Institute for
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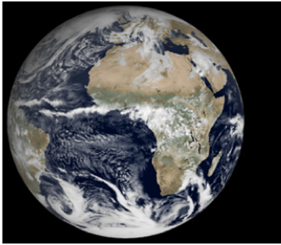
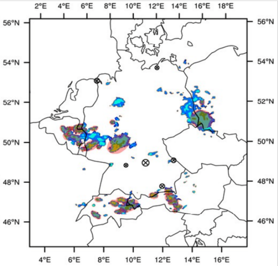
Can we improve rain prediction?

Global Model forecast (36hrs)

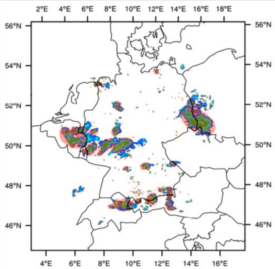


Verifying satellite data

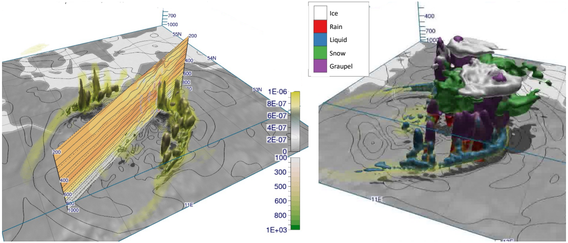
Local Model forecast (1hr)



Verifying radar data



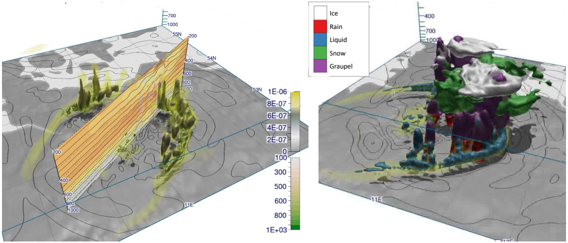
Convective/Storm scale application



Met3D

- High resolution NWP models of atmosphere that incorporate our knowledge of the dynamics and physics.

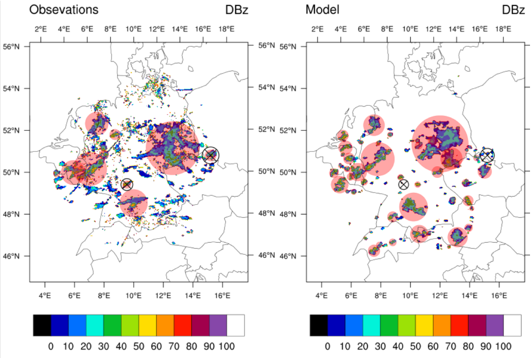
Convective/Storm scale application



Met3D

- High resolution NWP models of atmosphere that incorporate our knowledge of the dynamics and physics.
- In addition to dynamical variables, prognostic hydrometeors variables (rain, graupel, snow, ...) at all grid points

Uncertainty of geophysical models



- These models are not perfect. We need to specify background uncertainty including model error uncertainty.
- Forecast error for convective storms is often location and amplitude error

- In addition to global observing system, radar is the primary new observation.

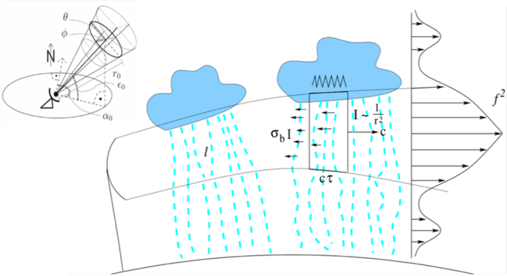


Figure 1 - Radars emit microwave energy from a transmitter into the atmosphere in a rapid succession of short pulses (i.e., from tens of nanoseconds to tens of microseconds). When these pulses impinge on objects in the atmosphere, such as raindrops, hailstones, snowflakes, cloud droplets, birds, insects, or dust particles, part of the energy bounces back towards the radar. The receiver on the radar then collects the reflected energy. The radar scans the area up to 250 km in the horizontal and 15 km in the vertical directions every 5 to 10 min [99].

Data assimilation algorithms

- Both ensemble and variational DA methods in use operationally (Gustafsson et al. 2018).
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- Much of the effort put in modelling of background error covariance and model error representation (Zeng et al. 2018, 2019, JAMES, Zeng et al. 2020 MWR, Feng et al 2021, JAMES)
- But also observational error covariance for radar data (Waller et al. 2019, MWR; Zeng et al. 2021, AMT).

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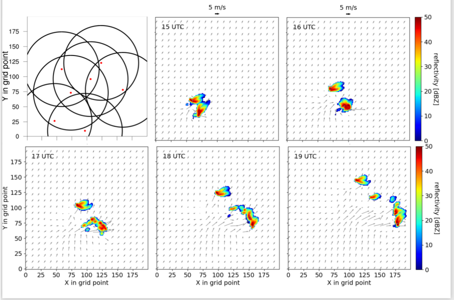
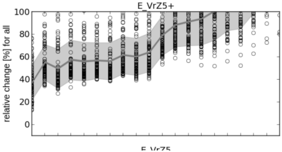
- But also observational error covariance for radar data (Waller et al. 2019, MWR; Zeng et al. 2021, AMT).
- Overview of challenges in Gustafsson et al. 2018 and Bannister et al. 2019, QJRMS.

A problem on convective scale

Idealized setup for radar DA



Zeng et al. 2021: Assimilating radar radial wind and reflectivity data in an idealized setup of the COSMO-KENDA system, Atmospheric Research, 249, 105282, https://doi.org/10.1016/j.atmosres.2020.105282.



Analysis mass of all hydrometeors compared to truth during DA (Janjic and Zeng, 2021).

Convective/Storm Scale DA Challenges

Problem high-dimensional, nonlinear and highly non-Gaussian.

1. Uncertainty quantification is a challenge for both model and observation error.
2. Forecast error for convective storms is often location and amplitude error.
3. In addition to dynamical variables, the state vector $\mathbf{x}_k^b$ at time k should consist of prognostic hydrometeors variables (rain, graupel, snow, ...) at all grid points
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4. Depending on microphysical scheme in model, even higher dimensional problem with third of the variables (one order of magnitude) that need to be nonnegative
5. We need fast algorithms applicable for large scale problems due to the computational costs of producing non-negative estimates, the frequency of their estimation, and the need to initialize probabilistic prediction.

DA methods would require a better way to estimate (multivariately) prognostic hydrometeors. This is a challenging task, in particular, due to spurious convections triggered.

— Some solutions —

- Physical properties/Conservation laws
- Weak formulation for mass/positivity constraints

QPEns

Propagation step. Propagate the mean and the covariance with the dynamics between observations. Prior to new observation we have $\mathbf{w}_k^b$ and its covariance $\mathbf{P}_k^b$.

$$\mathbf{w}_k^{b,i} = \mathcal{M}\mathbf{w}_{k-1}^{a,i} + \mathbf{q}_k^i \quad i = 1, \dots N$$

$$\mathbf{P}_k^b = \frac{1}{N-1}\sum_{i=1}^{N}[\mathbf{w}_k^{b,i} - \mathbf{w}_k^b][\mathbf{w}_k^{b,i} - \mathbf{w}_k^b]^T.$$

Kalman analysis.

$$\begin{aligned}
\mathbf{w}_k^{a,i} &= \mathbf{w}_k^{b,i} + \mathbf{K}_k(\mathbf{w}_k^o + r^i - \mathbf{H}_k\mathbf{w}_k^b), \\
\mathbf{K}_k &= \mathbf{P}_k^b\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^b\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \\
\mathbf{P}_k^a &= (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T\mathbf{P}_k^b
\end{aligned}$$

Derived using $q^i \sim \mathcal{N}(0, \mathbf{Q})$, $r^i \sim \mathcal{N}(0, \mathbf{R})$, $\mathbf{w}_0^b \sim \mathcal{N}(0, \mathbf{P_0^b})$ and all uncorrelated.

QPEns algorithm

$$\mathbf{w}_k^{a,i} = \mathbf{w}_k^{b,i} + \arg\min_{\delta w^i} \frac{1}{2}[\delta \mathbf{w}^{i\,T} (\mathbf{P}^b)^{-1} \delta \mathbf{w}^i + \mathbf{f}^{i\,T} \mathbf{R}^{-1} \mathbf{f}^i]$$

subject to

$$\delta \mathbf{w}^i \geq -\mathbf{w}_k^{b,i}. \quad i = 1, \ldots N$$

$$\delta \mathbf{w}^i = \mathbf{w}_k^{a,i} - \mathbf{w}_k^{b,i}, \mathbf{f}^i = \mathbf{w}_k^{o,i} - \mathbf{H}_k \mathbf{w}_k^{b,i} - \mathbf{H}_k \delta \mathbf{w}^i - \bar{\mathbf{r}}_k^o.$$

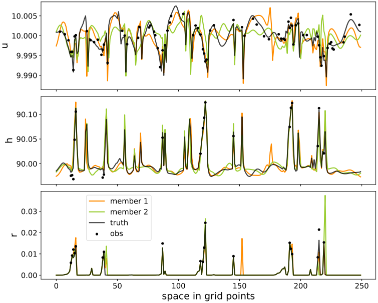
Janjic, T., D. McLaughlin, S. E. Cohn, M. Verlaan, 2014: Conservation of mass and preservation of positivity with ensemble-type Kalman filter algorithms, Mon. Wea. Rev., 142, No. 2, 755-773.

Modified shallow water model

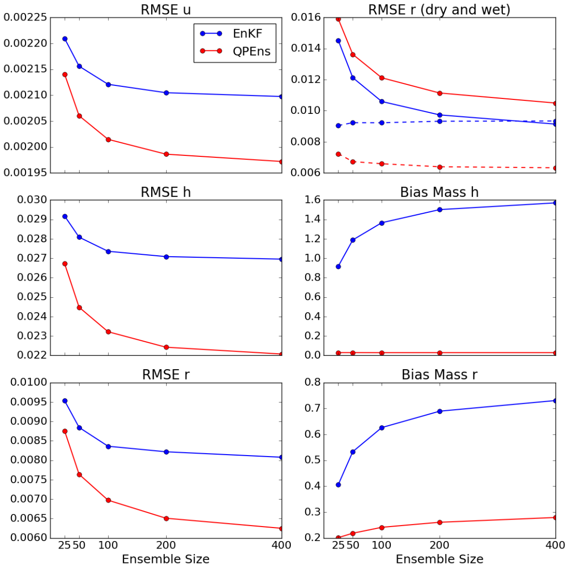
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial(\phi + \gamma^2 r)}{\partial x} = \beta_u + D_u\frac{\partial^2 u}{\partial x^2}, \phi = \begin{cases} \phi_c & \text{if } h > h_c \\ gh & \text{otherwise}, \end{cases}$$

$$\frac{\partial r}{\partial t} + u\frac{\partial r}{\partial x} = D_r\frac{\partial^2 r}{\partial x^2} - \alpha r - \begin{cases} \delta\frac{\partial u}{\partial x}, & \text{if } h > h_r \text{ and } \frac{\partial u}{\partial x} < 0 \\ 0 & \text{otherwise}, \end{cases}$$

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = D_h\frac{\partial^2 h}{\partial x^2}.$$



Wuersch and Craig 2014: A simple dynamical model of cumulus convection for data assimilation research., Meteorol. Z., 23, 483-490.



Ruckstuhl and Janjic 2018: Parameter and state estimation with ensemble Kalman filter based algorithms for convective scale applications. Q.J.R. Meteorol. Soc.. 144:712, 826–841, doi:10.1002/qj.3257.

Application of QPEns to high dimensional systems

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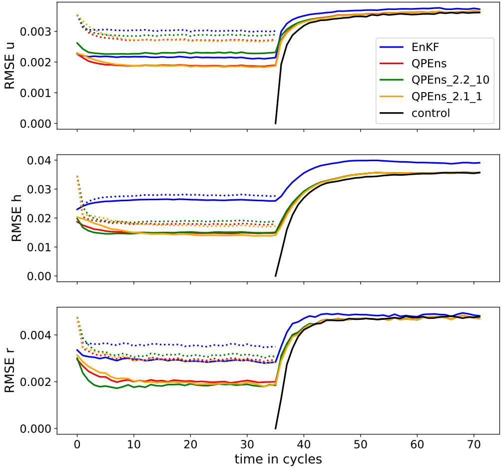
- High dimensional problem with third of the variables that need to be nonnegative and high update frequency
- We use the fact that constraints on mass and positivity are disjoint
- Active-set algorithm whose feature is to maintain feasibility with respect to the linear mass equality constraints on at each iteration, while at the same time using classical projection techniques to enforce positivity:

Alg1 still requires solving the Karush–Kuhn–Tucker (KKT) system
Alg2 exploits the low rank of the linear equality constraints (mass) and uses a well-known iterative approach to compute a possibly approximate solution while ensuring satisfaction of the constraint.

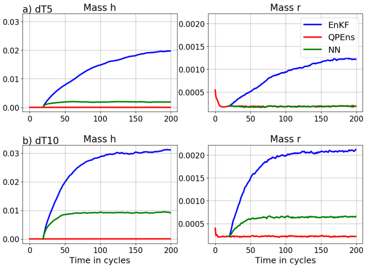
Janjić, T., Y. Ruckstuhl and P. Toint, 2021: A data assimilation algorithm for predicting rain, Q. J. R. Meteorol. Soc., 1949 –1963, doi:10.1002/qj.4004

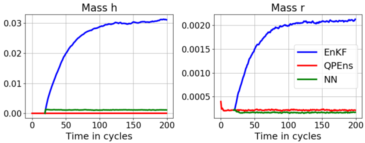
Modified shallow water model tests



Janjić, T., Y. Ruckstuhl and P. Toint, 2021: A data assimilation algorithm for predicting rain, Q. J. R. Meteorol. Soc., doi:10.1002/qj.4004

Alternative approach via NN





Left: Deviation in total mass and total rain for NN trained on QPEns analysis obtained with 5 min and 10 min updates. Also EnKF and QPEns results are shown for comparison.

Right: For 10 minutes update if in addition penalty on mass is imposed during training.

Ruckstuhl, Y., T. Janjić, S. Rasp, 2021: Training a convolutional neural network to conserve mass in data assimilation, Nonlin. Processes Geophys., 28, **111–119, https://doi.org/10.5194/npg-28-111-2021**.

Weak Constraint Convective scale

- DA on convective scales should update hydrometeors
- Operationally hydrometeors are not updated (Gustafsson et al. 2018)
- Assimilation of observations such as radar reflectivity or cloud products are important for prediction on convective scales

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- DA on convective scales should update hydrometeors
- Operationally hydrometeors are not updated (Gustafsson et al. 2018)
- Assimilation of observations such as radar reflectivity or cloud products are important for prediction on convective scales
- When DA algorithms update hydrometeors they clip negative values to zeros, modifying mass
- Janjic et al. 2014 show that is important to preserve both positivity and mass with DA algorithm when estimating variables that should be non-negative
- Here, we propose a fast, easy to implement modification of LETKF that is able to weakly preserve both properties of mass conservation for each hydrometeor variable and non-negativity.

Weakly Constrained LETKF

$$
\begin{aligned}
\min_{x} \quad J(\mathbf{x}) &= J_b(\mathbf{x}) + J_o(\mathbf{x}) + J_m(\mathbf{x}) \\
&= \frac{1}{2}(\overline{\mathbf{x}}_k^b - \mathbf{x})\mathbf{P}_k^{b^{-1}}(\overline{\mathbf{x}}_k^b - \mathbf{x})^T + \frac{1}{2}\left[\mathbf{y}_k^o - \mathbf{H}(\mathbf{x})\right]\mathbf{R}_k^{-1}\left[\mathbf{y}_k^o - \mathbf{H}(\mathbf{x})\right]^T \\
&+ \frac{1}{2}\left[\mathbf{m}_k - \mathbf{S}(\mathbf{x})\right]\mathbf{M}_k^{-1}\left[\mathbf{m}_k - \mathbf{S}(\mathbf{x})\right]^T
\end{aligned}
$$

- $\mathbf{m}$ is a vector quantity, whose elements are the domainwise (global) integral of hydrometeors.
- $\mathbf{S}$ is operator which calculates the domainwise (global) integral for each of the microphysical spieces,

Constraint on mass is up to accuracy $\mathbf{M}_k$

$$
\mathbf{M}_k = \frac{1}{N_{ens} - 1} \sum_{i=1}^{N_{ens}} \left[\mathbf{m}_k^* - \mathbf{S}\left(\mathbf{x}_k^{b(i)}\right)\right]\left[\mathbf{m}_k^* - \mathbf{S}\left(\mathbf{x}_k^{b(i)}\right)\right]^T
$$

Weakly Constrained LETKF

For mass:

$$\overline{\mathbf{x}}_k^{a,M} = \quad = \overline{\mathbf{x}}_k^b + \mathbf{P}_k^{a,M}\mathbf{H}^T\mathbf{R}_k^{-1}\left(\mathbf{y}_k^o - \overline{\mathbf{y}}_k^b\right) + \frac{1}{N_{ens}-1}\mathbf{X}_k^{a,M}\left(\mathbf{S}\mathbf{X}_k^{a,M}\right)^T\mathbf{M}_k^{-1}\left[\mathbf{m}_k - \mathbf{S}\left(\overline{\mathbf{x}}_k^b\right)\right]$$

$$\mathbf{W}_k^{a,M} = \left[(N_{ens}-1)\mathbf{I} + \left(\mathbf{H}\mathbf{X}_k^b\right)^T\mathbf{R}_k^{-1}\mathbf{H}\mathbf{X}_k^b + \left(\mathbf{S}\mathbf{X}_k^b\right)^T\mathbf{M}_k^{-1}\mathbf{S}\mathbf{X}_k^b\right]^{-1},$$

Note:

1. Corrections to weights implemented locally
2. After analysis ensemble is calculated correction to mass of each member

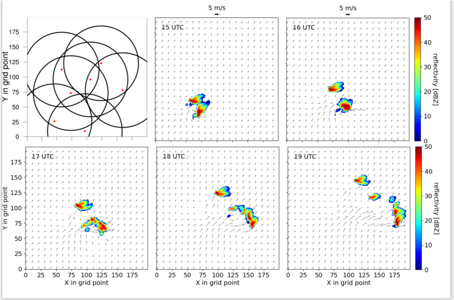
Weakly Constrained LETKF

For positivity:

- To avoid spurious convection (Aksoy et al., 2009) clear-air reflectivity data are assimilated, i.e. non-negative threshold value is set for very small reflectivities.
- Radar reflectivity data depend nonlinearly on hydrometeors. Further, reflectivity data (including clear-air reflectivity data) are available at radar observation locations, therefore not in every grid point of the model
- By assimilating additional clear-air reflectivity data, we are asking in the approximate weak sense that non-negativity is preserved in the analysis of hydrometeors

Experimental setup

Idealized setup for radar DA



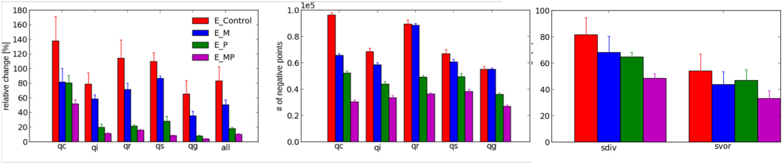
Zeng et al. 2021: Assimilating radar radial wind and reflectivity data in an idealized setup of the COSMO-KENDA system, Atmospheric Research, 249, 105282, https://doi.org/10.1016/j.atmosres.2020.105282.

- COSMO model with a 2-km horizontal resolution
- Efficient Modular VOlume scanning RADar Operator (EMVORADO, Zeng et al., 2014, 2016)
- Both radial wind and reflectivity data are assimilated
- Ensemble size is 80
- Assimilated observations are perturbation of nature run with Gaussian noise with a standard deviation of 5.0 dBZ and 1.0 m/s

Results: Impact of the different constraints

1. $E_{Control}$ Radar reflectivity and wind assimilated
2. E_M With mass constraint
3. E_P With positivity constraint (clear-air reflectivity data)
4. E_{MP} Both constraint

If clear-air reflectivity data are assimilated, a threshold value of 5 dBZ is set, that is, all reflectivity values smaller than 5 dBZ are set to 5 dBZ. If clear-air reflectivity data are not assimilated, all reflectivity values smaller than 5 dBZ are set to missing values.



Janjic, T. and Y. Zeng, 2021, Weakly constrained LETKF for estimation of hydrometeor variables in convective-scale data assimilation, Geophysical Research Letters, 48, e2021GL094962, https://doi.org/10.1029/2021GL094962.

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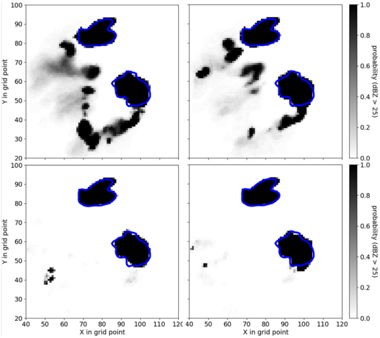
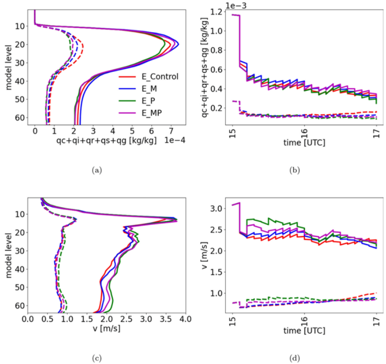
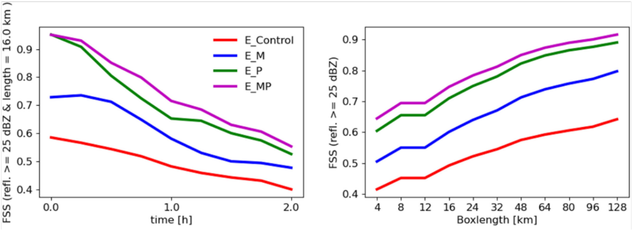


True storm plotted with blue line. From upper left to lower right: $E_{control}$, E_M, E_P, E_{MP}.



RMSEs calculated within storms only (full) and for full domain (dashed).

Accuracy



Accuracy of short term forecasts.

FSS score through time (left) and in dependence of grid box (right).

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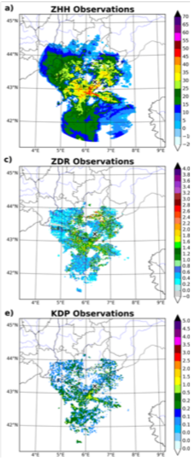
- By including mass and positivity, we improve on prediction of convective events.
- Some methods proposed require only minor changes to the already existing implementation.
- The inclusion of model error and observation error are still required.

— Bright Future —

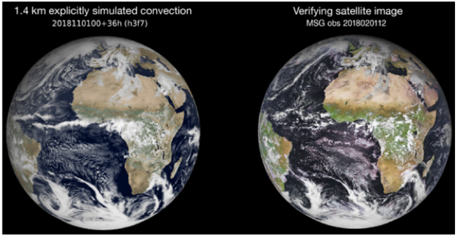
Bright future

New data



Thomas et al. 2020, Atmos. Meas. Tech.

Higher resolution models



From https://www.ecmwf.int/en/about/media-centre/

New tools

- AI
- New uncertainty quantification methods