

# Unit 11 Patterns and Algebra: Patterns and Equations

## Introduction

In this unit, students will make connections between different representations of patterns—numeric and geometric patterns, patterns on number lines, and patterns in hundreds charts and calendars. They will use patterns to skip count by 5s forwards and backwards, starting at any number within 1000 (not just multiples of 5).

Students will learn that a number sentence with an equal sign is called an equation, and equations can include some unknown numbers. Students will learn to represent these unknown numbers with blanks, boxes, symbols, or letters. Students will also explore various methods to solve an equation for the unknown number. Methods will include:

- performing calculations
- drawing a picture
- guessing and checking

## Meeting Your Curriculum

Alberta—All lessons in this unit are required.

British Columbia—All lessons in this unit are required.

Manitoba—All lessons in this unit are required.

Ontario—All lessons in this unit are required.

**NOTE:** Some exercises and mental math minutes in this unit use multiplication within  $9 \times 9$ . If you have limited multiplication facts to  $5 \times 5$  or  $7 \times 7$ , you will need to adjust the numbers in the examples given.

**Materials.** In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section V, are used in Unit 11:

**BLM Empty Spinners** (p. V-1)

**BLM Multiplication Chain** (pp. V-2–7)

## Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

	AB	BC	MB	ON
<b>Quiz</b>	PA3-13 to 14	PA3-13 to 14	PA3-13 to 14	PA3-13 to 14
<b>Quiz</b>	PA3-15 to 19	PA3-15 to 19	PA3-15 to 19	PA3-15 to 19
<b>Test</b>	PA3-13 to 19	PA3-13 to 19	PA3-13 to 19	PA3-13 to 19

# PA3-13 Geometric Patterns

Pages 31–33

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

array  
column  
decreasing  
difference  
gap  
geometric pattern  
growing  
increasing  
pattern  
perimeter  
row  
rule  
shrinking  
T-table

## Goals

Students will describe geometric patterns and represent them using number patterns.  
Students will represent number patterns using geometric patterns.  
Students will determine the pattern rule for both number and geometric patterns.

## PRIOR KNOWLEDGE REQUIRED

Can extend growing and shrinking patterns made by adding or subtracting a constant gap  
Can determine the rule for a growing or shrinking pattern  
Can determine the perimeter of a shape  
Can draw a T-table and extend a pattern in a T-table

## MATERIALS

blocks of different shapes, including many cubes or cylinders (optional)  
3 shapes of pattern blocks per student (at least 4 blocks of each shape)  
**BLM Patterns with Increasing Gaps** (p. N-50, see Extension 3)

**Mental math minute.** Have students stand in a line. Give the first student an addition problem that does not need regrouping, such as  $21 + 13$ . Students in line repeatedly add a number, in this case 13, with each student saying one addition aloud. When a student says an addition that involves regrouping, emphasize that this addition is a bonus. Example: Student 1 says, “ $21 + 13 = 34$ ”; Student 2 says, “ $34 + 13 = 47$ ”; Student 3 says, “ $47 + 13 = 60$ ” (note that this is a bonus). Continue for a few questions before starting a new chain.

**Introduce geometric patterns.** Draw the following sequence of figures on the board and tell students that the pictures show several stages in the construction of a castle made of blocks. Alternatively, if blocks are available, build a similar pattern from actual blocks. Use cubes or cylinders for towers.



Figure 1



Figure 2

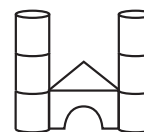


Figure 3

Ask students to imagine that they want to keep track of the number of blocks used in each stage of the construction of the castle. SAY: We can use a T-table to keep track of how many blocks are needed for each stage of construction. Draw the following table on the board and ask students to help you fill in the number of blocks used in each figure.

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Figure Number	Number of Blocks
1	4
2	6
3	8

ASK: What patterns do you see in the columns of the table? (the figure number grows by 1, and the number of blocks grows by 2 each time) To prompt students to see the second pattern, draw circles on the right side of the table and remind students that they can write the gap between the numbers in the second column in the circles, as shown below:

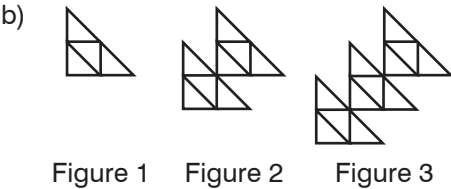
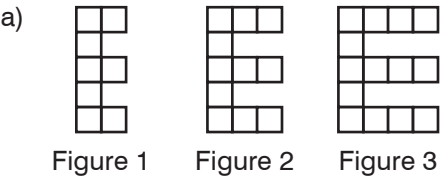
Figure Number	Number of Blocks	
1	4	
2	6	+2
3	8	+2

Add a few more rows to the table and have volunteers fill them in. If you are working with blocks for demonstration, invite other volunteers to build the next figures in the pattern to check the numbers predicted in the table. Keep the table on the board for later use.

**NOTE:** Have students complete Exercise 1 below, and alert them that in Exercise 2 the patterns are shrinking, or decreasing. This means that students will need to subtract the number each time. Remind students that in such cases they can write the difference in the circles with a minus sign in front.

**Exercises**

1. Make a T-table for the number of blocks in the pattern. Extend the table to show how many blocks will be in Figure 6.



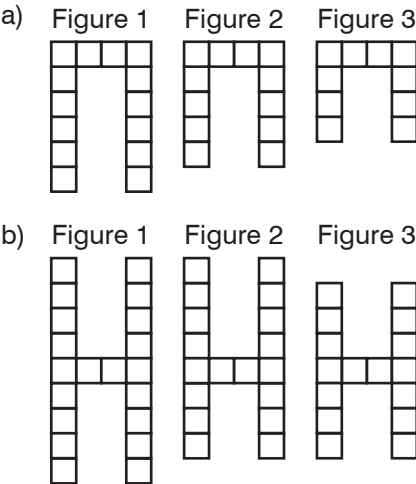
**Bonus:** Draw the figures in the pattern to check your answers in the table.

**Selected answers**

a)	Figure Number	Number of Blocks
	1	8
	2	11
	3	14
	4	17
	5	20
	6	23

b)	Figure Number	Number of Blocks
	1	4
	2	8
	3	12
	4	16
	5	20
	6	24

2. Make a T-table for the number of blocks in the pattern. Extend the table to show how many blocks will be in Figure 6.



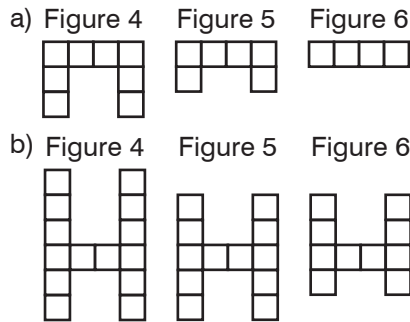
**Bonus:** Draw the figures in the pattern to check your answers in the table.

**Answers**

a)	Figure Number	Number of Blocks
	1	14
	2	12
	3	10
	4	8
	5	6
	6	4

b)	Figure Number	Number of Blocks
	1	20
	2	18
	3	16
	4	14
	5	12
	6	10

### Bonus



**Review writing rules for number patterns.** Remind students that to write a rule for a number pattern, you need to say what number to start with and what number to add or subtract. Draw students' attention to the table on the board for the number of blocks in the castle. ASK: What number do you start with? (4) Do you add or subtract to get the next number of blocks? (add) How many blocks do you add each time? (2) How do you see that from the pictures? (there are two towers in the castle, each time we add one block to each tower) What is the rule for the number pattern? (start at 4 and add 2 each time) Write the rule on the board.

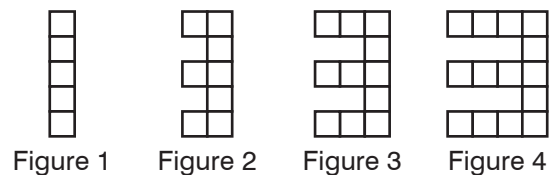
**Exercises:** Write the rule for the number patterns in Exercises 1 and 2 above.

### Answers

1. a) start at 8 and add 3 each time, b) start at 4 and add 4 each time;
2. a) start at 14 and subtract 2 each time, b) start at 20 and subtract 2 each time

**Describing the geometric pattern.** SAY: I want to tell my friend in Quebec City about the castle we built. I would like to describe the castle for her so that she can build a castle just like it. Remind students that the blocks that make the towers are called cylinders. Ask students to try to describe the first castle in the pattern. (each castle is made from a gate, a triangular roof, and two towers—one on each side; the first castle in the pattern has towers that are 1 block tall) ASK: How do you make each next castle? (add 1 block on top of each castle tower)

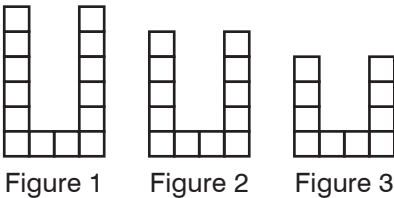
Draw on the board:



Ask students to try to describe the pattern. PROMPT: Are the squares arranged in a row, a column, or an array of several columns or rows? (a column) How many cubes are in the column? (5) How many cubes do you add each time? (3) Do you add the cubes to the left side or to the right side of the column? (to the left side) Do you add the cubes at the top, at the bottom, or at the middle? How many cubes do you add at each place? (1 at the top, 1 at the bottom, 1 in the middle) Summarize: Start with a

column of 5 cubes. Each time, add 3 cubes to the left side, 1 at the top, 1 at the bottom, and 1 at the middle cube. Point out to students that sometimes patterns resemble familiar shapes and it makes sense to mention the resemblance when describing a pattern. For example, starting from Figure 2, the figures resemble a backwards capital E.

**Exercise:** Describe the pattern.



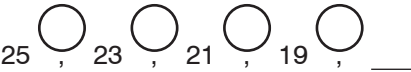
**Sample answer:** Start with 14 blocks that form the letter U. The figure is 4 blocks wide and 6 blocks tall. Remove 2 blocks each time, one block from the top of each column of the letter U.

Draw on the board:

Figure Number	Number of Blocks
1	13
2	18
3	23

SAY: This T-table shows how many blocks I used at each stage of building another castle. The castle has several towers, and I added one block to the top of each tower at each stage. There is a gate with a roof between each pair of towers. ASK: How many towers does the castle have? (5) How do you know? (the gap is 5, so if 1 block is added to each tower each time, there must be 5 towers) If you are using blocks, have a volunteer build the first figure, and check that the correct number of blocks is used. The first castle will have towers 1 block tall, with 4 gates and 4 triangular roofs. You can also ask students to sketch a picture of the castle. Then ask students to help you extend the T-table to five terms by adding the gap to successive terms. (28, 33)

**Review writing rules for number patterns that are not in a table.** Write on the board:



ASK: Is this a growing or a shrinking number pattern? (shrinking) How do you know? (the numbers get smaller each time) Should you add or subtract to continue the pattern? (subtract) What number do you subtract each time? (2) Have volunteers check different pairs of numbers and write “–2” in the corresponding circle. ASK: What is the rule for the pattern? (start at 25 and subtract 2 each time)

**Exercises:** Which number is added or subtracted each time? Write the rule for the pattern.

- a) 78, 74, 70, 66      b) 32, 37, 42, 47      c) 107, 102, 97, 92, 87

**Answers:** a) 4 is subtracted, start at 78 and subtract 4 each time; b) 5 is added, start at 32 and add 5 each time; c) 5 is subtracted, start at 107 and subtract 5 each time

**Creating geometric patterns for a number pattern.**

### ACTIVITIES 1–2

Provide each student with a large number of different pattern blocks.

1. Create a growing pattern from pattern blocks. Build the first three or four figures in the pattern. Write the number pattern showing the number of blocks in the pattern and predict the number of blocks in the next figure. Create the next figure in the pattern to check your answer. Describe the pattern you made.
2. Write a rule for the number pattern. Make a pattern of blocks that matches the number pattern.

- a) 7, 11, 15      b) 17, 14, 11, 8      c) 1, 5, 9

#### Selected sample answer

a) Start at 7 and add 4 each time.



**Answers:** b) start at 17 and subtract 3 each time, c) start at 1 and add 4 each time

**Review perimeter.** Draw on the board:



Remind students that the distance around a shape is called perimeter.  
**ASK:** If each side of a triangle is 1 unit long, how long is the perimeter of this figure? (5 units) Invite a volunteer to show how to find the perimeter. (count the outer edges or add the side lengths)

**Producing different number patterns from geometric patterns.** Draw on the board:



Figure 1



Figure 2



Figure 3

**SAY:** I want to find the perimeter of each of the three figures. Imagine that each triangle is made from 3 toothpicks, and all the toothpicks are the same length. Have students sketch the figures in their notebooks and then

find the perimeter of each figure in toothpicks. Have a volunteer write the perimeter underneath the figure on the board. (5, 6, 7 toothpicks)

ASK: Do the perimeters make a number pattern? (yes) Explain that you can make number patterns from geometric patterns not only by counting the number of shapes, but also by counting or measuring other things, such as perimeter. For example, if you make this pattern by placing toothpicks to form triangles, you can also count the total number of toothpicks, or the number of triangles. Have students write the pattern for the number of toothpicks (7, 9, 11) and the number of triangles for each figure (3, 4, 5).

SAY: Imagine I have 20 toothpicks. I want to find the largest figure that I can make in this pattern. ASK: Should I draw larger and larger figures and count all the toothpicks, or is there a shorter, easier way to find it out? PROMPT: Can one of the number patterns we wrote help me find what the figure number will be? (yes) Which pattern? (the number of toothpicks)

Start a T-table on the board, as shown below:

Figure Number	Number of Toothpicks
1	7
2	9
3	11

Have students copy the table and extend it. Do the same on the board and have students help you fill it in, until you reach the 8<sup>th</sup> row—Figure 8 and 21 toothpicks. ASK: If I have 20 toothpicks, can I make Figure 8? (no) Why not? (you need 21 toothpicks, and you only have 20 toothpicks) Can I make Figure 7? (yes) Will I have any toothpicks leftover? (yes) How many triangles will I have in this figure? (9) How do you know? (answers may vary; students might notice that the number of triangles is always 2 more than the figure number, or they might extend the pattern for the number of triangles to 7 terms)

**Exercises:** Anika designs a pattern of long rectangles with toothpicks.



Figure 1      Figure 2      Figure 3

- a) Make a T-table for the number of toothpicks in each figure.
- b) Anika has 27 toothpicks. How many toothpicks long is the longest rectangle she can make in her pattern?

**Bonus:** Make a T-table for the perimeter of Anika’s rectangles. What is the perimeter of the longest rectangle she can make with 27 toothpicks?



### Answers

a-b)

Figure Number	Number of Toothpicks
1	5
2	9
3	13
4	17
5	21
6	25
7	29

The longest rectangle Anika can make is Figure 6, which uses 25 toothpicks. Figure 1 is 1 toothpick long, Figure 2 is 2 toothpicks long, and so on, so Figure 6 is 6 toothpicks long.

Bonus:

Figure Number	Perimeter
1	4
2	6
3	8
4	10
5	12
6	14

The perimeter of the longest rectangle she can make is 14 toothpicks long.

## Extensions

- Matt makes a castle by adding 1 block at a time to each of 4 towers. He has a gate with a triangular roof between each pair of towers. Matt uses 22 blocks altogether.
  - How many blocks are not used in the towers?
  - How many blocks are used in the towers?
  - How tall is each tower?

**Answers:** a) 6 blocks, b) 16 blocks, c) 4 blocks

- Cathy uses one kind of block to build a pattern. She adds the same number of blocks to make each new figure. She writes the number of blocks in the figure in a T-table. Cathy makes one mistake in the table. Find and correct her mistake.

Figure Number	Number of Blocks
1	5
2	7
3	11
4	14

**Answer:** Figure 2 should have 8 blocks.

- Have students complete **BLM Patterns with Increasing Gaps**.

### Answers

1. a) gaps: +2, +3, +4, +5, +6, next terms: 17, 23; b) gaps: +1, +2, +3, +4, +5, +6, next terms: 19, 25; c) gaps: +2, +4, +6, +8, +10, +12, next terms: 36, 48; d) gaps: +3, +5, +7, +9, +11, next terms: 34, 45

2. a–c)

Figure Number	Number of Squares
1	1
2	3
3	6
4	10
5	15
6	21

+2  
+3  
+4  
+5  
+6

3.

Figure Number	Number of Squares
1	1
2	4
3	9
4	16
5	25

+3  
+5  
+7  
+9

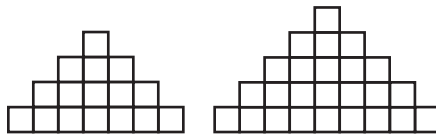


Figure 4

Figure 5

4. Armand makes a pattern starting at 2. He multiplies each term by the same number to get the next number in the pattern. His pattern is 2, 4, 8, 16.
- What number does Armand multiply each term by?
  - Write 3 more numbers in Armand's pattern.
  - Find the gaps between the numbers in Armand's pattern. What do you notice about the pattern in the gaps?

**Answers:** a) 2; b) 32, 64, 128; c) The pattern in the gaps is 2, 4, 8, 16, 32, 64. It is the same as the pattern itself.

# PA3-14 Patterns on Number Lines

Pages 34–36

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

decreasing  
difference  
gap  
geometric pattern  
growing  
increasing  
multiples of 10  
pattern  
rule  
shrinking  
sum

## Goals

Students will represent number patterns, including numeric representations of geometric patterns, on number lines.  
Students will represent patterns given on number lines as number patterns and describe the pattern rule for number patterns.

## PRIOR KNOWLEDGE REQUIRED

Can add and subtract two numbers within 1000  
Can extend a number pattern by adding or subtracting the same number  
Can extend a geometric pattern  
Can write a number pattern based on a geometric pattern  
Can write a rule for a number pattern  
Can represent an addition or subtraction sentence on a number line  
Can create a geometric pattern based on a number pattern

## MATERIALS

ball  
transparency of **BLM Number Lines to 100** (p. N-51)  
overhead projector  
**BLM Number Lines** (p. N-52), several copies per student  
paper clip, pencil, and **BLM Empty Spinners** (p. V-1) per pair of students  
**BLM Number Lines with Large Numbers** (p. N-53)  
pencil crayons  
large number of beads, cubes, or pattern blocks

**Mental math minute.** Give students problems that require subtracting one-digit numbers from one- and two-digit numbers, such as  $35 - 8$ . Students can use any method, such as counting up using 1s and multiples of 10, or using number facts within 20. Toss a ball to the student you want to answer the question and have the student toss the ball back to you after answering.

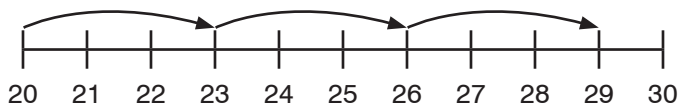
**Review adding and subtracting on a number line.** Draw a number line starting at 20 and ending at 30 on the board. Draw an arrow from 20 to 23 and ASK: What addition sentence does this picture show? ( $20 + 3 = 23$ ) How does the arrow show this? (the start of the arrow is the number we start with, or the first addend; the length of the arrow itself shows the number we are adding, or the second addend; the end of the arrow shows the answer, or the sum)

Reverse the direction of the arrow so that it points from 23 to 20, and ask students to describe the subtraction sentence the model shows. ( $23 - 3 = 20$ ; the start of the arrow is the number we subtract from; the length of

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the arrow is the number subtracted; the end of the arrow is the result, or the difference)

**Writing a pattern shown on a number line.** Draw on the board:



SAY: This picture shows adding 3 repeatedly. ASK: What number do we start with? (20) Point to the first arrow and SAY: First we add 3 and get 23. Then we add another 3 (point to the second arrow) and get 26. Then we add another 3 (point to the third arrow) and get 29. Write on the board underneath the number line:

20  $\xrightarrow{+3}$  23  $\xrightarrow{+3}$  26  $\xrightarrow{+3}$  29

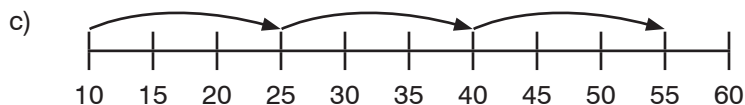
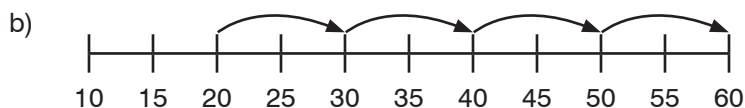
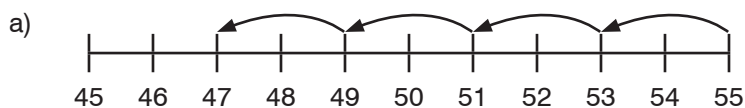
SAY: The picture on the number line shows a pattern. ASK: Is this a growing pattern or a shrinking pattern? (growing) How do you know? (we add 3, the numbers get larger) How do you see that from the picture on the number line? (the arrows point to the right, towards larger numbers) SAY: We usually make growing patterns by adding the same number over and over. ASK: How do we see from the picture that we add the same number over and over? (the arrows are the same length and point in the same direction)

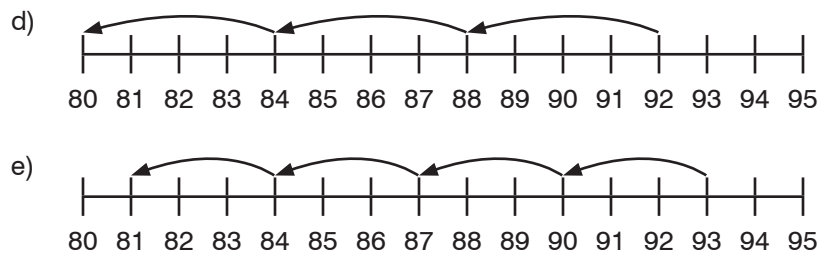
Repeat the discussion with the picture below, showing the shrinking pattern 29, 27, 25, 23, 21.



Point out that when you add or subtract the same number over and over, you are actually skip counting forwards or backwards.

**Exercises:** Write the number pattern the picture shows.





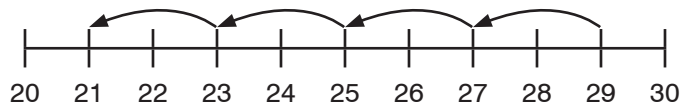
**Answers:** a) 55, 53, 51, 49, 47; b) 20, 30, 40, 50, 60; c) 10, 25, 40, 55; d) 92, 88, 84, 80; e) 93, 90, 87, 84, 81

**Representing number patterns on a number line.** Project **BLM Number Lines to 100** on the board. Explain that all the numbers on the number lines are marked, but only the multiples of 10 are labelled. Point to several different marks that are not numbered and have students say what number the mark shows. Have volunteers explain how they know. One possible strategy is to look at the multiple of 10 before the number and count up by 1s marks. Students can also look at the next multiple of 10 and count back. Discuss when to use each strategy. The second strategy is best when the mark is closer to the larger multiple of 10. However, if the mark is about the same distance from either side, it is better to count up, because people tend to make more mistakes when counting back than when counting on.

Write the pattern “0, 25, 50, 75, 100” on the board and invite volunteers to place dots on those numbers on the number line. Ask another volunteer to draw the arrows to show the pattern. Repeat with 91, 86, 81, 76, 71. Then draw arrows showing the pattern 22, 32, 42, 52 and have students write the pattern the number line shows. Keep the patterns displayed for the next explanation.

**Review rules for patterns.** Remind students that to describe a pattern, we need to say what number we start with and what number we add or subtract each time. For example, the pattern 0, 25, 50, 75, 100 has the rule “Start at 0 and add 25 each time.” Write the description underneath the pattern on the board and have volunteers describe the other two patterns on the board. (start at 91 and subtract 5 each time, start at 22 and add 10 each time)

**Writing rules for patterns represented on a number line.** Draw the pattern below on the board again:



ASK: Can you find the rule for a pattern if the pattern is shown on a number line? (yes) What is the starting number? (29) How does the pattern on a number line show the starting number? (this is the start of the first arrow) Write “start at 29” underneath the picture. ASK: How do you know if you need to add or to subtract? (the arrows point to the left, so you need to subtract) Write “subtract \_\_\_ each time” underneath the picture.

ASK: How do you know which number to subtract? (the arrow is 2 units long, so we need to subtract 2) Write “2” in the blank.

**Exercises:** Write a rule for the patterns in the previous exercises.

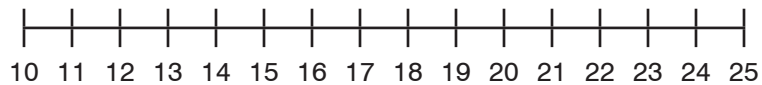
**Answers:** a) start at 55 and subtract 2 each time; b) start at 20 and add 10 each time; c) start at 10 and add 15 each time; d) start at 92 and subtract 4 each time; e) start at 93 and subtract 3 each time

### ACTIVITY 1

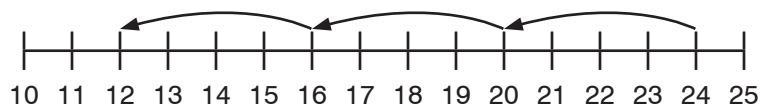
1. Give each pair of students several copies of **BLM Number Lines**, one copy of **BLM Empty Spinners**, a paper clip to act as a pointer for the spinner, and a pencil to anchor the paper clip. Have students use the spinner with 8 regions and label the regions “+ 2, + 3, + 4, + 5, - 2, - 3, - 4, - 5.” Player 1 spins the spinner so that Player 2 does not see the result. Player 1 uses the result of the spin as the pattern, chooses the first number of the pattern, and draws the pattern on a number line. Player 2 writes the rule for the pattern that Player 1 drew. Players switch roles and repeat. Students will use the same materials in Activity 2.

**Representing patterns on a number line.** Draw on the board:

Start at 24 and subtract 4 each time.



Explain that you want to show this pattern on the number line. Have students tell you how to do so. **PROMPT:** What number should the arrows start at? (24) How long should the arrows be? (4 units) How do you know? (you need to subtract 4 each time, the gap in the pattern is 4) Should the arrows point right or left? (left) How do you know? (you need to subtract each time) How many arrows do you need to draw to show the pattern? (3) Draw the arrows, as shown below:



### ACTIVITY 2

2. Use the same materials as in Activity 1. Player 1 spins the spinner to determine what number to add or subtract each time and picks the number the pattern starts at. Player 1 then writes a rule for the pattern. Player 2 draws the pattern on the number line. Both players write the numerical pattern and compare answers. Players switch roles after each turn.

Distribute **BLM Number Lines with Large Numbers** and have students do the exercises below. Students might use different-coloured pencil crayons for patterns that appear on the same number lines.

**Exercises:** Show the first 5 numbers of each pattern on the number line.

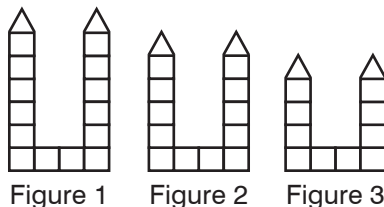
- a) Use Line 1. Start at 219 and subtract 2 each time.
- b) Use Line 1. Start at 202 and add 2 each time.
- c) Use Line 2. Start at 481 and add 5 each time.
- d) Use Line 2. Start at 498 and subtract 5 each time.
- e) Use Line 3. Start at 227 and add 5 each time.
- f) Use Line 4. Start at 718 and add 5 each time.
- g) Use Line 5. Start at 889 and subtract 5 each time.

**Bonus**

- h) Start at 818 and add 10 each time.
- i) Start at 996 and subtract 10 each time.

**Answers:** a) 219, 217, 215, 213, 211; b) 202, 204, 206, 208, 210; c) 481, 486, 491, 496, 501; d) 498, 493, 488, 483, 478; e) 227, 232, 237, 242, 247; f) 718, 723, 728, 733, 738; g) 889, 884, 879, 874, 869, 864; Bonus: h) 818, 828, 838, 848, 858; i) 996, 986, 976, 966, 956

**Representing a geometric pattern on a number line.** Draw on the board:



Remind students that there are many ways to make a number pattern from a geometric pattern. For example, you can write how many shapes are in each figure of the pattern or how many of a specific type of shape, such as squares, are in each figure of the pattern. Other ways include finding the lengths, heights, or perimeters of each figure. Explain that as soon as you produce a number pattern, you can show it on a number line.

Point to the drawing on the board and SAY: Let's make a pattern for the number of blocks in this pattern. Have students count the blocks and have a volunteer write the number pattern underneath the geometric pattern, clearly labelling it as the pattern in the number of blocks. (number of blocks in each figure: 16, 14, 12) Draw a number line from 0 to 20, have students copy it in their notebooks, and have them draw the pattern on the number line using arrows.



ASK: Can you use the number line to check how many blocks will be in the 6<sup>th</sup> figure? (yes, extend the pattern on the number line) SAY: The number of blocks in the first figure is the beginning of the first arrow, and the number of blocks in the second figure is the end of the first arrow. The number of blocks in the third figure is the end of the second arrow. ASK: How many arrows do you need to draw to find the number of blocks in the 6<sup>th</sup> figure? (5 arrows) Have students draw the arrows and find how many blocks will be in the 6<sup>th</sup> figure. (6 blocks) Invite volunteers to draw the next figures in the pattern and check the answer.

### ACTIVITIES 3–4

Provide students with a copy of BLM Number Lines and a large number of pattern-making materials, such as beads, cubes, or pattern blocks.

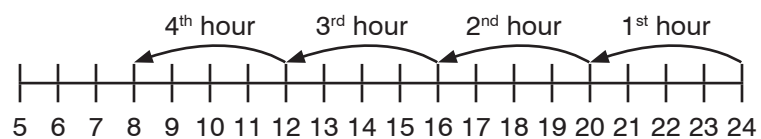
3. Player 1 makes a growing or a shrinking pattern with the pattern-making materials. Player 2 shows the pattern on a number line. Players switch roles.
4. Player 1 draws a growing or a shrinking pattern on a number line. Player 2 makes the pattern that has the same number of shapes in each figure as the pattern on the number line.

## Extensions

1. Teach students to use number lines to solve word problems. Write on the board:

A caterpillar on a branch is 24 cm away from the tree trunk. The caterpillar crawls towards the trunk, 4 cm every hour. How far from the trunk is the caterpillar after 1 hour? 2 hours? 4 hours?

SAY: We could make a T-table to solve this problem or we could make a number pattern for how far the caterpillar is from the tree trunk. We could also show this pattern on a number line and use this number line to solve the problem. Draw the number line below on the board, but do not draw the arrows.



SAY: This number line shows the distance from the tree trunk. At the start, the caterpillar is 24 cm away from the trunk. Let's start the arrows at 24. ASK: How long should each arrow be? (4 cm) How do you know? (the caterpillar travels 4 cm every hour) Should we draw arrows pointing to the left or right? (left) How do you know? (the caterpillar is crawling towards the trunk, so the distance to the trunk gets smaller) Have a volunteer draw the arrows, and have other volunteers decide how far from the trunk the caterpillar is after 1 hour (20 cm), 2 hours (16 cm), 4 hours (8 cm).

Have students use the same method to solve the problems below.

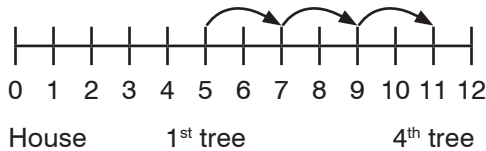
- a) Jin can walk 5 km in 1 hour. He is 20 km away from home. Jin starts walking home. How far from home is Jin after 3 hours?
- b) Anna is 15 km away from her campsite. She starts hiking towards her campsite. She can hike 4 km each hour. How far from her campsite is Anna after 3 hours?

**Answers:** a) 5 km, b) 3 km

2. Carl plants 4 apple trees in a row. The first tree is 5 m away from his house. Each tree after that is 2 m farther from the house than the tree before it. Draw a number line for the distance from the house. Put the house at 0. How far from the house is the 4<sup>th</sup> tree?

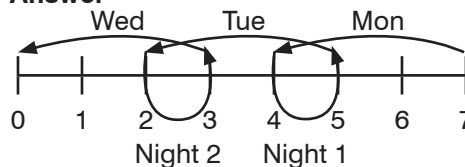
**Answer**

The 4<sup>th</sup> tree is 11 m away from the house.



3. A snail is at the bottom of a well on Monday morning. Every day the snail climbs up 3 m and every night the snail slides back 1 m. The well is 7 m deep. On what day does the snail reach the top of the well? Draw a number line for the depth of the well and show the snail's journey.

**Answer**



The snail climbs out of the well on Wednesday evening.

# PA3-15 Patterns in Charts

Pages 37–39

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

addend  
column  
decreasing  
**diagonal**  
difference  
gap  
growing  
hundreds chart  
increasing  
multiple  
number pattern  
ones digit  
product  
row  
rule  
shrinking  
sum  
tens digit

## Goals

Students will identify and describe number patterns in hundreds charts and in calendars.  
Students will use the patterns seen on hundreds charts to skip count forwards and backwards by 5s starting at any number within 1000.

## PRIOR KNOWLEDGE REQUIRED

Can multiply one-digit numbers  
Can extend a number pattern by adding or subtracting the same number  
Can extend a geometric pattern  
Can write a number pattern based on a geometric pattern  
Can write the rule for a number pattern  
Can represent an addition or subtraction sentence on a number line  
Can create a geometric pattern based on a number pattern

## MATERIALS

transparency of a hundreds chart or **BLM Hundreds Charts** (p. N-54)  
transparency of a calendar or **BLM Calendars** (p. N-55)  
overhead projector  
**BLM Multiplication Chain** (pp. V-2–7)  
hundreds chart or **BLM Hundreds Charts** (p. N-54) per student  
pencil crayons  
small token per pair of students  
calendar or **BLM Calendars** (p. N-55) per student  
**BLM Empty Calendar** (p. N-56)

**NOTE:** Throughout this lesson you will need to shade different patterns on a hundreds chart and a calendar page. If you do not have convenient manipulatives (such as a commercial erasable hundreds chart), you can photocopy **BLM Hundreds Charts** and **BLM Calendars** onto transparencies and either display new charts each time, or project enlarged copies on the board and shade the numbers on the board. This would allow you to erase the shading or circles without erasing the charts themselves.

**Mental math minute.** Give each student a card from **BLM Multiplication Chain**. Call a volunteer to the front of the class. The volunteer reads the card (e.g., I have  $3 \times 4$  and 25). Students who have 12 or  $5 \times 5$  on their cards should come to the front of the class and stand beside the volunteer, showing their cards. If there is more than one student with a card that matches (for example, 12 appears on multiple cards), pick who joins the chain at this moment and who will join the chain later. The students who just joined the chain read the unmatched halves of their cards, and new students with matches join the chain. If the number called from one side

of the chain matches the multiplication sentence on the other side of the chain, and there is no third student who can join either side of the chain, the chain is complete. The remaining students should try to make a new chain of their own. The game ends when everyone has come to the front.

**Review patterns in charts.** Draw on the board:

2	4	6	8	10
12	14	16	18	20
22	24	26	28	30
32	34	36	38	40
42	44	46	48	50

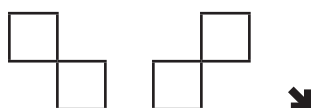
Shade the top row. ASK: What pattern do you see in this row? (2, 4, 6, 8, 10) What kind of pattern is that? (increasing pattern, growing pattern) What is the rule for this pattern? (start at 2 and add 2 each time, skip count by 2s) How do you know? (the difference, or the gap between the numbers, is always 2) Have a volunteer write the rule on the board. Repeat with the pattern in the third row. (start at 22 and add 2 each time) ASK: Do you think in this chart all rows are made by adding 2? (yes) Have students check different rows.

Repeat the discussion with columns, concluding that the columns are all made by adding 10 each time.

**Identifying patterns in rows and columns of a hundreds chart.** Display a large hundreds chart and shade the third row. ASK: If I look at these numbers from left to right, do they show a pattern? (yes) What rule does this pattern have? (start at 21 and add 1 each time) Repeat with the 6<sup>th</sup> column going down. (start at 6 and add 10 each time)

Distribute **BLM Hundreds Charts**. Have students work with a partner. Each person uses pencil crayons to shade a row and a column on the first hundreds chart on the BLM. (Ask students not to shade the same column and the same row as you did, and to choose a row and a column different from that of their partners.) Have each student write the rules for their pattern going to the right and their pattern going down. Partners trade BLMs and write the rules for the patterns in the rows their partners shaded, this time going to the left, and the columns their partners shaded, this time going up from the bottom row.

ASK: What is the gap in the patterns in the rows of a hundreds chart? (1) Did anyone get a different gap? (no) Does everyone have the same starting number? (no) Point out that a hundreds chart is made by counting up by 1s in rows, so it makes sense that the patterns all have a gap of 1. ASK: What is the gap in the patterns in the columns of a hundreds chart? (10) Did anyone get a different gap? (no) Does everyone have the same starting number? (no) Why is the gap always 10 in any column? (there are 10 columns in the chart, so to get to the number directly below any number, you need to count up 10, or add 10) PROMPT: If you fill in the



hundreds chart counting by 1s, how many numbers are between a number and the number right below it? Do you need to count a whole row to get to the number right below? How many cells, or boxes, are in the whole row? How many columns are in the hundreds chart? Students should keep their copies of the BLM for later in the lesson.

**Identifying diagonal patterns on a hundreds chart.** Explain that when you go 1 row down and 1 column right or left on a chart, the cells are *diagonal* from each other. Diagonal cells have only one corner in common. Draw two diagonal arrangements on the board to illustrate, as shown in the margin.

Display a fresh hundreds chart. ASK: Which cells are diagonally beside 23? (12, 14, 32, 34) Point out that you can go both right and left, and up and down. However, when you want to make a pattern on a hundreds chart, you need to choose a pair of directions, say, right and down, and go only in that direction to make a pattern. Shade 23, then shade the cells diagonally, one down and one right, to illustrate. (34, 45, 56, 67, 78, 89, 100) Ask students to write the shaded numbers in order, from top to bottom, and write on the board:

23   ,   34   ,   45   ,   56   ,   67   ,   78   ,   89   ,   100

SAY: Let's check if this is a pattern. Have students find the gaps between the numbers and help you to fill them in. SAY: All gaps are  $+11$ , so it is a pattern. On the second hundreds chart on BLM Hundreds Charts, have students shade another diagonal pattern that goes right and down, starting from a number of their choice. Have students check what the gaps are. Students will see that they all got another pattern that requires adding 11 each time.

Ask students why they all got gaps of 11 when going 1 cell down and 1 cell right. To prompt students to see the answer, have them recall how they get from any number to the number directly below. SAY: You need to add 10 to get to the number directly below, and then add 1 more to get the number to the right of it. This means that in total, you always add 11.

Point out that to make a pattern that decreases by 11, students need to start at the bottom of the chart and go in the opposite direction. ASK: Do you need to go up and right or up and left? (up and left)

Repeat the whole discussion with patterns that go 1 row down and 1 column left. Conclude that these patterns are made by adding 9 each time—when you go down a row, you add 10, but when you go 1 column to the left, you subtract 1.

**Multiples of 9 on a hundreds chart.** Remind students that multiples of, say, 4 are numbers we say when counting up by 4s starting at 0. So the multiples of 4 are 0, 4, 8, 12, and so on. ASK: What are multiples of 9? (numbers you say when counting up by 9s when starting at 0) Have students write down the multiples of 9 up to 90 and have a volunteer write them on the board. (0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90)

SAY: The word “multiple” reminds us of the word “multiply.” ASK: Do you think there is a connection between multiples and multiplying? (yes) Write on the board:

$$1 \times 9 =$$

$$2 \times 9 =$$

$$3 \times 9 =$$

Have students help you fill in the numbers. Have more volunteers continue the pattern of multiplication. ASK: Are the answers you are writing down multiples of 9? (yes) Explain that multiples of 9 are also numbers that are products of any number—like 1, 2, 3, and so on—multiplied by 9. SAY: Zero is also a multiple of 9. ASK: What number should we multiply by 9 to get zero? (0) Remind students that multiplication is adding the same number again and again, and that zero times any number is zero because you are simply not adding any numbers.

ASK: Where on a hundreds chart are all multiples of 9? Ask students to shade all the multiples of 9 on the third hundreds chart, if they have not done so already. Ask students to describe where the multiples of 9 are located on a hundreds chart. (diagonally, from 9 down and left)

**Skip counting forwards by 5s on a hundreds chart.** Write “3, 8, 13, 18, 23” on the board. Ask students to describe this pattern. (start at 3 and add 5 each time) Explain that another way to describe this pattern is “Start at 3 and skip count forwards by 5s.” Point out that these numbers are not the multiples of 5, because you started at a different number, not at 0, and not at 5. Ask students to shade the numbers in the pattern on the fourth hundreds chart, and have a volunteer do that on a fresh hundreds chart on the board. Have them continue for a few more numbers. Ask students to describe the location of the shaded numbers on the chart. (they are all in two columns, the 3<sup>rd</sup> and the 8<sup>th</sup> column) ASK: What do you know about the ones digits of numbers in the same column of a hundreds chart? (they are all the same) What do you notice about the ones digits of the numbers in this pattern? (they make a pattern: 3, 8, repeat)

Have a volunteer write out the tens digits of the numbers in the pattern 3, 8, 13, 18, 23. You might need to remind students that a one-digit number has a tens digit of 0. The pattern in the tens digits is 0, 0, 1, 1, 2, 2, and so on. Remind students that such patterns repeat and grow at the same time: you repeat the number once, then add 1, then repeat the number again. ASK: Did we skip any numbers in the tens digits? (no) If I continue to count up by 5s in this pattern, will I eventually say the number 93? (yes) How do you know? (you will say 90, because we do not skip numbers in the tens digit, and you will say 3 or 8 in the ones digit) Will I eventually say the number 87? (no) Why not? (the ones digits are 3 and 8, never 7) Do you think this pattern will continue beyond 100? (yes) What number will you say after 93? (98) After 98? (103) Have students continue writing out a few more numbers in the pattern. ASK: Will you eventually say 139? (no) Why not? (the ones digit does not fit the pattern) Will you eventually say 763? (yes) How do you know? (we say all numbers with ones digits of 3 and 8)

### Exercises

- Skip count forwards by 5s starting at 1. Write the first 10 numbers of the pattern.
- Describe the pattern in the ones digits.
- Describe the pattern in the tens digits.
- Circle the numbers you say in the pattern if you count long enough.

71    86    90    91    99    101    105    106    276    394

**Bonus:** 996    1000

**Answers:** a) 1, 6, 11, 16, 21, 26, 31, 36, 41, 46; b) 1, 6, repeat; c) 0, 0, 1, 1, 2, 2, repeat a number, then add 1; d) these numbers should be circled: 71, 86, 91, 101, 106, 276, 996

**Skip counting backwards by 5s on a hundreds chart.** Repeat the discussion, this time for counting backwards, using the pattern 99, 94, 89, 84, and so on. You can have students circle or underline the numbers on the same hundreds chart as before, since the patterns also are in two columns. Students should see that the pattern in the ones digits is similar, 9, 4, repeat, and the pattern in the tens digits is a repeating and shrinking pattern, where we say all the numbers two times.

### Exercises

- Skip count backwards by 5s starting at 97. Write the first 10 numbers of the pattern.
- Describe the pattern in the ones digits.
- Describe the pattern in the tens digits.
- Circle the numbers you say in the pattern if you count long enough.

54    37    21    12

**Bonus:** What is the smallest number in this pattern?

**Answers:** a) 97, 92, 87, 82, 77, 72, 67, 62, 57, 52; b) 7, 2, repeat; c) repeat a number, then subtract 1; d) these numbers should be circled: 37, 12; Bonus: 2

### Missing numbers on a hundreds chart.

#### ACTIVITY 1

- Have students work with a partner. Students use a hundreds chart from BLM Hundreds Chart and a small token that can cover a number on a hundreds chart. Player 1 closes their eyes. Player 2 covers one of the numbers on a hundreds chart. Player 1 opens their eyes and says what number is covered. Players switch roles after each round.

**Identifying patterns on calendars.** Display a calendar or BLM Calendars. Distribute a copy of the BLM to students as well. Discuss how calendars are similar to a hundreds chart, and how they are different. (months start at different days of the week; there are only 7 columns in a calendar, not 10 as on a hundreds chart; the columns in a calendar have labels for weekdays; a calendar can have a different number of days and sometimes a different number of rows)

Shade a row and a column on the calendar and have students describe the pattern in the row and the pattern in the column. ASK: Why are the gaps in the row 1? (a calendar is made by writing the numbers in order, each next number to the right of the previous one, until a row ends, similar to a hundreds chart) Why are the gaps in the column 7? (there are 7 columns on a calendar, so you need to add 7 to get to the number directly below) Have students shade a column and a row on the first calendar on the BLM and check that the patterns are as discussed.

Have students make a list of the multiples of 7 and find them on a calendar. The multiples of 7 are always in one column. Have students check that in all four calendars on the BLM. Then have students make a list of the multiples of 6 and multiples of 8. Have them describe the locations of these multiples on different calendars. Students should notice that these multiples are located diagonally on a calendar, though sometimes the diagonal breaks and starts over at the other side of the calendar. Have students explain why multiples of 8 go 1 row down and 1 column to the right (add 7 to go 1 row down and add 1 to go 1 column right, so add 8 in total) and why multiples of 6 go 1 row down and 1 column to the left. (add 7 to go 1 row down and subtract 1 to go 1 column left, so add  $7 - 1 = 6$  in total)

## ACTIVITY 2

2. Give students **BLM Empty Calendar**. Students create a calendar for the current month or the next month. They write the dates matching the month, and mark the dates of personal events, such as lessons, chores, and family activities. Encourage students to think of both special events, like birthdays or parties, and recurring events, such as cleaning their room, feeding a pet, going to the library, taking out the garbage, or visiting a relative.

## Extensions

**NOTE:** Students should use BLM Calendars for the extensions.

1. Rob cleans his hamster's cage on November 4<sup>th</sup>, and every 4<sup>th</sup> day after that. How many times does he clean the cage in November?

**Answer:** 7 times

2. Liz gets her allowance of 2 dollars every Monday. How much money does she get in November?

**Answer:** 8 dollars



- Ivan brings in firewood every 4<sup>th</sup> day, starting on December 4<sup>th</sup>. He goes ice fishing every 6<sup>th</sup> day starting on December 6<sup>th</sup>. What days in December does he do both?

**Answer:** December 12<sup>th</sup> and 24<sup>th</sup>

- On any calendar, draw a box around 4 days as shown below.


Add the numbers in the diagonal cells in the box. What do you notice about the sums? Try this 3 more times. Try a different month. Explain what happens and why.

**Solution:** The numbers always add to the same sum. The sum depends on the location of the box. Imagine placing counters in the cells, so that we have as many counters in each cell as the number written in it. For example, if we have a table, as shown below, we place 2 counters in the top-left cell, 3 counters in the top-right cell, and so on.

2	3
9	10

Let's place 2 and 10 counters in the shaded cells. We now have 12 counters to place in the unshaded cells. We need to place  $2 + 1$  counter in the top-right cell, because that number is always 1 more than the number in the top-left shaded cell. This means we have 1 counter less than in the bottom-right cell, but that is exactly the number we need in the bottom-left cell, because it is always 1 less than the number in the bottom-right. This means we need the same number of counters for both the shaded cells and the unshaded cells.

## CONNECTION

### Calendars of the world

- Explain to students that different calendars are used in many parts of the world. Have students pick a calendar from another part of the world and find more information about it. They can make a poster and report their findings. Questions to consider: How many months are in the year? Is the number of months the same every year? How long is the year? How long are the months? What defines the months and the year (movement of the sun, the moon, or anything else)? Is there a leap year? What is a leap year—an additional day or an additional month? How often does a leap year happen?

# PA3-16 Equal and Not Equal

Pages 40–41

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

equal  
equal sign (=)  
**equation**  
**false**  
**not equal**  
**not equal sign ( $\neq$ )**  
number sentence  
value

## Goals

Students will learn that an equation is a number sentence with an equal sign.  
Students will identify when two simple expressions are equal or not equal and when an equation is true or false.

## PRIOR KNOWLEDGE REQUIRED

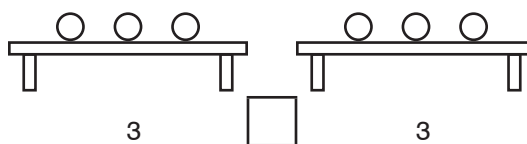
Can add and subtract two numbers within 100  
Can multiply and divide two numbers involving factors no larger than 10  
Understands what a number sentence is

## MATERIALS

ball (optional)  
2 small tables or desks (or 1 table or desk with a dividing line made from masking tape)  
about 12 identical objects to use as counters

**Mental math minute.** Ask students to solve multiplication questions within the range of  $1 \times 1$  to  $5 \times 5$  and corresponding division questions. For each number, go through the questions in order, such as  $1 \times 3$ ,  $3 \div 1$ ,  $2 \times 3$ ,  $6 \div 3$ , and so on to  $5 \times 3$  and  $15 \div 3$ . Then progress to a different number. Next try questions out of order, but keep multiplication and corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

**Introduce equal sides.** Place two small tables or empty desks (or one table with a dividing line) at the front of the class. Place three counters on the left table and three counters on the right table. Point to the left table and ASK: How many counters are on this side? (3) Point to the table on the right and ASK: How many are on this side? (3) Do we have the same number on both sides? (yes) Draw on the board:

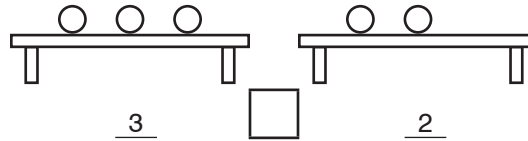


SAY: Because the left side and the right side have the same number, we can say the left side and right side are equal. We write an equal sign to show that the amount on the left equals the amount on the right. Have a volunteer draw an equal sign in the box on the board.

**Introduce unequal sides.** Now place three counters on the left table and two counters on the right table. As before, ask students to identify the number of counters on each side. ASK: Do we have the same number

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on both sides? (no) Modify the picture on the board to create the one shown below:



SAY: Because the left side and the right side do not have the same number, we say the left side and right side are *not equal*. Mathematicians have a special sign to show when two amounts are not equal. Write “ $\neq$ ” in the box.

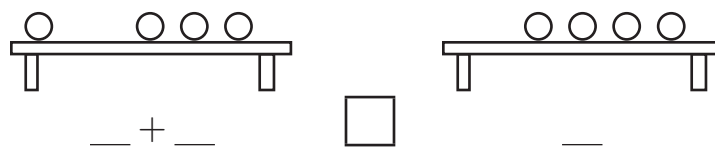
SAY: The *not equal sign* looks like a crossed out equal sign.

Draw several similar pictures on the board, one at a time, with equal and not equal sides. For example: 2 and 2 (equal), 5 and 4 (not equal), 1 and 3 (not equal), 5 and 5 (equal). For each picture, ask students to signal thumbs up for “equal” or thumbs down for “not equal” and then have a volunteer write the correct sign in the box.

**Identifying equal and unequal sides that include addition.** Turn back to the tables at the front of the class and place four counters on the left table in two groups, and four counters on the right table in one group, as shown below:



Point to the left table and ASK: How many counters are in the first group? (1) How many are in the second group? (3) How many counters in total are on the left table? (4) PROMPT: What is  $1 + 3$ ? (4) Point to the table on the right and ASK: How many counters are on the right table? (4) Do we have the same number on both sides? (yes) Draw on the board:



Have students signal which numbers belong in the blanks. (1, 3, 4) ASK: Are the two sides equal? (yes) What sign can we put in the box? (equal sign,  $=$ ) Have a volunteer write an equal sign in the box. Repeat with the following examples, using only a picture on the board and not physical counters on tables; ask students to signal the numbers for the blanks and either thumbs up for “equal” or thumbs down for “not equal.” Examples:  $4 + 1$  and 4 (not equal),  $3 + 2$  and 5 (equal), 6 and  $4 + 2$  (equal), 3 and  $4 + 1$  (not equal).

**Identifying correct and incorrect addition sentences.** Write on the board:

$$10 = 9 + 3 \qquad 10 \neq 9 + 3$$

Point to the first addition sentence and ASK: Is 10 the same as  $9 + 3$ ? (no) PROMPTS: What is  $9 + 3$ ? (12) Is 10 the same as 12? (no) ASK: Is this addition sentence correct? (no) Point to the second addition sentence and

ASK: Is this addition sentence correct? (yes) PROMPT: Is 10 different from  $9 + 3$ ? (yes) Point to each addition sentence and ASK: So, is it correct to say 10 is equal to  $9 + 3$ ? (no) Is it correct to say 10 is not equal to  $9 + 3$ ? (yes) Circle " $10 \neq 9 + 3$ ."

Write on the board:

$$8 + 5 = 13 \qquad 8 + 5 \neq 13$$

Point to the first addition sentence and ASK: Is  $8 + 5$  the same as 13? (yes) PROMPT: What is  $8 + 5$ ? (13) ASK: Is this addition sentence correct? (yes) Then point to the second addition sentence and ASK: Is this addition sentence correct? (no) PROMPT: Is  $8 + 5$  different from 13? Point to each sentence in turn and ASK: So, is it correct to say  $8 + 5$  is equal to 13? (yes) Is it correct to say  $8 + 5$  is not equal to 13? (no) Circle " $8 + 5 = 13$ ."

**Exercises:** Circle the correct addition sentence in the pair.

- |                  |                  |                  |
|------------------|------------------|------------------|
| a) $15 = 13 + 2$ | b) $11 + 2 = 15$ | c) $21 + 3 = 25$ |
| $15 \neq 13 + 2$ | $11 + 2 \neq 15$ | $21 + 3 \neq 25$ |

**Bonus**

- |                   |                      |
|-------------------|----------------------|
| d) $21 + 34 = 55$ | e) $513 + 201 = 724$ |
| $21 + 34 \neq 55$ | $513 + 201 \neq 724$ |

**Answers:** a)  $15 = 13 + 2$ , b)  $11 + 2 \neq 15$ , c)  $21 + 3 \neq 25$ ,  
Bonus: d)  $21 + 34 = 55$ , e)  $513 + 201 \neq 724$

**Introduce the word "equation."** SAY: A number sentence that has an equal sign is called an *equation*. Write on the board:

$$4 + 5 = 9$$

Have a volunteer read the number sentence: "Four plus five equals nine." ASK: Does this number sentence have an equal sign? (yes) So, is this number sentence called an equation? (yes) Point to the equal sign and SAY: The equal sign tells you that the part of the number sentence on the left side of the equal sign,  $4 + 5$  (point to  $4 + 5$ ), has the same value as the part of the number sentence on the right side of the equal sign, 9 (point to the 9).

Write the words "equal" and "equation" on the board. ASK: How many starting letters do these two words have in common? Have students signal the answer. (4) Underline the common starting letters on the board, as shown below:

equal                  equation

**Identifying equations.** Write on the board:

$$5 + 3 < 11 \qquad 16 - 2 \neq 19 \qquad 3 \times 4 = 12$$

Point to the first number sentence and ASK: Is this number sentence an equation? (no) Why not? (because it does not have an equal sign) Point to the next number sentence and repeat the questions. (no, because it does

not have an equal sign) Emphasize that this number sentence has a “not equal” sign, which is different from an equal sign. Point to the final number sentence and repeat the questions. (yes, because it has an equal sign)

**Exercises:** Circle the number sentences that are equations.

- A.  $8 \times 6 \neq 50$       B.  $35 + 2 < 40$       C.  $23 - 4 = 19$   
 D.  $9 = 72 \div 8$       E.  $100 > 42$       F.  $25 \neq 30 - 4$

**Answers:** C, D

**Equations can be true or false.** SAY: When something is not true, we can say that it is *false*. For example, “pigs can fly” is a false sentence. Write on the board:

$$3 + 4 = 10$$

ASK: Is this number sentence correct? (no) PROMPT: Is  $3 + 4$  the same amount as 10? (no,  $3 + 4$  is 7) ASK: Is this number sentence an equation? (yes) PROMPT: Does the number sentence have an equal sign? (yes) SAY: Even though the number sentence is incorrect, or false, it is still called an equation because it has an equal sign. An equation can be true or false.

Write on the board:

$$2 + 3 = 6 \quad 8 + 3 = 11 \quad 5 + 6 = 12$$

SAY: Let’s check each equation to see if it is true or false. Point to the first equation and ASK: Is this equation true or false? (false) PROMPT: What is  $2 + 3$ ? (5) Is 5 the same as 6? (no) Have a volunteer write “F” for false in the blank. Repeat with the remaining equations, writing “T” for true and “F” for false. (T, F)

**Exercises:** Write “T” if the equation is true or “F” if the equation is false.

- a)  $6 + 4 = 12$       b)  $5 + 3 = 6$       c)  $7 + 2 = 9$

**Bonus**

- d)  $4 + 3 = 185$       e)  $20 + 20 = 40$

**Answers:** a) F, b) F, c) T, Bonus: d) F, e) T

Repeat the process with equations that involve multiplication or division. Write on the board:

$$5 \times 7 = 36 \quad 18 \div 3 = 6 \quad 15 \div 4 = 12$$

Point to the first equation and ASK: Is this equation true or false? (false) PROMPT: What is  $5 \times 7$ ? (35) Is 35 the same as 36? (no) Have a volunteer write “F” for false in the blank. Repeat with the remaining equations. (T, F)

**Exercises:** Write “T” if the equation is true or “F” if the equation is false.

- a)  $26 - 14 = 12$       b)  $9 + 3 = 6$       c)  $15 - 4 = 19$   
 d)  $15 \div 3 = 18$       e)  $24 \div 3 = 8$       f)  $6 \times 9 = 54$

### Bonus

- g)  $14 + 16 = 3 \times 10$  \_\_\_\_ h)  $25 \div 5 = 5 + 1$  \_\_\_\_  
i)  $18 - 12 = 48 \div 8$  \_\_\_\_

**Answers:** a) T, b) F, c) F, d) F, e) T, f) T, Bonus: g) T, h) F, i) T

### Extensions

1. Which number sentence in the pair is an equation?

- a)  $15 + 9 = 8 \times 7$        $24 - 13 > 81 \div 9$   
b)  $34 - 25 \neq 3 + 7$        $35 = 7 + 21 + 8$   
c)  $25 - 19 \neq 60 \div 10$        $7 \times 3 = 40 - 19$   
d)  $600 - 1 < 999$        $10 + 2 = 14 - 3$

**Answers:** a)  $15 + 9 = 8 \times 7$ , b)  $35 = 7 + 21 + 8$ , c)  $7 \times 3 = 40 - 19$ ,  
d)  $10 + 2 = 14 - 3$

2. For each part of Extension 1, which number sentence is correct?

**Answers:** a)  $24 - 13 > 81 \div 9$ , b)  $34 - 25 \neq 3 + 7$ , c)  $7 \times 3 = 40 - 19$ ,  
d)  $600 - 1 < 999$

3. Write “T” if the equation is true or “F” if the equation is false.

- a)  $326 - 214 = 112$  \_\_\_\_  
b)  $189 + 203 = 501 - 109$  \_\_\_\_  
c)  $25 \times 2 = 10 \times 6$  \_\_\_\_  
d)  $321 + 200 + 289 = 990 - 108$  \_\_\_\_  
e)  $15 \div 3 = 583 - 578$  \_\_\_\_  
f)  $9 \times 8 = 801 - 654$  \_\_\_\_

**Answers:** a) T, b) T, c) F, d) F, e) T, f) F

4. Use each of the four signs  $=$ ,  $\neq$ ,  $<$ ,  $>$  once in the boxes below to make all the number sentences true. There are two solutions.

- 8   $3 \times 2$   
7   $5 + 2$   
6   $8 + 4$   
9   $56 \div 7$

**Answers:**  $\neq$ ,  $=$ ,  $<$ ,  $>$ ; or  $>$ ,  $=$ ,  $<$ ,  $\neq$

# PA3-17 Addition Equations

Pages 42–44

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

addend  
equal  
equation  
**fact family**  
**guessing and checking**  
not equal  
number sentence  
**solve**  
**solving**  
sum  
**unknown number**

## Goals

Students will use pictures, guessing and checking, and subtraction to write and solve simple addition equations that include an unknown.

## PRIOR KNOWLEDGE REQUIRED

Can add and subtract within 20 mentally  
Can add and subtract two-digit numbers  
Understands the connection between addition and subtraction

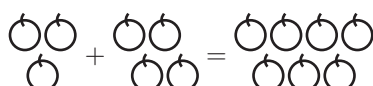
## MATERIALS

ball or relay race baton (optional)  
about 12 identical objects to use as counters  
one small table or desk with a dividing line (made from masking tape, for example)  
cards with plus (+), minus (−), and equal (=) signs  
cardboard box or opaque bag

**NOTE:** Demonstrations throughout this lesson and others in the unit feature apples (to match the pictures in AP Book 3.2). In place of real apples, you could use paper cut-outs of apples, counters, connecting cubes, or any other roughly identical objects.

**Mental math minute.** Arrange students in a line and give them addition problems within 20. Students can pass a ball or a relay race baton to each other, so that the person who receives the baton answers the next question.

**Review equality of two sides.** Set up a table at the front of the class with a dividing line. Place 5 apples on one side of the line and 3 apples on the other side. Show the card with the equal sign and ASK: Are the sides equal? (no) When students say that the sides are not equal, make a point of moving the card with the equal sign away from the demonstration; in other examples, when the sides are equal, place the card upright on the table on top of the dividing line so that students can see it clearly. Repeat with 4 apples on one side and 3 apples on the other, and then with 3 apples arranged differently on both sides. Then repeat with other similar situations, this time placing the apples on one side into two separate piles, with the plus card in between the piles. Draw pictures on the board to model the equality or inequality. For example, you would represent  $3 + 4 = 7$  like this:


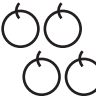


$$\begin{array}{c} \text{Apple} & & \text{Apple} \\ & \diagdown & / \\ & \text{Apple} \end{array} + \begin{array}{cc} \text{Apple} & \text{Apple} \\ \text{Apple} & \text{Apple} \end{array} = \begin{array}{cccc} \text{Apple} & \text{Apple} & \text{Apple} & \text{Apple} \\ \text{Apple} & \text{Apple} & \text{Apple} & \end{array}$$




Remind students that in an addition equation, the numbers you add are called addends and the result of adding the two numbers is called the sum. ASK: What are the addends in this equation? (3 and 4) What is the sum? (7)





**Solving addition equations presented as models.** Have 5 apples ready in a cardboard box or an opaque bag. Show students the box or bag and explain that sometimes we do not know how many apples there are in an addition equation. Place the box or bag on the table on the left side of the dividing line. On the right side, place a group of 2 apples, the card with the plus sign, and then a group of 3 apples to represent the addition  $2 + 3$ . Explain to students that the number of apples is the same on the left side of the dividing line as on the right side. ASK: So, can you tell how many apples are in the box? (yes) How many? (5) Have a volunteer verify the answer by removing the apples from the box and counting them. Repeat with  $5 + 2$  apples on one side and 7 apples in the box on the other side, and then repeat again with similar examples.

When students have mastered this, increase the challenge by making three groupings of apples on one side: for example,  $2 + 4 + 3$  apples on one side and 9 apples in the box on the other side. ASK: Since both sides have the same number of apples, what symbol can we put between the two sides to show this? (equal sign,  $=$ ) Place the card with the equal sign on the dividing line and do a few more examples in which students have to figure out how many apples are in the box.

**Exercises:** How many apples are in the box? Write the number.




a)  +  = 

b)  +  = 

**Bonus:**  +  +  = 

**Answers:** a) 7, b) 9, Bonus: 10

Present the following picture using the table, apples, and cards:

 +  = 

ASK: Can you tell how many apples are in the box? (yes) How many? (4) How did you figure this out? (several solutions are possible: counting up from 3 to 7, subtracting  $7 - 3$ , matching apples in the picture on one side to the other and circling the extra) Repeat with a few more examples, placing the box or bag on different sides of the dividing line. When students have





mastered such questions, increase the challenge by including an extra addend on one side of the equal sign, as shown in the example below:

$$\begin{array}{c} \circ \circ \circ \circ \\ \circ \circ \circ \end{array} + \boxed{\phantom{000}} = \begin{array}{c} \circ \circ \circ \\ \circ \circ \circ \end{array} + \begin{array}{c} \circ \circ \\ \circ \circ \end{array}$$

Guide students to solve examples such as this one by first adding the apples on the right-hand side. ( $6 + 4 = 10$ ) Students can then count up to 10 from 7 to find the missing quantity on the left-hand side of the equal sign ( $7 + 3 = 10$ ) or use subtraction ( $10 - 7 = 3$ ) to find the answer.

**Exercises:** How many apples are in the box? Write the number.

a)  +  $\boxed{\phantom{000}}$  = 

b)  =  $\boxed{\phantom{000}}$  + 

**Bonus**

$$\begin{array}{c} \circ \circ \circ \circ \\ \circ \circ \circ \end{array} + \boxed{\phantom{000}} = \begin{array}{c} \circ \circ \circ \\ \circ \circ \end{array} + \begin{array}{c} \circ \circ \circ \\ \circ \circ \end{array} + \begin{array}{c} \circ \circ \\ \circ \end{array}$$

**Answers:** a) 6, b) 2, Bonus: 6

**Using pictures to represent problems.** Draw attention to the pictures you have drawn on the board to represent the addition equations. Explain that people draw different pictures for different purposes. SAY: In art, we might try to draw apples as realistically as possible. We would pay attention to colour, shape, and other details. ASK: Does colour help us to answer the mathematical problem of how many apples are in the box? (no) Does including leaves on the apples help us to answer the mathematical problem? (no) SAY: These details do not help us to answer mathematical questions, so we do not need to include them. In mathematics, we want to use simple pictures that help us to answer problems but that do not take too much time to draw.

ASK: What should we pay attention to in the pictures we draw to help answer mathematical questions? (the number of objects; creating a picture that is not messy; drawing the objects so they are easy to count; in these specific examples, drawing circles that are about the same size from each other and not bigger than the box so that we are not distracted)

Ask students to copy the pictures from the previous exercises with the apples and complete the pictures by drawing the necessary number of apples in the box. Encourage students to draw circles or large dots for apples.

**Writing addition equations from pictures.** Point out that it is inconvenient to draw apples or circles all the time. ASK: What if you have a box and 79 apples on one side of the line and 125 apples on the other side? What would be more convenient to use than a picture? (numbers)

Remind students that a number sentence is called an equation because it has an equal sign. Draw on the board:



ASK: How can we show this picture as an equation with numbers? (write the number of apples instead of drawing apples) Have students tell you the number for each group as you write the equation, as shown below:

$$4 + \square = 9$$

Explain that the box in the equation with numbers can be smaller since you are not drawing the apples inside, you are just writing the number of apples. SAY: Think of how you have found missing numbers in equations so far. You have counted on from one number to the next, you have used subtraction, or you have matched pictures to show the extra. Here, count up from 4 until you get to 9, or subtract  $9 - 4$ . Pointing to the picture, ASK: How many apples should we draw in the box? (5) Draw the 5 apples in the box. Point to the box underneath in the numerical equation and ASK: What number should we write in this box? (5)

**Exercises:** Draw the missing apples in the box and then write the missing number in the box.

a)

$$4 + \square = 8$$

b)

$$7 + \square = 10$$

**Answers:** a) 4, b) 3

**Drawing pictures to solve addition equations.** Write on the board:

$$6 + \square = 8$$

Explain that there is a number missing in the equation and it is shown by the box. SAY: The missing number is called the *unknown number*, because we don't know what it is right away. I want to draw a picture for the equation that will help find the unknown number. ASK: How many apples should

I draw under the number 6? (6) Draw the 6 apples. Write a plus sign underneath the plus sign of the numerical equation, and then draw a large box under the small box of the numerical equation. ASK: Why should we draw a larger box for the picture? (because we need more space to draw apples) Write an equal sign under the equal sign of the numerical equation, and then ASK: How many apples should I draw for the number 8? (8) The final picture should look like this:

$$\begin{array}{ccccccc}
 6 & + & \boxed{\phantom{00}} & = & 8 \\
 \begin{array}{c} \circ \circ \circ \\ \circ \circ \circ \end{array} & + & \boxed{\phantom{000000}} & = & \begin{array}{c} \circ \circ \circ \circ \circ \\ \circ \circ \circ \end{array}
 \end{array}$$

Point to the large box and ASK: How many apples should I draw here? (2) Point to small box in the equation and ASK: What number should I write here? (2) Write “2” in the box. SAY: We just found the missing number in the equation. Finding the missing number in an equation is called *solving* the equation. When you are asked to *solve* an equation, it means you need to find the missing number in the equation. Remind students that for these drawings they should keep the apples simple: just circles or circles with a small line to show the stem if they wish.

**Exercises:** Draw a picture for the equation. Use your picture to solve the equation.

a)  $7 + \boxed{\phantom{00}} = 9$

b)  $8 = 1 + \boxed{\phantom{00}}$

**Bonus**

c)  $5 + \boxed{\phantom{00}} = 5$

d)  $10 = 10 + \boxed{\phantom{00}}$

**Answers:** a) 2, b) 7, Bonus: c) 0, d) 0

**Using guessing and checking to solve an equation.** Write on the board:

$$8 + \boxed{\phantom{00}} = 17$$

SAY: We can use a picture to solve this equation, but we will need to draw 8 apples on one side, and 17 apples on the other side. That’s a lot of drawing! Let’s try to solve this equation without drawing. ASK: What number do you need to add to 8 to get 17? (9) How do you know? (answers will vary: use doubles,  $8 + 8 = 16$  so  $8 + 9 = 17$ ; count on; use memorized addition facts) Write “9” in the box and ASK: Does  $8 + 9$  equal 17? (yes) So, is the equation true? (yes)

Write on the board:

$$8 + \boxed{\phantom{00}} = 15$$

SAY: I think the missing number is 6. Write “6” in the box and ASK: Is this equation true? (no) Why not? ( $8 + 6 = 14$ , not 15) ASK: Should I try a larger number or a smaller number next? (larger) How do you know? (14 is too small; we need 15, so we need a larger addend) Write “7” in the box and

ASK: Is this equation true? (yes) SAY: The method we are using to solve the equation is called *guessing and checking*. You first try to guess the correct number, then you check if your guess is correct. Use your knowledge of number facts because the closer your guess is to the answer, the better. Although it is best if you guess the correct number right away because you know your number facts.

**Exercises:** Solve by guessing and checking.

- a)  $\square + 4 = 10$       b)  $5 + \square = 11$       c)  $15 = 7 + \square$   
d)  $6 + 8 = \square$       e)  $\square = 7 + 9$       f)  $18 = \square + 9$   
g)  $19 = 10 + \square$       **Bonus:**  $100 = 20 + \square$

**Answers:** a) 6, b) 6, c) 8, d) 14, e) 16, f) 9, g) 9, Bonus: 80

**Introduce fact families.** Draw on the board:



SAY: This is another picture we can draw for addition and subtraction sentences. ASK: How many dark circles do we have? (3) How many light circles do we have? (5) How many circles do we have in total? (8) How do you get 8 from 5 and 3? (add) What addition or subtraction equations can you write for this model, using the total number of circles? ( $3 + 5 = 8$ ,  $5 + 3 = 8$ ,  $8 - 5 = 3$ ,  $8 - 3 = 5$ ) If students need help thinking about subtraction equations, SAY: There are 8 circles. Three of them are dark. ASK: How many light circles do you have? (5) What equation can you write for that problem? ( $8 - 3 = 5$ )

Write all four equations underneath the picture. SAY: These four equations together are called a *fact family*. The fact family shows the addition and the subtraction equations you can write for a picture. You can have the first addend be the number of dark circles (point to  $3 + 5 = 8$ ) or the number of light circles (point to  $5 + 3 = 8$ ). Point to  $8 - 3 = 5$  and ASK: Is this subtraction giving us the number of light circles or the number of dark circles? (the number of light circles) Repeat with  $8 - 5 = 3$ , showing the number of dark circles.

Draw on the board:



Have volunteers come to the board and write the equations forming the fact family for the picture. ( $4 + 6 = 10$ ,  $6 + 4 = 10$ ,  $10 - 6 = 4$ ,  $10 - 4 = 6$ ) Point out the structure of the equations: the parts can come in any order in the addition equations, and the two subtraction equations can have either part as the number that is being subtracted.

**Exercise:** Write the fact family for the picture.



**Answers:**  $3 + 6 = 9$ ,  $6 + 3 = 9$ ,  $9 - 3 = 6$ ,  $9 - 6 = 3$

Reverse the task. Write “ $2 + 1 = 3$ ” on the board and ask students to draw a model for the equation in their notebooks. Have a volunteer draw the model on the board. The model should show two circles of one colour and one circle of another colour. Have students write the rest of the equations in the fact family in their notebooks. ( $1 + 2 = 3$ ,  $3 - 2 = 1$ ,  $3 - 1 = 2$ )

Write “ $4 - 3 = 1$ ” on the board. ASK: How many circles should be in the model for this equation? (4) How do you know? (the total, the largest number in the equation, is 4) Draw 4 circles and ASK: How many circles should I shade? (3 or 1) Does it matter for the fact family if I shade 3 circles or 1 circle? (no) Why not? (in a fact family you will have two subtraction equations, showing how to find the parts; both pictures will produce the same fact family) Shade 1 circle and have students write the fact family for the model in their notebooks. Have a volunteer write the fact family on the board. ( $4 - 3 = 1$ ,  $4 - 1 = 3$ ,  $3 + 1 = 4$ ,  $1 + 3 = 4$ )

**Exercises:** Draw the model for the equation. Write the rest of the equations in the fact family.

a)  $2 + 3 = 5$

b)  $7 - 2 = 5$

**Answers**



$3 + 2 = 5$ ,  $5 - 2 = 3$ ,  $5 - 3 = 2$



$2 + 5 = 7$ ,  $5 + 2 = 7$ ,  $7 - 5 = 2$

**Using subtraction to find the missing addend.** SAY: We can use pictures with circles to solve equations, too. Imagine that some circles are covered by a box. Draw on the board:



SAY: There are 8 circles in total in this picture; you can see 3 of the circles and the rest are hidden in the box. I can write an equation for this picture. Write on the board:

$3 + \square = 8$

ASK: What is the missing number here? (5) How can you get 5 from 8 and 3? (subtract) Can you always subtract when you need to find the missing addend? (yes) Write “5” in the box and have volunteers write the fact family for the equation  $3 + 5 = 8$  on the board. ( $5 + 3 = 8$ ,  $8 - 5 = 3$ ,  $8 - 3 = 5$ )

SAY: For each number in each of these equations you can make a problem

where this is the missing number. For example, there are 8 circles, 3 you can see, and the rest are in the box. How many circles are in the box? To solve this problem, you can write “ $3 + \text{box} = 8$ ” (point to the equation on the board) or you can write “ $8 - 3 = \text{box}$ .” Write “ $8 - 3 = \text{box}$ ” on the board as well.

Point to the two equations with the boxes,  $3 + \text{box} = 8$  and  $8 - 3 = \text{box}$ , and ASK: How are the equations the same? (they describe the same situation or picture, they have the same numbers, in both of them 5 is missing) How are the equations different? (in  $3 + \text{box} = 8$  you need to guess the number, in  $8 - 3 = \text{box}$  you just need to calculate) SAY: For any problem where an addend, meaning the number you add, is missing, you can write a subtraction equation. You just need to subtract the other addend from the total.

$$\square + 2 = 5$$

Write the equation in the margin on the board. SAY: An addend is missing here. ASK: What subtraction equation can I write so that I can find the missing number? ( $5 - 2 = \text{box}$ ) Write “ $5 - 2 = \text{box}$ ” on the board. ASK: What is  $5 - 2$ ? (3) Does writing 3 in the box make the first equation true as well? (yes)

**Exercises:** Write the subtraction equation to find the missing number.

a)  $\square + 4 = 12$       b)  $2 + \square = 11$       c)  $15 = 9 + \square$   
d)  $18 = \square + 9$       **Bonus:**  $100 = 50 + \square$

**Answers:** a)  $12 - 4 = 8$ , b)  $11 - 2 = 9$ , c)  $15 - 9 = 6$ , d)  $18 - 9 = 9$ ,  
Bonus:  $100 - 50 = 50$

SAY: Let’s try this method for larger numbers. Write “ $\square + 36 = 52$ ” on the board. SAY: It would take a lot of time to solve this equation by drawing circles. Let’s write a subtraction equation to find the missing number. Have a volunteer write the subtraction equation ( $52 - 36 = \square$ ) and have students solve it. (16) You may want to remind students of some mental math strategies they learned, such as counting up by 1s to get to 40 and then by 10s to get to 50 and by 1s to get to 52. Write “16” in the blank of the initial equation and have students check that the addition equation is true. Emphasize that checking the answer by doing the addition is important because it allows students to find out if they are correct without depending on anybody to check their answers.

**Exercises:** Write the subtraction equation to find the missing number.

a)  $\square + 43 = 72$       b)  $52 + \square = 99$   
c)  $75 = 9 + \square$       d)  $88 = \square + 79$   
**Bonus:**  $999 = 520 + \square$

**Answers:** a)  $72 - 43 = 29$ , b)  $99 - 52 = 47$ , c)  $75 - 9 = 66$ , d)  $88 - 79 = 9$ ,  
Bonus:  $999 - 520 = 479$

## Extensions

1. Write +, −, or = in each blank to make a true equation.

a)  $5 \underline{\quad} 4 \underline{\quad} 9$

b)  $12 \underline{\quad} 2 \underline{\quad} 10$

c)  $16 \underline{\quad} 20 \underline{\quad} 4$

d)  $35 \underline{\quad} 22 \underline{\quad} 57$

### Bonus

e)  $416 \underline{\quad} 515 \underline{\quad} 99$

f)  $82 \underline{\quad} 12 \underline{\quad} 90 \underline{\quad} 20$

**Answers:** a)  $5 + 4 = 9$ , b)  $12 - 2 = 10$  or  $12 = 2 + 10$ , c)  $16 = 20 - 4$ , d)  $35 + 22 = 57$ , Bonus: e)  $416 = 515 - 99$ , f)  $82 - 12 = 90 - 20$

2. Which part in Extension 1 has two possible answers? Write the two equations.

**Answer:** part b),  $12 - 2 = 10$  and  $12 = 2 + 10$

3. Draw a picture for the equation. Use your picture to solve the equation.

a)  $7 + 2 + \square = 19$

b)  $11 = 1 + 5 + \square$

c)  $3 + 2 + \square = 14$

d)  $13 = 1 + 5 + 3 + \square$

**Selected answers:** a) 10, b) 5, c) 9, d) 4

4. Beth shows an equation using apples and two boxes:

$$\begin{array}{c} \text{Box A} \\ \begin{array}{c} \text{Box A} \end{array} \\ \begin{array}{c} \text{Box B} \end{array} \end{array}$$

The diagram shows an equation: 5 apples + Box A = 6 apples + Box B. The apples are represented by small circles with a stem and leaf.

There are fewer than 10 apples on each side of the equal sign. List the number of apples that could go into Box A and Box B that would make the equation true.

**Answers:** 2, 0; 3, 1; 4, 2

# PA3-18 Subtraction Equations

Pages 45–47

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

difference  
equal  
equation  
fact family  
not equal  
number sentence  
solve  
solving  
sum  
unknown number

## Goals

Students will use pictures, guessing and checking, addition, and subtraction to write and solve simple subtraction equations that include an unknown.

## PRIOR KNOWLEDGE REQUIRED

Can add and subtract within 20 mentally  
Can add and subtract two-digit numbers  
Can solve equations using guessing and checking  
Knows that a box can represent an unknown number  
Can write the equations in a fact family

## MATERIALS

ball  
about 12 identical objects to use as counters  
cardboard box or opaque bag

**NOTE:** Demonstrations at the beginning of this lesson and others in the unit feature apples (to match the pictures in AP Book 3.2). In place of real apples, you could use paper cut-outs of apples, counters, connecting cubes, or any other roughly identical objects.

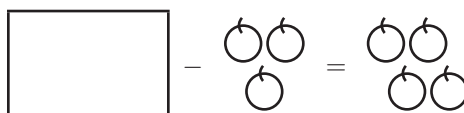
**Mental math minute.** Give students subtraction problems within 20. Toss a ball to a student who you want to answer, and have the student toss the ball back to you as he or she answers the question. Repeat until all students have had a chance to answer a subtraction problem.

**Review vocabulary.** Write on the board:

$$\square - 3 = 4$$

ASK: Is this number sentence an equation? (yes) How do you know? (it has an equal sign) Point to the box and ASK: What does this box stand for? (a missing number or an unknown number) Tell students that you would like to find the missing number. ASK: What is it called when we find the missing number in an equation? (solving the equation)

**Introducing subtraction equations with unknowns.** Place 7 apples in a box so that students do not see how many apples are in the box. Show them the box and SAY: There are some apples in this box, but I won't tell you how many. Remove 3 apples from the box. SAY: Now there are 4 apples left in the box. Draw on the board:


$$\square - 3 = 4$$

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SAY: There were some apples in the box. I took away 3. Now there are 4 apples left in the box. ASK: How can you find how many apples were in the box before you took away 3 apples? Students might suggest guessing and checking, or adding the number of apples that were taken away to the number left over. Ensure both ideas arise from the discussion.

**Solving subtraction equations using guessing and checking.** Point to the equation you wrote on the board and ASK: What could be the number in the box? Suggest 6 as a first guess. Write “6” in the box and SAY: We guess that  $6 - 3 = 4$ . So let’s check our guess. ASK: Does  $6 - 3 = 4$ ? (no,  $6 - 3 = 3$ ) SAY: So, we erase the 6 in the box, mark our first guess of 6 to the side, and cross that 6 out because it does not solve the equation. ASK: Now, what number can you guess next? Students will likely say 7. Write “7” in the box and ASK: Does  $7 - 3 = 4$ ? (yes) SAY: So the missing number is 7. Write “ $\boxed{7} - 3 = 4$ ” on the board to demonstrate that 7 is the unknown number. Emphasize that it is okay to guess the correct number right away; moreover, if students learn the addition and subtraction facts up to  $9 + 9$ , they will be able to say what the answer is right away without guessing. However, they still need to check their answers.

**Exercises:** Solve the equation by guessing and checking.

a)  $\boxed{\phantom{00}} - 3 = 6$

b)  $8 - \boxed{\phantom{00}} = 2$

c)  $9 = \boxed{\phantom{00}} - 3$

d)  $\boxed{\phantom{00}} = 13 - 6$

**Bonus**

e)  $0 = \boxed{\phantom{00}} - 5$

f)  $7 = 7 - \boxed{\phantom{00}}$

**Answers:** a) 9, b) 6, c) 12, d) 7, Bonus: e) 5, f) 0

**Drawing a picture to solve an equation with a missing total.** Write on the board:

$$\boxed{\phantom{00}} - 5 = 6$$

SAY: I want to draw a picture showing this equation, similar to the picture we had at the beginning of the lesson. The first number is the total number of apples, before I took some out, and it is the number we do not know. So let’s draw a box for it. Draw a large box and have students copy the equation and draw a large box in their notebooks.

SAY: The second number shows the apples I took out of the box. We drew them with a minus sign. Draw 5 circles or symbolic apples with a minus sign in front of them. SAY: The last number, the number after the equal sign, shows how many apples are left. Draw the equal sign and 6 apples. Have students do the same. ASK: How many apples were in the box in the beginning? (11) How do you know? (this is the total, the apples you took out plus the apples that are left in the box) Point out that you add the numbers to find the missing total number of apples.

**Exercise:** Draw a picture for the equation  $7 = \square - 5$ .

**Answer**



**Using addition to solve subtraction equations with a missing total.**

ASK: How do you know how many apples to draw in a box? (you add the apples that are left and the apples that were taken out) SAY: When the missing number in an equation is the total number of apples, you use addition to find the missing number. Write on the board:

$$\square - 15 = 6$$

ASK: What quantities do we know in this equation? (the number of apples left in the box and the number of apples taken out) Is the missing number the total? (yes) SAY: Let's add the numbers in the equation. Write " $15 + 6 = \underline{\quad}$ " on the board. ASK: What is  $15 + 6$ ? (21) Write "21" in the blank and in the box. SAY: Let's check if this equation is true. ASK: Does  $21 - 15$  equal 6? (yes) SAY: We checked, so we know we solved the equation correctly.

**Exercises:** Solve the equation by writing an addition sentence. Check your answer.

a)  $\square - 23 = 61$

b)  $\square - 35 = 12$

c)  $9 = \square - 73$

d)  $38 = \square - 46$

**Bonus**

e)  $0 = \square - 125$

f)  $\square - 200 = 777$

**Answers:** a)  $61 + 23 = 84$ , check:  $84 - 23 = 61$ ; b)  $35 + 12 = 47$ , check:  $47 - 35 = 12$ ; c)  $73 + 9 = 82$ , check:  $82 - 73 = 9$ ; d)  $46 + 38 = 84$ , check:  $84 - 46 = 38$ ; Bonus: e)  $0 + 125 = 125$ , check:  $125 - 125 = 0$ ; f)  $777 + 200 = 977$ , check:  $977 - 200 = 777$

**Review fact families.** Draw on the board:



ASK: What addition and subtraction equations can we write for this picture? ( $2 + 3 = 5$ ,  $3 + 2 = 5$ ,  $5 - 2 = 3$ ,  $5 - 3 = 2$ ) Have volunteers write the equations on the board. ASK: What do we call these four equations together? (a fact family)

**Exercise:** Write the fact family for the picture.



**Answer:**  $2 + 6 = 8$ ,  $6 + 2 = 8$ ,  $8 - 2 = 6$ ,  $8 - 6 = 2$

**Using subtraction to solve subtraction equations.** Draw on the board:



SAY: Some circles are hidden in the box. The total number of circles is 7. Write “Total = 7” on the board. SAY: I want to write the equations this picture shows before we figure out how many circles are hidden. ASK: What addition equations did we write for pictures like this in the last lesson? ( $5 + \text{box} = 7$ ,  $\text{box} + 5 = 7$ ) Invite volunteers to write the equations on the board. SAY: I want to write subtraction equations from the addition equations. ASK: What number do we start with? (the total) Write “7 – ” on the board twice, and ASK: What number do we subtract? (5 or the number of circles hidden in the box) Write both options on the board. Point to the first option and ASK: What is this subtraction equal to? (the number of circles hidden in the box) Finish writing the equation. Then have students help you finish writing the second equation. The equations are shown in the margin.

$$\begin{array}{l} 7 - 5 = \square \\ 7 - \square = 5 \end{array} \quad \blacktriangledown$$

ASK: Which of these equations is just a calculation? (the first equation) Which is easier to solve? (the first equation) What is the missing number? (2) Have a volunteer draw the circles in the box in the picture above and check that there are indeed 7 circles in total.

Ask students to look at all four equations with the unknown number. ASK: How are the equations the same? (they describe the same situation, they have the same numbers, in all of them the unknown number is 2) How are the equations different? (in three of them you need to guess the number, in the fourth you just need to calculate) SAY: A subtraction equation shows a situation with a total and one part is subtracted. If the subtracted part is missing, you can just subtract the other part from the total. If you do not know the total, add the parts.

Write “ $12 - \square = 4$ ” on the board. ASK: What is the unknown, the total or a part? (part) What is the total in this situation? (12) What subtraction should you write to find the missing part? ( $12 - 4 = \text{box}$ ) Write that equation underneath the first one. Write “8” in the box for the missing number and ASK: Is this equation true? (yes) SAY: This means we solved the equation correctly.

**Exercises:** Solve the equation by writing the subtraction sentence. Check your answer.

a)  $12 - \square = 9$

b)  $35 - \square = 12$

c)  $9 = 46 - \square$

d)  $38 = 75 - \square$

e)  $35 = 48 - \square$

f)  $100 - \square = 77$

**Bonus:**  $450 - \square = 120$

**Answers:** a)  $12 - 9 = 3$ , b)  $35 - 12 = 23$ , c)  $46 - 9 = 37$ , d)  $75 - 38 = 37$ , e)  $48 - 35 = 13$ , f)  $100 - 77 = 23$ , Bonus:  $450 - 120 = 330$

**Solving different equations.** Emphasize again that if a total is unknown in a subtraction equation, you can add the parts to find the total. If a part is missing, you can subtract the other part from the total to find the unknown number. Remind students to check their answers.

**Exercises:** Solve the equation.

a)  $\square - 37 = 62$

b)  $78 - \square = 2$

c)  $49 = \square - 7$

d)  $\square = 93 - 36$

**Bonus**

e)  $0 = \square - 45$

f)  $700 = 800 - \square$

**Answers:** a)  $62 + 37 = 99$ , b)  $78 - 2 = 76$ , c)  $49 + 7 = 56$ , d)  $93 - 36 = 57$ , Bonus: e)  $0 + 45 = 45$ , f)  $800 - 700 = 100$

## Extensions

1. You can write a question for an equation. Example:

For  $\square - 37 = 62$ , you can ask: What number is 37 more than 62?

Write a question for the equation.

a)  $\square - 17 = 6$

b)  $7 - \square = 2$

c)  $9 = \square - 7$

d)  $\square = 93 - 39$

**Sample answers:** a) What number is 6 more than 17? b) How much more than 2 is 7? c) What number is 7 more than 9? d) How much more than 39 is 93?

2. You can make a story for any equation. Example

For  $\square - 37 = 62$ , you can make a problem: Emma has some stickers. She gives 37 stickers to her brother. She has 62 stickers left. How many stickers did Emma start with?

Write a story for the equation.

a)  $\square - 17 = 6$

b)  $7 - \square = 2$

c)  $9 = \square - 7$

d)  $\square = 93 - 39$

**Sample answers**

- a) Ren has some marbles. He loses 17 of them and has 6 marbles left. How many marbles did he start with?
- b) Alice has 7 dollars. She pays some money for lunch and has 2 dollars left. How much did she pay for lunch?
- c) Eric has some apples. He makes a pie with 7 apples and has 9 apples left. How many apples did he start with?
- d) Jasmin's book has 93 pages. She reads 39 pages. How many pages are left?

3. Make your own subtraction equation and create a story for it.

# PA3-19 Using Letters for Unknown Numbers

Pages 48–49

## CURRICULUM REQUIREMENT

AB: required  
BC: required  
MB: required  
ON: required

## VOCABULARY

equal sign (=)  
equation  
number sentence  
operation  
solve  
solving  
symbol  
unknown number

## Goals

Students will represent an unknown number in an equation with a letter or a symbol.  
Students will solve simple addition and subtraction equations.

## PRIOR KNOWLEDGE REQUIRED

Can add and subtract numbers within 20 mentally  
Can add and subtract two-digit numbers  
Can solve an addition or subtraction equation involving an unknown

## MATERIALS

ball (optional)  
about 12 identical objects to use as counters  
cardboard box

**Mental math minute.** Ask students to solve multiplication questions within the range of  $1 \times 1$  to  $9 \times 9$  and corresponding division questions. For each number, go through the questions in order, such as  $1 \times 3$ ,  $3 \div 3$ ,  $2 \times 3$ ,  $6 \div 3$ , and so on to  $9 \times 3$  and  $27 \div 3$ . Then progress to a different number. Next try questions out of order, but keep multiplication and corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

**Review equations.** Write on the board:

$$\text{a) } \square - 7 = 6 \quad \text{b) } 29 - \square = 12 \quad \text{c) } 30 = \square + 17$$

ASK: What are these number sentences called? (equations) How do you know? (they all have an equal sign) What do the boxes stand for? (unknown numbers) In equation a), is the unknown one of the parts or the total? (total) How do you know? (because you are subtracting from the unknown) How do you find the unknown total? (by adding the parts) What is the unknown number? (13) Have a volunteer write the addition equation ( $13 = 7 + 6$ ) underneath the initial equation, then have students check that 13 is the correct solution to the subtraction equation.

Repeat with the two other equations. In both equations, a part is missing so students will need to subtract to find the number. (b) 17, c) 13

**Using letters to stand for unknown numbers.** ASK: What did we use to show the unknown number in the equation? (a box) SAY: Mathematicians also use letters to stand for the unknown numbers in an equation. Write on the board:

$$\square + 33 = 65 \quad x + 33 = 65$$

SAY: I can write “box + 33 = 65” or I can replace the box with the letter  $x$  and write “ $x + 33 = 65$ .” I can use the letter  $x$  or any other letter. For example, I could use the letter  $m$ . Write on the board:

$$m + 33 = 65$$

Use subtraction to solve this equation. The solution should look like this:

$$\begin{aligned} m + 33 &= 65 \\ 65 - 33 &= 32 \\ m &= 32 \end{aligned}$$

Have students check the solution. SAY: You do not have a box to write the solution in, but you can write the equation again, writing the number instead of the letter or the symbol. Show how to do this:

$$\begin{aligned} \text{Check: } m + 33 &= 65 \\ 32 + 33 &= 65 \end{aligned}$$

ASK: Is this equation true? (yes) Place a checkmark beside the equation. Repeat with the equation  $y - 37 = 9$ . ( $y = 46$ )

Have students solve the following exercises the same way. Remind them to write the final answer with the letter, for example  $x = 3$ .

**Exercises:** Solve the equation.

- |                  |                  |                  |
|------------------|------------------|------------------|
| a) $x + 8 = 11$  | b) $n - 8 = 2$   | c) $7 = y + 6$   |
| d) $9 + n = 16$  | e) $b - 8 = 5$   | f) $7 = x - 6$   |
| g) $11 = m - 28$ | h) $32 - x = 18$ | i) $37 = a + 26$ |

**Bonus**

- |                    |                    |                    |
|--------------------|--------------------|--------------------|
| j) $100 + w = 350$ | k) $799 - u = 799$ | l) $123 = r - 654$ |
|--------------------|--------------------|--------------------|

**Answers:** a)  $x = 3$ , b)  $n = 10$ , c)  $y = 1$ , d)  $n = 7$ , e)  $b = 13$ , f)  $x = 13$ , g)  $m = 39$ , h)  $x = 14$ , i)  $a = 11$ , Bonus: j)  $w = 250$ , k)  $u = 0$ , l)  $r = 777$

**Solving equations that require rewriting before solving.** Explain that sometimes you need to rewrite an equation before you can solve it. Write on the board:

$$13 + 4 + x = 22$$

SAY: Imagine that I have 13 apples, another 4 apples, a box with some apples, and in total there are 22 apples. ASK: How many apples are outside the box? (17) How do you know? ( $13 + 4 = 17$ ) SAY: We need to rewrite this equation before we can properly solve it. Write on the board:

$$\begin{array}{r} 13 + 4 + x = 22 \\ \underbrace{\phantom{13 + 4}}_{17} + x = 22 \end{array}$$

ASK: How can you find the unknown number? (subtract  $22 - 17$ ) What is the unknown number? (5) PROMPT: What number makes the equation true? (5)

$$22 - 17 = 5 \quad \blacktriangleright$$

$$x = 5$$

How do you know? ( $17 + 5$  is 22) Write the solution on the board.  
(see margin)

Repeat with the equation  $35 - a = 23 - 3$ . ( $35 - a = 20$ ,  $35 - 20 = 15$ ,  
 $a = 15$ )

**Exercises:** Rewrite the equation, then solve it.

a)  $6 + 2 + y = 18$       b)  $n - 6 = 15 + 7$       c)  $21 + a = 56 - 20$

**Bonus:**  $n - 9 = 4 \times 5$

**Answers:** a)  $8 + y = 18$ ,  $18 - 8 = 10$ ,  $y = 10$ ; b)  $n - 6 = 22$ ,  $22 + 6 = 28$ ,  
 $n = 28$ ; c)  $21 + a = 36$ ,  $36 - 21 = 15$ ,  $a = 15$ ; Bonus:  $n - 9 = 20$ ,  
 $20 + 9 = 29$ ,  $n = 29$

**Using symbols in equations.** SAY: I can also use other symbols to represent unknown numbers, such as smiley faces, question marks, or any other things that are easy to draw and will not be confusing in an equation. Write on the board:

$$\square + 3 = 22 \quad x + 3 = 22 \quad \text{☺} + 3 = 22 \quad ? + 3 = 22$$

SAY: All these equations are the same. They all have the same solution, the same numbers, and the same operation; the only difference is the symbol they use for the unknown number. We solve equations with symbols exactly the same way we solve equations with boxes or letters. Write on the board:

$$\text{☺} + 3 = 22$$

$$22 - 3 = 19$$

$$\text{☺} = 19$$

SAY: If you use a question mark instead of a smiley face, write a question mark in the last line instead.

### Exercises

1. Rewrite the equation using ☺ instead of the letter.

a)  $x + 8 = 11$       b)  $n - 8 = 2$       c)  $7 = y + 6$

**Answers:** a) ☺ + 8 = 11, b) ☺ - 8 = 2, c) 7 = ☺ + 6

2. Solve the equation.

a)  $11 = \text{☺} - 5$       b)  $12 - \text{☀} = 8$       c)  $17 = \text{☆} + 16$

d)  $61 + \text{△} = 96$       e)  $5 = ? - 38$       f)  $77 = \text{⊗} - 9$

### Bonus

g)  $100 + \text{☆} = 100$       h)  $100 - \text{☺} = 100$       i)  $0 = ? - 654$

**Answers:** a) ☺ = 16, b) ☀ = 4, c) ☆ = 1, d) △ = 35, e) ? = 43,

f) ⊗ = 86, Bonus: g) ☆ = 0, h) ☺ = 0, i) ? = 654

**Discuss how many solutions an equation has.** Write on the board:

$$4 + x = 7$$

Invite a volunteer to solve the equation. ASK: Is there any other number besides 3 that will solve this equation? (no) How do you know? (sample answers:  $x$  should be equal to  $7 - 4 = 3$ , any other number added to 4 will give a different sum—not 7) If students suggest numbers other than 3, check the sum of all answers and point out that to get a sum of 7, there is only one number that can be added to 4.

Write on the board:

$$\text{☺} + 0 = \text{☺}$$

SAY: I have to use the same number for both smiley faces. Can you tell me a number that will solve the equation? Have students make suggestions. For each number that students suggest, check by writing the equation, such as  $2 + 0 = 2$ . ASK: Is the equation true? (yes) Does any other number make this equation true? (yes) Point out that this equation is very different from the other equation,  $4 + x = 7$ , because the equation with the two smiley faces can have many solutions, but the other equation can have only one solution.

**Exercises:** How many solutions, one or many, does the equation have? If there is only one, write the solution.

a)  $5 - a = 3$       b)  $25 + \text{☺} = 25$       c)  $\text{☺} - \text{☺} = 0$

**Answers:** a) one,  $a = 2$ ; b) one,  $\text{☺} = 0$ , c) many

## Extensions

1. Nina writes two equations. She uses  $x$  for the unknown number in the first equation and  $y$  for the unknown number in the second equation.

$$x + 6 = 9$$

$$3 + y = 8$$

- a) What does  $x$  stand for? Solve the first equation by guessing and checking. Write your answer as  $x = \underline{\hspace{1cm}}$ .
- b) What does  $y$  stand for? Solve the second equation by guessing and checking. Write your answer as  $y = \underline{\hspace{1cm}}$ .
- c) Which number is larger,  $y$  or  $x$ ?
- d) What is  $x + y$ ? Add the two unknown numbers to find out.
- e) What is  $y - x$ ?

**Answers:** a)  $x = 3$ , b)  $y = 5$ , c)  $y$ , d)  $x + y = 8$ , e)  $y - x = 2$



2. Bill has some apples in a box and 7 apples outside the box. Altogether he has 11 apples. He writes an equation, using the letter  $b$  for the unknown number of apples in his box.

$$b + 7 = 11$$

Rani has 3 apples outside a box and some apples in the box. Altogether she has 12 apples. She writes an equation using the letter  $r$  for the unknown number of apples in her box.

$$3 + r = 12$$

- a) How many apples are in Bill's box? Solve Bill's equation by guessing and checking. Write your answer as  $b = \underline{\hspace{1cm}}$ .
- b) How many apples are in Rani's box? Solve Rani's equation by guessing and checking. Write your answer as  $r = \underline{\hspace{1cm}}$ .
- c) Are there more apples in Bill's box or Rani's box? How many more are there?
- d) How many apples are in both boxes in total?

**Answers:** a)  $b = 4$ ; b)  $r = 9$ ; c) there are more apples in Rani's box, 5;  
d)  $r + b = 9 + 4 = 13$

3. In the equation, the letters  $x$  and  $y$  stand for two unknown numbers. The numbers  $x$  and  $y$  can be different, or they can be the same.

$$x + y = 4$$

Each unknown number is a whole number between 0 and 4. List all pairs of whole numbers that make the equation true.

**Answers:** 0, 4; 1, 3; 2, 2; 3, 1; 4, 0

## Patterns with Increasing Gaps

In some patterns, the gap between the numbers makes a growing pattern.

$\textcircled{+1}$   $\textcircled{+2}$   $\textcircled{+3}$   $\textcircled{+4}$   
1, 2, 4, 7, 11

The next gap is +5. The next number in the pattern is 16.  $\textcircled{+1}$   $\textcircled{+2}$   $\textcircled{+3}$   $\textcircled{+4}$   $\textcircled{+5}$   
1, 2, 4, 7, 11, 16

1. Find the pattern in the gaps. Extend the number pattern.

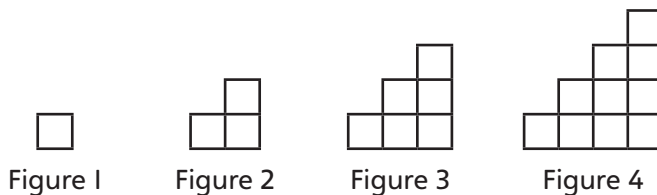
a)  $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   
3, 5, 8, 12, \_\_\_\_\_, \_\_\_\_\_

b)  $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   
4, 5, 7, 10, 14, \_\_\_\_\_, \_\_\_\_\_

c)  $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   
6, 8, 12, 18, 26, \_\_\_\_\_, \_\_\_\_\_

d)  $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   $\textcircled{\phantom{0}}$   
10, 13, 18, 25, \_\_\_\_\_, \_\_\_\_\_

2. a) Complete the T-table for Figure 3 and Figure 4.



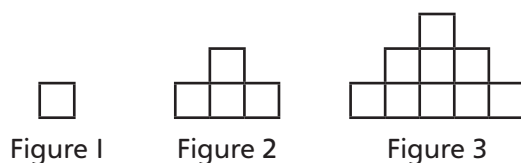
b) Write the number of squares added each time in the circles.

c) Use the pattern in the circles to find the number of squares in Figure 5 and Figure 6.

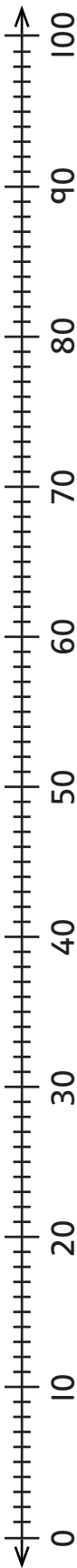
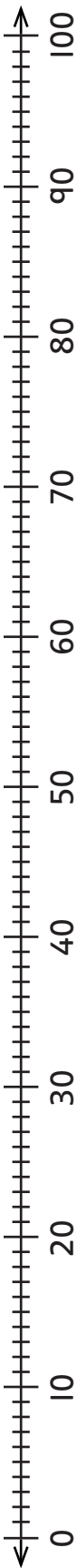
Figure Number	Number of Squares
1	1
2	3
3	
4	
5	
6	



3. Make a T-table to predict how many squares are needed for Figure 5 in the pattern. Draw Figure 4 and Figure 5 to check your answer.

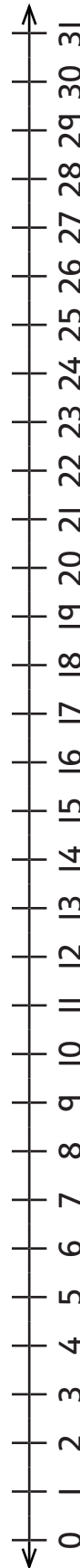
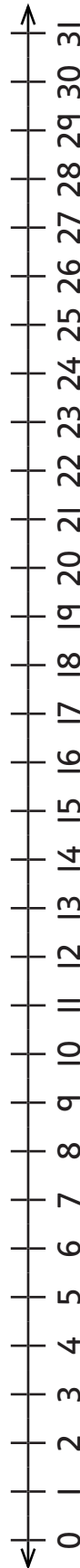
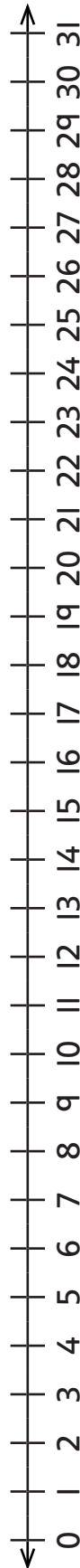
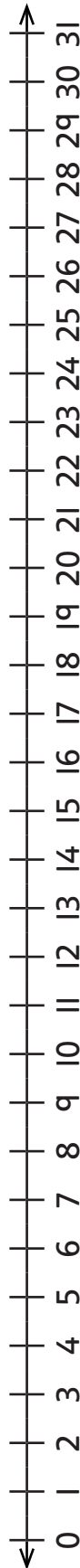
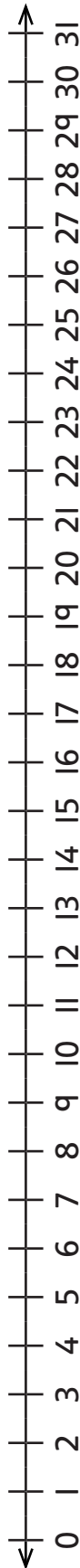
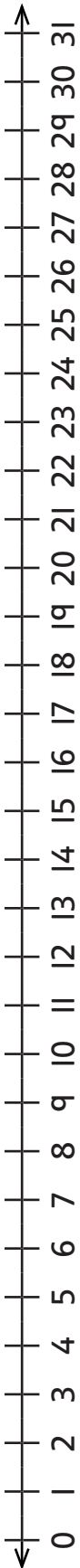


# Number Lines to 100



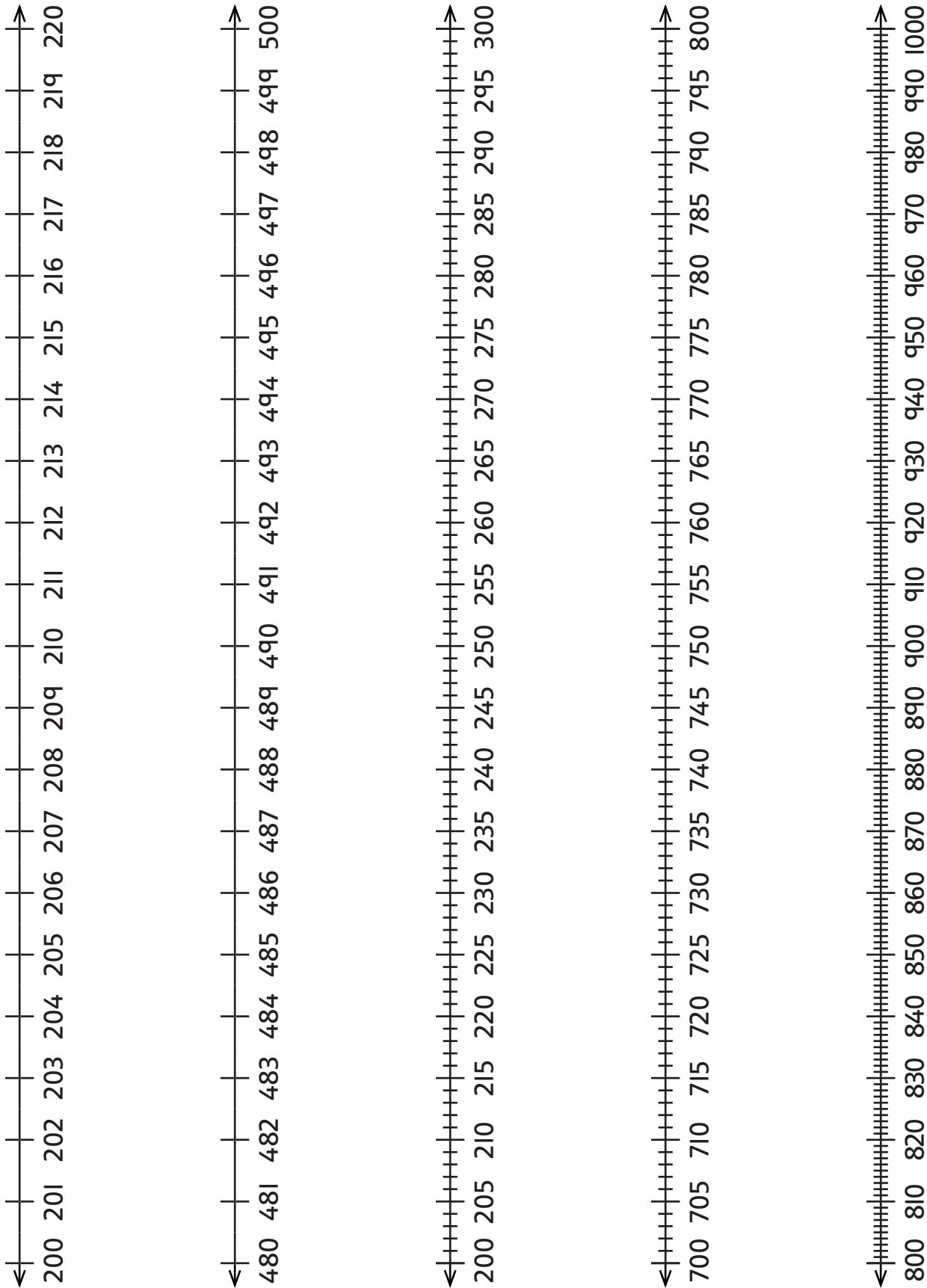
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# Number Lines



# Number Lines with Large Numbers

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# Hundreds Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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# Calendars

December

Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

February

Sun	Mon	Tue	Wed	Thu	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28		

November

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

January

Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

# Empty Calendar

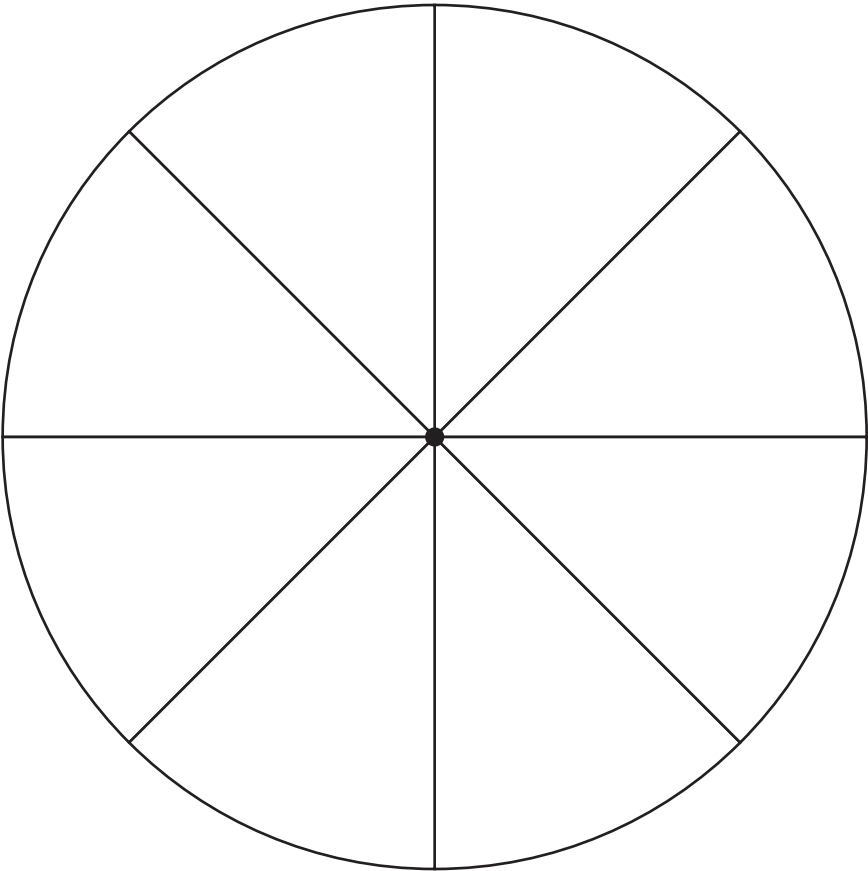
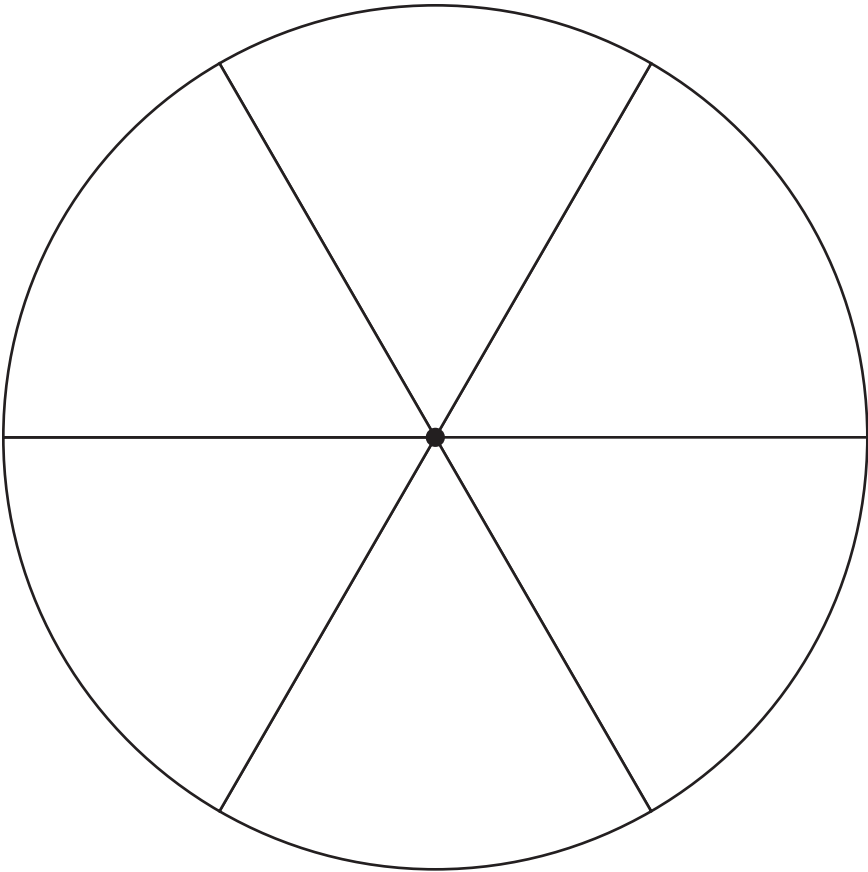
Month: \_\_\_\_\_

Sunday							
Monday							
Tuesday							
Wednesday							
Thursday							
Friday							
Saturday							

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# Empty Spinners



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NAME \_\_\_\_\_ DATE \_\_\_\_\_

## Multiplication Chain (I)

$1 \times 5$

9

$2 \times 5$

12

$1 \times 3$

3

$2 \times 3$

4

$1 \times 1$

12

$2 \times 1$

16

$1 \times 4$

6

$2 \times 4$

8

$1 \times 2$

15

$2 \times 2$

20

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NAME \_\_\_\_\_ DATE \_\_\_\_\_

## Multiplication Chain (2)

$3 \times 5$

15

$4 \times 5$

2

$3 \times 3$

5

$4 \times 3$

5

$3 \times 1$

20

$4 \times 1$

3

$3 \times 4$

10

$4 \times 4$

1

$3 \times 2$

25

$4 \times 2$

4

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NAME \_\_\_\_\_ DATE \_\_\_\_\_

## Multiplication Chain (3)

$$5 \times 5$$

$$4$$

$$5 \times 3$$

$$10$$

$$5 \times 1$$

$$6$$

$$5 \times 4$$

$$2$$

$$5 \times 2$$

$$8$$

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NAME \_\_\_\_\_ DATE \_\_\_\_\_

## Multiplication Chain (4)

$1 \times 6$

$42$

$2 \times 6$

$35$

$6 \times 6$

$7$

$6 \times 5$

$7$

$1 \times 7$

$35$

$2 \times 7$

$42$

$6 \times 7$

$24$

$7 \times 7$

$30$

$7 \times 6$

$6$

$7 \times 5$

$12$

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NAME \_\_\_\_\_ DATE \_\_\_\_\_

## Multiplication Chain (5)

$3 \times 6$

$28$

$4 \times 6$

$21$

$6 \times 4$

$14$

$6 \times 3$

$21$

$3 \times 7$

$6$

$4 \times 7$

$12$

$6 \times 1$

$36$

$7 \times 1$

$49$

$7 \times 4$

$18$

$7 \times 3$

$24$

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NAME \_\_\_\_\_ DATE \_\_\_\_\_

## Multiplication Chain (6)

$$5 \times 6$$

$$14$$

$$6 \times 2$$

$$28$$

$$5 \times 7$$

$$18$$

$$7 \times 2$$

$$30$$

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