

Unit 3 Number Sense: Multiplying Whole Numbers

Introduction

This unit furthers understanding of multiplication of whole numbers using:

- pictures,
- mental math strategies,
- base ten models,
- expanded form,
- the standard algorithm, and
- estimation.

Meeting Your Curriculum

ALBERTA		
Required	NS5-14 to 18, 20, 21	
Optional	NS5-19, 22	
BRITISH COLUMBIA		
Required	NS5-14 to 22	including Extension 1 in NS5-14
MANITOBA		
Required	NS5-14 to 22	including Extension 1 in NS5-19
ONTARIO		
Required	NS5-15 to 18, 20, 21	
Recommended	NS5-14	
Optional	NS5-19, 22	

Mental Math Minutes

The mental math minutes in this unit:

- use strategies to multiply single-digit numbers by multiples of 10 and 100
- practise doubling and halving

Generic BLMs

The Generic BLM used in this unit is:

BLM 1 cm Grid Paper (p. I-3)

This BLM can be found in Section I.

Assessment

The lessons covered by a quiz or test are as follows:

	AB	BC	MB	ON
Quiz	NS5-14 to 16	NS5-14 to 16	NS5-14 to 16	NS5-14 to 16
Quiz	NS5-17, 18	NS5-17 to 19	NS5-17 to 19	NS5-17, 18
Quiz	NS5-20, 21	NS5-20 to 22	NS5-20 to 22	NS5-20, 21
Test	NS5-14 to 18, 20, 21	NS5-14 to 22	NS5-14 to 22	NS5-15 to 18, 20, 21

NS5-14 Introduction to Multiplication

Pages 47–50

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: recommended

VOCABULARY

adding on
addition
array
column
horizontal
multiplication
product
row
vertical

Goals

Students will write a product as repeated addition.
Students will recognize the commutative property of multiplication.
Students will write a product for a given array.
Students will find products by adding on to smaller products.

PRIOR KNOWLEDGE REQUIRED

Can add strings of single-digit numbers
Can add two-digit numbers
Can multiply one-digit numbers
Can represent multiplication different ways

MATERIALS

BLM Filling a Blank Multiplication Chart (p. D-53)
grid paper or **BLM 1 cm Grid Paper** (p. I-3)
calculators
BLM Multiplication Charts (pp. D-54–56)
BLM 10 × 10 Multiplication Chart (p. D-57)
BLM Multiplication Luck (p. D-58, see Extension 1)
6–10 dice per group (see Extension 1)

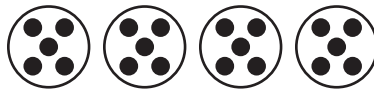
Mental math minute. Give students **BLM Filling a Blank Multiplication Chart**. Have them fill in the chart as much as they can in three minutes. Have students write their name on their chart, then collect them for use in Lesson NS5-21.

Review the concept of multiplication as repeated addition. Write on the board:

$$5 \times 4 = ?$$

Ask students for the answer. (20) ASK: How can we draw a diagram to show that $5 \times 4 = 20$?

Although a diagram such as:



that has 4 groups with 5 items in each group is correct, try to direct students toward a diagram with 5 groups of 4 items in each group, such as:



Students should think of 5×4 as adding five 4s. The first number is the number of times we are adding the second number.

ASK: What addition statement could we write for this diagram? ($4 + 4 + 4 + 4 + 4 = 20$) Tell students that 5×4 is really just a short form for adding five 4s.

Draw the following diagram (which students may have already suggested for the previous question) on the board:



ASK: What addition statement could we write for this diagram? ($5 + 5 + 5 + 5 = 20$) Remind students that this is a way of adding four 5s to get 20.

ASK: What multiplication statement can you write for $5 + 5 + 5 + 5 = 20$? Although $5 \times 4 = 20$ is correct, direct students toward $4 \times 5 = 20$ instead. They should think of 4×5 as adding four 5s.

Exercises: Write each multiplication statement as an addition statement.

a) $5 \times 3 = 15$ b) $4 \times 7 = 28$ c) $5 \times 1 = 10$ d) $3 \times 0 = 0$

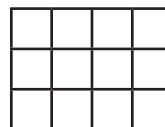
Answers: a) $3 + 3 + 3 + 3 + 3 = 15$, b) $7 + 7 + 7 + 7 = 28$,
c) $1 + 1 + 1 + 1 + 1 = 5$, d) $0 + 0 + 0 = 0$

Writing a product for a given array. Draw the following diagrams on the board:

Using dots:

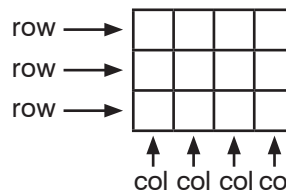
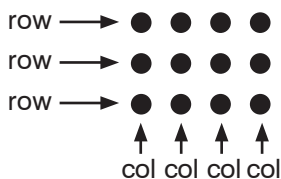


Using squares:



ASK: What product is represented by each diagram?

If students are having difficulty, have them count the number of rows and number of columns. Remind them that rows are horizontal, while columns are vertical.

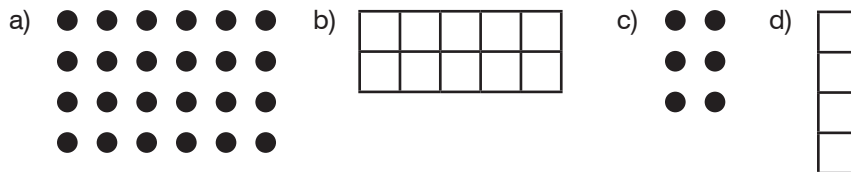


Tell students that the product is created by writing the number of rows first, then the number of columns. Write on the board:

product = rows \times columns

ASK: How many rows are there? (3) How many columns are there? (4)
SAY: We write the product as 3×4 .

Exercises: Write a product for the diagram.

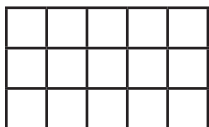


NOTE: You do not need to use the word “commutative” with students.

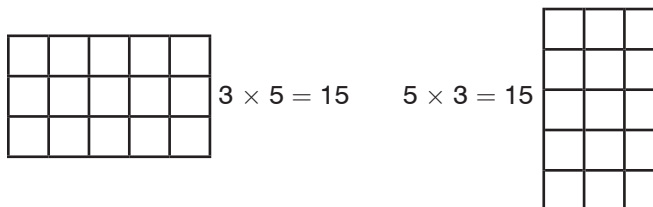
Answers: a) 4×6 , b) 2×5 , c) 3×2 , d) 4×1

Demonstrating the commutative property of multiplication. SAY: In multiplication you can reverse the numbers in a product, and the answer will not change. For example, $5 \times 4 = 4 \times 5$.

Using a ruler, draw a 3 by 5 array on a piece of paper. It should look like this:



Show students the array. ASK: What multiplication is represented by this array? (3×5) Rotate the array 90 degrees left or right. SAY: Now there are 5 rows and 3 columns. ASK: What multiplication should we write for the array now that I have turned it? (5×3) Draw on the board:



SAY: When we rotate the array on the left, the total number of squares doesn't change. So, $3 \times 5 = 5 \times 3$.

Have students draw an array diagram for 4×5 , then for 5×4 on grid paper or **BLM 1 cm Grid Paper**. ASK: Does rotating the diagram on the left give you the diagram on the right? (yes) What can we say about 4×5 and 5×4 ? (they both equal 20)

Ask students for other examples that show this is true. (sample answers: $7 \times 4 = 28$ and $4 \times 7 = 28$, $9 \times 3 = 27$ and $3 \times 9 = 27$)

Bonus: If $13 \times 15 = 195$, what is the answer for 15×13 ? (195)

Have students use their calculators to create similar bonus questions with greater numbers. (sample answers: $234 \times 85 = 19,890$ and $85 \times 234 = ?$)

Have students complete **BLM Multiplication Charts**. (1. b) $3 \times 4 = 12$, c) $5 \times 2 = 10$, d) $7 \times 1 = 7$, e) $1 \times 6 = 6$, f) $2 \times 5 = 10$; 2. a) 6, b) 12, c) 8, d) 10; 3. a) 8, b) 15, c) 8, d) 20; 4. parts a) and c) are the same because $2 \times 4 = 4 \times 2$; 5. a) 14, b) 18, c) 32, d) 35, e) 24, f) 24; 6. b) counting by 3s,

c) 2nd, 3rd, 4th) Students are now ready to complete **Questions 1–5** on AP Book 5.1 pp. 47–48.

Brackets tell us what to do first. Write on the board:

$$3 \times 5 + 2$$

Point out that in this expression there is multiplication and there is addition. SAY: Brackets show us what to do first. Write on the board:

$$(3 \times 5) + 2$$

$$3 \times (5 + 2)$$

Explain that in the expression on the left the brackets tell us to multiply 3×5 first and then add 2 and in the expression on the right they tell us to add $5 + 2$ first and then multiply by 3. Calculate each expression, describing aloud what you do:

$$\begin{aligned}(3 \times 5) + 2 &= 15 + 2 \\ &= 17\end{aligned}$$

$$\begin{aligned}3 \times (5 + 2) &= 3 \times 7 \\ &= 21\end{aligned}$$

NOTE: Students who do not know their multiplication facts can refer to the multiplication chart on **BLM 10 × 10 Multiplication Chart**.



Point out that we get different answers depending on which operation we do first.

Exercises: Do the operation in brackets first and then calculate each expression.

- a) $(2 + 3) \times 4$ and $2 + (3 \times 4)$ b) $7 - (3 \times 2)$ and $(7 - 3) \times 2$

Answers: a) $5 \times 4 = 20$ and $2 + 12 = 14$, b) $7 - 6 = 1$ and $4 \times 2 = 8$

Finding a product by adding on to a smaller product using arrays.

SAY: If we know how to multiply by a smaller number, we can find a larger product by adding on from the smaller product. Have students use arrays to practise representing products as smaller products and sums. Begin by providing an array with blanks, and have volunteers come up and fill in the blanks, as shown below. SAY: Four times five is the same as three times five, plus one more five. Three times five is 15, plus five is 20. So four times five is 20. Remind students that brackets tell us which operation to do first.

$$\begin{array}{c} 4 \times 5 \\ \hline \end{array} \left\{ \begin{array}{c} \bullet \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \bullet \end{array} \right\} \begin{array}{c} 3 \times 5 \\ \hline 15 \\ + 5 \\ \hline 20 \end{array}$$

Exercises: Draw an array to show the equation. You can draw dots or squares.

- a) $3 \times 6 = (2 \times 6) + 6$ b) $5 \times 3 = (4 \times 3) + 3$
c) $3 \times 8 = (2 \times 8) + 8$

Finding a product by adding on to a smaller product without using arrays. Do parts a) and b) of the following exercises as a class, and then let students complete the remainder on their own.

Exercises: Find the product without using arrays.

- a) If $10 \times 2 = 20$, what is 11×2 ? b) If $5 \times 4 = 20$, what is 6×4 ?
 c) If $11 \times 5 = 55$, what is 12×5 ? d) If $8 \times 4 = 32$, what is 9×4 ?
 e) If $6 \times 3 = 18$, what is 7×3 ? f) If $8 \times 2 = 16$, what is 9×2 ?
 g) If $2 \times 7 = 14$, what is 3×7 ?

Answers: a) 22, b) 24, c) 60, d) 36, e) 21, f) 18, g) 21

Finally, have students turn products into a smaller product and a sum without using arrays. Begin by giving students statements with some of the blanks filled in, then move to statements where students must fill in all the blanks themselves.

Exercises

1. Write the product as a smaller product and a sum without using arrays.

- a) $5 \times 8 = 4 \times 8 + \underline{\hspace{1cm}}$ b) $9 \times 4 = \underline{\hspace{1cm}} \times 4 + \underline{\hspace{1cm}}$
 c) $7 \times 4 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$

Answers: a) 8; b) 8, 4; c) 6, 4, 4

2. Write the product as a smaller product and a sum.

- a) 7×6 b) 8×7 c) 9×3

Answers: a) $(6 \times 6) + 6 = 36 + 6 = 42$, b) $(8 \times 6) + 8 = 48 + 8 = 56$,
 c) $(8 \times 3) + 3 = 24 + 3 = 27$

NOTE: Extension 1 is required to cover the British Columbia curriculum.

Extensions

1. **Multiplication Luck.**

Objective: To maximize the score by rolling as many of a unique chosen number as possible on each of 6 turns.

Preparation: Provide each group of 4 or 5 students with 6–10 dice and each student with a score sheet from **BLM Multiplication Luck**.

Instructions: Each turn involves 3 rolls of the dice, and each player takes 6 turns during the game. First, the player rolls all the dice and chooses one number to be a “keeper.” See example on the following page. The keeper does not need to be shown on any of the dice. Any dice showing the keeper are set aside, and the player rolls the remaining dice two more times, setting aside the keepers. Then, on a score sheet, the player records the product of the keeper by the number of dice that show the keeper, and calculates the score. Play then moves to the next person. The game is over when all players have had 6 turns and have filled in every row of their score sheet. Players must choose a different keeper number on each of their turns, and

once a row on a score sheet is filled in, it cannot be changed. Players add their scores for each number to calculate a total score.

For example, suppose a player rolls 1, 2, 4, 4, 4, 5, 6 on the first roll. That player would choose 4 as the keeper, and set aside the 3 dice showing a 4. Only the dice showing 1, 2, 5, and 6 are rolled next, and if any 4s turn up on this second roll, they are also put aside. The dice not showing 4 are rolled a third and final time. Assume the dice show 1, 3, 4, 4, 4, 4, 4 after the third roll. The player would record $4 \times 5 = 20$ in the “4” row on their score sheet. On the remaining five turns, the player cannot choose 4 as the keeper again.

	Multiplication	Score
1		
2		
3		
4	$4 \times 5 = 20$	20
5		
6		
Total		

2. The rows and columns of the times tables have been mixed up. Fill in the missing numbers.

a)

×	7	5	6	
4	28			
		25		10
3				
			12	

b)

×		6	4		7
5	40	30	20		
2				10	
					42
3			12		
			4		

Answers

a)

×	7	5	6	2
4	28	20	24	8
5	35	25	30	10
3	21	15	18	6
2	14	10	12	4

b)

×	8	6	4	5	7
5	40	30	20	25	35
2	16	12	8	10	14
6	64	36	24	30	42
3	24	18	12	15	21
1	8	6	4	5	7

3. Use all the numbers from 1 to 5 to fill in the missing numbers.

$$\begin{array}{r} \square \\ \times \square \\ \hline \square 0 \end{array} \quad \begin{array}{r} \square \\ \times \square 4 \\ \hline \square 2 \end{array}$$

Answers: $4 \times 5 = 20$ or $5 \times 4 = 20$; $3 \times 4 = 12$

4. Use all the numbers from 0 to 9 to fill in the missing numbers.

$$\begin{array}{r} \square \\ \times \square \\ \hline 24 \end{array} \quad \begin{array}{r} \square \\ \times \square 6 \\ \hline 5 \square \end{array} \quad \begin{array}{r} \square \\ \times \square \\ \hline 35 \end{array} \quad \begin{array}{r} \square \\ \times \square 5 \\ \hline 3 \square \end{array} \quad \begin{array}{r} \square 6 \\ \times \square \\ \hline \square 2 \end{array}$$

Answers: $8 \times 3 = 24$ or $3 \times 8 = 24$, $9 \times 6 = 54$, $7 \times 5 = 35$ or $5 \times 7 = 35$, $6 \times 5 = 30$, $6 \times 2 = 12$

5. Tell students that when two operations in an equation are both subtraction, doing them in different orders can produce different answers. (When the two operations are both addition or multiplication, the answers are the same.) Write on the board:

$$(9 - 4) - 3 \qquad 9 - (4 - 3)$$

Demonstrate getting different answers:

$$\begin{array}{r} (9 - 4) - 3 = 5 - 3 \\ = 2 \end{array} \qquad \begin{array}{r} 9 - (4 - 3) = 9 - 1 \\ = 8 \end{array}$$

Calculate the expressions.

a) $(7 - 2) - 1$ and $7 - (2 - 1)$

b) $15 - (10 - 1)$ and $(15 - 10) - 1$

Answers: a) $5 - 1 = 4$ and $7 - 1 = 6$, b) $15 - 9 = 6$ and $5 - 1 = 4$

6. Circle the correct statement.

$$(3 \times 7) + 7 = 3 \times 8 \quad \text{or} \quad (3 \times 7) + 7 = 4 \times 7$$

Answer: $(3 \times 7) + 7 = 4 \times 7$

7. a) For which expressions do you get the same answer whether you add first or multiply first?

$$1 \times 3 + 4 \qquad 3 \times 1 + 4 \qquad 4 + 1 \times 3 \qquad 4 + 3 \times 1$$

- b) Write another expression that produces the same answer whether you add first or multiply first. Check your work.

Answers

- a) $1 \times 3 + 4 = 3 + 4$ or 1×7 , so they give the same answer.
 $3 \times 1 + 4 = 3 + 4$ or 3×5 , so they give different answers.
 $4 + 1 \times 3 = 4 + 3$ or 5×3 , so they give different answers.
 $4 + 3 \times 1 = 4 + 3$ or 7×1 , so they give the same answer.

Sample answers: b) $1 \times 4 + 9$, $8 + 3 \times 1$, $1 \times 7 + 6$, $8 + 9 \times 1$

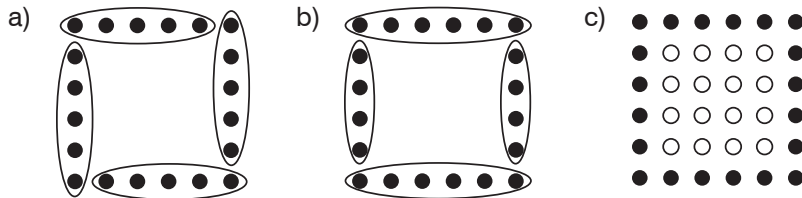
NOTE: Although students do not need to articulate this, in order for adding first or multiplying first to give the same answer, the 1 needs to be on the left or on the right, not in the middle, and it needs to be involved in multiplication only, not addition.

8. Sara counts the dots in different ways. Show Sara's groupings and explain her thinking to fill in the blank.



- a) $4 \times 5 = \underline{\hspace{2cm}}$ dots
b) $(2 \times 6) + (2 \times 4) = 12 + 8 = \underline{\hspace{2cm}}$ dots
c) $(6 \times 6) - (4 \times 4) = 36 - 16 = \underline{\hspace{2cm}}$ dots

Answers



Explanation for part c): There are 6 rows of 6 dots, and 4 rows of 4 white dots. Take away the white dots to get the number of black dots.

NS5-15 Multiplying by Multiples of 10, 100, and 1000

Pages 51–53

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

multiple

Goals

Students will multiply by multiples of 10, 100, and 1000.

PRIOR KNOWLEDGE REQUIRED

Can use base ten materials to represent numbers
Can multiply single-digit numbers

MATERIALS

base ten blocks

Mental math minute—number talk. Present this problem: Double 169. (338)
The following strategies could arise:

$$(2 \times 100) + (2 \times 60) + (2 \times 9)$$

$$(2 \times 160) + (2 \times 9), (2 \times 170) - 2$$

Multiplying one-digit numbers by 10. Give each student 9 ones blocks, 9 tens blocks, and 9 hundreds blocks. ASK: Which block is equal in value to 10 ones blocks? (1 tens block) Remind students:

$$\square = 1$$

$$\text{rod} = 10$$

$$\text{cube} = 100$$

Ask students to take 4 ones blocks. Write on the board:

$$10 \times 4 = 10 \times \square\square\square\square$$

SAY: Since each ones block gets multiplied by 10, replace each ones block by a tens block. Continue writing on the board:

$$10 \times 4 = 10 \times \square\square\square\square = \text{4 rods} = 40$$

Exercises: Use base ten blocks to model the multiplication. Multiply.

a) 10×3

b) 6×10

c) 10×9

Answers: a) 3 tens blocks, 30; b) 6 tens blocks, 60; c) 9 tens blocks, 90

For part b), you may have to remind students that $6 \times 10 = 10 \times 6$.

Write on the board:

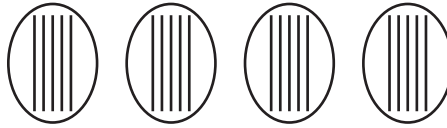
$$10 \times \underline{3} = \underline{30}$$

$$10 \times \underline{6} = \underline{60}$$

$$10 \times \underline{9} = \underline{90}$$

ASK: What pattern is there when we multiply a one-digit number by 10?
(the answer is the number with a 0 after it)

Multiplying one-digit numbers by multiples of 10. Draw on the board:



Explain that the straight lines represent tens blocks. ASK: How many groups are there? (4) How many tens are in each group? (5) What is 5 tens equal to? (50) What multiplication does this show? (4×50) Under the picture draw:

$$4 \times 50 = 4 \times \begin{array}{|l|} \hline \text{|||||} \\ \hline \end{array} = 4 \times \underline{5} \text{ tens}$$

ASK: How many tens are there after we multiply by 4? (20) What number is 20 tens equal to? (200) Repeat for 3×40 .

Exercises: Draw base ten blocks to represent the multiplication. Multiply.

- a) 3×60 b) 7×30 c) 8×40

Answers: a) 3 groups of 6 tens, 180; b) 7 groups of 3 tens, 210;
c) 8 groups of 4 tens, 320

ASK: What shortcut can we use for multiplying a one-digit number by multiples of 10? (multiply the digits that aren't 0, then write a 0 to the right of the number) If students are struggling for the answer, write on the board:

$$\begin{array}{r} \underline{3} \times \underline{60} = \underline{180} \\ \underline{7} \times \underline{30} = \underline{210} \\ \underline{8} \times \underline{40} = \underline{320} \end{array}$$

Multiplying 10 by multiples of 10. Write on the board:

$$10 \times 30$$

ASK: How many tens are in 30? (3) Which block is the same as 10 tens blocks? (a hundreds block) Continue writing on the board:

$$10 \times 30 = 10 \times \begin{array}{|l|} \hline \text{|||} \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} = 300$$

Exercises: Model using base ten blocks. Multiply.

- a) 10×40 b) 80×10 c) 10×60

Answers: a) 10×4 tens = 4 hundreds blocks, 400; b) 10×8 tens = 8 hundreds blocks, 800; c) 10×6 tens = 6 hundred blocks, 600

For part b), you may have to remind students that $80 \times 10 = 10 \times 80$.

ASK: What is a shortcut for multiplying a multiple of 10 by 10? (multiply the digits that are not zero, then write 2 zeros to the right of the answer)
If students are struggling for the answer, write on the board:

$$\underline{10} \times \underline{40} = \underline{400}$$

$$\underline{80} \times \underline{10} = \underline{800}$$

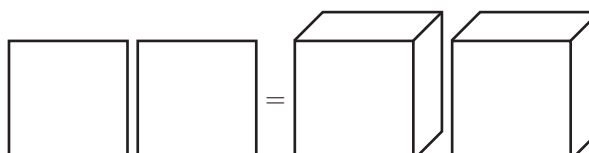
$$\underline{10} \times \underline{60} = \underline{600}$$

Multiplying 10 by multiples of 100. Write on the board:

$$10 \times 200$$

ASK: How many hundreds are in 200? (2) What block is the same as 10 hundreds blocks? (a thousands block)

Draw on the board:

$$10 \times 200 = 10 \times \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} = 2000$$


The diagram illustrates the multiplication of 10 by 200 using base ten blocks. On the left, the equation $10 \times 200 = 10 \times$ is followed by two 10x10 blocks. An equals sign follows, then two 10x10x10 blocks, and finally $= 2000$. The 10x10 blocks represent 200, and the 10x10x10 blocks represent 2000.

Provide students with thousands blocks for the following exercises.

Exercises: Model using base ten blocks. Multiply.

a) 10×800

b) 10×400

c) 10×500

Answers: a) 10×8 hundreds = 8 thousands, 8000; b) 10×4 hundreds = 4 thousands, 4000; c) 10×5 hundreds = 5 thousands, 5000

Multiplying a multi-digit number by multiples of 10. Write on the board:

$$10 \times 3000$$

ASK: How many thousands are in 3000? (3) What is 10 times 3? (30)

SAY: So there are 30 thousands blocks. Complete the multiplication sentence on the board:

$$10 \times 3000 = 30\,000$$

Write on the board:

$$10 \times 14 = 140$$

$$100 \times 14 = 1400$$

$$1000 \times 14 = 14\,000$$

Ask students if they see a pattern. SAY: When you multiply by 10, you write 1 zero to the right of the number, and when you multiply by 100, you write 2 zeros to the right of the number, and when you multiply by 1000, you write 3 zeros.

Exercises: Multiply.

- a) 10×14 b) 100×24 c) 1000×7
d) 1000×52 e) 100×67 f) 1000×43

Bonus

- g) 1000×583 h) 98×1000

Answers: a) 140, b) 2400, c) 7000, d) 52 000, e) 6700, f) 43 000,
Bonus: g) 583 000, h) 98 000

Multiplying without blocks. Write on the board:

$$30 \times 40 =$$

ASK: How many tens are in 30? (3) How many tens are in 40? (4) How much is 3×4 ? (12) Write “12” on the board. SAY: There is a shortcut for answering multiplications in which both numbers end in zero, like 30 times 40. First, we multiply the digits that are not zero and write the answer down, so in this case 3 times 4 equals 12. Then we write all the zeros from the numbers being multiplied. In this example 30 and 40 have two zeros altogether, so we write two zeros after 12 and the answer is 1200. Finish writing “1200” on the board. Do another example, this time multiplying 500 times 50. (25 000)

Exercises: Multiply.

- a) 20×80 b) 30×50 c) 50×600

Answers: a) 1600, b) 1500, c) 30 000

Identifying patterns when multiplying powers of 10. Write on the board:

$$\begin{aligned} 10 \times 10 &= 100 \\ 10 \times 100 &= 1000 \\ 10 \times 1000 &= 10\,000 \end{aligned}$$

ASK: What do you notice about the total number of zeros in the answer of each equation compared to the total number of zeros in the question? (they are the same)

Write on the board:

$$100 \times 1000$$

ASK: How many zeros are in 100? (2) How many zeros are in 1000? (3) How many zeros are there in total? (5) What number starting with 1 has 5 zeros at the end? (100 000) SAY: So, $100 \times 1000 = 100\,000$.

Exercises: Multiply.

- a) 100×100 b) 1000×100
c) $10\,000 \times 100$ d) $100 \times 100\,000$
e) 1000×1000 f) $1\,000\,000 \times 100\,000$

Answers: a) 10 000, b) 100 000, c) 1 000 000, d) 10 000 000, e) 1 000 000, f) 100 000 000 000

Extensions

1. Find the missing number.

a) $\underline{\hspace{2cm}} \times 200 = 60\,000$

b) $\underline{\hspace{2cm}} \times 400 = 80\,000$

c) $\underline{\hspace{2cm}} \times 1000 = 500\,000$

Bonus: $400 \times \underline{\hspace{2cm}} = 2\,400\,000$

Answers: a) 300, b) 200, c) 500, Bonus: 6000

2. Find at least ten answers using multiples of 10 for each question.

a) $\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = 40\,000$

b) $\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = 120\,000$

Sample answers

a) $1 \times 40\,000$, 5×8000 , 8×5000 , $4 \times 10\,000$, 10×4000 , 50×800 ,
 80×500 , 100×400 , 200×200 , 250×160

b) $1 \times 120\,000$, $2 \times 60\,000$, $3 \times 40\,000$, $4 \times 30\,000$, $5 \times 24\,000$,
 $6 \times 20\,000$, $8 \times 15\,000$, $10 \times 12\,000$, $12 \times 10\,000$, 15×8000 ,
 20×6000 , 24×5000 , 25×4800 , 30×4000 , 40×3000 , 48×2500 ,
 50×2400 , 60×2000 , 80×1500 , 100×1200 , 120×1000 , 150×800 ,
 200×600 , 240×500 , 250×480 , 300×400

3. How many dimes are in the dollar amount?

a) \$4

b) \$100

c) \$10 000

Answers: a) 40, b) 1000, c) 100 000

4. Write the number in expanded form.

a) 45 321

b) 1 052 670

Answers: a) $(4 \times 10\,000) + (5 \times 1000) + (3 \times 100) + (2 \times 10) + (1 \times 1)$,

b) $(1 \times 1\,000\,000) + (5 \times 10\,000) + (2 \times 1000) + (6 \times 100) + (7 \times 10)$

5. Circle the correct answer.

$(500 \times 300) + 300 = 600 \times 300$ $(500 \times 300) + 300 = 501 \times 300$

$(500 \times 300) + 300 = 500 \times 600$

Answer: $(500 \times 300) + 300 = 501 \times 300$

NS5-16 Easier Ways to Multiply

Pages 54–56

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

column
product
row

Goals

Students will use doubles and doubling and halving to multiply.

PRIOR KNOWLEDGE REQUIRED

Can double one-digit numbers
Can represent a two-digit number as the sum of tens and ones

MATERIALS

BLM Doubling and Halving (p. D-59)
BLM Using Triples to Multiply (p. D-60, see Extension 4)

Mental math minute. Have students skip count by tens, hundreds, or thousands within 10 000 starting at different numbers.

Using doubles to multiply. SAY: We know that 2×6 is 12. ASK: What is 4×6 ? (24) Point out that 4 is double 2, so 4 sixes is double 2 sixes. Write on the board:

3×7 is _____ 4×8 is _____
So 6×7 is _____ So 8×8 is _____

Have volunteers answer each question successively. (21, 42, 32, 64)
Point out that the first number in the multiplication, in these examples 3 and 4, doubles each time, so the product doubles as well. Whenever either number in the multiplication expression doubles, so does the product.

Review doubling two-digit numbers with a ones digit less than 5.

Remind students that they can double two-digit numbers by doubling the digits separately. Write on the board:

double 23 = double 20 + double 3
double 23 = 40 + 6
double 23 = 46

Exercises: Double the number.

a) 32 b) 41 c) 13 d) 24 e) 52 f) 34 g) 63 h) 54

Answers: a) 64, b) 82, c) 26, d) 48, e) 104, f) 68, g) 126, h) 108

Doubling two-digit numbers with a ones digit of 5 or more. SAY: When the ones digit is 5 or more, regrouping is required. Write on the board:

$36 = 30 + 6$
So the double of 36 is _____ + _____ = _____

To fill in the blanks, ASK: What is double 30? (60) What is double 6? (12)
So, what is double 36? ($60 + 12 = 72$) Write on the board:

3×9 is _____
So 6×9 is _____
So 12×9 is _____
So 24×9 is _____

Have volunteers answer each question successively. (27, 54, 108, 216)

Exercises: Use doubling to solve the problem.

- a) 8×3 is _____, so 16×3 is _____
b) 4×9 is _____, so 8×9 is _____
c) 3×9 is _____, so 3×18 is _____

Bonus

2×13 is _____, so 4×13 is _____, so 8×13 is _____, so 16×13 is _____

Answers: a) 24, 48; b) 36, 72; c) 27, 54; Bonus: 26, 52, 104, 208

Multiplying using pairs that make multiples of 10. Write on the board:

$$5 \times 16 \times 2$$

SAY: It is not straightforward to multiply 5×16 and then double it, but there is a method that makes this question easier. Reorder the numbers and write on the board:

$$5 \times 2 \times 16$$

ASK: What is 5×2 ? (10) What is 10×16 ? (160) Write “= 160” on the board. SAY: Sometimes you can find pairs that make a multiple of 10, like 20. ASK: What pairs multiply to make 20? (4 and 5, 2 and 10) Explain to students that the pair 2 and 10 already has 10 in it. Write on the board:

$$4 \times 23 \times 5$$

Reorder the numbers and continue writing on the board:

$$4 \times 23 \times 5 = 4 \times 5 \times 23$$

ASK: What is 4×5 ? (20) Ask a volunteer to find 20×23 . (460)

Exercises: Multiply by finding a multiple of 10.

- a) $5 \times 25 \times 2$ b) $4 \times 34 \times 5$ c) $2 \times 97 \times 5$ d) $5 \times 52 \times 6$

Answers: a) 250, b) 680, c) 970, d) 1560

Using doubling and halving to find the product. Give each student two arrays from **BLM Doubling and Halving**. Challenge them to move some of the dots in the array on the right so there are 2 rows of 12 by crossing out and redrawing dots, as shown on the following page.



ASK: Did moving some dots change how many there are? (no) Write on the board:

$$4 \times 6 = 2 \times 12$$

Have students find a different product that also equals 4×6 by moving the dots in a different way. ($4 \times 6 = 8 \times 3$)

Draw an array with 5 rows of 6 dots. ASK: If I double the number of rows what do I have to do to the number of columns to keep the total number of dots the same? (cross out half of them)

Exercises: Double the first number and halve the second number to find an equal product. Use the new product to multiply.

- a) 12×20 b) 15×6 c) 23×20 d) 27×4

Bonus: 50×28

Answers: a) $24 \times 10 = 240$, b) $30 \times 3 = 90$, c) $46 \times 10 = 460$, d) $54 \times 2 = 108$, Bonus: $100 \times 14 = 1400$

Extensions

1. Fill in the missing number.

- a) $4 \times 6 \times 12 = \underline{\quad} \times 12 \times 12$
 b) $9 \times 4 \times 12 = \underline{\quad} \times 12 \times 12$
 c) $25 \times 7 \times 35 = \underline{\quad} \times 35 \times 35$
 d) $36 \times 8 \times 48 = \underline{\quad} \times 48 \times 48$
 e) $49 \times 2 \times 14 = \underline{\quad} \times 14 \times 14$

Bonus: $100 \times 510 \times 5100 = \underline{\quad} \times 5100 \times 5100$

Answers: a) 2, b) 3, c) 5, d) 6, e) 7, Bonus: 10

2. Multiply by finding and then adding all the pairs of numbers that add to 10. Hint: The brackets mean find all pairs that add to 10 before multiplying.

- a) $70 \times (2 + 1 + 8 + 9)$
 b) $120 \times (3 + 4 + 5 + 5 + 6 + 7)$

Bonus: $2300 \times (8 + 3 + 1 + 7 + 6 + 2 + 9 + 4)$

Answers

$$\text{a) } 70 \times (2 + 8 + 1 + 9) = 70 \times (10 + 10) = 70 \times 20 = 1400$$

$$\text{b) } 120 \times (3 + 7 + 4 + 6 + 5 + 5) = 120 \times (10 + 10 + 10) = 120 \times 30 = 3600$$

$$\text{Bonus: } 2300 \times (8 + 2 + 3 + 7 + 1 + 9 + 6 + 4) = 2300 \times (10 + 10 + 10 + 10) = 2300 \times 40 = 92\,000$$

3. Multiply by finding and then multiplying all the pairs of numbers that multiply to multiples of 10.

$$\text{a) } 3 \times (2 \times 4 \times 5 \times 5)$$

$$\text{b) } 2 \times (2 \times 3 \times 5 \times 10)$$

$$\text{Bonus: } 45 \times (4 \times 4 \times 5 \times 25)$$

Answers

$$\text{a) } 3 \times (2 \times 5 \times 4 \times 5) = 3 \times (10 \times 20) = 3 \times 200 = 600$$

$$\text{b) } 2 \times (2 \times 5 \times 3 \times 10) = 2 \times (10 \times 30) = 2 \times 300 = 600$$

$$\text{Bonus: } 45 \times (4 \times 5 \times 4 \times 25) = 45 \times (20 \times 100) = 45 \times 2000 = 90\,000$$

4. Teach students to use triples to multiply. For example, 3×7 is 21 and 9 sevens is triple 3 sevens, so 9 sevens is 63. Have students complete

BLM Using Triples to Multiply.

Answers: 1. a) 6, b) 9, c) 15; 2. a) 69, b) 96, c) 93, d) 126, e) 159, f) 123, Bonus: 639 966 396; 3. b) $10 + 7, 30 + 21 = 51$, c) $20 + 6, 60 + 18 = 78$, d) $60 + 24 = 84$, e) $90 + 12 = 102$, f) $150 + 27 = 177$, Bonus: $12\,000 + 900 + 60 + 18 = 12\,978$; 4. a) 8, 24; b) 10, 30; c) 12, 36; d) 18, 54; e) 24, 72; f) 16, 48; 5. a) 21, 63; b) 24, 72; c) 36, 108

NS5-17 Arrays and Multiplication

Pages 57–59

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

array
expanded form
product

Goals

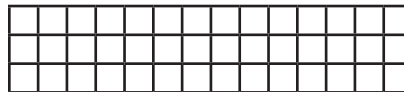
Students will write a product with or without an array using the distributive property.

PRIOR KNOWLEDGE REQUIRED

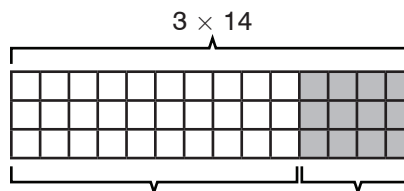
Can write a product for a given array
Can add multi-digit numbers
Can multiply a single-digit number by a multiple of 10 or 100
Can express numbers in expanded form

Mental math minute. Have students multiply one-digit numbers by multiples of 10. Write a list of one-digit numbers and a single multiple of 10 on the board, for example, 6, 9, 5, 3, 7, 8 and 60. The first student multiplies $6 \times 60 = 360$; the next student multiplies $9 \times 60 = 540$. After all the one-digit numbers have been multiplied by 60, repeat with a new multiple of 10.

Using the distributive property to write a product for an array. Draw a 3 by 14 grid on the board:



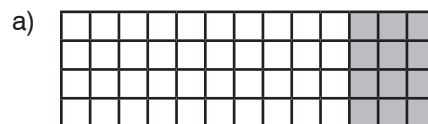
ASK: What is a product that represents this array of squares? (3×14)
Shade the last 4 columns of the array and continue drawing on the board:



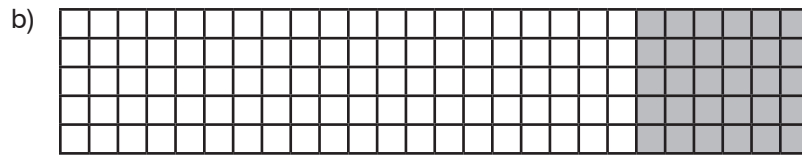
ASK: What products should we write for the unshaded part and the shaded part of the array? (3×10 and 3×4) Did the total number of squares change? (no) So what can we say about the relationship between expressions 3×14 and $(3 \times 10) + (3 \times 4)$? (they are equal) Write on the board:

$$3 \times 14 = (3 \times 10) + (3 \times 4)$$

Exercises: Write a product for the entire diagram, the unshaded part, and the shaded part. Then write an equation for the diagram.



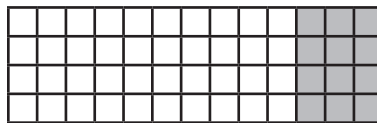
NOTE: Students do not need to know the term “distributive property.”



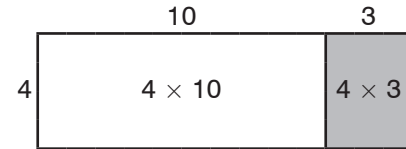
Answers: a) 4×13 , 4×10 , 4×3 , $4 \times 13 = (4 \times 10) + (4 \times 3)$;
 b) 5×26 , 5×20 , 5×6 , $5 \times 26 = (5 \times 20) + (5 \times 6)$

SAY: We don’t have to draw all the squares. Draw on the board:

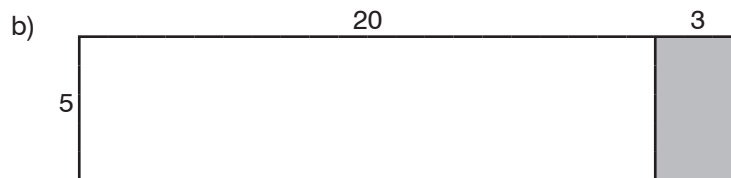
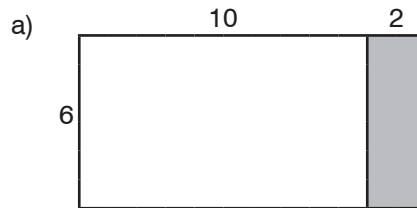
Instead of



We draw



Exercises: Write a product for the entire diagram, the unshaded part, and the shaded part. Then write an equation for the diagram.



Answers: a) 6×12 , 6×10 , 6×2 , $6 \times 12 = (6 \times 10) + (6 \times 2)$;
 b) 5×23 , 5×20 , 5×3 , $5 \times 23 = (5 \times 20) + (5 \times 3)$

Using the distributive property to write a product without using an array. SAY: Let’s look at the answers from our last two exercises. Write on the board:

$$6 \times 12 = (6 \times 10) + (6 \times 2)$$

$$5 \times 23 = (5 \times 20) + (5 \times 3)$$

ASK: What pattern do you see in how the right side of the equation was created? (the first number is shared in each bracket; the second number is broken into a multiple of 10 and a one-digit number) If students have difficulty seeing the pattern, write on the board:

$$\underline{6} \times (\underline{12}) = (\underline{6} \times \underline{10}) + (\underline{6} \times \underline{2})$$

$$\underline{5} \times (\underline{23}) = (\underline{5} \times \underline{20}) + (\underline{5} \times \underline{3})$$

SAY: The products 6×12 and 5×23 are now in expanded form. Point to 6×12 and ASK: What is 6×10 ? (60) What is 6×2 ? (12) What is $60 + 12$? (72) Continue writing on the board:

$$6 \times 12 = (6 \times 10) + (6 \times 2) = 60 + 12 = 72$$

Have students calculate 5×23 the same way. ($5 \times 23 = (5 \times 20) + (5 \times 3) = 100 + 15 = 115$)

Exercises: Rewrite the product in expanded form. Solve.

- a) 6×53 b) 7×64 c) 9×45

Answers

- a) $6 \times 53 = (6 \times 50) + (6 \times 3) = 300 + 18 = 318$
 b) $7 \times 64 = (7 \times 60) + (7 \times 4) = 420 + 28 = 448$
 c) $9 \times 45 = (9 \times 40) + (9 \times 5) = 360 + 45 = 405$

Bonus: Write the product as a sum of three products and then solve.

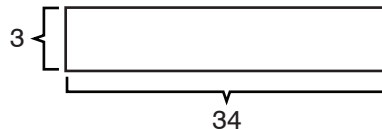
- a) 7×125 b) 5×348 c) 4×875 d) 9×686 e) 3×953

Answers

- a) $7 \times 125 = (7 \times 100) + (7 \times 20) + (7 \times 5) = 700 + 140 + 35 = 875$
 b) $5 \times 348 = (5 \times 300) + (5 \times 40) + (5 \times 8) = 1500 + 200 + 40 = 1740$
 c) $4 \times 875 = (4 \times 800) + (4 \times 70) + (4 \times 5) = 3200 + 280 + 20 = 3500$
 d) $9 \times 686 = (9 \times 600) + (9 \times 80) + (9 \times 6) = 5400 + 720 + 54 = 6174$
 e) $3 \times 953 = (3 \times 900) + (3 \times 50) + (3 \times 3) = 2700 + 150 + 9 = 2859$

Applying multiplication. Write on the board:

The dimensions of a hallway are shown. Write a product in expanded form and solve.



Work through the problem as a class. The solution should look like this:

$$\begin{aligned} 3 \times 34 &= (3 \times 30) + (3 \times 4) \\ &= 90 + 12 \\ &= 102 \end{aligned}$$

Repeat with a cricket pitch with dimensions 3 by 22, a driveway with dimensions 5 by 13, and an aisle in a theatre with dimensions 4 by 53. Draw the three rectangles with their dimensions on the board and have students work through the solution on their own. (66, 65, 212)

Extensions

1. Fill in the missing information.

a)

×	5
<hr/>	

 = $(5 \times \underline{\quad}) + (5 \times \underline{\quad})$
 = $350 + 10$
 = $\underline{\quad}$

b)

×	4
<hr/>	

 = $(\underline{\quad} \times 80) + (\underline{\quad} \times 1)$
 = $\underline{\quad} + 4$
 = $\underline{\quad}$

c)

7	
×	
<hr/>	

 = $(\underline{\quad} \times \underline{\quad}) + (2 \times \underline{\quad})$
 = $140 + 8$
 = $\underline{\quad}$

Answers: a) $72 \times 5 = 360$, b) $81 \times 4 = 324$, c) $74 \times 2 = 148$

2. Fill in the missing digits.

a)

		0
×		4
<hr/>		
2	8	0

b)

		0
×		7
<hr/>		
5		0

c)

		0
×		9
<hr/>		
	7	0

Answers

a)

	7	0
×		4
<hr/>		
2	8	0

b)

	8	0
×		7
<hr/>		
5	6	0

c)

	3	0
×		9
<hr/>		
2	7	0

NS5-18 The Standard Method for Multiplication

Pages 60–61

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

product
regrouping

Goals

Students will use the standard algorithm to multiply two-digit numbers by one-digit numbers.

PRIOR KNOWLEDGE REQUIRED

Can multiply one-digit numbers by one-digit numbers
Can group ones into tens

MATERIALS

grid paper or **BLM 1 cm Grid Paper** (p. I-3)

Mental math minute. Have students multiply one-digit numbers by multiples of 100. Write a list of one-digit numbers and a single multiple of 100 on the board, for example, 6, 9, 5, 3, 7, 8 and 300. The first student multiplies $6 \times 300 = 1800$; the next student multiplies $9 \times 300 = 2700$. After all the one-digit numbers have been multiplied by 300, repeat with a new multiple of 100.

Review the standard algorithm for multiplication without regrouping ones. Write on the board:

	4	2
×		3
1	2	6

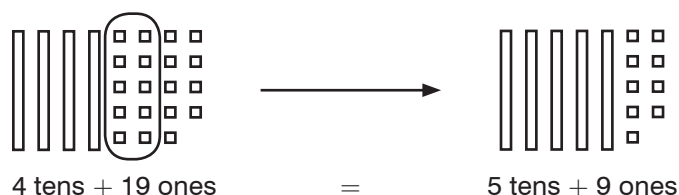
Point to the 6 and ASK: Where does this six come from? (multiplying 3×2)
Point to the 12 and ASK: Where does the 12 come from? (multiplying 3×4)
As a class repeat with 53×3 using a grid. Provide students with grid paper or **BLM 1 cm Grid Paper** for the following exercises.

Exercises: Write the multiplication on grid paper. Find the product.

- a) 41×2 b) 23×3 c) 51×5

Answers: a) 82, b) 69, c) 255

Review regrouping. Draw base ten models with up to 9 tens and more than 10 ones and have students practise trading 10 ones blocks for a tens block. They should draw models to record their trades in their notebooks. Example:



Review the standard algorithm for multiplication with regrouping. Write on the board:

		3
	4	5
×		7
		5

ASK: What is 7×5 ? (35) How many of those 35 ones can we regroup? (30) How many ones are left over? (5) Point to the 3 and ASK: What does this 3 stand for? (3 tens) Point to the 5 at the bottom and ASK: What does this 5 stand for? (the 5 ones that are left after 30 have been regrouped) SAY: We multiplied the ones digits, 7 and 5, and we grouped as many ones as we could. Leave the diagram on the board.

Exercises: Using grid paper, multiply the ones digits and regroup.

- a) 34×3 b) 68×2 c) 27×3 d) 45×3 **Bonus:** 78×4

Answers

		1
	3	4
×		3
		2

		1
	6	8
×		2
		6

		2
	2	7
×		3
		1

		1
	4	5
×		3
		5

Bonus

		3
	7	8
×		4
		2

Refer to the diagram on the board and SAY: This shows that the ones digits have been multiplied: 30 of them are being regrouped as 3 tens and 5 are left. ASK: What is 7×4 tens? (28 tens) SAY: So now we have 28 tens plus 3 more tens that we regrouped. ASK: How many tens is 28 tens plus 3 tens? (31 tens) The completed multiplication should look like this:

		3
	4	5
×		7
3	1	5

Exercises

1. Using grid paper, complete the multiplication. The ones have already been multiplied.

a)

		1
	3	4
×		3
<hr/>		
		2

b)

		1
	6	8
×		2
<hr/>		
		6

c)

		2
	2	7
×		3
<hr/>		
		1

d)

		1
	4	5
×		3
<hr/>		
		5

Bonus:

		3
	7	8
×		4
<hr/>		
		2

Answers: a) 102, b) 136, c) 81, d) 135, Bonus: 312

2. Complete all steps of the multiplication using grid paper.

a) 29×5 b) 16×7 c) 46×5 d) 54×6

Bonus: 86×3

Answers

a)

		4
	2	9
×		5
<hr/>		
1	4	5

, b)

		4
	1	6
×		7
<hr/>		
1	1	2

, c)

		3
	4	6
×		5
<hr/>		
2	3	0


, d)

		2
	5	4
×		6
<hr/>		
3	2	4

Bonus

		1
	8	6
×		3
<hr/>		
2	5	8

Extensions

NOTE: Drawings do not have to be at all accurate. 

1. Draw five different rectangles with dimensions so that the product of the length and width equals 120.

Answers: 1×120 , 2×60 , 3×40 , 4×30 , 6×20

2. All 15 multiplications below are equal to 720. Twelve of them can be grouped into six pairs because of a feature they have. This feature makes the pairing possible. Three cannot be paired up because they do not have this feature.

1×720	2×360	3×260	4×180	5×144
6×120	8×90	10×72	15×48	16×45
20×36	30×26	40×18	60×12	80×9

- Which 12 multiplications can be paired up?
- What do the 12 multiplications have in common?
- What is the strategy for pairing them up?
- Which three multiplications cannot be paired up?

Answers

- 1×720 and 10×72 , 2×360 and 20×36 , 3×260 and 30×26 , 4×180 and 40×18 , 6×120 and 60×12 , 8×90 and 80×9
- one of the numbers is divisible by 10
- multiply one number by 10 and divide the other number by 10. For example, in 1×720 if you multiply 1 by 10 and divide 720 by 10 you get its pair which is 10×72
- 5×144 , 15×48 , 16×45

NS5-19 Multiplying Large Numbers by 1-Digit Numbers

Pages 62–63

CURRICULUM REQUIREMENT

AB: optional
BC: required
MB: required
ON: optional

VOCABULARY

expanded form
regrouping

Goals

Students will multiply a multi-digit number by a one-digit number using the standard algorithm.

PRIOR KNOWLEDGE REQUIRED

Can multiply a two-digit number by a one-digit number using the standard algorithm

MATERIALS

base ten materials
BLM Circle Magic (p. D-61, see Extension 6)

Mental math minute. Remind students that they can double twice to multiply by 4 and double three times to multiply by 8. For example: to multiply 4×6 , do $2 \times 6 = 12$, then do $2 \times 12 = 24$. Then you can double 24 to get $8 \times 6 = 64$. Remind students also that order does not matter in multiplication, so they can find an answer to 9×4 by doubling 9 twice. Ask students multiplication questions where one of the factors is 4 or 8; for example, 7×4 .

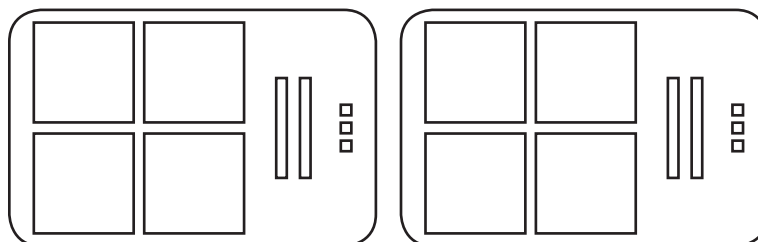
Using expanded form to multiply a 3-digit number by a 1-digit number without regrouping. Tell students you want to multiply 423×2 . ASK: How can we write 423 in expanded form? (4 hundreds + 2 tens + 3 ones) Write on the board:

$$\begin{array}{r} 4 \text{ hundreds} + 2 \text{ tens} + 3 \text{ ones} \\ \times 2 \\ \hline 8 \text{ hundreds} + 4 \text{ tens} + 6 \text{ ones} \end{array}$$

ASK: What is 4 hundreds $\times 2$? (8 hundreds) What is 2 tens $\times 2$? (4 tens) What is 3 ones $\times 2$? (6 ones) What is the product? (846)

Using base ten materials to multiply a 3-digit number by a 1-digit number without regrouping. Draw on the board:

$$423 \times 2$$



ASK: How many hundreds are there altogether? (8) How many tens are there altogether? (4) How many ones are there altogether? (6) What is the product? (846)

Using the standard algorithm to multiply a 3-digit number by a 1-digit number without regrouping. Write on the board:

4	2	3
×		2
8	4	6

ASK: What is 3×2 ? (6) What is 2×2 ? (4) What is 4×2 ? (8) SAY: So $423 \times 2 = 846$.

Using expanded form to multiply a 3-digit number by a 1-digit number with regrouping. SAY: Sometimes regrouping may be involved. Write on the board:

$$467 \times 2$$

ASK: How can we write 467 in expanded form? (4 hundreds + 6 tens + 7 ones) Write on the board:

$$\begin{array}{r} 4 \text{ hundreds} + 6 \text{ tens} + 7 \text{ ones} \\ \times 2 \\ \hline 8 \text{ hundreds} + 12 \text{ tens} + 14 \text{ ones} \end{array}$$

ASK: What is 4 hundreds $\times 2$? (8 hundreds) What is 6 tens $\times 2$? (12 tens) What is 7 ones $\times 2$? (14 ones) Write on the board:

$$\begin{array}{l} 8 \text{ hundreds} + 12 \text{ tens} + 14 \text{ ones} \\ = 8 \text{ hundreds} + (\text{ } \text{ hundred} + \text{ } \text{ tens}) + (\text{ } \text{ ten} + \text{ } \text{ ones}) \end{array}$$

Ask a volunteer to come up to fill in the blanks. (1, 2, 1, 4) Write on the board:

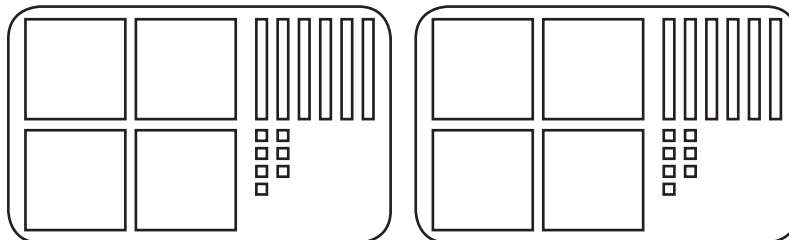
$$= \text{ } \text{ hundreds} + \text{ } \text{ tens} + \text{ } \text{ ones}$$

Ask another volunteer to gather up the hundreds, tens, and ones to fill in the blanks. (9, 3, 4) Write the answer on the board:

$$= 934$$

Using base ten materials to multiply a 3-digit number by a 1-digit number with regrouping. Draw on the board:

$$467 \times 2$$



ASK: How many hundreds are there altogether? (8) How many tens are there altogether? (12) How many ones are there altogether? (14) SAY: If we exchange 10 ones for a tens block, and 10 tens for a hundreds block, the diagram will look like this. Draw on the board:

	12 tens become 1 hundred + 2 tens	14 ones become 1 ten + 4 ones

ASK: How many hundreds are there altogether? (9) How many tens are there altogether? (3) How many ones are there altogether? (4) What is the final product? (934)

Using the standard algorithm to multiply a 3-digit number by a 1-digit number with regrouping. Write on the board, without the top row filled in:

4	6	7
×		2
		4

ASK: What is 7×2 ? (14) How do we write 14 in expanded form? (1 ten + 4 ones) SAY: We write the 1 ten in the row above the grid, and we write the 4 in the ones column in the bottom row of the grid. As you fill in the numbers, ask students what each number represents and add their answer to the algorithm as shown below:

1 hundred → 1 1 ← 1 ten

4	6	7
×		2
<hr/>		
	3	4

3 tens → 3 4 ← 4 ones

ASK: What are 6 tens \times 2? (12 tens) SAY: But we have 1 ten from multiplying 7×2 , so we really have 13 tens altogether. ASK: How do we write 13 tens in expanded form? (1 hundred + 3 tens) SAY: So we write the 3 in the tens column in the bottom row, and we write the 1 in the hundreds column in the row above the grid.

1	1	
4	6	7
×		2
	3	4

ASK: What is 4 hundreds \times 2? (8 hundreds) SAY: But we have 1 hundred from multiplying 6 tens \times 2, so really we have 9 hundreds. Write “9” in the hundreds column in the bottom row of the grid.

Together, solve the following:

- problems that require regrouping ones to tens (examples: 219×3 , 312×8 , 827×2)
- problems that require regrouping tens to hundreds (examples: 391×4 , 282×4 , 172×3)
- problems that require regrouping both ones and tens (examples: 479×2 , 164×5 , 129×4)

Have students solve additional problems in their notebooks.

Exercises: Use base ten materials, expanded form, and the standard algorithm to solve the problem.

a) 112×5 b) 321×8 c) 215×7 d) 312×9

Answers: a) 560, b) 2568, c) 1505, d) 2808

Tell students to be sure that they get the same answer all three ways. If they do not, they should check their work to find the mistake.

Bonus: Find the product.

a) 2456×3 b) $5\,234\,562 \times 7$

Answers: a) 7368, b) 36 641 934

Exploring the special case in which the 3-digit number has a 0 digit.

Write on the board:

			5
	3	0	6
\times			9
2	7	5	4

Describe each step of the process, pointing to each digit as you say it:

- 6 ones \times 9 is 54 ones, so that’s 5 tens and 4 ones
- 0 tens \times 9 is 0 tens, then add the 5 tens
- 3 hundreds \times 9 is 27 hundreds, so that’s 2 thousands and 7 hundreds

Exercises: Find the product.

a) 406×9 b) 460×8 c) 807×6 d) 870×5 e) 708×3

Bonus: $12\,009 \times 7$

Answers: a) 3654, b) 3680, c) 4842, d) 4350, e) 2124, Bonus: 84 063

NOTE: Extension 1 is required to cover the Manitoba curriculum.

Extensions

1. a) Find the product using expanded form.

i) 3125×4 ii) 1828×3 iii) 6742×6 iv) 5297×4

Bonus: $46\,913 \times 5$

- b) Find the products from part a) using a grid.

Answers

a) i) $(4 \times 3000) + (4 \times 100) + (4 \times 20) + (4 \times 5) = 12\,000 + 400 + 80 + 20 = 12\,500$

ii) $(3 \times 1000) + (3 \times 800) + (3 \times 20) + (3 \times 8) = 3000 + 2400 + 60 + 24 = 5484$

iii) $(6 \times 6000) + (6 \times 700) + (6 \times 40) + (6 \times 2) = 36\,000 + 4200 + 240 + 12 = 40\,452$

iv) $(4 \times 5000) + (4 \times 200) + (4 \times 90) + (4 \times 7) = 20\,000 + 800 + 360 + 28 = 21\,188$

Bonus: $(5 \times 40\,000) + (5 \times 6000) + (5 \times 900) + (5 \times 10) + (5 \times 3) = 200\,000 + 30\,000 + 4500 + 50 + 15 = 234\,565$

- b)

i)

	1	2			
	3	1	2	5	
×				4	
	1	2	5	0	0

ii)

	2	2			
	1	8	2	8	
×				3	
	4	0	4	5	2

iii)

	4	2	1		
	6	7	4	2	
×				6	
	4	0	4	5	2

iv)

	1	3	2		
	5	2	9	7	
×				4	
	2	1	1	8	8

Bonus

	1	3	2		
	5	2	9	7	
×				4	
	2	1	1	8	8

2. Using only the digits 2, 3, 4, and 6, find the greatest product that can be made by multiplying a 3-digit number by a 1-digit number.

Answer: 2592

3. Using only the digits 4, 5, 6, and 9, find the least product that can be made by multiplying a 3-digit number by a 1-digit number.

Answer: 2276

4. What is the greatest product possible when multiplying a 3-digit number by a 1-digit number?

Answer: $999 \times 9 = 8991$

NS5-20 Estimation in Multiplication

Pages 64–66

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

approximately equal to (\approx)
estimate
product
round

Goals

Students will multiply up to four-digit numbers by one-digit numbers, using estimation to judge the reasonableness of their answers.

PRIOR KNOWLEDGE REQUIRED

Can round whole numbers to the nearest ten, hundred, or thousand
Can multiply by multiples of 10, 100, and 1000
Can use the standard algorithm to multiply up to four-digit numbers by one-digit numbers

MATERIALS

calculators

Mental math minute. Give a student a large even number, with tens- and ones-digits both even, such as 144, to halve. Successive students repeatedly halve the previous answer, the first student saying 72, the next 36, and so on. Occasionally ask a student to explain how they got the answer. When students reach an odd number, start with a new large even number.

Review rounding to the nearest ten. Have students round two-digit numbers to the nearest ten. At first, do not include numbers that have 5 in the ones digit. For example, have students round 37. Ask students how they know which multiples of 10 it is between. ASK: How many tens are in 37? (3) How many would be one more ten? (4) SAY: So, 37 is between 3 tens and 4 tens, which means it's between 30 and 40. ASK: Which multiple of 10 is 37 closer to? (40) Have students round 94. ASK: How many tens are in 94? (9) How many would be one more ten? (10) What number is 10 tens? (100) So, 94 is between which two multiples of 10? (90 and 100) Which multiple of 10 is it closer to? (90) Tell students that when the ones digit is 5, it is not closer to either the lesser or the greater ten, but we always round up. Give them many examples to practise with: 25, 45, 95, 35, 15, 5, 85, 75, 55, 65.

Exercises: Round to the nearest ten.

a) 11 b) 8 c) 45 d) 52 e) 88 f) 96

Answers: a) 10, b) 10, c) 50, d) 50, e) 90, f) 100

Review rounding to the nearest hundred. Move on to rounding to the nearest hundred. For each example, ASK: Which multiple of 100 is this number closest to? What do we round to? Start with examples that are multiples of 10: 230, 640, 790, 60, 450 (450 is not closest to either, but we round up to 500). Then move on to examples that are not multiples of 10, such as 236, 459, 871, 548, and so on.

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ASK: When rounding to the nearest 100, what digit do we look at? (the tens digit) When do we round down? (when the tens digit is 1, 2, 3, or 4) When do we round up? (when the tens digit is 5, 6, 7, 8, or 9) SAY: Look at these numbers: 240, 241, 242, 243, 244, 245, 246, 247, 248, 249. ASK: What do these numbers all have in common? (three digits, hundreds digit is 2, tens digit is 4) Are they closer to 200 or 300? (200) How can you tell without even looking at each ones digit? (look at tens digit)

Exercises: Round to the nearest hundred.

a) 201 b) 58 c) 148 d) 352 e) 951 f) 850

Answers: a) 200, b) 100, c) 100, d) 400, e) 1000, f) 900

Estimating two-digit by one-digit products by rounding to the

nearest 10. ASK: When rounding to the nearest 10, what digit do we look at? (the ones digit) When do we round down? (when the ones digit is 1, 2, 3, or 4) When do we round up? (when the ones digit is 5, 6, 7, 8, or 9) Write “ \approx ” on the board. ASK: What does this symbol mean? (approximately equal to) Write on the board:

81 29 8 3 52 47

SAY: We’re going to round to the nearest ten. Do the first one together. (81 \approx 80) Have students do the rest on their own and take up the answers. (29 \approx 30, 8 \approx 10, 3 \approx 0, 52 \approx 50, 47 \approx 50)

Write “ 52×3 ” on the board. Have students calculate the answer. (156) Write “ $= 156$ ” on the board.

ASK: What will the two numbers in this multiplication become if we round them both to the nearest 10? (50 and 0) What is 50×0 ? (0) Write on the board:

$$50 \times 0 = 0$$

ASK: Is 0 a good estimate for 52×3 ? (no) If instead of rounding 3 down to 0 we round it up to 10, what would our estimate be? (500) Is 500 a good estimate for 52×3 ? (no) Write on the board:

$$50 \times 10 = 500$$

Write “ 50×3 ” on the board. ASK: What is 50×3 ? (150) Continue writing on the board:

$$50 \times 3 = 150$$

ASK: Is 150 a good estimate for 52×3 ? (yes) How do we know it’s a good estimate? (it’s close to the actual answer) Write on the board:

$$52 \times 3 \approx 50 \times 3 = 150$$

$$52 \times 3 = 156$$

SAY: When we multiply by a one-digit number, we don’t round the one-digit number to the nearest 10 because we get very bad estimates.

Exercises

1. Estimate by rounding the two-digit number to the nearest 10. Calculate the product.

- a) 48×6 b) 73×4 c) 19×3
d) 35×8 e) 76×6 **Bonus:** 99×9

Answers: a) $50 \times 6 = 300$, $48 \times 6 = 288$; b) $70 \times 4 = 280$, $73 \times 4 = 292$;
c) $20 \times 3 = 60$, $19 \times 3 = 57$; d) $40 \times 8 = 320$, $35 \times 8 = 280$;
e) $80 \times 6 = 480$, $76 \times 6 = 456$; Bonus: $100 \times 9 = 900$, $99 \times 9 = 891$

2. Estimate by rounding the two-digit number to the nearest 10. Calculate the product.

- a) The average 13-year-old has 28 teeth. How many teeth do seven 13-year-olds have?
b) Each shelf in a bookstore can hold 86 books.
i) How many books can 4 shelves hold?
ii) How many books can 7 shelves hold?
c) A school provides 5 extra-large pizzas for every class at the year-end party. If there are 18 classes in the school, how many pizzas do they need?

Answers: a) $30 \times 7 = 210$, $28 \times 7 = 196$; b) i) $90 \times 4 = 360$, $86 \times 4 = 344$; ii) $90 \times 7 = 630$, $86 \times 7 = 602$; c) $20 \times 5 = 100$, $18 \times 5 = 90$

NOTE: Students in Alberta and Ontario do not have to multiply numbers with more than two digits. Students in British Columbia do not have to multiply numbers with more than three digits. Students in Manitoba have to multiply numbers up to four digits.

Estimating three-digit by one-digit multiplications by rounding to the nearest 100. Repeat the explanation above for the multiplication 583×6 , rounding 583 to the nearest hundred. If students have learned to multiply three-digit numbers by one-digit numbers, have them calculate the actual product. Otherwise, they can use calculators. The multiplication should look like this:

$$583 \times 6 \approx 600 \times 6 = 3600$$

$$583 \times 6 = 3498$$

Exercises: Estimate by rounding the three-digit number to the nearest 100. Calculate the product.

- a) Ethan is saving up to go on a trip to South America. He saves \$165 a month. How much will he have saved in 8 months?
b) Sound travels at about 343 metres per second. How many metres does sound travel in 4 seconds?

Answers: a) $200 \times 8 = \$1600$, $165 \times 8 = \$1320$; b) $300 \times 4 = 1200$ m, $343 \times 4 = 1372$ m

Estimating four-digit by one-digit multiplication by rounding to the nearest 1000. Repeat the explanation above for the multiplication 7099×7 , rounding 7099 to the nearest thousand. If students have learned to multiply four-digit numbers by one-digit numbers, have them calculate the actual product. Otherwise, they can use calculators. The multiplication should look like this:

$$7099 \times 7 \approx 7000 \times 7 = 49\,000$$

$$7099 \times 7 = 49\,693$$

Exercises: Estimate by rounding the four-digit number to the nearest 1000. Calculate the product.

A mid-sized jet burns about 2153 litres of fuel every hour. How much fuel is burned in 4 hours?

Answers: $2000 \times 4 = 8000$, $2153 \times 4 = 8612$ litres

Estimating two-digit by two-digit products. Show students how to estimate 52×29 by rounding each number to the nearest ten: $50 \times 30 = 1500$. SAY: Since 52 is close to 50 and 29 is close to 30, 52×29 will be close to, or approximately, 50×30 . We can write $52 \times 29 \approx 1500$. It would be incorrect to write $52 \times 29 = 1500$ because they are not actually equal—they are just close to, or approximately, equal.

Tell students that when they round up or down before multiplying, they are not finding the exact answer, they are just estimating. They are finding an answer that is close to the exact answer. ASK: When do you think it might be useful to estimate answers? (to have a quick guess, to have a good enough answer, to check an answer)

Exercises: Estimate the product of two-digit numbers by rounding each number to the nearest ten. Remember to use the \approx sign.

- | | | |
|-------------------|-------------------|-------------------|
| a) 37×42 | b) 51×12 | c) 74×16 |
| d) 81×11 | e) 93×35 | f) 84×21 |

Answers: a) $\approx 40 \times 40 = 1600$, b) $\approx 50 \times 10 = 500$, c) $\approx 70 \times 20 = 1400$, d) $\approx 80 \times 10 = 800$, e) $\approx 90 \times 40 = 3600$, f) $\approx 80 \times 20 = 1600$

SAY: Now we're going to estimate and then calculate products in which both numbers have two digits.

Checking the estimated product with the actual product. Write on the board:

$$\begin{array}{r} 71 \\ \times 68 \\ \hline \end{array}$$

SAY: We are going to estimate and then calculate this product. That way we will see how close the estimated product is to the actual product.

NOTE: Simon's answer, 1357, is intentionally incorrect.



Ask a volunteer to estimate the product by rounding each number to the nearest ten. ($70 \times 70 = 4900$) Then ask another volunteer to complete the multiplication on the board. ($71 \times 68 = 4828$) SAY: 4900 is a good estimation for 4828 because if you round 4828 to the nearest hundred you get 4900. Ask students to check parts b) and c) of the previous exercises using a calculator. ($51 \times 12 = 612$, $74 \times 16 = 1184$)

Teach students how they can use rounding to check if the product is reasonable. Write on the board:

Simon multiplied 43 and 61 and got the answer 1357.
Does this answer seem reasonable?

Students should see that rounding both numbers down gives a product of 2400. SAY: Both numbers are rounded down, so the actual answer should be even larger than 2400. ASK: Is Simon's answer reasonable? (no) Have a volunteer calculate the actual answer to confirm that 1357 is not a reasonable answer. ($43 \times 61 = 2623$)

Exercises: Is the product reasonable? Explain.

a) $21 \times 82 = 1852$ b) $18 \times 19 = 41\ 490$ c) $53 \times 28 = 1934$

Answers

- a) yes, since $20 \times 80 = 1600$ and both factors are rounded down, the actual answer should be greater than the estimate so the product 1722 is reasonable
b) no, since $20 \times 20 = 4000$ and both factors are rounded up so the product cannot be greater than 4000
c) no, since $50 \times 43 = 1500$, even with rounding both factors up, the product cannot be greater than 1800

Remind students that rounding to the first digit on the left of a number, sometimes called front-end estimation, can change answers dramatically. Write on the board:

54 students are going to the theatre. The theatre manager reserved 3 rows with 16 seats in each row. Will there be enough seats?

SAY: If you round the number of seats in each row to the first digit of 16 you get 20 and it appears that 3 rows of 20 seats will be enough for the 54 students. ASK: With this rounding how many seats did they calculate they will need to reserve? ($3 \times 20 = 60$) SAY: 54 is less than 60, so it seems enough seats were reserved. Ask a volunteer to find the exact number of seats in 3 rows by multiplying 3×16 . (48) ASK: Did the theatre reserve enough seats? (no) SAY: Just as we saw in the last unit when you round a number, you need to pay close attention to the context or details of the situation.

Exercises: Eileen is preparing a bowl of fruit salad for each of 39 students. She wants to put one slice of pineapple in every bowl. If a package of pineapple contains 14 slices how many packages does Eileen need to buy? Explain how you know.

Answers: 3 packages because $3 \times 14 = 42$. If Eileen rounds the number of slices in each package to 10, then she'll buy 4 packages which is too many.

Extensions

1. Estimate the product by rounding the four-digit number to the nearest 10, 100, and 1000. Calculate the product.

a) 6518×6

Bonus: 8549×4

Answers: a) $6520 \times 6 = 39\ 120$, $6500 \times 6 = 39\ 000$, $7000 \times 6 = 42\ 000$, $6518 \times 6 = 39\ 108$; Bonus: $8550 \times 4 = 34\ 200$, $8500 \times 4 = 34\ 000$, $9000 \times 4 = 36\ 000$, $8549 \times 4 = 34\ 196$

2. Sometimes it is better to always round down or always round up. Which way would you round? Explain.

- a) You are estimating how much food you need for a camping trip.
- b) You are estimating how many buses will be needed to take students on a school trip.
- c) You are estimating how much time you will need to get to an appointment.
- d) You are estimating how much money you actually have to spend.

Sample answers: a) round up so you don't run out of food, b) round up so everyone is sure to get a seat, c) round up to make sure you don't get there late, d) round down to be sure you have enough money to cover your costs

3. Look at the following statement:

If you multiply a whole number by 10 000, the number of zeros in the product will be _____.

- a) Rani says the blank should be 4, since there are 4 zeros in 10 000. Do you agree with Rani? Explain why or why not.
- b) Try examples of different whole numbers to find a pattern. Complete the sentence using this pattern.

Sample answers: a) I disagree, because there might be zeros already in the whole number (the other factor); b) the number of zeros in the first factor plus 4

NS5-21 Multiplying 2-Digit Numbers by 2-Digit Numbers

Pages 67–69

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

expanded form

Goals

Students will use the standard algorithm to solve two-digit by two-digit multiplication problems.

PRIOR KNOWLEDGE REQUIRED

Can multiply two-digit numbers by one-digit numbers using the standard algorithm

MATERIALS

BLM Filling a Blank Multiplication Chart (p. D-53) from Lesson NS5-14 grid paper or **BLM 1 cm Grid Paper** (p. I-3)
scissors (see Extension 4)
BLM Patterns in Multiplication (p. D-62, see Extension 4)

Mental math minute. Give students the copy of **BLM Filling a Blank Multiplication Chart** that they started in Lesson NS5-14. Have them continue to fill in the chart as much as they can in three minutes, using any strategies they want.

Multiplying two-digit numbers by multiples of 10 using the standard algorithm. Write on the board:

$$37 \times 20$$

	1		
	3	7	
×		2	
	7	4	

$\times 10 \rightarrow 740$

Students should work on grids to start, to help ensure correct alignment. Have grid paper or **BLM 1 cm Grid Paper** available. SAY: We can solve 37×20 by first multiplying 37 by 2 and then multiplying the answer by 10.

Exercises: Solve by first multiplying by a one-digit number and then multiplying the answer by 10.

- a) 45×30 b) 38×70 c) 32×50

Answers: a) $45 \times 3 = 135$, $135 \times 10 = 1350$; b) $38 \times 7 = 266$, $266 \times 10 = 2660$; c) $32 \times 5 = 160$, $160 \times 10 = 1600$

ASK: Why can we do the multiplication 37×20 as though it is 37×2 and then just write a 0 at the end? (writing the zero at the end is what would happen if we actually multiplied 37×2 and then by 10)

Write on the board:

		4	8
	×	3	0

SAY: You can solve this without first having to multiply 48 by 0. You can just write a zero to show that you are multiplying by 30 rather than 3. Continue writing on the board:

		4	8
	×	3	0
			0

Then have a volunteer multiply 48 by 3. We regroup the 2 in the hundreds place because we are really multiplying by 30, not 3. The final picture should look like this:

NOTE: There are different notations for where to regroup the 2. Some teachers may have learned a different notation. ➡

	2		
		4	8
	×	3	0
1	4	4	0

Exercises: Multiply.

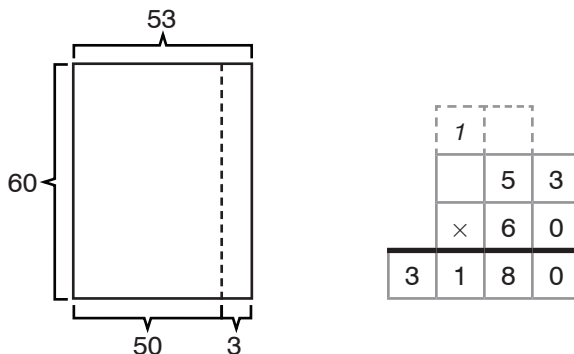
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Answers

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Multiplying by a multiple of 10 using expanded form. Explain that the exercises students just did can be understood with a drawing like ones they used before. Draw a large, undivided rectangle on the board and label the top 53 and the left side 60. Then draw a dotted line and ask students what to label the bottom left (50) and the bottom right (3). Write the answer to

part a) from the previous exercises beside the drawing. The picture should look like this:



Point to the smaller rectangle on the right and ASK: What multiplication can we write for this rectangle? (60×3) What multiplication can we write for the larger rectangle on the left? (60×50) SAY: It's easy to see the two parts of the multiplication when we use expanded form. Write on the board:

$$60 \times 53 = (60 \times 50) + (60 \times 3)$$

ASK: What is 60×50 ? (3000) Write "3000" below (60×50). ASK: What is 60×3 ? (180) Write "+ 180" below (60×3). ASK: What is $3000 + 180$? (3180) The final answer should look like this:

$$\begin{aligned} 60 \times 53 &= (60 \times 50) + (60 \times 3) \\ &= 3000 + 180 \\ &= 3180 \end{aligned}$$

Exercises: Multiply using expanded form.

- a) 53×60 b) 67×40 c) 75×30 d) 82×50

Answers: a) $(60 \times 50) + (60 \times 3) = 3000 + 180 = 3180$, b) $(40 \times 60) + (40 \times 7) = 2400 + 280 = 2680$, c) $(30 \times 70) + (30 \times 5) = 2100 + 150 = 2250$, d) $(50 \times 80) + (50 \times 2) = 4000 + 100 = 4100$

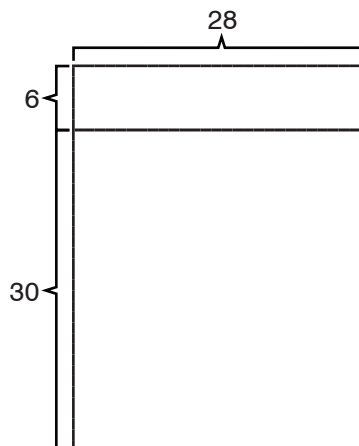
Have students compare these answers with their answers to the previous exercises that use the same four multiplications.

Multiplying two-digit numbers by two-digit numbers. Write " 28×36 " on the board. ASK: How is this multiplication different from the previous kind? (it is two-digits times two-digits but with no multiples of ten) Can we break this problem into two problems that we already know how to do? Suggest that they change one of the two numbers, either 28 or 36 into a multiple of ten and a one-digit number. ASK: What 28 would look like if we broke it down that way? ($28 = 20 + 8$) Repeat for 36.

Ask students to multiply 28×6 and 28×30 the way they do when they multiply on a grid. Draw the picture shown below and ask students to identify which multiplication goes with which part of the picture. (top is 28×6 and bottom is 28×30) Write “ 28×6 ” in the top rectangle and “ 28×30 ” in the bottom rectangle.

$$\begin{array}{r} 4 \\ 28 \\ \times 6 \\ \hline 168 \end{array}$$

$$\begin{array}{r} 2 \\ 28 \\ \times 30 \\ \hline 840 \end{array}$$



ASK: How can we find 28×36 from 28×6 and 28×30 ? (add them together) SAY: 6 twenty-eights plus 30 twenty-eights equals 36 twenty-eights. Then show the addition on the board:

$$\begin{array}{r} 168 \\ + 840 \\ \hline 1008 \end{array}$$

Have students use the same approach to solve the multiplication again but this time have them split 28 into two parts.

Exercises: Multiply.

a) 43×27

b) 54×45

c) 36×44

Answers: a) 1161, b) 2430, c) 1584

Then show students how to use this interim notation (it helps to explain where the standard notation comes from later):

$$\begin{array}{r} 28 \\ \times 36 \\ \hline 168 \\ + 840 \\ \hline 1008 \end{array}$$

Have students use this notation for the exercises above. Now show students the standard algorithm:

$$\begin{array}{r} 4 \\ 28 \\ \times 36 \\ \hline 168 \\ + 840 \\ \hline 1008 \end{array} \longrightarrow \begin{array}{r} 24 \\ 28 \\ \times 36 \\ \hline 168 \\ + 840 \\ \hline 1008 \end{array} \longrightarrow \begin{array}{r} 24 \\ 28 \\ \times 36 \\ \hline 168 \\ + 840 \\ \hline 1008 \end{array}$$

Have students practise the first step of the standard algorithm (i.e., multiplying the first number by the ones digit of the second number) for several problems.

Exercises: Multiply.

- a) 23×17 b) 46×19 c) 25×32 d) 42×26

Answers: a) 391, b) 874, c) 800, d) 1092

Ensure that students leave enough space in their notebook to solve the entire problem, not just the first step. Example:

$$\begin{array}{r} 2 \\ 23 \\ \times 17 \\ \hline 161 \end{array} = 23 \times 7$$

For multiplying by 7, ensure that any digits “carried” at this stage are put in the tens column and that students can explain why this makes sense. When students are finished the first step, have them practise the second step of the standard algorithm (i.e., multiplying the first number by the tens digit of the second number) for the same problems they started above. Example:

$$\begin{array}{r} 2 \\ 23 \\ \times 17 \\ \hline 161 \\ 230 \end{array} = 23 \times 10$$

When students are finished, have them complete the final step of the standard algorithm (adding the two results to find the total answer).

Extensions

- ASK: Which is larger: 36×5 or 35×4 ? How can you tell without actually multiplying the numbers? Encourage students to do the calculations and then to reflect back on how they could have known which was larger before doing the calculations. For the pairs below, challenge students to first decide which product will be larger and then do the actual calculations to check their predictions. Hint: For part a), compare multiplying the one-digit number by the tens digit.

- a) 26×3 or 23×6 b) 51×2 or 52×1
c) 93×2 or 92×3 d) 59×7 or 57×9

Sample solution: a) 6×20 is more than 3×20 , so 6×23 is larger than 3×26 .

2. Fill in the missing numbers. Hint: Starting from the beginning of the problem is not necessarily the easiest way to work out the answer.

a)

	3	
×	1	4
<hr/>		
3	2	0
<hr/>		
4	4	8

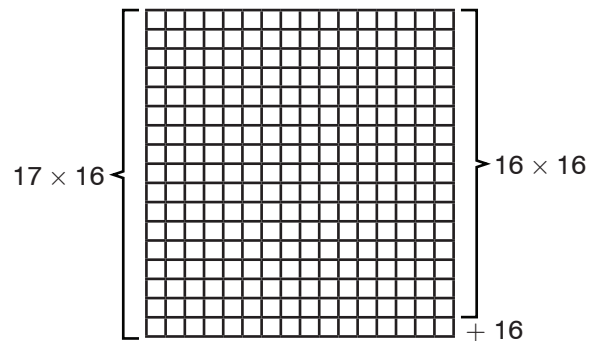
b)

	4	
×		4
<hr/>		
1	3	5
<hr/>		
1	5	3

Answers: a) $32 \times 14 = 128 + 320 = 448$, b) $45 \times 34 = 180 + 1350 = 1530$

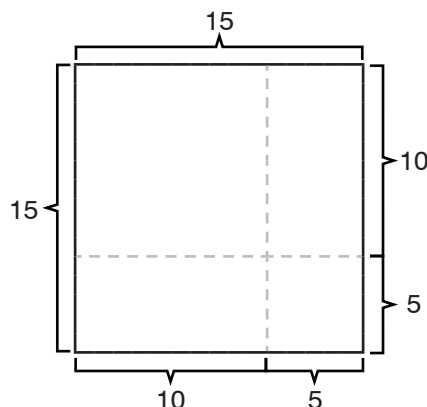
3. Earlier in the unit you used array models to demonstrate that $(4 \times 6) + 6 = 5 \times 6$. On grid paper draw a rectangle model to show that $17 \times 16 = (16 \times 16) + 16$.

Answer:



4. Distribute **BLM Patterns in Multiplication**. Students will discover an easy way to find the product of a two-digit number and itself when the number has ones digit 5 (for example, $35 \times 35 = 1225$). The trick is that the number always ends in 25. To find the leading digits of the number, start with the tens digit (for 35×35 , start with 3) and multiply it by itself-plus-one ($3 \times 4 = 12$). Write “12” before the 25 to get $35 \times 35 = 1225$. When students have finished the BLM, have them use rectangles to explain why the trick works.

- a) On grid paper, draw a 15 by 15 rectangle and divide it into 4 smaller rectangles using $15 = 10 + 5$ (as shown below).

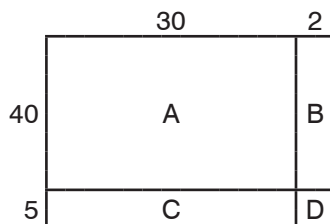


Cut out the four rectangles. Arrange them to show 15×15 , then move them to show $(10 \times 20) + 25$.

- b) Repeat part a) for 25×25 . Arrange the four rectangles to show 25×25 , then move them to show $(20 \times 30) + 25$.
- c) How did you move the rectangles in parts a) and b)? Did you arrange both sets of rectangles the same way?

Selected sample answer: c) I arranged both sets of rectangles the same way. In part a), I placed the two 5×10 rectangles beside the 10×10 rectangle to make a 10×20 rectangle. Likewise in part b), I placed the two 5×20 rectangles beside the 20×20 rectangle to make a 20×30 rectangle.

5. The rectangle for 45×32 is divided into four smaller rectangles as shown.



- a) i) Use $45 = 40 + 5$ and $32 = 30 + 2$ to write a multiplication statement for each rectangle.
- ii) Add up the products of the individual rectangles to calculate 45×32 .
- b) Use the technique from part a) to find the product.
- i) 56×38 ii) 82×41 iii) 73×39
- c) Check the reasonableness of your answers by rounding the original factors to the nearest 10.

Answers: a) i) A: 40×30 , B: 40×2 , C: 5×30 , D: 5×2 ; ii) 1440;
b) i) $(50 \times 30) + (50 \times 8) + (6 \times 30) + (6 \times 8) = 2128$, ii) $(80 \times 40) + (80 \times 1) + (2 \times 40) + (2 \times 1) = 3362$, iii) $(70 \times 30) + (70 \times 9) + (3 \times 30) + (3 \times 9) = 2847$; c) i) $56 \times 38 \approx 60 \times 40 = 2400$, ii) $82 \times 41 \approx 80 \times 40 = 3200$, iii) $73 \times 39 \approx 70 \times 40 = 2800$

NS5-22 Multiplying Large Numbers by 2-Digit Numbers

Pages 70–72

CURRICULUM REQUIREMENT

AB: optional
BC: required
MB: required
ON: optional

VOCABULARY

multiples

Goals

Students will use the standard algorithm to multiply multi-digit numbers by two-digit numbers.

PRIOR KNOWLEDGE REQUIRED

Can use the standard algorithm to multiply two-digit numbers by one-digit numbers
Can use the standard algorithm to multiply two-digit numbers by two-digit numbers
Can multiply two-digit numbers by two-digit multiples of 10

MATERIALS

grid paper or **BLM 1 cm Grid Paper** (p. I-3)

Mental math minute—number string.

String 1: $45 - 20$, $45 - 18$, $41 - 30$, $41 - 32$ (25, 27, 11, 9)

Use rounding and compensating to explain the strategy: 20 is 2 more than 18, so subtracting 20 is subtracting 2 too many; $45 - 20$ is 2 less than $45 - 18$. Similarly, $41 - 30$ is 2 more than $41 - 32$, because subtracting 30 is 2 less than needed.

NOTE: Students in Alberta and Ontario do not have to multiply numbers with more than two digits. Students in British Columbia do not have to multiply numbers with more than three digits. Students in Manitoba have to multiply numbers with up to four digits.

Introduce multiplying three-digit numbers by two-digit numbers. Write on the board:

$$321 \times 45$$

ASK: How is this multiplication different from any we have done so far? (one of the numbers has three digits and the other has two digits)

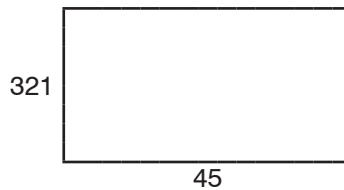
Splitting a problem into easier problems. Tell students that you would like to find a way to split the problem into two easier problems, both of which they already know how to do. Have students list all the types of problems they know how to do that might be helpful:

- Multiply a one-digit number by a one-digit number.
- Multiply a two-digit number by a one-digit number.
- Multiply a three-digit number by a one-digit number.
- Multiply a two-digit number by a two-digit multiple of 10.

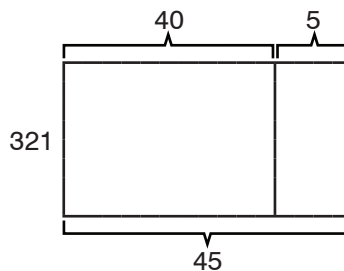
ASK: How can we break up the two-digit number so it involves a multiple of 10? ($45 = 40 + 5$)

Using the distributive property to multiply two-digit numbers by two-digit numbers in separate steps. Remind students that to find the area of a rectangle they can multiply one side by the other side. For example, the area of a rectangle with sides 2 and 7 is $2 \times 7 = 14$.

Draw on the board:



ASK: What multiplication statement gives the area? (321×45) SAY: Let's break up 45 into 40 and 5. Draw on the board:



ASK: What multiplication statement gives the area of the rectangle on the left? (321×40) What multiplication statement gives the area of the rectangle on the right? (321×5) Write on the board and SAY: Since the total area of the two small rectangles is the same as the large rectangle, we have:

$$\begin{aligned} 321 \times 45 &= 321 \times (40 + 5) \\ &= (321 \times 40) + (321 \times 5) \end{aligned}$$

Multiplying by a multiple of 10. Remind students that multiplying by a multiple of 10 requires you to write a 0 at the end of the number. Write on the board:

$$3 \times 4 = 12 \text{ so } 3 \times 40 = \underline{\hspace{2cm}}$$

$$7 \times 5 = 35 \text{ so } 7 \times 50 = \underline{\hspace{2cm}}$$

$$9 \times 6 = 54 \text{ so } 9 \times 60 = \underline{\hspace{2cm}}$$

Ask for volunteers to fill in the answers. (120, 350, 540)

Bonus: Multiply.

$$\text{a) } 71 \times 5 = 355 \quad \text{so } 71 \times 50 = \underline{\hspace{2cm}}$$

$$\text{b) } 234 \times 3 = 702 \quad \text{so } 234 \times 30 = \underline{\hspace{2cm}}$$

Answers: a) 3550, b) 7020

Write on the board:

$$321 \times 45 = 321 \times (40 + 5)$$

$$= (321 \times 40) + (321 \times 5)$$

SAY: We know how to use a grid to multiply each of these.

$$321 \times 5$$

	1	1		
	3	2	1	
×				5
	1	6	0	5

$$321 \times 40$$

		3	2	1
×			4	0
	1	2	8	4
				0

write a 0 because we are multiplying by 10

SAY: We've calculated 321×5 and 321×40 . All that's left to do is to add the two parts:

$$321 \times 5$$

	1	1		
	3	2	1	
×				5
	1	6	0	5

$$321 \times 40$$

		3	2	1
×			4	0
	1	2	8	4
				0

$$(321 \times 40) + (321 \times 5)$$

		1	1	
		3	2	1
×			4	5
	1	6	0	5
	1	2	8	4
				0
	1	4	4	5

Exercises: Multiply using three separate steps.

a) 438×36

b) 294×63

c) 518×42

Answers: a) 15 768, b) 18 522, c) 21 756

Multiplying three-digit numbers by two-digit numbers using one grid.

SAY: To save paper, we can combine the two grids we've used into one grid.

Write on the board:

Erase or cover the tens unit in 45 so that it looks like this:

		3	2	1
×			4	5

		3	2	1
×				5

SAY: Perform the multiplication 321×5 first, as though we were only multiplying a three-digit number by a one-digit number.

	1	1		
		3	2	1
×				5
	1	6	0	5

Replace the 4 you previously erased, and temporarily replace the ones digit in 45 with a 0. SAY: Now we multiply 321×40 .

	/	/		
		3	2	1
×			4	0
	1	6	0	5
				0

write a 0 because we are multiplying by 10

SAY: To avoid confusion, cross out the regrouping we did when we multiplied 321×5 .

	/	/		
		3	2	1
×			4	0
	1	6	0	5
1	2	8	4	0

	/	/		
		3	2	1
×			4	5
	1	6	0	5
1	2	8	4	0
1	4	4	4	5

SAY: All that's left to do is to add the answers from 321×5 and 321×40 . Do this in the same grid. Be sure to replace the 5 in 40.

Exercises

1. Multiply using grid paper.

a) 579×31

b) 648×24

c) 395×27

Answers: a) 17 949, b) 15 552, c) 10 665

2. Check the reasonableness of your answers to Exercise 1 by rounding the first factor to the nearest hundred, and the second factor to the nearest ten (Example: $579 \times 31 \approx 600 \times 30 \approx 18\,000$).

Multiplying multi-digit numbers by two-digit numbers. Write on the board:

		5	8	4	1
			×	2	7

SAY: In order to find the product, you need to do the multiplication 5841×7 first. Do the first-row multiplication:

		5	2		
		5	8	4	1
			×	2	7
		4	0	8	8
					7

SAY: In the same way you multiplied three-digit numbers by two-digit numbers in one grid, you can continue by multiplying 5841×20 .

		1			
		5	2		
		5	8	4	1
			×	2	7
		4	0	8	8
					7
		1	1	6	8
					2
					0



		1			
		5	2		
		5	8	4	1
			×	2	7
		4	0	8	8
					7
		1	1	6	8
					2
					0

Exercises: Multiply.

a) 2193×78

b) 6047×53

Bonus: $28\,124 \times 65$

Answers: a) 171 054, b) 320 491, Bonus: 1 828 060

Extensions

1. Find the missing numbers.

a)

					4
		×			2
			9	6	8
				0	

b)

		×		4	7
			2		
				2	
1				1	6

Answers

a)	<div> <div> <div>4</div> <div>2</div> </div> <div> <div> <div>4</div> <div>8</div> <div>4</div> </div> <div> <div> <div>×</div> <div>5</div> <div>2</div> </div> <div> <div>9</div> <div>6</div> <div>8</div> </div> <div> <div>2</div> <div>4</div> <div>2</div> <div>0</div> <div>0</div> </div> <div> <div>2</div> <div>5</div> <div>1</div> <div>6</div> <div>8</div> </div> </div> </div> </div>	b)	<div> <div> <div>1</div> <div>3</div> </div> <div> <div> <div>3</div> <div>2</div> <div>8</div> </div> <div> <div> <div>×</div> <div>4</div> <div>7</div> </div> <div> <div>2</div> <div>2</div> <div>9</div> <div>6</div> </div> <div> <div>1</div> <div>3</div> <div>1</div> <div>2</div> <div>0</div> </div> <div> <div>1</div> <div>5</div> <div>4</div> <div>1</div> <div>6</div> </div> </div> </div> </div>
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2. Multiply a three-digit number by a two-digit number using the rectangle method. For example:

$$\begin{array}{r}
 321 \\
 \times 45 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 5 \leftarrow 5 \times 1 \\
 100 \leftarrow 5 \times 20 \\
 1500 \leftarrow 5 \times 300 \\
 40 \leftarrow 40 \times 1 \\
 800 \leftarrow 40 \times 20 \\
 + 12\,000 \leftarrow 40 \times 300 \\
 \hline
 14\,445
 \end{array}$$

Have students use this method to check their answers from the previous exercises multiplying three-digit numbers by two-digit numbers.

Filling a Blank Multiplication Chart

×	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Sample strategies

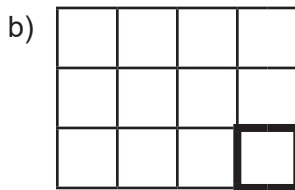
- Fill in all the facts that you have memorized or use numbers that you can skip count by easily, such as 1s, 2s, 3s, 4s, 5s, and 10s.
- Use doubling to fill in the 6s, 8s, and 12s. For example, 6×8 is double 3×8 because 3×8 is $8 + 8 + 8$ and 6×8 is $(8 + 8 + 8) + (8 + 8 + 8)$.
- Use the 5s and the 2s to fill in the 7s. For example, $7 \times 8 = (8 + 8 + 8 + 8 + 8) + (8 + 8)$, which is $(5 \times 8) + (2 \times 8)$.
- Use the 10s and the 1s to fill in the 9s and the 11s.
- Check that the same two numbers always multiply to the same number.

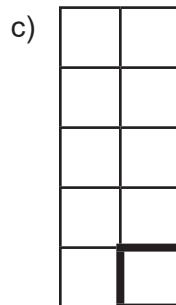
Multiplication Charts (1)

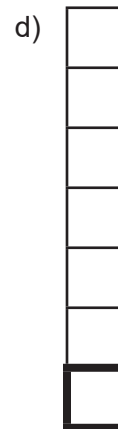
1. Count the number of squares in each rectangle. Write your answer in the bottom right square. Then write the multiplication equation. Hint: Write number of rows \times number of columns.

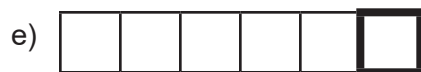


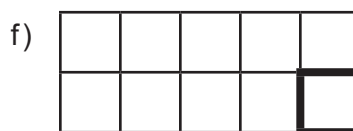
$$2 \times 3 = 6$$











2. Draw a rectangle for the product. Count the number of squares in the rectangle. Write the answer in the bottom right square of the rectangle.

a) 2×3

\times	1	2	3	4	5
1					
2					
3					
4					
5					

b) 3×4

\times	1	2	3	4	5
1					
2					
3					
4					
5					

c) 4×2

\times	1	2	3	4	5
1					
2					
3					
4					
5					

d) 2×5

\times	1	2	3	4	5
1					
2					
3					
4					
5					

Multiplication Charts (2)

Carl wants to find 3×4 using the chart.

He draws a rectangle starting at the dot.

He draws the rectangle with 3 rows and 4 columns.

The answer is the number in the bottom right corner of the rectangle.

So $3 \times 4 = 12$.

\times	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

3. Use Carl's method to multiply.

a) 2×4

\times	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

So $2 \times 4 =$ _____

b) 5×3

\times	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

So $5 \times 3 =$ _____

c) 4×2

\times	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

So $4 \times 2 =$ _____

d) 5×4

\times	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

So $5 \times 4 =$ _____

4. Which two answers from Question 3 are the same? Why are they the same?

Multiplication Charts (3)

5. Use the multiplication chart to multiply.

×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40

a) $2 \times 7 =$ _____

b) $3 \times 6 =$ _____

c) $4 \times 8 =$ _____

d) $5 \times 7 =$ _____

e) $4 \times 6 =$ _____

f) $3 \times 8 =$ _____

6. a) Finish the multiplication chart.

b) Describe the pattern in the third row you filled in.

c) Fill in the blanks.

The _____ row is the same as the 2nd column.

The _____ row is the same as the 3rd column.

The _____ row is the same as the 4th column.

×	1	2	3	4	5
1	1		3		
2		4		8	
3					
4			12		
5	5				25

7. Use the completed half of the chart to quickly finish the empty half.

×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2		4	6	8	10	12	14	16
3			9	12	15	18	21	24
4				16	20	24	28	32
5					25	30	35	40
6						36	42	48
7							49	56
8								64

BONUS ►

×	1	2	3	4	5	6	7	8
1	1	2		4	5			8
2		4		8		12		16
3	3	6	9		15		21	24
4			12	16	20	24		
5		10			25		35	40
6	6		18		30	36		
7	7	14		28		42	49	
8				32		48	56	64

10 × 10 Multiplication Chart

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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Multiplication Luck

	Multiplication	Score
1		
2		
3		
4		
5		
6		
Total		

	Multiplication	Score
1		
2		
3		
4		
5		
6		
Total		



	Multiplication	Score
1		
2		
3		
4		
5		
6		
Total		

	Multiplication	Score
1		
2		
3		
4		
5		
6		
Total		

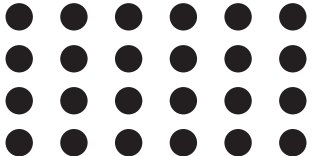
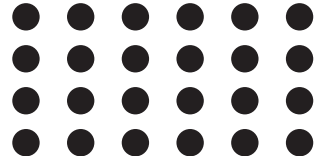
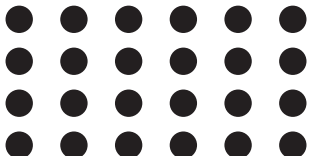
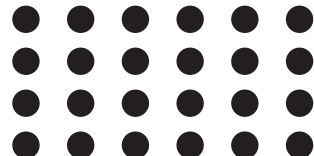
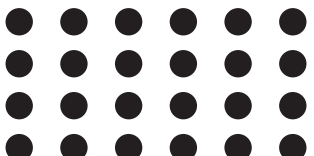
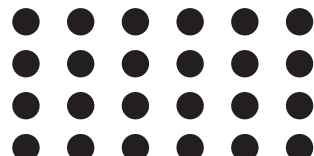
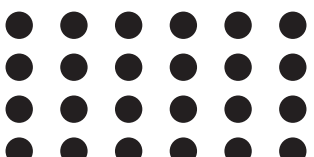
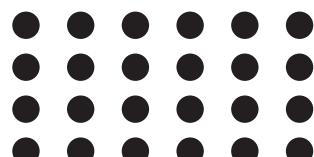

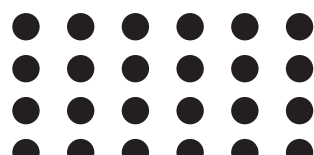

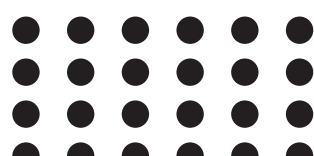
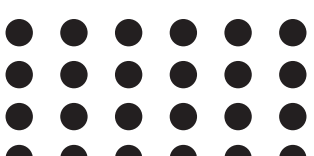
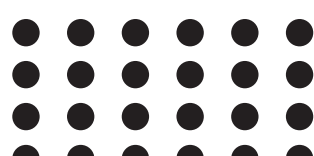
	Multiplication	Score
1		
2		
3		
4		
5		
6		
Total		

	Multiplication	Score
1		
2		
3		
4		
5		
6		
Total		

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NAME _____ DATE _____

Doubling and Halving

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Using Triples to Multiply

Remember, to double a number means to multiply the number by 2.

In the same way, to **triple** a number means to multiply the number by 3.

1. Triple the number.

a) triple 2 is _____

b) triple 3 is _____

c) triple 5 is _____

2. Triple the number mentally by tripling the tens digit and the ones digit separately.

a) triple 23 is _____

b) triple 32 is _____

c) triple 31 is _____

d) triple 42 is _____

e) triple 53 is _____

f) triple 41 is _____

BONUS ► triple 213 322 132 is _____

3. Triple the tens and ones separately and add the result.

a) 14 is $10 + 4$ so

b) 17 is _____ so

c) 26 is _____ so

triple 14 is $30 + 12 = 42$

triple 17 is _____

triple 26 is _____

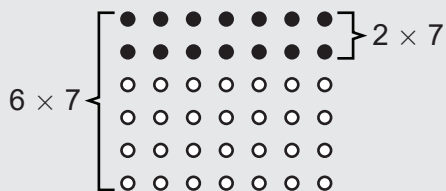
d) triple 28 is _____

e) triple 34 is _____

f) triple 59 is _____

BONUS ► triple 4326 is _____

If you know 2 times a number, you can triple it to find 6 times the number.



$$\begin{aligned} 2 \times 7 &= 14 = 10 + 4 \\ \text{So } 6 \times 7 &= 30 + 12 \\ &= 42 \end{aligned}$$

4. Triple 2 times the number to find 6 times the number.

a) $2 \times 4 =$ _____

b) $2 \times 5 =$ _____

c) $2 \times 6 =$ _____

So $6 \times 4 =$ _____

So $6 \times 5 =$ _____

So $6 \times 6 =$ _____

d) $2 \times 9 =$ _____

e) $2 \times 12 =$ _____

f) $2 \times 8 =$ _____

So $6 \times 9 =$ _____

So $6 \times 12 =$ _____

So $6 \times 8 =$ _____

5. Triple 3 times the number to find 9 times the number.

a) $3 \times 7 =$ _____

b) $3 \times 8 =$ _____

c) $3 \times 12 =$ _____

So $9 \times 7 =$ _____

So $9 \times 8 =$ _____

So $9 \times 12 =$ _____

Circle Magic

The number 142 857 is one of those numbers in mathematics that just seems to be magical. Perform the following multiplications:

a)

	1	4	2	8	5	7
×						1
<hr/>						

b)

	1	4	2	8	5	7
×						2
<hr/>						

c)

	1	4	2	8	5	7
×						3
<hr/>						

d)

	1	4	2	8	5	7
×						4
<hr/>						

e)

	1	4	2	8	5	7
×						5
<hr/>						

f)

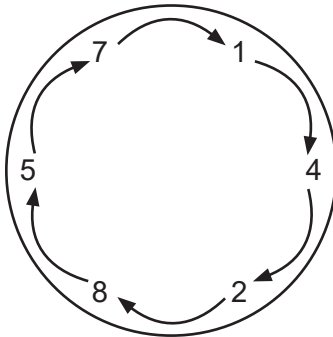
	1	4	2	8	5	7
×						6
<hr/>						

What do you notice about the digits in each of the answers? _____

Now here's the magic!

Place the digits in 142 857 in a circle. We can find any of the products above in a split second!

Example: To find $142\,857 \times 6$, use the chart below to find the starting point:



Factor	× 1	× 2	× 3	× 4	× 5	× 6
Starting Point	1	2	4	5	7	8

The starting point for $142\,857 \times 6$ is 8. To get the answer, read the numbers in the circle starting at 8 and travelling clockwise: 8, 5, 7, 1, 4, 2. So $142\,857 \times 6$ is 857 142.

Check your answers by using the circle magic!

Patterns in Multiplication

1. Multiply.

a)

	1	5
×	1	5

$1 \times 2 = \underline{\hspace{2cm}}$

$15 \times 15 = \underline{\hspace{2cm}}$

b)

	2	5
×	2	5

$2 \times 3 = \underline{\hspace{2cm}}$

$25 \times 25 = \underline{\hspace{2cm}}$

c)

	3	5
×	3	5

$3 \times 4 = \underline{\hspace{2cm}}$

$35 \times 35 = \underline{\hspace{2cm}}$

d)

	4	5
×	4	5

$4 \times 5 = \underline{\hspace{2cm}}$

$45 \times 45 = \underline{\hspace{2cm}}$

e)

	5	5
×	5	5

$5 \times 6 = \underline{\hspace{2cm}}$

$55 \times 55 = \underline{\hspace{2cm}}$

f)

	6	5
×	6	5

$6 \times 7 = \underline{\hspace{2cm}}$

$65 \times 65 = \underline{\hspace{2cm}}$

2. Predict these products without calculating them.

a)

	7	5
×	7	5

b)

	8	5
×	8	5

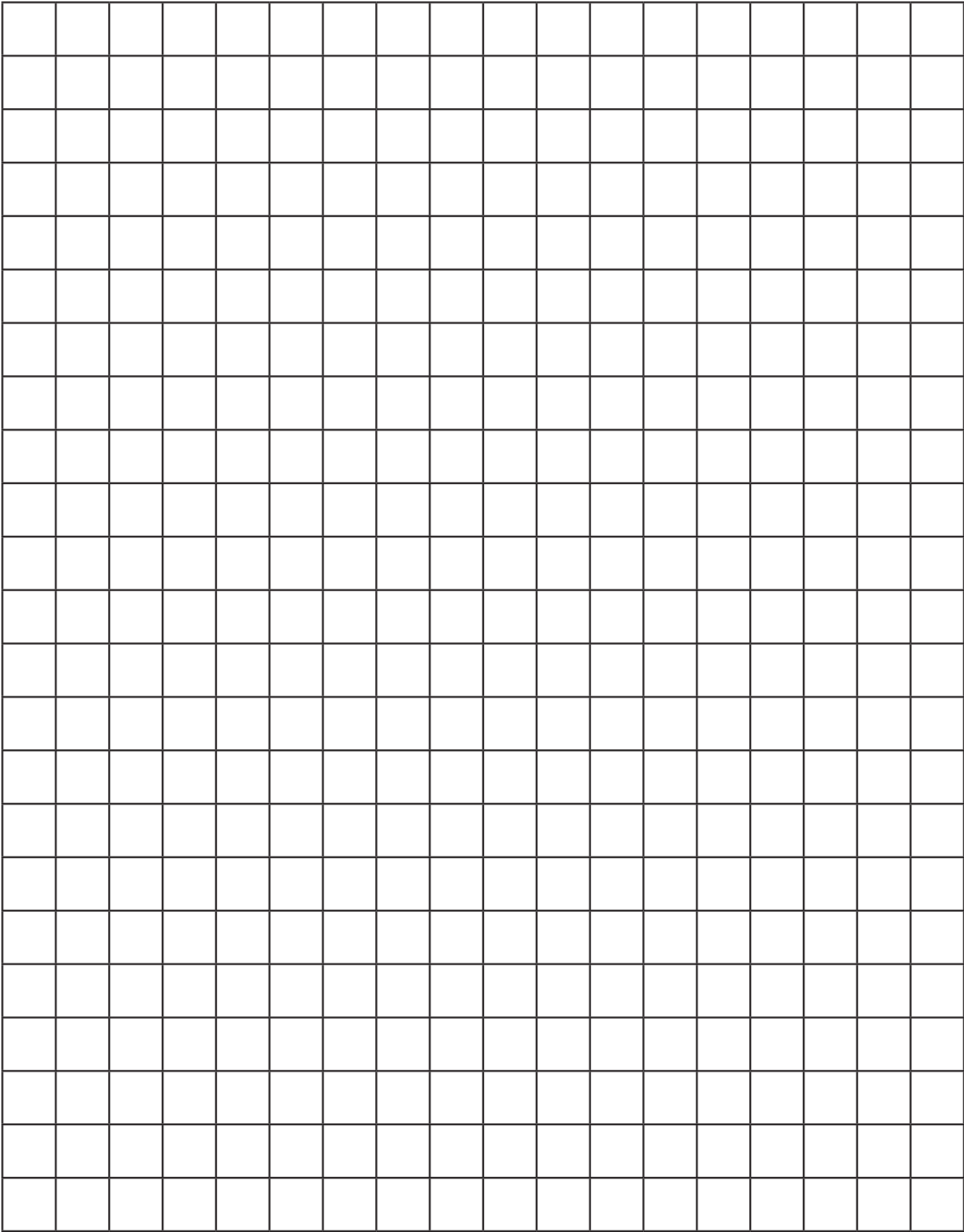
c)

	9	5
×	9	5

3. Explain your predictions in Question 2.

4. Check your predictions in Question 2 using a calculator.

1 cm Grid Paper



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