

Unit 8 Patterns and Algebra: Variables, Expressions, and Equations

Introduction

This unit focuses on numerical expressions, variables, and equations. This unit describes how to:

- verify equations by calculating expressions on both sides of the equal sign;
- translate words into expressions;
- represent variables and algebraic expressions;
- solve easy equations using guess and check and writing the variable by itself; and
- solve word problems involving addition, subtraction, multiplication, and division using equations.

Meeting Your Curriculum

ALBERTA	
Required	PA5-9 to PA5-16
Recommended	PA5-8
BRITISH COLUMBIA	
Required	PA5-9 to PA5-16
Recommended	PA5-8
MANITOBA	
Required	PA5-9 to PA5-16
Recommended	PA5-8
ONTARIO	
Required	PA5-9 to PA5-16
Recommended	PA5-8

Mental Math Minutes

The mental math minutes in this unit are dedicated to:

- multiplication and division skills using skip counting
- solving addition and subtraction equations by comparing the sides of an equation

Assessment

The lessons covered by a quiz or test are as follows:

	AB	BC	MB	ON
Quiz	PA5-8 to 12	PA5-8 to 12	PA5-8 to 12	PA5-8 to 12
Quiz	PA5-13 to 16	PA5-13 to 16	PA5-13 to 16	PA5-13 to 16
Test	PA5-9 to 16	PA5-9 to 16	PA5-9 to 16	PA5-9 to 16

PA5-8 Numerical Expressions

Page 1

CURRICULUM REQUIREMENT

AB: recommended
BC: recommended
MB: recommended
ON: recommended

VOCABULARY

brackets
equation
numerical expression
operations
verify

Goals

Students will verify equations by calculating the expressions on both sides of the equal sign and verifying that they are equal.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, multiply, and divide whole numbers
Knows operations inside brackets have priority

Mental math minute. Write “ $115 \div 5 = \underline{\quad}$ ” on the board. SAY: Doubling both numbers in a division does not change the answer. If you double both numbers in this division, you get $230 \div 10 = 23$. Write the second equation underneath the first and write “23” in the blank.

Exercises: Double both numbers in the division to find the answer.

a) $75 \div 5$ b) $225 \div 5$ c) $430 \div 5$ d) $540 \div 5$

Answers: a) 15, b) 45, c) 86, d) 108

Introduce numerical expressions. Explain that an expression shows or represents something. For example, a facial expression is when a person's face shows or represents how that person feels. Demonstrate by making one or two facial expressions to show emotions and ask students to guess which emotions you are showing. SAY: A *numerical expression* represents calculations with numbers. For example, $2 + 3$ is a numerical expression.

Exercises: Calculate the numerical expression.

a) $2 + 5 + 3$ b) $5 \times 3 \times 2$ c) $17 + 14 + 20$ d) $5 + 9 - 3$

Answers: a) 10, b) 30, c) 51, d) 11

Review brackets. Remind students that brackets in an expression tell you to do everything inside them before you do all the other operations. For example, $8 - (2 \times 3)$ is calculated as $8 - 6 = 2$, but $(8 - 2) \times 3 = 6 \times 3 = 18$.

NOTE: Students will learn the correct order of operations in a later grade. For now, only assign questions where doing the operations from left to right will give the correct order, or include brackets to ensure that the correct order is used.

Invite volunteers to solve these problems on the board:

$$(2 \times 3) + 5 \qquad 2 \times (3 + 5)$$

$$(6 + 5 = 11, 2 \times 8 = 16)$$

Exercises: Evaluate the expression.

a) $(20 - 5) \div 3$ b) $5 \times (3 + 2)$ c) $7 - (4 + 2)$ d) $18 \div (2 + 4)$

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Answers: a) 5, b) 25, c) 1, d) 3

Introduce equations. Write on the board:

$$8 + 2 \quad 50 \div 5$$

SAY: Both expressions are equal to 10. When two equal expressions are separated by an equal sign, we call it an *equation*. Add an equal sign between the expressions on the board:

$$8 + 2 = 50 \div 5$$

SAY: To *verify* that an equation is true, you can calculate the numerical expressions on both sides of the equal sign and see whether they have the same value. For example, $1 + 2 = 7 - 4$ is a true equation because both sides have the same value; $2 \times 3 = 2 + 3$ is not true because the left side is equal to 6 and the right side is equal to 5.

Exercises

1. Verify that the equation is true.

- a) $20 - 5 = 5 \times 3$ b) $5 \times (3 + 2) = 20 + 5$
c) $9 + (4 \times 2) = 26$ d) $1 + 3 + 5 = 3 \times 3$

Answers: a) true, b) true, c) not true, d) true

2. What's the mistake in Exercise 1, part c)?

Answer: the multiplication in the brackets has to be done first:
 $9 + (4 \times 2) = 9 + 8 = 17$

Bonus

3. a) Verify that each equation is true.

$$\begin{aligned} 1 + 3 + 5 + 7 &= 4 \times 4 \\ 1 + 3 + 5 + 7 + 9 &= 5 \times 5 \\ 1 + 3 + 5 + 7 + 9 + 11 &= 6 \times 6 \end{aligned}$$

- b) Look for a pattern in part a). Use the pattern to predict and then check:

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = \underline{\quad} \times \underline{\quad}$$

Selected answer: b) 8×8

4. a) Verify that each equation is true.

$$\begin{aligned} 7 + 6 + 10 &= (7 + 3) + (6 - 2) + (10 - 1) \\ 8 + 5 + 12 &= (8 + 4) + (5 + 3) + (12 - 7) \end{aligned}$$

- b) Look for a pattern in part a). Use the pattern to write another similar equation. Evaluate the expressions on both sides of the equal sign to verify that your equation is true.

Selected sample answer: b) $12 + 31 + 14 = (12 - 2) + (31 - 1) + (14 + 3)$

Extensions

1. Add brackets where necessary to the equation to make it true. Hint: In some equations, you might need to add one set of brackets inside another.

a) $3 + 1 \times 7 - 2 = 20$ b) $3 + 1 \times 7 \times 2 = 56$
c) $8 - 4 \times 2 + 5 = 28$ d) $5 \times 4 - 3 + 2 = 7$

Answers: a) $(3 + 1) \times (7 - 2)$, b) $(3 + 1) \times 7 \times 2 = 56$,
c) $(8 - 4) \times (2 + 5) = 28$, d) $(5 \times (4 - 3)) + 2 = 7$

2. Verify that the equation is true.

a) $(5 - 2) \times (1 + 7) = 24$
b) $(2 + 5) \times (8 - 3) = (6 \times 6) - 1$
c) $(3 + 2) \times (9 - 1) \div 4 = 10$
d) $(2 \times 5) + (2 \times 2) = (6 \div 3) \times (4 + 3)$

3. Each blank represents a single digit. Find the missing digits.

a) $(2 \times 1000) + (6 \times 100) + (___ \times 10) + 5 = 2645$
b) $2___7 = (___ \times 100) + (3 \times 10) + ___$
c) $(3 \times 100\,000) + (9 \times 10\,000) + (___ \times 1000) + (___ \times 100) + ___$
 $= ___ 90\,7___ 2$

Answers: a) 4; b) 3, 2, 7; c) 0, 7, 2, 3, 0

PA5-9 Unknown Quantities and Equations

Pages 2–4

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

equation
in total
numerical expression
unknown

Goals

Students will write and solve easy addition and subtraction equations using models.
Students will solve easy multiplication and division equations using the guess-and-check method and by writing the unknown by itself.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, multiply, and divide
Understands multiplication as repeated addition
Knows multiplication and division up to 10×10

MATERIALS

masking tape, string, or ruler
counters
paper bag
calculators

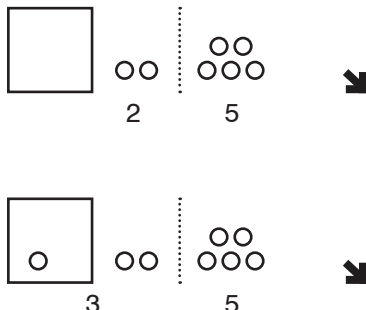
Mental math minute—number talk. Present this problem: Is the equation $11 \times 7 = 10 \times 8$ true? (no) The following strategies could arise:

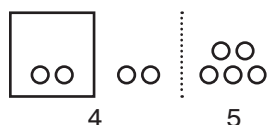
The left side is 77, and the right side is 80.
The right side is a multiple of 10, but the left side is not.
The right side is a multiple of 5, but the left side is not.
The left side is 7 more than 10×7 , but the right side is 10 more than 10×7 .

Solving algebraic equations. Divide a desk in half using masking tape, string, or a ruler. Put five counters on one side of the dividing line and put two counters and a paper bag containing three more counters on the other side. (Don't let students see how many counters are in the bag.) Tell students that the number of counters in total is the same on both sides of the line. Have students guess how many counters are in the bag. (3) Repeat the example with different numbers of counters, but don't use more than one bag.

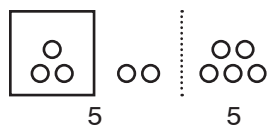
Draw a representation of the first concrete model of the equation on the board. ASK: How many counters are on the right side? (5) How many counters can you see on the left side? (2) Write the numbers 2 and 5 under the corresponding number of circles (see example in margin).

SAY: The box represents the bag. To find how many circles are in the box, I put a circle in the box. Draw a circle inside the box. ASK: How many circles can you see on the left side in total? (3) Erase the number 2 and write 3 instead, but write 3 a little to the left, between the box and the two counters (see example in margin).

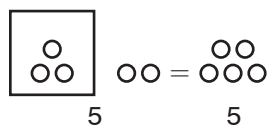




ASK: Is the number of circles on both sides equal? (no) SAY: I'm going to add another circle in the box. Draw another circle inside the box. ASK: How many circles can you see on the left side in total? (4) Erase the number 3 and write 4.



SAY: The number of circles on both sides is still not equal, so I'm going to put another circle in the box. Draw another circle inside the box. Erase the number 4 and write 5 (see example in margin).



ASK: How many circles are on the left side in total? (5) Is the number of circles on both sides equal? (yes) How many circles are in the box? (3) SAY: We can replace the dotted line with an equal sign because the two sides are equal. Erase the dotted line and draw an equal sign in its place, as shown in the margin, and SAY: This shows that the two sides are equal.

Have a student open the bag to check that there are three counters inside. Students should also see that they can find the number of hidden counters either by counting up from 2 to 5 or by subtracting 2 from 5. Have students check that the equation was solved correctly. ASK: Does the number of counters drawn in the box make the equation true? (yes)

Exercises: Draw circles in the box until the number of circles is the same on both sides.

a) $\boxed{} \bigcirc = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$

b) $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc = \bigcirc \bigcirc \boxed{}$

Explain that it is inconvenient to draw counters all the time. SAY: We can use numbers to represent all the quantities instead. Have students practise writing equations that represent pictures, similar to **Question 2** on AP Book 5.2 p. 2. For example, the equation for the first example in this lesson is $\boxed{3} + 2 = 5$.

Challenge students to solve several more examples. Students can create models for equations that involve addition using counters. They can also draw models; in this case, ask students to use a box for the *unknown* (the "hidden" number or the number we don't know) and use a set of circles to model the numbers in the equation. For example, a model for the equation $\boxed{5} + 2 = 7$ is shown in the margin.



Ask students to make a model with boxes and circles to solve the following exercises. They should explain how many circles they would put in each box to make the equation true.

Exercises

a) $7 + \boxed{} = 11$

b) $6 + \boxed{} = 13$

c) $4 + \boxed{} = 10$

d) $9 + \boxed{} = 12$

Answers: a) 4, b) 7, c) 6, d) 3

Equations with addition. Read several word problems to students. Invite volunteers to draw models, write equations using boxes and numbers, and solve the equations. Some examples are shown on the next page.

- a) There are 10 trees in the garden. Three of them are apple trees. All the rest are cherry trees. How many cherry trees are in the garden?
- b) Jane has 12 T-shirts. Three of them are plain. All the rest have designs. How many of Jane's T-shirts have designs?

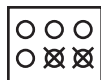
Exercises: Write an equation to solve the problem.

- a) There are 15 plants in the flowerbed. Six are lilies. All the rest are peonies. How many peonies are in the flowerbed?

Bonus

- b) There are 150 pirates on two ships, a galleon and a schooner. Forty of the pirates are on the schooner. How many are on the galleon?
- c) A dragon has 15 heads. A knight cut off some of the heads. The dragon has seven remaining heads. How many heads did the knight remove?

Equations with subtraction. Present this word problem: Simon has a box of apples. He took two apples from the box. Four were left. How many apples were in the box before he removed any?



Draw the box with four apples in it. Draw two more apples in the box and cross them out to show that they have been taken away (see example in margin).



$$- \text{OO} = \text{OOOO}$$



ASK: How many apples were in the box at the beginning? (6) Explain that, when we write a subtraction equation, we draw it a bit differently than the addition equations we drew earlier. We draw a box for the number we don't know (the original number), show a minus sign, and then draw the apples we took out of the box. We show the four apples that were left in the box on the other side of the equation (see example in margin).

SAY: To solve the equation, we have to put all the apples into the box—the ones that we took out and the ones that we left there. Remind students that they also learned to write equations using numbers. ASK: How could you write this equation using numbers? ($\square - 2 = 4$)

Draw several models for subtraction equations (like those in **Questions 4 and 5** on AP Book 5.2 pp. 2–3) and ask students to write the equations for them. Ask volunteers to present the answers on the board.

Exercises: Draw the model to solve the equation.

- a) $\square - 6 = 9$ b) $\square - 7 = 12$ c) $\square - 5 = 3$ d) $\square - 3 = 10$

Guessing and checking. Tell students that someone tried to solve the equation $9 + \square = 28$ by guessing and checking. The first guess was 16. Write "16" in the box and then ask students to add $9 + 16$ to see if the equation is true. (no, $9 + 16 = 25$) ASK: Should the next guess be more than 16 or less than 16? (more) Students can indicate their answer with thumbs up or thumbs down. Have a volunteer try 15 for the unknown. ASK: Did we get closer to 28? (no) Have another volunteer try 17. ASK: Did we get closer to 28? (yes) Have students make more guesses until they

find the answer. (19) Repeat with the equation $\square + 8 = 31$, with a first guess of 25.

Exercises: Solve the equation by guessing and checking.

a) $5 + \square = 13$

b) $\square + 7 = 22$

c) $\square - 8 = 4$

Bonus: $31 - \square = 14$

Answers: a) 8, b) 15, c) 12, Bonus: 17

Equations with multiplication. Tell students that they can also write equations for multiplication problems. Remind students that “ $2 \times$ ” means that some quantity will be two times as large—in other words, it will be doubled. Examples:

$$2 \times \square = \begin{array}{cccccc} \circ & \circ & \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ & \circ & \circ \end{array}$$

$$2 \times \circ \circ \circ = \begin{array}{cc} \circ \circ & \circ \\ \circ \circ & \circ \end{array} \quad \text{and} \quad 2 \times \square = \square \square$$

Present the problem shown in the margin and ask students to draw the appropriate number of circles in the box. (6) Students should solve the problem by dividing the circles on the right side into two equal groups. ASK: How would you write an equation with numbers for this problem? ($2 \times \square = 12$)

Solving multiplication equations (missing second factor) without using a diagram. ASK: What mathematical operation are you performing when you split circles into equal groups? (division) Have students write the division equations for the problem they solved above using a diagram. Is the number of circles they drew in the diagram the same as the number they wrote in the equation? (It should be!)

NOTE: Students can use a calculator to check their answers for the Bonus exercises below.

Exercises: Solve the multiplication equation by rewriting it using division.

a) $3 \times \square = 24$ b) $5 \times \square = 45$ c) $6 \times \square = 42$ d) $9 \times \square = 72$

Bonus

e) $3 \times \square = 240$

f) $5 \times \square = 4500$

g) $6 \times \square = 42\,000$

h) $9 \times \square = 720\,000$

Answers: a) 8, b) 9, c) 7, d) 8, Bonus: e) 80, f) 900, g) 7000, h) 80 000

Using division to solve multiplication equations of different types.

SAY: Multiplication is commutative. Write the two equations below on the board and ask students whether they should have the same answer.

$$3 \times \square = 27 \quad \square \times 3 = 27$$

Ask students to explain their thinking. Remind them that the box replaces a missing number, so they can do with the box all the same things they would do with a number. ASK: How would you solve the first equation? ($27 \div 3 = 9$)

Would the same solution work for the second equation? (yes) Why? (because order does not matter in multiplication, we can just switch the numbers we are multiplying together)

Exercises: Solve the multiplication equation by rewriting it using division.

a) $\square \times 3 = 21$ b) $\square \times 4 = 48$ c) $\square \times 7 = 63$ d) $\square \times 8 = 56$

Bonus

e) $\square \times 3 = 3000$ f) $\square \times 4 = 1200$
g) $\square \times 7 = 4200$ h) $\square \times 8 = 24\,000$

Answers: a) 7, b) 12, c) 9, d) 7, Bonus: e) 1000, f) 300, g) 600, h) 3000

Solving equations with a missing dividend. Present the equation

$\square \div 5 = 7$. ASK: What number divided by 5 gives you 7? (35) How did you get 35 from 5 and 7? (by multiplying) Point out that the equation describes this situation: You had some apples that you divided into 5 equal groups. There are 7 apples in each group. Draw 5 circles and 7 dots in one circle, then ASK: What would you draw to find the total number of apples? (7 dots in each of the remaining circles) How would you then find the total number of dots? (multiply 5×7) Emphasize that to find the number that is being divided into groups, you multiply the number of groups by the number in each group.

Exercises: Solve the division equation by rewriting it using multiplication.

a) $\square \div 4 = 10$ b) $\square \div 6 = 3$ c) $\square \div 4 = 8$ d) $\square \div 5 = 12$

Answers: a) 40, b) 18, c) 32, d) 60

Solving equations with a missing divisor. Have students solve several equations with a missing divisor, such as $45 \div \square = 5$. (9) Students who still have trouble with multiplication and division facts and use guessing and checking to find the answer can use skip counting instead.

Exercises: Solve the equation.

a) $25 = 5 \times \square$ b) $\square = 6 \times 3$ c) $35 \div \square = 7$
d) $\square \times 3 = 15$ e) $\square \div 2 = 8$ f) $21 \div 3 = \square$

Answers: a) $\square = 5$, b) $\square = 18$, c) $\square = 5$, d) $\square = 5$, e) $\square = 16$, f) $\square = 7$

Rewriting equations with the unknown by itself. Write on the board:

$$5 \times \square = 35$$

ASK: How can we rewrite this multiplication as division? ($\square = 35 \div 5$) Write " $\square = 35 \div 5$ " on the board to the right of the first equation. Ask students to find the unknown in each equation. (7, 7) Explain to students that solving the equation on the right is easier because the unknown is by

itself, so to find the unknown, they just need to do an easy division with two known numbers. SAY: Writing the unknown by itself makes the equation easier to solve because you just need to calculate a numerical expression.

Exercises: Solve the equation by rewriting it with the unknown by itself.

a) $3 \times \square = 12$

b) $\square \div 7 = 6$

c) $\square - 11 = 4$

Bonus: $36 \div \square = 3$

Selected answers: a) $\square = 12 \div 3$, $\square = 4$; c) $\square = 11 + 4$, $\square = 15$;

Bonus: $\square = 36 \div 3$, $\square = 12$

Extensions

1. The same symbol in the equation means the same number. What does each symbol represent?

a) $\square + \square = 12$

b) $\diamond + \diamond + \diamond = 9$

c) $5 + \bigcirc + \bigcirc = 13$

d) $9 + \square + \square + \square = 15$

Answers: a) 6, b) 3, c) 4, d) 2

2. Two birds each laid the same number of eggs. Seven eggs hatched, three did not. How many eggs did each bird lay?

Answer: 5

3. Sixty baby alligators hatched from three alligator nests of the same size. We know that only half of the total number of eggs hatched. How many eggs were in each nest? Hint: How many eggs were laid in total?

Answer: 40

PA5-10 Translating Words into Expressions

Pages 5–6

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

bracket
equation
numerical expression
operation

Goals

Students will write easy addition, subtraction, multiplication, and division equations.
Students will translate between mathematical phrases written in words and numerical expressions.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, multiply, and divide

Mental math minute. SAY: Remember, an equal sign means “the same as.” To check if an equation is true, you can use what you know about multiplication and division without calculating both sides. For example, you know that doubling both numbers in division does not change the quotient. And you know that doubling one factor and halving another factor keeps the product the same.

Present the equations in the following exercises one at a time and have students signal the answers using thumbs up for “yes” and thumbs down for “no.”

Exercises: Is the equation true?

- a) $24 \div 4 = 48 \div 8$ b) $12 \div 3 = 36 \div 9$ c) $4 \times 3 = 8 \times 6$
d) $20 \times 8 = 10 \times 4$ e) $34 \div 2 = 68 \div 4$ f) $11 \times 4 = 22 \times 2$

Answers: a) yes, b) yes, c) no, d) no, e) yes, f) yes

Associating words and phrases with operations. SAY: You can use clues to write expressions. The words give clues to the operations you need to use. On the board, make a table with four columns and these headings: Add, Subtract, Multiply, and Divide.

Have students discuss which operation each phrase makes them think of. Based on the class response, create a table on the board like the one below, with each phrase under its correct heading.

Add	Subtract	Multiply	Divide
increased by sum more than total	less than difference decreased by reduced by fewer than	product times twice as many	divided by divided share equally

Exercises: Translate the phrase into an expression.

- | | |
|---------------------|---------------------------|
| a) 5 more than 7 | b) 5 less than 7 |
| c) 5 times 7 | d) the product of 7 and 5 |
| e) 7 reduced by 5 | f) 7 divided by 5 |
| g) 5 divided by 7 | h) 7 decreased by 5 |
| i) 7 increased by 5 | j) the sum of 7 and 5 |
| k) 5 fewer than 7 | l) the product of 5 and 7 |

Bonus: 3 multiplied by 7 then increased by 5

Answers: a) $7 + 5$, b) $7 - 5$, c) 5×7 , d) 7×5 , e) $7 - 5$, f) $7 \div 5$, g) $5 \div 7$, h) $7 - 5$, i) $7 + 5$, j) $7 + 5$, k) $7 - 5$, l) 5×7 , Bonus: $(3 \times 7) + 5$

Associating phrases with expressions with brackets. Discuss expressions with more than one operation. Write on the board:

Multiply 2 and 3. Then subtract 1.

Ask a volunteer to write the numerical expression for the phrase. $((2 \times 3) - 1)$

Exercises: Translate the phrase into an expression with more than one operation. Use brackets to indicate which operation has to be done first.

- | | |
|---|---------------------------------------|
| a) Divide 6 by 2. Then add 3. | b) Add 4 and 6. Then divide by 5. |
| c) Multiply 5 and 4. Then add 2. | d) Divide 8 by 4. Then multiply by 3. |
| e) Subtract 2 from 5. Then multiply by 4. Then add 3. | |

Answers: a) $(6 \div 2) + 3$, b) $(4 + 6) \div 5$, c) $(5 \times 4) + 2$, d) $(8 \div 4) \times 3$, e) $((5 - 2) \times 4) + 3$

Writing mathematical expressions in words. Start with an easy expression. On the board, write " $3 + 5$ " and ask students to read the expression. If some students answer "three plus five," say that, rather than using the word "plus," we prefer to use the verb "add." Write on the board "add 3 and 5."

Exercises: Write the operation in words.

- | | | | |
|-----------------|------------|------------|----------------|
| a) 5×2 | b) $7 - 4$ | c) $4 + 7$ | d) $15 \div 3$ |
|-----------------|------------|------------|----------------|

Answers: a) multiply 5 by 2, b) subtract 4 from 7, c) add 4 and 7, d) divide 15 by 3

Teach students how to write a mathematical expression with two or more operations in words. Write on the board:

$(3 + 2) \times 4$

ASK: Which operation would you do first, addition or multiplication?

(addition, because it's in brackets) Write "Add 3 and 2" on the board.

SAY: The first operation is done, so we have to end the sentence. Put a

period to show the sentence is ended. ASK: What operation would you do

next? (multiplication) Ask a volunteer to write “Multiply by 4.” on the board, following your first sentence. (Add 3 and 2. Multiply by 4.)

Exercises: Write the mathematical expression in words.

a) $(5 + 1) \times 2$ b) $(9 - 3) \div 3$ c) $(4 \times 2) - 3 + 7$

Bonus: $(6 + (3 \times 4)) \div 9$

Answers: a) Add 5 and 1. Then multiply by 2. b) Subtract 3 from 9. Then divide by 3. c) Multiply 4 and 2. Then subtract 3. Then add 7. Bonus: Multiply 3 and 4. Then add 6. Then divide by 9.

Interpreting expressions. SAY: A parking lot charges \$3 per hour, so you need to pay 2×3 dollars for two hours. ASK: How much do you need to pay for four hours? (4×3) ASK: If the cost of parking is 5×3 , how many hours does that pay for? (5 hours)

Exercises: To rent skates, you must pay \$6 for each hour. Complete the meaning of the expression.

- a) 3×6 is the cost of renting skates for ____ hours.
b) 2×6 is the cost of renting skates for ____ hours.
c) 5×6 is the cost of renting skates for ____ hours.

Answers: a) 3, b) 2, c) 5

Writing word problems as mathematical expressions. SAY: A movie ticket costs \$8 for adults and \$5 for students. ASK: How much will it cost for two adults to watch the movie? (2×8) If some students answer “16,” say that we don’t want to calculate the expressions right now; instead, we just want to write a proper numerical expression that describes the problem. ASK: How much will it cost for three students? (3×5) How much will it cost for two adults and three students? $((2 \times 8) + (3 \times 5))$

Exercises

1. Six people can travel in one van. Sixteen students and two teachers go to the museum. Write an expression to show the number of vans that they need.

Answer: $(16 + 2) \div 6$

2. Kim wants to buy a video game that costs \$45. Kim has already saved \$9.

- a) Write an expression to show how much money she needs to save.
b) Kim decides to save the same amount of money each month for the next four months. Write an expression to show the amount of money that she has to save each month.

Bonus: Kim’s father agrees to give her \$8 per month for the next three months. Write an expression to show the amount of money that she has to save each month.

Answers: a) $45 - 9$, b) $(45 - 9) \div 4$, Bonus: $((45 - 9) - (3 \times 8)) \div 4$

Extensions

1. Ferry tickets cost \$5 for kids and \$8 for adults. Write an expression to represent the cost of tickets for three kids and seven adults.

Answer: $(3 \times 5) + (7 \times 8)$

2. Fifteen students from each class go on a trip. There are six classes as well as two teachers and three parents for each class. How many buses will be needed if 34 people can ride in each bus?

Encourage students to model the situation using a single equation with brackets and to explain how their equation shows the situation.

Sample answer: There are 6 classes and each class has 15 students, 2 teachers, and 3 parents, so the expression for the total number of people is $6 \times (15 + 2 + 3)$. Since 34 people can ride in each bus, I can find the number of buses needed by dividing the total number of people by 34: $(6 \times (15 + 2 + 3)) \div 34 = (6 \times 20) \div 34 = 120 \div 34 = 3 \text{ R } 18$. So 4 buses (3 full buses and 1 bus with 18 people) will be needed.

3. a) Is the equation $(35 \div 5) + 2 = 35 \div (5 + 2)$ true? Explain.
b) Use your answer to part a) to explain why brackets are important.

Answers: a) No. The left side of the equation is $7 + 2 = 9$ and the right side is $35 \div 7 = 5$, so the two expressions are not equal;
b) Brackets are important because they show which operations need to be performed first. The answer to part a) shows that changing where the brackets go can change the value of an expression.

4. Jake says that $(40 \div 4) \times 2 = 40 \div (4 \times 2)$ since the expressions on both sides of the equal sign have the same numbers and the operations are in the same order.

Mary says that $(30 \div 3) \times 1 = 30 \div (3 \times 1)$ since the expressions on both sides of the equal sign have the same numbers and the operations are in the same order.

- a) Explain why you agree or disagree with each person's reasoning.
- b) How are Mary's example and Jake's example the same? How are they different?

Answers: a) both Mary and Jake use incorrect reasoning because they are ignoring the brackets, which tell us that the order of operations in each pair of expressions is not the same; b) The examples are the same because both equations have division followed by multiplication on the left side and multiplication followed by division on the right side, and both Mary and Jake use the same incorrect reasoning. The examples are different because Mary's equation is true (even though her reasoning is incorrect), while Jake's equation is false.

PA5-11 Variables

Pages 7–8

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

algebraic expression

equation

evaluate

unknown

value

variable

$$2 \times (n + 4) \rightarrow$$



Figure 1



Figure 2



Figure 3 →

Goals

Students will substitute values for the variables in algebraic expressions and translate simple word problems into algebraic expressions.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, and multiply
Can find rules for patterns

Variables and algebraic expressions. Explain to students that today they will learn to write equations the way mathematicians write them. Instead of drawing a square or a diamond for the unknown, mathematicians usually write letters. We call the letters *variables*. Remind students that they have used letters in formulas before. Write “ $2 \times (3 + 4)$ ” on the board. SAY: This is a numerical expression. If I replace some numbers with variables, then I have an *algebraic expression*. Erase the number 3 and write n in its place on the board as shown in the margin.

Explain that when letters are used in an expression, the multiplication sign (\times) is often omitted to avoid confusion with the letter “x” and to make the notation shorter. SAY: In this case, instead of writing $2 \times (n + 4)$, we can simply write $2(n + 4)$.

Exercises

1. Rewrite the expression.

- a) $2 \times n$ b) $(2 \times n) + 3$ c) $2 \times (n + 3)$

Answers: a) $2n$, b) $(2n) + 3$ or $2n + 3$, c) $2(n + 3)$

2. Write the expressions.

- a) A boat travels at a speed of 10 km per hour. What distance will it cover in 2 hours? In 5 hours? In h hours?
b) A house has 12 windows. How many windows do 3 houses have? 7 houses? n houses?

Answers: a) 10×2 , 10×5 , $10 \times h$ or $10h$; b) 12×3 , 12×7 , $12 \times n$ or $12n$

Equations and tables. Draw the figures in the margin on the board and then make a table:

Figure Number	Number of Blocks
1	5
2	6
3	7

ASK: How can you get the number of blocks from the figure number?
 (Number of Blocks = Figure Number + 4) Change the headings of the T-table to A and B and ask a volunteer to write the equation for the new table. Explain to students that, even if the names of the columns change, the rule for the T-table will still have the same form. SAY: Now the rule is $B = A + 4$.

Draw on the board:

A	B
1	3
2	6
3	9

ASK: What is the difference between the two numbers in the first row? (2) In the second row? (4) Can you add the same number to the numbers in the first column to get the numbers in the second column? (no) SAY: So the numbers in columns B and A are not related by addition. Let's try multiplication. Point to the table and ASK: What number can I multiply by 1 to get 3? (3) By 2 to get 6? (3) By 3 to get 9? (3) SAY: The numbers in the second column are 3 times as much as the numbers in the first column. Write on the board "Second column = $3 \times$ First column," then " $B = 3 \times A$."

Exercises: Find the rule, then write an equation for the table.

a)

A	B
1	11
2	12
3	13

b)

A	B
1	2
2	4
3	6

c)

A	B
1	4
2	8
3	12

Answers: a) $B = A + 10$, b) $B = 2 \times A$, c) $B = 4 \times A$

Substituting numbers for variables and evaluating expressions. Write " $n + 4$ " on the board. Tell students that we can replace n with a number and get a numerical expression. For example, if we replace n with 3, then the expression becomes $3 + 4$, which is 7. Writing 7 is called "evaluating the expression" because we are saying the value of the expression. Write the words "evaluate" and "value" on the board with underlining as shown to emphasize the connection.

Exercises: Replace n with 3 in each expression and evaluate the expression.

a) $n + 2$ b) $n - 1$ c) $5 - n$ d) $7 + n$

Answers: a) 5, b) 2, c) 2, d) 10

Tell students you are going to try to trick them and write " $5n$ " on the board. Have students replace n with 3. Discuss the problem that students run into. SAY: The answer looks like the number 53, but $5n$ really means $5 \times n$, so we mean 5×3 , not 53. To avoid confusion, we write brackets around the

number that replaces the variable. Tell students that “5(3)” is another way to write “ 5×3 .”

Exercises: Evaluate.

- a) $5(4)$ b) $7(3)$ c) $6(2)$
 d) $9(6)$ **Bonus:** $9(2000)$

Answers: a) 20, b) 21, c) 12, d) 54, Bonus: 18 000

SAY: After evaluating an expression, we can add to it or subtract from it. Write “ $5(4) + 3$ ” on the board. Tell students that this means “Multiply 5 and 4, then add 3.” SAY: We should probably write it like this to show what we mean. Continue writing on the board:

$$(5(4)) + 3$$

SAY: But that’s awkward because there are too many brackets, so we’ll just write it like this. Continue writing on the board:

$$5(4) + 3$$

SAY: We’ll all understand that it means “do 5(4) first.”

Exercises: Evaluate the expression.

- a) $3(5) + 4$ b) $2(3) + 7$
 c) $3(4) - 5$ d) $2(4) - 7$

Have students combine the steps: replace the variable with a number and evaluate the resulting expression.

Exercises: Replace n with 5, then evaluate.

- a) $3n$ b) $10n$ c) $10n + 1$
 d) $10n - 2$ e) $10n + 4$ f) $8n - 7$

Answers: a) 15, b) 50, c) 51, d) 48, e) 54, f) 33



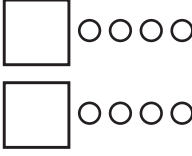
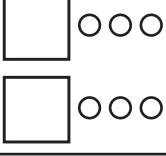


Interchangeable expressions. Write on the board:

$$2n + 3 \qquad 2p + 3 \qquad 2t + 3 \qquad 2w + 3$$

Explain to students that using different variables in the same expression doesn’t change the meaning of the expression. You can ask students to verify that all the expressions have the same value for the same number; for example, $n = 5$, $p = 5$, $t = 5$, and $w = 5$.

Extension

In the magic trick below, the magician can always predict the result of the sequence of operations performed on any chosen number. Try the trick with students, then encourage them to figure out how it works. Students can use blocks to represent the mystery number and counters to represent the ones that are added. Give students lots of hints as they manipulate the concrete materials.

The Trick	The Algebra	
Pick any number.		Use a square to represent the mystery number.
Add 4.		Use 4 circles to represent the 4 ones that were added.
Multiply by 2.		Create 2 sets of squares and circles to show the doubling.
Subtract 2.		Take away 2 circles to show the subtraction.
Divide by 2.		Remove one set of squares and circles to show the division.
Subtract the mystery number.		Remove the square.

The result is 3. No matter what number you choose, after performing the operations in the magic trick, you will always get the number 3. The model above shows why the trick works.

- Explain why the trick works using variables.
- In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

Sample solution: a) Let x be the mystery number. Adding 4 can be shown as $x + 4$. Multiplying by 2 gives $2 \times (x + 4) = 2x + 8$. Subtracting 2 gives $2x + 8 - 2 = 2x + 6$. Dividing by 2 gives $(2x + 6) \div 2 = x + 3$. Subtracting the original mystery number, x , gives $x + 3 - x = 3$.

PA5-12 Totals, Differences, and Equations

Pages 9–11

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

difference
equation
expression
part
solve
total
variable

Goals

Students will write and solve equations for one-step word problems involving sums or differences of parts.

PRIOR KNOWLEDGE REQUIRED

Can perform the four basic operations
Knows that the variable in an equation represents an unknown
Can solve simple, one-step addition and subtraction equations

Mental math minute. Present the equation $x + 5 = 5 + 3$. SAY: Let's compare the two sides of the equation. Cover the 3 with your hand and SAY: On the left side of the equal sign we have some number and 5 added to it. On the other side, we have something added to 5. The sides are equal, so it looks like the addends are simply switched. Remove your hand and ASK: What is the missing number? (3) Write " $3 + 5 = 5 + 3$ " underneath the first equation and ASK: Is this true? (yes) SAY: The answer is $x = 3$.

Exercises: Solve the equation by comparing the sides.

a) $2 + 8 = x + 8$

b) $11 + 32 = 32 + x$

c) $2 + 3 + 4 = 3 + 4 + x$

Bonus: $18 + 5 + 44 = 5 + 44 + x$

Answers: a) $x = 2$, b) $x = 11$, c) $x = 2$, Bonus: $x = 18$

Identifying parts, totals, and unknowns. Draw the table below on the board but fill in only the first column. Tell students that they can use the information in the first column to fill in two of the next three columns; the value that goes in the third column is unknown. ASK: What do we use to represent a number we are not given? (a variable) Point to the columns in turn for each row, and have students say which number goes in each column. They can make the letter x with their fingers when you point to the column that contains the unknown.

	Green Grapes	Purple Grapes	Total Number of Grapes	Another Way to Write the Total
3 green grapes 2 purple grapes	3	2	x	
7 green grapes 9 grapes altogether	7	x	9	
5 green grapes 3 purple grapes	5	3	x	
10 grapes altogether 6 green grapes	6	x	10	
5 purple grapes 9 grapes altogether	x	5	9	

Writing addition equations for the data. ASK: How can you get the total number of grapes from the number of green grapes and the number of purple grapes? (add them) Students can record the expression for each total in the last column. ($3 + 2$, $7 + x$, $5 + 3$, $6 + x$, $x + 5$)

Remind students that every equation has two parts and the equal sign between them shows that the parts are equal. So if you have two expressions showing the same thing (the total), you can write an equation. For example, the first row of the table would produce the equation $3 + 2 = x$. Have students write an equation for each row of the table.

Identifying more parts and totals. SAY: In all the questions about grapes above, there were parts—the number of green grapes and the number of purple grapes—and a total—the number of all grapes together. Write “parts” and “total” on the board. Explain that a total is always made up of parts, and you can find the total by adding the different parts together. SAY: If you have many different apples, the parts could be the number of green apples, yellow apples, and red apples, and “all apples” would be the total. If you look at the students in your class, students who have siblings and students who don’t have a sibling can be parts, and the whole class can be the total. The 10-year-olds and the 11-year-olds could also be parts (but the total would be the same: the whole class). Have students name more situations in which there are parts and a total. Other students can identify the parts and the total in each case.

Present the exercises below and have students identify the total in each one. (If you write the sentences on separate lines, students can raise the number of fingers corresponding to the number of the line that shows the total.)

Exercises: Identify the total.

- a) There are 7 red marbles. There are x blue marbles.
There are 13 marbles in total.
- b) There are x red apples. There are 9 green apples.
There are 35 apples in total.
- c) Liz has 51 marbles. Ronin has 36 marbles. Liz and Ronin have x marbles altogether.
- d) There are 25 hockey cards. There are x sports cards altogether.
There are 17 cards that are not hockey cards.
- e) Rayder had 59 baseball cards. He gave x cards away.
He has 37 cards left.

Answers: a) 13, b) 35, c) x , d) x , e) 59

Writing equations to find a total from parts. Have students write the equation that calculates the total from the parts for the exercises above.

- (a) $7 + x = 13$, b) $x + 9 = 35$, c) $51 + 36 = x$, d) $25 + 17 = x$,
e) $59 = x + 37$)

Exercises: Write the equation that calculates the total from the parts.

- Nora has 5 apples. Arsham has 6 apples. Nora and Arsham have x apples together.
- There are 15 birds in a tree. x birds are robins. 7 birds are not robins.
- Marko had 12 cookies. He gave 9 cookies to his friends. He has x cookies left.
- Shelly paid x dollars for a scarf and a hat. The scarf cost \$12. The hat cost \$15.

Answers: a) $5 + 6 = x$, b) $x + 7 = 15$, c) $12 = x + 9$, d) $12 + 15 = x$

Review solving equations. SAY: You can rewrite an equation with the variable by itself the same way you rewrote equations with the unknown by itself. Invite a volunteer to rewrite one of the equations above so that the variable, x , is by itself, and then solve the equation. (For example, in part b), rewrite $x + 7 = 15$ as $x = 15 - 7$ and solve it: $x = 8$.) Repeat with all the other equations from the two previous exercises.

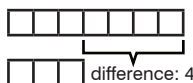
Writing and solving equations for stories (simple word problems) that involve parts and totals. Tell students that they will now solve problems from beginning to end: they will have to determine the parts and the total, write an equation to find the total from the parts, and solve it. For the exercises below, work through the first problem together, then have students solve the rest individually.

Exercises: Write an equation to solve the problem.

- Rani hiked 15 km on Saturday. She hiked x km on Sunday. She hiked 32 km over the weekend.
- There were 25 birds in a tree. x birds flew away. 17 birds were left.
- John raised \$35 for charity. Megan raised \$27. They raised x dollars together.
- Jun weighs 33 kg. Jun and his cat weigh 38 kg together. The cat weighs x kilograms.

Bonus: There were x black-footed ferrets at the end of 1988. Seventy-eight black-footed ferret kits were born in captivity in 1989. At the end of 1989, there were 120 black-footed ferrets.

Answers: a) $15 + x = 32$, $x = 17$; b) $25 = x + 17$, $x = 8$; c) $35 + 27 = x$, $x = 62$; d) $33 + x = 38$, $x = 5$; Bonus: $x + 78 = 120$, $x = 42$



The difference. Review the meaning of the word “difference”: how much larger one number is than another. Draw two rows of blocks on the board and have a volunteer show the difference between the two rows, as shown in the margin.

Identifying the larger number in a situation. Point out that many problems deal with a situation in which there is a larger number, a smaller number, and the difference between them. Write “There are 3 green apples. There are 2 more red apples than green apples.” on the board. ASK: What objects

are in this situation? (green apples and red apples) Draw a table with three columns on the board and label the first two columns “red apples” and “green apples.” ASK: Which piece of information is given: the number of green apples or the number of red apples? (green) Have students signal the number of green apples and add it to the table.

ASK: Which sentence tells us which kind of apples we have more of? (There are 2 more red apples than green apples.) Ask students to write down that sentence, then to cover the number 2 with their fingers. ASK: Which kind of apples are there more of? (red apples) Circle “red apples” in the sentence on the board. ASK: How many more red apples are there? (2) Label the third column “difference” and fill it in. Finally, write x in the empty (red apples) column.

Repeat the above for this situation: There are 3 fewer red apples than green apples. There are 12 red apples.

Provide the exercises below one at a time, and have students signal the number to put in each column. If the data for the column is not known, students can signal the letter x . Then have students signal which part is larger and circle it.

Exercises

- a) 5 red apples; 3 more green apples than red apples
- b) 6 more green apples than red apples; 2 red apples
- c) 9 green apples; 3 fewer green apples than red apples
- d) 7 green apples; 3 fewer red apples than green apples
- e) 9 red apples; 4 more red apples than green apples

Identifying the larger number and writing an equation. Remind students that to find the difference, they subtract the smaller number from the larger number. Remind students also that when they can express the same amount—the difference—in two different ways, they can write an equation. Add a fourth column to the table on the board, label it “another way to write the difference,” and have volunteers help you fill it in. Finally, have students write the equations for all the rows of the table. (a) $x - 5 = 3$, b) $x - 2 = 6$, c) $x - 9 = 3$, d) $7 - x = 3$, e) $9 - x = 4$)

Tell students that you will now make the task harder: they will need to write an equation without using the table. Present the following exercises one at a time. Have students identify the larger part and then write the equation.

Exercises

- | | |
|--|---|
| <p>a) 7 green apples
x red apples
3 more red apples than green apples</p> | <p>b) x spoons
9 forks
3 more forks than spoons</p> |
| <p>c) 3 cars
6 buses
x more buses than cars</p> | <p>d) There are 13 pears.
There are x apples.
There are 10 more apples than pears.</p> |
| <p>e) There are x rats.
There are 9 fewer mice than rats.
There are 2 mice.</p> | <p>f) There are 14 hats.
There are 4 fewer scarves than hats.
There are x scarves.</p> |
| <p>g) Zack has x hats.
Zack has 7 scarves.
He has 2 more scarves than hats.</p> | <p>h) A cat weighs 8 kg.
A dog weighs x kilograms less than the cat.
The dog weighs 5 kg.</p> |
| <p>i) Marla studied math for 30 minutes.
She read for 10 minutes less than she studied math.
She read for x minutes.</p> | |

Answers: a) $3 = x - 7$, b) $9 - x = 3$, c) $x = 6 - 3$, d) $x - 13 = 10$,
e) $x - 2 = 9$, f) $14 - x = 4$, g) $2 = 7 - x$, h) $8 - 5 = x$, i) $30 - x = 10$

Solving equations. Remind students how to solve equations by writing an equation in which x is by itself. Solve the first two equations students wrote in the exercises above as a class and then have students work individually on the rest. Students who struggle can use guessing and checking to solve the equations. (a) 10, b) 6, c) 3, d) 23, e) 11, f) 10, g) 5, h) 3, i) 20)

What parts are being compared? Present this situation: Ray spent \$12 on a book. He spent \$4 less than that on a snack. How much money did he spend on the snack? ASK: What objects or quantities appear in this problem? (cost of book, cost of snack) Write on the board:

cost of book
cost of snack

Then ask students how much the book costs. ASK: Are we given the cost of the snack? (no) Remind students that we can use a variable, x, for the unknown number. Write these amounts beside the labels.

Exercises: Write an equation for the word problem.

- a) A cat weighs 9 kg, and a dog weighs 12 kg more than the cat. The dog weighs x kilograms. How many kilograms does the dog weigh?
- b) Jasmin bikes 8 km before lunch and x km after lunch. She bikes 7 km less before lunch than after lunch. How many kilometres did Jasmin bike in total?
- c) On Monday, 27 people came to the book fair. On Tuesday, 34 people came to the book fair. How many more people came on Tuesday than on Monday?
- d) Cameron has 41 Canadian stamps. He has 26 more Canadian stamps than stamps from Brazil. How many stamps from Brazil does he have?
- e) A flight attendant served 18 vegetarian meals. There were 69 fewer vegetarian meals than meat meals. How many meat meals were there?

As a class, find the solutions to the equations in the exercises above.

(a) 21 kg, b) 15 km, c) 7 people, d) 15 stamps, e) 87 meat meals)

Writing and solving equations for simple word problems that involve differences. Tell students that the next task is more challenging. Now they will solve the problems from beginning to end: determine which part is larger, write an equation, and solve the equation.

Exercises: Write an equation for the word problem, then solve it.

- a) Jennifer read for 35 minutes. She spent 10 minutes less than that on math. She spent x minutes on math.
- b) Main Street is 30 m wide. King Street is x metres narrower than Main Street. King Street is 25 m wide.
- c) Ren has 27 stamps from Canada. He has x stamps from Mexico. He has 15 fewer stamps from Mexico than from Canada.
- d) Ansel has 29 blue marbles and x red marbles. He has 17 more blue marbles than red marbles.

Bonus: Kate weighs 42 kg with her cat in her arms and 36 kg without it. The cat weighs x kilograms.

Answers: a) $35 - 10 = x$, $x = 25$; b) $30 - 25 = x$, $x = 5$; c) $27 - x = 15$, $x = 12$; d) $29 - x = 17$, $x = 12$; Bonus: $36 + x = 42$, $x = 6$

Extensions

1. Matt asked Grade 4 and 5 students about the sports that they like. He wrote the results in a table, but his baby brother spilled water on some parts of the table. Help Matt find the missing numbers in the table.

	Hockey Fans	Soccer Fans	Baseball Fans
Grade 4	13	7	
Grade 5	16		14
Total		22	25

Answers: Total hockey fans: 29, Grade 5 soccer fans: 15, Grade 4 baseball fans: 11

2. Have students solve the following problem that involves three quantities instead of two.

There are 3 red apples, x green apples, and 7 more yellow apples than red apples in a large basket of fruit. There are 21 apples in total inside the basket. How many green apples are there?

Answers: $3 + x + (3 + 7) = 21$, so there are 8 green apples

PA5-13 Problems and Equations—Addition and Subtraction

Pages 13–14

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

difference
equation
part
solve
total

Goals

Students will solve word problems involving differences or totals using addition and subtraction equations.

PRIOR KNOWLEDGE REQUIRED

Can write an equation for finding the total or the difference from the parts given a one-step word problem
Knows that the variable in an equation represents an unknown
Can solve addition and subtraction equations

MATERIALS

BLM Word Problem Cards (p. K-43)

Mental math minute. Have students complete the exercises below, which practise the skills learned in the mental math minute in Lesson PA5-12.

Exercises: Solve the equation by comparing sides.

- a) $79 - 53 = x - 53$ b) $489 + 5 = 5 + x$ c) $62 - 43 = 62 - x$
d) $8 + 35 = 35 + x$ **Bonus:** $12\,356 - 736 = 12\,356 - x$

Answers: a) $x = 79$, b) $x = 489$, c) $x = 43$, d) $x = 8$, Bonus: $x = 736$

Organizing data. Explain that when a word problem is long, it is convenient to write out the data in point form. Write on the board:

Armand spent 25 minutes doing his math homework. He spent 15 minutes more on his science project than on math homework. How much time did Armand spend on his science project?

25 minutes on math homework
15 minutes more on science project than on math homework
 x minutes on science project

ASK: Is it easier to write an equation using the original problem or the data in point form? (the data in point form) Where did I get the last point? (this sentence answers the question in the problem)

Exercises: Write out the data in point form.

- a) Iva has 16 marbles. Seven of them are red. How many are not red?
b) There are 6 rats and 3 hamsters in a store. How many rats and hamsters altogether are in the store?
c) There are 18 cars in a parking lot. There are 7 fewer vans than cars in the lot. How many vans are in the lot?

Selected answer: b) 6 rats, 3 hamsters, x rats and hamsters altogether

7:

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3:

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Review totals and differences. Draw the picture in the margin on the board. Remind students that there are two things they can find given these two numbers: the difference between the two numbers and the total. Ask volunteers to show both in the model and to find what each is equal to. (the difference is 4 and the total is 10) Review writing the equation for the total and the difference. Repeat with this situation: larger number 8, smaller number x , difference 3, total 13.

Difference or total? Create a table with the headings “Parts,” “Total,” and “Difference” on the board. Look at the exercises below as a class and have students identify which piece of data belongs in which column. In parts f) and g), students need to decide which piece of data is the unknown (x).

Exercises: Which piece of data belongs in which column?

- a) x spoons, 8 forks, 19 forks and spoons altogether
- b) 12 cars, x buses, 18 cars and buses in a parking lot
- c) There are 5 bananas. There are x kiwis. There are 3 fewer bananas than kiwis.
- d) A cat weighs 8 kg. A dog weighs 2 kg less than the cat. The dog weighs x kg.
- e) Jayden paid \$7 for a hat. He paid \$9 for a pair of mitts. He paid x for the mitts and the hat.
- f) Alexa studied math for 30 minutes. Math and reading took 50 minutes altogether. How long did she read for?
- g) A salad recipe calls for 2 onions and 3 more tomatoes than onions. How many tomatoes are needed?

ACTIVITY (Essential)

Give students cards from **BLM Word Problem Cards**. Have them sort the cards according to the problems (the cards belonging to the same problem have the same picture). Then ask them to write an answer sentence, with x in place of the answer, below the question. For example, “How much does Hero weigh?” should have the sentence “Hero weighs x kilograms.” written below it. Have students place the cards in the table they created during the previous exercises.

Writing and solving an equation for a word problem. Have students write the equation for each situation or problem above, including the problems from BLM Word Problem Cards. Work together through the first two. Then solve the first two equations together and have students solve the equations for the rest of the problems individually. You can show students how to do the problem using the guess-and-check method or by finding an equation that has the variable by itself. For example, for part a), write “ $x + 8 = 19$ ” on the board. ASK: How can we get a related equation that has x all by itself on one side of the equation? (19 is 8 more than x , so x is 8 fewer than 19)

Write " $x = 19 - 8$ " on the board. ASK: How can we find the value of x now? (evaluate the expression on the right, $19 - 8$) Write " $x = 11$ " underneath the previous equation on the board.

(a) $x + 8 = 19$, $x = 11$; b) $12 + x = 18$, $x = 6$; c) $x - 5 = 3$, $x = 8$;
 d) $8 - x = 2$, $x = 6$; e) $7 + 9 = x$, $x = 16$; f) $30 + x = 50$, $x = 20$;
 g) $2 + 3 = x$, $x = 5$; BLM: $5 + 7 = x$, $x = 12$; $16 - 12 = x$, $x = 4$;
 $12 + 16 = x$, $x = 28$; $55 - 20 = x$, $x = 35$; $16 + 9 = x$, $x = 25$)

Organizing data, writing an equation, and solving it. Work through the first two exercises below as a class, then have students work individually.

Exercises: Write an equation and solve it.

- Ken bought 9 books and 3 magazines. How many books and magazines did he buy altogether?
- A book costs \$8 and a poster is \$3 cheaper than the book. How much does the poster cost?
- A pet store sells parrots and canaries. There are 16 canaries in the store. There are 6 fewer parrots than canaries. How many parrots are in the store?
- Avril read 9 pages on Sunday. She read 4 pages more on Sunday than on Monday. How many pages did she read on Monday?
- A cake recipe calls for 5 cups of berries. Jack has 3 cups of raspberries and some blueberries. How many cups of blueberries will he need for the cake?

Answers: a) 12, b) \$5, c) 10, d) 5, e) 2

Extension

Write an equation and solve it.

- There are 32 students in a class. 14 of them are playing soccer, and the rest are playing baseball. How many more students are playing baseball than are playing soccer?
- There are 29 students in a class. 7 of them don't wear eyeglasses. How many more students wear eyeglasses than do not wear eyeglasses?
- Braden baked 24 oatmeal cookies and 32 chocolate chip cookies. He brought 42 cookies to a bake sale. How many cookies did he leave at home?
- Jessica got \$100 for her birthday. She paid \$39 for a construction toy and \$12 for beads. How much money does she have left?

Bonus: Jin bought a book for \$9 and two magazines for \$7 each. He paid with a \$20 bill and a \$10 bill. How much change did he get?

Answers: a) 4, b) 15, c) 14, d) \$49, Bonus: \$7

PA5-14 Models and “Times as Many”

Pages 14–15

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

difference
part
tape diagram
total

Raj's stickers

Sun's stickers ➡

Goals

Students will solve word problems with “times as many” using models.

PRIOR KNOWLEDGE REQUIRED

Can solve a one-step word problem requiring addition or subtraction
Understands the expression “times as many”
Can identify the parts, total, and/or difference in a problem

Drawing a model for “times as many” situations. Tell students that two people, Sun and Raj, have some stickers. Write “Sun has four times as many stickers as Raj” on the board. SAY: I want to draw a model to represent this situation. ASK: Who has more stickers, Sun or Raj? (Sun) Draw a small rectangle or bar on the board and explain that it represents Raj’s stickers. Label the bar “Raj’s stickers.” ASK: How can we show that Sun has four times as many stickers as Raj? Accept all reasonable answers. Then explain that you are going to draw a model a specific way. You have drawn a bar to represent all of Raj’s stickers, and now you will draw the same bar four times to represent all of Sun’s stickers. Draw the picture in the margin on the board and keep it for future reference. Explain that this type of model is called a *tape diagram*. (This model is similar to the model students used for problems such as “Raj has 4 stickers; Sun has 2 more stickers than Raj.”)

Present the following situation: Karen has three times as many nickels as dimes. Draw on the board:

- | | |
|--|---|
| a) number of dimes <input type="text"/> | b) number of dimes <input type="text"/> |
| number of nickels <input type="text"/> <input type="text"/> <input type="text"/> | number of nickels <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> |
| c) number of dimes <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> | d) number of dimes <input type="text"/> |
| number of nickels <input type="text"/> | number of nickels <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> |

Ask which would fit the situation and which would not. Have students explain why the models that do not fit the situation do not work. (a) yes; b) yes; c) no, there are more dimes than nickels; d) no, there are 4 times as many nickels as dimes)

ASK: How do you know that the short bar should be the number of dimes? (there are more nickels than dimes)

Present this situation: Lyn is twice as old as Abella. ASK: Whose age will be the smaller bar? (Abella’s) Why? (because Lyn is older, so her age is larger) Ask students to draw a model for this situation and make sure students understand the meaning of “twice.”

Repeat with the exercises below. For each part, ask students to first identify the smaller number and remind them that this should be the shorter bar.

Exercises: Draw a tape diagram for the situation.

- Bill is three times as tall as his baby brother.
- Tina's full name is four times as long as Josh's.
- There are eight times as many students in the school as in our class.
- A book is twice as thick as a notebook.

Answers

a) Bill

 b) Tina

 Brother

 Josh

 c) school

 d) book

 class

 notebook

Raj's stickers

 Sun's stickers

 ➡

Finding the length of the bars when the smaller part is given. Return to the situation and model with Raj and Sun. Tell students that Raj has 3 stickers. Write "3" in Raj's block. Remind students that the blocks are the same and write "3" in each of Sun's blocks. (see margin) ASK: Can you tell from the model how many stickers Sun has? (yes, 12) How do you know? (there are 4 blocks of 3) Have students write the multiplication statement for the length of the longer bar. ($3 \times 4 = 12$)

Have students draw a model and find the lengths of the bars for this situation: Ella has 3 red marbles. She has twice as many green marbles as red marbles. Invite volunteers to show the answers.

Exercises: Draw a model and find the length of each bar.

- A car holds 5 people. A van holds three times as many people.
- Don's apartment building is 3 storeys high. Hanna's building is five times as high as Don's.
- Ethan is 5 years old. David is four times as old as Ethan.

Bonus: A sparrow has 4 eggs in its nest. A duck has three times as many eggs in its nest as a sparrow. An ostrich has five times as many eggs in its nest as a sparrow.

Answers

a) car

 b) Don's building

 van

 Hanna's building

 c) Ethan

 Bonus: sparrow

 David

 duck

 ostrich

Solving problems when the larger part is given. Write on the board:

Tasha has 20 stickers. Tasha has four times as many stickers as Eric.

Invite a student to draw the bars for the situation, without writing the numbers. ASK: How many blocks are in Tasha's bar? (4) SAY: Tasha has 20 stickers. ASK: How many stickers does each block represent? (5) How do you know? ($20 \div 4 = 5$) How many stickers does Eric have? (5)

Have students draw bars and find the length of each block for the exercises below. Work through the first one as a class and have students work individually on the rest.

Exercises: Draw a block diagram to solve the problem.

- There are 6 apples on the table. There are twice as many apples as pears. How many pears are there?
- A mini-bus holds 16 people. The mini-bus holds twice as many people as a van. How many people can the van hold?
- Jay's apartment building is 30 storeys high. Jay's building is five times as high as Vicky's building. How tall is Vicky's building?
- Luc is 14 years old. Luc is 7 times as old as Lily. How old is Lily?


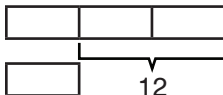
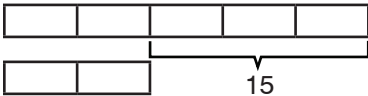
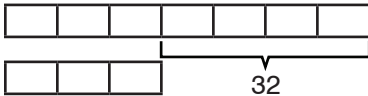
Bonus: A sugar pinecone is 45 cm long. It is three times as long as an eastern pinecone. The sugar pinecone is nine times as long as a jack pinecone. How long are the eastern pinecone and the jack pinecone?

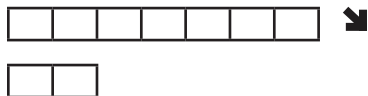
Answers: a) each block is 3, there are 3 pears; b) each block is 8, the van can hold 8 people; c) each block is 6, Vicky's building is 6 storeys; d) each block is 2, Lily is 2 years old; Bonus: the eastern pinecone is 15 cm long (the block is 15), and the jack pinecone is 5 cm long (the block is 5)

Finding the size of a single block when the difference is given.

Explain that a student you know drew the model in the margin for some word problem. In the problem, the student was given that the difference between the parts was 18. ASK: What does this mean? (the longer bar is 18 more than the shorter bar) Show how to mark this on the diagram by adding a bracket below the difference and marking it as 18, as shown in the margin. ASK: How many blocks is the difference? (3) What is the length of each block? (6) How do you know? ($18 \div 3 = 6$)

Exercises: What is the size of one block?

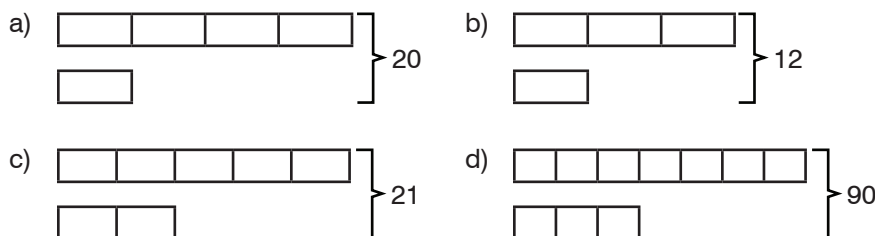
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Answers: a) 4, b) 6, c) 5, d) 8

Finding the size of a single block when the total is given. Explain that another student you know drew the model in the margin for a different word problem. Again, all the blocks are the same size, and in the problem the student was given, the total was 18. Show how to mark this on the diagram using a vertical bracket. ASK: How many blocks are there in total? (9) What is the length of each block? (2) How do you know? ($18 \div 9 = 2$)

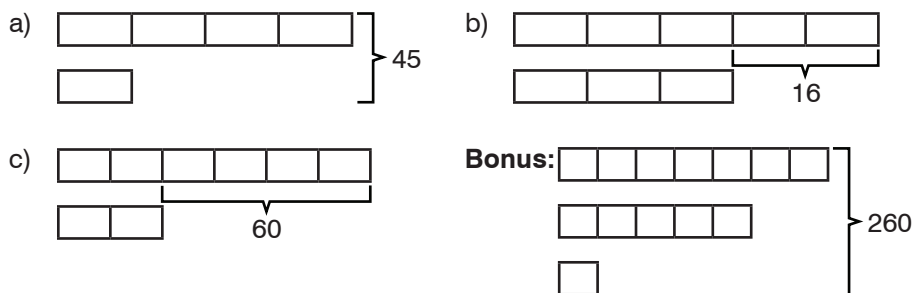
Exercises: What is the size of one block?



Answers: a) 4, b) 3, c) 3, d) 9

Now combine the two types of problems: problems with the total given and problems with the difference given.

Exercises: What is the size of one block?



Answers: a) 9, b) 8, c) 15, Bonus: 20

Solving problems with the difference or total given. Tell students that now they will need to draw the models themselves. Write on the board:

Rick is four times as old as Sara. Sara is 15 years younger than Rick. How old is Sara?

Ask students to draw a model that fits the first sentence. Have a volunteer present the answer. ASK: What does the second sentence give us: the difference, the total, or one of the parts? (the difference) Have students mark that on the diagram. ASK: How large is one block? (5) How do you know? ($15 \div 3 = 5$) How many blocks long is Sara's bar? (1 block) How old is Sara? (5) How long is Rick's bar? (4 blocks) How old is Rick? (20 years old)

Work through the first two exercises on the following page as a class, then have students work individually.

Exercises

- a) Zara saved three times as much pocket money as Anton. Anton saved \$18 less than Zara. How much money do they have together?
- b) Rob and Clara used all their pocket money to buy a common present for their grandmother. They had \$60 together. Rob had twice as much money as Clara had. How much money did each of them have?
- c) The number of students in the school who are not in Grade 5 is eight times as large as the number of students in Grade 5. There are 248 students in the school who are not in Grade 5. How many students are in the school altogether?
- d) A number is five times as large as another number. If you add the two numbers together, you get 54. What are the numbers?

Answers: a) Zara saved \$27, Anton saved \$9; b) Clara had \$20, Rob had \$40; c) 279 students in the school in total; d) 9 and 45

Extensions

- 1. Kyle reads the same number of pages every weekday and twice as many pages every weekend day. He finished a book of 108 pages in a week. How many pages did he read on Monday? How many pages did he read on Sunday?

Answer: 12 pages on Monday, 24 pages on Sunday

- 2. Choose any model from the lesson and invent a word problem that would fit the model. Have a partner solve the problem.

PA5-15 Problems and Equations—Multiplication and Division

Pages 16–17

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

equation

part

scale factor

set

unknown

Goals

Students will use equations to solve multiplication and division word problems.

PRIOR KNOWLEDGE REQUIRED

Knows that a variable can replace a number in an equation
Understands the expression “times as many/much”

Mental math minute. Ask students to solve multiplication questions within the range of 0×1 to 10×10 . For each number, first go through the questions in order, such as 0×3 , 1×3 , and so on to 10×3 , then in reverse order. After that, go through the same questions out of order. Then progress to a different number.

NOTE: Students who find it difficult to write equations without first drawing a diagram can draw diagrams as needed.

Scale factor. SAY: Sally has 3 marbles. Sam has four times as many marbles as Sally. Have a volunteer draw a diagram for this situation. Explain that the number that tells us how many times as large one part is than the other is called the *scale factor*. SAY: In this situation, the scale factor is 4.

Exercises: Identify the scale factor.

- There are 3 green apples and four times as many red apples as green apples.
- Glen is three times as old as Nina. Nina is 2 years old.
- cat’s weight
dog’s weight
- Kathy’s savings
Marcel’s savings

Answers: a) 4, b) 3, c) 2, d) 4

Writing an equation using a scale factor to find parts. Explain that you can use the scale factor to write two equations that show how to use one part to find the other part. Write on the board:

$$\text{Larger Part} = \text{Smaller Part} \times \text{Scale Factor}$$

$$\text{Smaller Part} = \text{Larger Part} \div \text{Scale Factor}$$

Look at the diagrams from the previous exercises with the class to make sure that both equations make sense.

Exercises: Write two equations for the situation, one equation telling how to get the larger part and the other telling how to get the smaller part.

- a) w mice, 4 rats, two times as many mice as rats
- b) 6 blue marbles, w green marbles, four times as many green marbles as blue marbles
- c) 8 bananas, w oranges, twice as many bananas as oranges

Answers: a) $w = 4 \times 2$, $4 = w \div 2$; b) $w = 6 \times 4$, $6 = w \div 4$;
c) $w = 8 \div 2 = 4$, $8 = w \times 2$

Remind students that equations in which the unknown number is by itself are very easy to solve—you only need a calculation. Have students identify the equations in which w is by itself in the previous exercises.

Tell students that in the next exercises, you want them to write only one equation, the one that has w by itself.

Exercises: Write an equation with w by itself.

- a) Neka earned \$36 babysitting. Jen earned w dollars mowing lawns. Neka earned three times as much as Jen.
- b) Amir hiked 16 km on Monday. He hiked w km on Tuesday. He hiked twice as far on Monday as on Tuesday.
- c) A recipe calls for 3 cups of oatmeal and twice as much flour. How much flour is needed?
- d) A dog weighs three times as much as a rabbit. The dog weighs 12 kg. How much does the rabbit weigh?

Answers: a) $w = 36 \div 3$, b) $w = 16 \div 2$, c) $w = 3 \times 2$, d) $w = 12 \div 3$

If students have trouble with problems that include units (distance, weight, etc.), point out that they can treat units such as kilometres the same way as they treat objects such as marbles. Three times as many as 4 marbles is 12 marbles, and three times as far as 4 km is 12 km. ASK: If Lewis is three times as old as Mandy, and Mandy is 4 years old, how old is Lewis? (12 years old) If a table is three times as heavy as a chair, and the chair weighs 4 kg, how heavy is the table? (12 kg)

Review the connection between sets and multiplication. Remind students that we use multiplication to find the total number of objects in equal sets. SAY: For example, 5 people can sit in each car. There are 3 cars. ASK: How many people are in 3 cars? (15) How do you know? ($3 \times 5 = 15$) Remind students that to find the total number of objects, they need to multiply the number in each set by the number of sets. To find either of the other two numbers, they need to divide the total by one of those numbers.

Write an equation for a story with an unknown number. Present a few situations and have students identify which number in the situation shows the total, which number shows the number of objects in a set, and which number shows the number of sets.

Draw on the board:

Number in Each Set	Number of Sets	Total Number of Objects
--------------------	----------------	-------------------------

Point at the Total Number of Objects column and SAY: If we don't know the total, we multiply. When we don't know either of the other two numbers, we divide. Have students use the headings on the board in the following exercises.

Exercises: Fill in a row in the table for the situation.

- a) There are 7 people in each van. There are w vans. There are 21 people altogether.
- b) There are w pears in each basket. There are 5 baskets. There are 45 pears altogether.
- c) There are 4 juice boxes in each pack. There are 15 packs. There are w juice boxes altogether.
- d) There are 6 pens in each pack. Aputik bought w packs. Aputik bought 42 pens altogether.

Bonus: An octopus has 8 arms. There are w octopuses. There are 88 arms altogether.

Answers

	Number in Each Set	Number of Sets	Total Number of Objects
a)	7	w	21
b)	w	5	45
c)	4	15	w
d)	6	w	42
Bonus	8	w	88

Add a column to the table with the heading "Equation". Go through the problems in the previous exercises one by one and ask students to write a multiplication or a division equation in which the unknown is by itself. Remind students who are struggling that they need to multiply to find the total number of objects and divide in the other two cases.

Have students solve the equations and find the unknown numbers. (a) $w = 21 \div 7 = 3$, b) $w = 45 \div 5 = 9$, c) $w = 4 \times 15 = 60$, d) $w = 42 \div 6 = 7$, Bonus: $w = 88 \div 8 = 11$)

Present a few word problems and have students use the whole process together to solve them: writing the information (in a table as needed), writing and solving the equation, and writing the answer statement.

Exercises

- a) Alex is 9 years old. Yu is three times as old as Alex. How old is Yu?
- b) Ivan is 7 years old. Tess is four times as old as Ivan. How old is Tess?
- c) A book costs \$16. The book is twice as expensive as a notebook. How much does a notebook cost?
- d) A pen costs \$7. A book costs three times as much as the pen. How much does the book cost?
- e) A pair of eyeglasses costs \$75. The pair of eyeglasses is three times as expensive as a pair of pants. How much do the pants cost?
- f) A magazine costs \$11. A book costs three times as much as the magazine. How much does the book cost?
- g) Anna spends three times as much time reading as she spends on math. She spends 45 minutes reading. How much time does she spend on math?
- h) Fred spends twice as much time reading as he spends on science. He spends 20 minutes on science. How much time does he spend reading?

Bonus

- i) Sumatran tigers are endangered. Only about 500 Sumatran tigers are left in the wild. There are about five times as many Bengal tigers as there are Sumatran tigers left in the wild. How many Bengal tigers are left in the wild?
- j) There are about ten times as many Sumatran tigers as Indo-Chinese tigers left in the wild. How many Indo-Chinese tigers are left in the wild?

Sample solution: a) $9 \times 3 = w$, $27 = w$, Yu is 27 years old

Answers: b) 28, c) \$8, d) \$21, e) \$25, f) \$33, g) 15 minutes, h) 40 minutes, Bonus: i) 2500, j) 50

Extensions

1. A subway station has two parking lots. There are 30 rows of 25 parking spots in the east lot and 15 rows of 30 parking spots in the west lot.
 - a) How many cars can park in each lot? Which lot has more parking spots?
 - b) How many cars can park at the subway station in total?

Answers: a) 750 in the east lot and 450 in the west lot, b) 1200
2. Someone wants to donate money to 15 different charities for a total of \$30 000. If each charity gets the same amount, how much money will each charity get?

Answer: \$2000

3. Seven people can ride in a mini-van. Six times as many people can ride on a regular bus. A double-decker bus can hold ten times as many people as a mini-van. How many people altogether can ride in a mini-van, a bus, and a double-decker bus?

Solution: $7 + 42 + 70 = 119$

PA5-16 More Problems and Equations

Pages 18–19

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

difference
part
scale factor
total
twice

Goals

Students will solve word problems with additive and multiplicative comparisons.

PRIOR KNOWLEDGE REQUIRED

Can solve a single-step word problem requiring addition, subtraction, multiplication, or division
Can identify the scale factor and the difference in a single-step word problem
Understands the expression “times as many/much”

Identifying scale factor or difference. Remind students what a difference between two quantities is (for example, 5 is 2 more than 3, so 2 is the difference). Then remind them what a scale factor is (for example, 12 is 4 times as many as 3, so 4 is the scale factor).

Ask students to say which words in a problem let them know what is given in it—a scale factor or a difference. Present the problems in the exercises below one at a time and have students say if they are given the difference or the scale factor. Students can use crossed fingers to signal the scale factor (as in multiplication) and a horizontal finger to signal the difference (as in subtraction). Fill in the table for the first two exercises as a class, then have them work individually on the rest.

Exercises: Fill in the table for each problem. Write x for the unknown number. Cross out the cell that does not apply to the problem.

Parts	How Many?	Scale Factor: _____
		Difference: _____

- An apple-pear cake recipe calls for 16 apples. The recipe calls for four times as many apples as pears. How many pears are needed for the cake?
- There are 50 cars in a parking lot. There are 10 fewer vans than cars. How many vans are at the parking lot?
- A van holds 9 people. A bus holds five times as many people as the van. How many people can the bus hold?
- It takes Tristan 20 minutes to do math homework. He spends twice as much time reading as he spends on math. How much time does he spend reading?
- Lela decorates her house with balloons and stars. She uses 70 balloons. She uses 10 more balloons than stars. How many stars are there?

- f) A table costs \$80. A chair costs \$40 less. How much does the chair cost?
- g) Amy is 12 years old. Amy is three times as old as Ben. How old is Ben?

Answers

a)	apples	16	Scale Factor: 4
	pears	x	Difference:
b)	cars	50	Scale Factor:
	vans	x	Difference: 10
c)	people in a van	9	Scale Factor: 5
	people in a bus	x	Difference:
d)	math time	20	Scale Factor: 2
	reading time	x	Difference:
e)	balloons	70	Scale Factor:
	stars	x	Difference: 10
f)	cost of table	80	Scale Factor:
	cost of chair	x	Difference: 40
g)	Amy's age	12	Scale Factor: 3
	Ben's age	x	Difference:

Identifying which part is the larger quantity. Ask students to decide which quantity is larger in each exercise above. Students can signal the answer. Circle the larger quantity each time and underline the smaller quantity. For each problem, ask students to say whether the quantity they need to find is the larger quantity. Underline or circle the quantity that needs to be found depending on whether it is the larger quantity or the smaller quantity.

Finally, have students find the missing quantity for each of the exercises above. They can either use an equation or a simple computation to solve the problem. Have students find the difference between the quantities in parts a), c), d), and g) and the totals in parts a) to f). Ask them to try to find a word that describes both quantities in the problem. For example, in part a), there are 20 fruits, and in part b), there are 90 vehicles.

Exercises: Find the missing quantity in the previous exercises.

Answers: a) 4 pears, b) 40 vans, c) 45 people, d) 40 minutes, e) 60 stars, f) \$40, g) 4 years old; differences: a) 12 apples, c) 36 cars, d) 20 minutes, g) 8 years; totals: a) 20 fruits, b) 90 vehicles, c) 54 people, d) 60 minutes, e) 130 decorations, f) \$120

Problems using total and difference. Remind students that in many problems they need to find the total or the difference between amounts, so the problem will have several steps.

Exercises

- a) Cody reads 7 books over the holidays. Emma reads three times as many books as Cody. How many more books did Emma read?
- b) There are 12 bicycles parked at the entrance. There are twice as many bicycles as scooters parked there. How many bicycles and scooters are parked at the entrance?
- c) A movie pass costs \$24. A concert ticket costs \$10 more than the movie pass. How much do both tickets cost together?

Bonus

- d) A jack pine is 11 m tall. A red pine is three times as tall as the jack pine. A red cedar is six times as tall as the jack pine. A giant sequoia (a giant redwood tree) is eight times as tall as the jack pine. How tall is each tree?
- e) A truck driver drives 810 km on Monday. On Tuesday, she drives 50 km more than on Monday, and that is twice as much as she drives on Wednesday. How many kilometres did she drive in three days?
- f) Another driver needs to travel the same distance as the driver in part e). He decides to drive the same distance each day for three days. How many kilometres did he drive each day?

Answers: a) 14 books, b) 18 bicycles and scooters, c) \$58,

Bonus: d) red pine: 33 m, red cedar: 66 m, giant sequoia: 88 m;

e) $810 + 860 + 430 = 2100$ km; f) 700 km

Solving problems involving division with remainders. Remind students that there is sometimes a remainder when they divide. SAY: For example, $22 \div 5 = 4 \text{ R } 2$. ASK: If 5 children share 22 grapes, how many does each child get? (4 grapes) What do we do with the remainder? (disregard it) If 22 children need to go on a boat trip, and each boat can hold only 5 children, how many boats will they need? (5) Why 5 and not 4? (we need to add another boat for the remainder) Tell students that the next exercises will be the hardest they have done yet: there will be several steps, and when dividing, they will need to decide what to do with the remainder.

Exercises

- a) Kyle has 12 marbles, Grace has 16 marbles, and Jax has 10 marbles. Can they share all the marbles equally? How many marbles will each person get? What can they do with the remainder?
- b) There are 135 apples in a crate. 29 apples are spoiled. Alice packs the rest into bags of 6 apples each. How many full bags can she make?
- c) Four classes go on a field trip. There are 29 children in one class, 31 in the second class, 35 in the third class, and 28 in the fourth class. Each bus has 45 seats. How many buses do they need?

Bonus: On another field trip for the classes in part c), three volunteers and one teacher go with each class. Now how many buses do they need?

Answers: a) no, each person gets 12 marbles if they share equally and the remainder is disregarded, or one person gets 12 marbles and the other two people get 13 marbles if the remainder is shared; b) 17; c) 3; Bonus: 4

NOTE: Tell students that some problems in the AP Book will require them to answer one or more questions before the question in the problem can be answered. Remind them to look for comparisons with “times” that require division or multiplication and comparisons that do not have the words “times” or “twice” that require addition or subtraction. Students can use equations, tape diagrams, or simple computations to solve the problems.

Extension

Ed is training Dory on how to put tires on cars at a factory. They have to put all 4 tires on 354 cars. If Ed puts on five times as many tires as Dory, how many tires does Dory put on? Show your work using a tape diagram and labels.

Answer

Tires Ed puts on:

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 } $354 \times 4 = 1416$ tires

Tires Dory puts on:

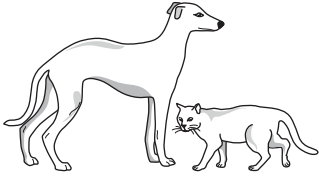
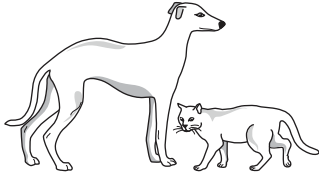
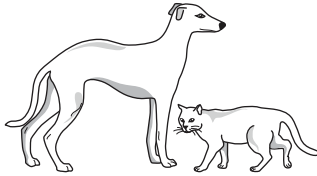









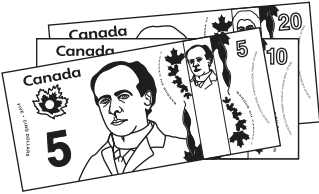


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The total number of tires that Dory and Ed put on is 1416. From the tape diagram, that’s six times the number that Dory put on, so I found $1416 \div 6 = 236$. Dory put on 236 tires.

Word Problem Cards



<p>Fluffy the cat weighs 5 kg.</p> 	<p>Hero the dog weighs 7 kg more than Fluffy the cat.</p> 	<p>How much does Hero weigh?</p>  <p>_____</p>
<p>There are 12 girls in Ms. A's class.</p> 	<p>There are 16 boys in Ms. A's class.</p> 	<p>How many more boys than girls are in Ms. A's class?</p>  <p>_____</p>
<p>There are 12 girls in Ms. A's class.</p> 	<p>There are 16 boys in Ms. A's class.</p> 	<p>How many boys and girls are in Ms. A's class?</p>  <p>_____</p>
<p>The lunch period is 55 minutes long.</p> 	<p>Sandy spent 20 minutes eating.</p> 	<p>Sandy went to the library for the rest of the period. How much time did she spend in the library?</p>  <p>_____</p>
<p>Randi earned \$16 by tutoring.</p> 	<p>Evan earned \$9 more than Randi by babysitting.</p> 	<p>How much money did Evan earn?</p>  <p>_____</p>