Unit 2 Number Sense: Addition and Subtraction

Introduction

This unit is dedicated to multi-digit whole numbers and integers. It describes how to:

- represent, compare, and order multi-digit whole numbers and integers;
- add and subtract multi-digit whole numbers using the standard algorithm; and
- use estimation in addition and subtraction of multi-digit whole numbers.

Meeting Your Curriculum

ALBERTA		
Required	NS6-1 to 4, 6 to 8	
Recommended	NS6-5	supports material in next lesson
BRITISH COLUMBIA		
Required	NS6-1 to 6	
Optional	NS6-7, 8	studied in later grades
MANITOBA		
Required	NS6-1 to 4, 6 to 8	
Recommended	NS6-5	supports material in next lesson
ONTARIO		
Required	NS6-1 to 5	
Recommended	NS6-6	preparation for estimation with decimals
Optional	NS6-7, 8	studied in later grades
NOTE: This unit covers place value concepts to billions. Students in Ontario are required to learn place value only to 1 000 000. However, higher numbers frequently appear in scientific contexts, so we recommend including numbers greater than 1 000 000.		

Mental Math Minutes

The mental math minutes in this unit:

- review strategies for addition and subtraction

Generic BLMs

The Generic BLMs used in this unit are:

BLM 1 cm Grid Paper (p. J-4)
BLM Hundreds Charts (p. J-1)

These BLMs can be found in Section J.

Materials

In this unit you will need:

- 1 cm grid paper (or **BLM 1 cm Grid Paper**)
- A grid on the board (or a transparency of BLM 1 cm Grid Paper projected on the board)

Assessment

The lessons covered by a quiz or test are as follows:

	AB	BC	MB	ON
Quiz	NS6-1 to 4	NS6-1 to 4	NS6-1 to 4	NS6-1 to 4
Quiz	NS6-5 to 8	NS6-5, 6	NS6-5 to 8	NS6-5, 6
Test	NS6-1 to 4, 6 to 8	NS6-1 to 6	NS6-1 to 4, 6 to 8	NS6-1 to 5

NS6-1 Place Value

Pages 21–23

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
billions
hundred thousands
hundreds
millions
ones
place value
ten thousand
tens
thousands

Goals

Students will identify the place value and the actual value of digits in numbers with up to seven digits.
Students will write numbers with up to nine digits using words.

PRIOR KNOWLEDGE REQUIRED

Can write number words—one, ten, hundred, thousand, ten thousand, hundred thousand—and their corresponding numerals

MATERIALS

BLM Place Value Cards (p. C-48)
magazines or science books

NOTE: This lesson covers place value concepts to billions. If you are following the Ontario curriculum, students are required to learn place value to only 1 000 000. However, higher numbers frequently appear in scientific contexts, so we recommend teaching numbers greater than 1 000 000 to students in Ontario as well.

Identifying place value. Write “65 321” on the board, leaving extra space between all the digits, and hold the “ones” card from **BLM Place Value Cards** under the 3.

ASK: Have I put the card in the right place? Is 3 the ones digit? Have a volunteer put the card below the correct digit. Invite volunteers to position the other cards correctly.

Now erase the 6 and take away the “ten thousands” card. ASK: Are these cards still in the right place? (yes) Write the 6 back in, put the “ten thousands” card back beneath the 6, erase the 1, and remove the “ones” card. ASK: Are these cards still in the right place? (no) Have a volunteer reposition the cards correctly.

Repeat this process with 6 5 2 1 (erase the 3), 6 5 3 1 (erase the 2), and 6 3 2 1 (erase the 5).

Write “23 989” on the board. Underline different digits and ask students to identify the place value of the underlined digit. If necessary, count aloud the place values starting from the right: ones, tens, and so on. Give each student one set of place value cards from BLM Place Value Cards (five cards). Students can hold up their answers to allow you to see all answers simultaneously. Repeat with several numbers. Include whole numbers with one to five digits. Include examples where you ask for the place value of the digit 0.

Exercises: Find the place value of the digit 4 in each number.

a) 24 001 b) 42 305 c) 34 d) 432 e) 32 847

Large place values—hundred thousands and millions. Introduce the place values "hundred thousands" and "millions." Write out all the place values to millions on the board.

Exercises: Write the place value of the digit 3 in the number.

a) 312 607 b) 453 207 c) 3 762 906 d) 7 401 235

e) 8 435 241 f) 8 004 312 g) 9 041 003

Answers: a) hundred thousands, b) thousands, c) millions, d) tens, e) ten thousands, f) hundreds, g) ones

Actual value. Write "28 306" on the board. SAY: 28 306 is a five-digit number. What is the place value of the digit 2? (ten thousands). SAY: The 2 is in the ten thousands place, so it stands for 2 ten thousands, or 20 000. ASK: What does the digit 8 stand for? (eight thousands or 8000) The 3? (3 hundreds or 300) The zero? (zero tens or just zero) The 6? (6 ones or 6) What is the value of the zero in 340? (zero) In 403? (zero) In 8097? (zero) SAY: Zero always has a value of zero, no matter what its place value is.

Which digit is worth more? Write on the board:

2350 5031 1435 44 573

Ask students to identify which digit, the 5 or the 3, is worth more in each number. Students should be using the phrases introduced in the lesson: stands for, has a value of, is short for. Example: In 2350, the 5 stands for 50 and the 3 stands for 300, so the digit 3 is worth more. ASK: Did you need to know the value of each digit to know which one is worth more, the 3 or the 5? PROMPT: Is there a shortcut? (the one on the left is worth more) Have students signal, by holding up the correct number of fingers, whether the 3 or the 5 is worth more in each of these numbers: 30 500, 753 414, 3 015 462. Now point out that the zero is to the left of the 5 in 3 015 462. ASK: Is it worth more than the 5 because it comes before, or to the left of, the 5? (no, zero is worth zero, and 5 is worth 5000) SAY: Only zero has no numerical value; all other digits will be worth more if they appear in a higher place value.

Reading and writing numbers up to 999. Write on the board:

325 = three hundred twenty-five

SAY: Notice that we write how many hundreds are in the number, but when there is more than one hundred, we don't write an "s" at the end of "hundred."

NOTE: We do not write "and" when writing words for whole numbers. The word "and" is reserved for decimal numbers and fractions. For example, 3.2 is written as "three and two tenths;" 3 1/2 is written as "three and one half."

Exercises: Write the number in words.

a) 412 b) 836 c) 794 d) 860 e) 703

Answers: a) four hundred twelve, b) eight hundred thirty-six, c) seven hundred ninety-four, d) eight hundred sixty, e) seven hundred three

Reading and writing multiples of 1000. Write on the board:

2000 = two thousand

Point out that the word "thousand" doesn't have an "s" even though there is more than one thousand in the number. We write how many thousands there are and then the word "thousand." Write on the board:

17 000 = __________

ASK: How many thousands are in this number? (17) Have a volunteer write the number out in the blank: seventeen thousand.

Exercises: Write the number using words.

a) 18 000 b) 20 000 c) 489 000 d) 704 000

Reading and writing numbers up to 999 999. Write "17 541" on the board. ASK: How many thousands are in this number? (17) If students answer 7, emphasize that while 7 is the thousands digit, the original number is greater than ten thousand, so 7 can't be the right answer. Write "seventeen thousand" on the board. Cover up the first two digits (17) in the numeral and ask students how they would write the rest of the number. Then finish writing the number: seventeen thousand, five hundred forty-one. Point out that we write the comma in the same place as the space appears in the number.

Exercises: Write the number using words.

a) 23 802 b) 254 006 c) 30 109 d) 140 019 e) 632 540

Reading and writing numbers up to 999 999 999 999. Write on the board these place value words:

ones	tens	hundreds
thousands	ten thousands	hundred thousands
millions	ten millions	hundred millions
billions	ten billions	hundred billions

Point out that after the thousands, there is a new word every three place values. SAY: This is why we put breaks between every three digits in our numbers—so that we can see when a new word will be used. This helps us to identify and read large numbers quickly. Write on the board:

___ ___ ___	___ ___ ___	___ ___ ___	___ ___ ___
billions	millions	thousands	

Fill in the blanks with the number 3 456 720 603 and demonstrate how to read it: three *billion*, four hundred fifty-six *million*, seven hundred twenty thousand, six hundred three. Erase the number in the blanks. Then write another large number using the same blanks—42 783 089 320—and ASK: How many billions are in this number? (42) How many more millions? (783) How many more thousands? (89) Then read the whole number together. Have students practise reading more large numbers.

The value of using spaces. Write a large number without any spaces and ASK: What makes this number hard to read? Emphasize that when the digits are not separated, you can't see at a glance how many hundreds, thousands, millions, or billions there are. Instead, you have to count the digits to identify the place value of the leftmost digit. ASK: How can you figure out where to put the spaces in this number? Should you start counting from the left or the right? (From the right, otherwise you have the same problem: you don't know what the leftmost place value is, so you don't know where to put the spaces. You always know the rightmost place value is the ones place, so start counting from the right.)

Exercise: Write the number with the proper spaces, then write the number word: 87301984387.

Answer: 87 301 984 387 is eighty-seven billion, three hundred one million, nine hundred eighty-four thousand, three hundred eighty-seven

Point out that we use spaces only in numbers with at least five digits. Sometimes, people also use spaces in four-digit numbers.

Reading and writing large numbers in context. ASK: Which contexts can involve very large numbers? (population of countries, distances in astronomy, number of stars in a galaxy, age of fossils, number of cells in a body, etc.)

Exercises

1. Write the number using numerals.

   a) The population of Ontario in 2016 was thirteen million, four hundred forty-eight thousand, four hundred ninety-four people.

   b) There are about two hundred fifty billion stars in our galaxy.

   c) Scientists say that life first appeared on Earth about three billion, eight hundred million years ago.

   d) The temperature in the core of the sun is about thirteen million, eight hundred eighty-nine thousand degrees Celsius.

   Answers: a) 13 448 494, b) 250 000 000 000, c) 3 800 000 000, d) 13 889 000

2. Write the number using words.

   a) The population of Canada in 2016 was 35 151 728 people.

   b) It takes light about 26 000 years to travel from the centre of our galaxy to Earth.

   c) Earth formed about 4 600 000 000 years ago.

   d) The temperature inside the hottest stars can reach 111 111 000°C.

   Answers: a) thirty-five million, one hundred fifty-one thousand, seven hundred twenty-eight; b) twenty-six thousand; c) four billion, six hundred million; d) one hundred eleven million, one hundred eleven thousand

CONNECTION

Real World

ACTIVITY (Essential)

Have students search the internet, magazines, or science books to find examples of large numbers. Have them write the numbers in words and in numerals. Students can make posters showing the large numbers in contexts that they found.

Extensions

1. Have students identify and write numbers given specific criteria. Examples: a two-digit number with a tens digit eight less than its ones digit; a three-digit number where the digits add to 15; a four-digit number with all digits the same and the digits add to 20.

2. **How many times as much?** ASK: What is the value of the first 1 in the number 1312? (1000) What is the value of the second 1? (10) How many times as much as the second 1 is the first 1 worth? To guide students, write on the board:

   $1000 = 10 \times$ ______,

   so 1000 is ______ times as much as 10.

   a) How many times as much as the second 1 is the first 1 worth?

   i) 1341 ii) 11 iii) 101 iv) 21 314

   b) The first 3 in 3231 is worth 3000. The second 3 is worth 30, so the first 3 in 3231 is worth 100 times as much as the second. How many times as much as the second 3 is the first 3 worth?

   i) 320 135 ii) 28 331 iii) 24 303 iv) 3 789 453

   c) Use your answers in part b) to fill in the table below:

Number	How Many Places Over	How Many Times as Much
***3**2**3**1*	*2*	*100*
***3**20 1**3**5*	*4*	*10 000*

   d) Use the pattern to find how many times as much the first 3 is worth than the second 3.

   i) 23 812 342 ii) 36 072 034

   Bonus: 823 450 917 863 401

   Selected answers: i) 10 000, ii) 1 000 000, Bonus: 1 000 000 000

3. a) Which is worth more in the number, the 3 or the 6? How many times as much is it worth?

   i) 63   ii) 623   iii) 6342   iv) 36   v) 376   vi) 3006

   Selected answers: i) 20, ii) 200, iii) 20, iv) 5, v) 50, vi) 500

   Hint: Pretend the 6 is a 3, compare the first 3 to the second 3, then solve the harder problem. Example: in part ii), look at the number 323. The first 3 is worth 100 times as much as the second 3. Since 6 is 2 times as much as 3, the 6 in 623 is worth 200 times as much as the 3.

   b) How many times as much is the 2 worth than the 5?

   i) 25   ii) 253   iii) 2534   iv) 342 580   v) 3 472 508

   (As long as the numbers are immediately next to each other, the 2 is always worth 4 times as much as the 5.)

   vi) 2345   vii) 23 457   viii) 234 576   ix) 2 345 768

   (As long as the numbers are three digits apart, the 2 is always worth 400 times as much as the 5.)

CONNECTION

Real World

4. Explain that years are usually written in terms of hundreds instead of thousands, unless the hundreds digit is 0, and the word "hundred" is omitted. SAY: So 1927 is written as "nineteen twenty-seven," but 2007 is written as "two thousand seven."

   Have students research the correct year for an event of their choice and write the year both as a numeral and by using words. Possible topics could be sports (In what year did Montreal Canadiens last win the Stanley Cup?), history (In what year was Canada founded?), family (In what year was your grandmother born?), and so on.

CONNECTION

Real World

5. Show students a copy of a cheque and explain why it's important to write the amount using both words and numerals. Show students how easy it is to change a number such as "348.00" to "1348.00" by adding the digit 1. SAY: On the other hand, it would be an obvious forgery if someone then tried to add "one thousand" before "three hundred forty-eight."

NS6-2 Representation in Expanded Form

Pages 24–25

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
digit
expanded form
placeholder
place value

Goals

Students will represent numbers in expanded form.

PRIOR KNOWLEDGE REQUIRED

Can identify place value and actual value to billions

MATERIALS

base ten blocks
BLM Place Value Cards (p. C-48)
BLM Hundreds Charts (p. J-1)

Mental math minute—number string.

String 1: $10 + 7$, $9 + 8$ (17, 17)

Present the strategy by splitting 10 as 9 and 1 and moving the 1 to the 7, as shown below:

$$\begin{aligned}10 + 7 &= (9 + 1) + 7\\ &= 9 + (1 + 7)\\ &= 9 + 8\end{aligned}$$

String 2: $10 + 37$, $9 + 38$, $8 + 39$ (47, 47, 47)

String 3: $10 + 467$, $9 + 468$, $8 + 469$ (477, 477, 477)

Expanded form. ASK: How much is the 4 worth in 43? (40) The 3? (3) Write "$43 = 40 + 3$" on the board. Tell students that this form of writing numbers (as a sum of the actual value of each digit) is called the *expanded form* of a number.

Exercises: Write the number in expanded form.

a) 52 b) 81 c) 76 d) 92

Tell students that if one of the digits is a zero, they don't need to include it in the expanded form—70 is just 70, not $70 + 0$.

Move on to numbers with three or more digits. Have volunteers write the expanded form for 459 and 32 126.

Exercises: Write the number in expanded form.

a) 352 b) 3896 c) 51 784

d) 234 984 e) 345 782 194 **Bonus:** 21 258 671 859

Expanded form of numbers containing zero. Ask a volunteer to write 350 in expanded form on the board. ($350 = 300 + 50$) Have another volunteer write the expanded form of 305. ($300 + 5$)

Exercises: Write the number in expanded form.

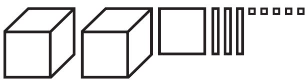
a) 207 b) 270 c) 702 d) 720

e) 6103 f) 1003 g) 29 031 h) 7 032 530

i) 408 097 j) 900 850 k) 3 070 004 l) 704 021 600

Bonus: 87 900 030 002

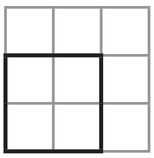
Base ten representations of numbers. Write "2135" on the board. Have a volunteer write the expanded form: 2000 + 100 + 30 + 5. Show students base ten blocks: thousands blocks, hundreds blocks, tens blocks, and ones blocks. ASK: How many thousands blocks do I need to make this number? (2) Repeat for hundreds blocks (1), tens blocks (3), and ones blocks (5). Demonstrate drawing the base ten representation on the board, as shown in the margin.



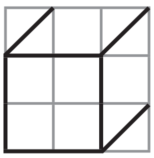
Exercise: Make a base ten representation for 1243 and then draw it.

NOTE: When drawing base ten representations, some students may need help drawing a cube:

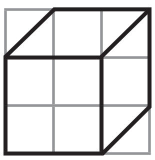
Step 1
Draw a square on a grid.



Step 2
Draw lines from the vertices.



Step 3
Join the lines.



Expanded form using place value words. Write on the board:

32 427 = _____ ten thousands + _____ thousands + _____ hundreds + _____ tens + _____ ones

Have a volunteer fill in the blanks. Repeat with the number 4589. Point out that when numbers include zeros, we only need to write place values for the other digits. For example:

30 061 = _____ ten thousands + _____ tens + _____ ones

Have a volunteer fill in the blanks.

Exercises: Write the expanded form using place value words. Only write the place values that you need to.

a) 5770 b) 804 c) 40 300

Answers: a) 5 thousands + 7 hundreds + 7 tens, b) 8 hundreds + 4 ones, c) 4 ten thousands + 3 hundreds

Proceed to six- and seven-digit numbers, adding the words "millions" and "hundred thousands."

Exercises: 308 910, 120 403, 6 079 820. **Bonus:** 17 000 000 015

Writing numbers from the expanded form. Invite volunteers to write two- and three-digit numbers given the expanded forms, or sum.

Exercises

a) 30 + 7 b) 40 + 3 c) 90 + 9

d) 300 + 50 + 7 e) 400 + 20 + 9 f) 800 + 60 + 1

ASK: What is 500 + 7? Is there a zero in the number? How do you know? What would happen if we didn't write the digit zero because we thought the zero didn't matter, and we just wrote the 5 and the 7 as "57"?

Does $500 + 7 = 57$? Emphasize that in expanded form we don't need to write the zero because expanded form is an addition statement (or sum) and the zero doesn't add anything, but in multi-digit numbers, it does mean something. It ensures that each digit's place value is recorded properly. Mathematicians call zero a *placeholder* because of this.

Exercises

1. Write the number for the expanded form.

   a) $300 + 2$ b) $200 + 30$ c) $5000 + 60 + 1$

   d) $7000 + 3$ e) $80\,000 + 300$ f) $60\,000 + 40 + 9$

   Bonus: $30\,000 + 8\,000\,000 + 10 + 400$

2. Fill in the blank.

   a) $200 + \_\_\_\_\_ + 3 = 253$

   b) $50\,000 + \_\_\_\_\_ + 20 = 54\,020$

   c) $7\,000\,000 + \_\_\_\_\_ + 300 = 7\,040\,300$

   Bonus: $4327 = 300 + 7 + \_\_\_\_\_ + 4000$

Connecting the sum of the digits in a number to the number of blocks in its base ten representation. ASK: What is the sum of the digits in 234? (9) In 4121? (8) Now have students make the numbers using base ten blocks. ASK: How many blocks did you need? (9 for 234, 8 for 4121) How can you find the number of blocks from the digits? (the sum of the digits is the number of blocks) Ask students to make the base ten representation of a number that has digits that add to 11. Encourage students to find more than one answer.

Numbers with special properties. ASK: Which digit is twice the ones digit in 432? (the hundreds digit) How do you know? (multiplying the ones digit by 2 gives the hundreds digit) Students can use the cards from **BLM Place Value Cards** to signal their answer. Repeat with 6723, 45 621, and 67 090.

Exercises

a) In the number 265 321, which digit is three times the tens digit?

b) Write a number whose hundreds digit is twice its ones digit.

c) Write an odd number whose ten thousands digit is twice its hundreds digit.

Bonus

d) In the number 4923, find a digit that is twice another digit. How many times as much as the smaller digit is the larger digit worth?

e) Make up a problem like part d) and solve it.

f) In the number 26 953, find a digit that is worth:

   i) 300 times as much as another.

   ii) 400 times as much as another.

Selected answers: a) ten thousands digit, b) sample answers: 412, 10 874, 3040; c) sample answers: 28 103, 43 299; d) the 4 is twice 2 and is worth 200 times as much, f) the 9 is worth 300 times as much as the 3, and the 2 is worth 400 times as much as the 5.

NOTE: Some students will find it easier to complete **Question 8** on AP Book 6.1 p. 25 using a hundreds chart (e.g., from **BLM Hundreds Charts**).

Representing large numbers by using a ones block to represent a thousand. Ask students to imagine that each thousands block has shrunk to the size of a ones block. Tell them that they will use the ones block as a thousands block. Ask students which base ten block could be used to represent a million. (the thousands block) PROMPTS: Which base ten block would represent ten thousand? (the tens block) A hundred thousand? (the hundreds block) Have students model the following numbers using this system: 873 000, 984 000, 6 014 000. ASK: Why would it be hard to model 984 315 using base ten blocks?

Extensions

1. What would a regular base ten block for 10 000 look like?

2. Here are descriptions of three-digit numbers. One of them does not produce a number. Which one? Justify your answer.

   a) The difference between my hundreds and tens digit is twice my ones digit. I have no zero digit. Make me with 12 blocks. (732)

   b) My hundreds and ones digits multiply to equal my tens digit. All my digits are different. My smallest digit is my ones digit. Make me with 11 blocks. (362)

   c) The sum of my ones and tens digits equals my hundreds digit. Make me with 11 blocks. (not possible: If the ones and tens digits add to the hundreds digit, then the sum of all the digits is twice the hundreds digit. The sum of all the digits is thus an even number, and so cannot equal 11.)

   d) My digits multiply to equal 8. All my digits are the same. (222)

   e) My digits increase from left to right. My digits multiply to equal 12. Make me with 8 blocks. (134)

If students know how to add multi-digit numbers (see Lesson NS6-4), they can double-check their answers by adding:

	1
2 hundreds	200
+ 9 tens	90
+ 22 ones	+ 22
	312

3. Expand a number in as many different ways as you can. Example:

   312 = 3 hundreds + 1 ten + 2 ones or
   2 hundreds + 9 tens + 22 ones
   and so on.

   Students could use base ten blocks. They can exchange blocks of one denomination for blocks of smaller denominations to find different representations.

4. A palindrome is a number that looks the same written forward or backward (Examples: 212, 3773). Find as many palindromes as you can where the sum of the digits is 10.

NS6-3 Comparing and Ordering Numbers

Pages 26–27

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
expanded form
greater than (>)
inequality signs
less than (<)

Goals

Students will order sets of two or more numbers, and will find the greatest and least number possible using the digits given.

PRIOR KNOWLEDGE REQUIRED

Can compare pairs of numbers
Knows place values to billions
Knows the number words for place values to billions and their corresponding numerals

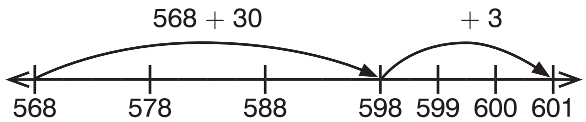
MATERIALS

grid paper or **BLM 1 cm Grid Paper** (p. J-4)
base ten blocks
3 dice per pair of students
3 small boxes, bags, or other containers per pair of students
one-digit to five-digit number cards, 1 per student

Mental math minute—number string.

String 1: $568 + 30$, $568 + 33$ (598, 601)

Present the strategy using an open number line, counting up or down by hundreds, tens, or ones, as shown below:



String 2: $438 + 200$, $438 + 199$ (638, 637)

String 3: $678 + 200$, $678 + 230$, $678 + 229$ (878, 908, 907)

Comparing numbers that differ by only one digit. Write the numbers 435 and 425 in expanded form on the board. ASK: Which number is greater? (435) How can you easily tell from the expanded form that 435 is greater than 425? (30 is more than 20, and everything else is the same) Circle the 30 and 20 on the board to emphasize this, as shown below:

$435 = 400 + 30 + 5$
$425 = 400 + 20 + 5$

Exercises: Use expanded form to identify the greater number.

a) 352 or 452 b) 405 or 401 c) 398 or 358 d) 541 or 241

Answers: a) 452, b) 405, c) 398, d) 541

Point out that you can compare the numbers 435 and 425 just by looking at the digits that are different. SAY: Since the tens digits are the only ones that are different, the number with the greater tens digit is greater. Show this by circling the 3 and 2 in the original numbers on the board:

4 3 5
4 2 5

Exercises: Circle the digits that are different, then write the greater number.

a) 542 642	b) 706 705	c) 3240 3540	d) 432 706 412 706
e) 43 207 932 43 247 932	f) 6 308 004 975 1 308 004 975		

Answers: a) hundreds digit, 642; b) ones digit, 706; c) hundreds digit, 3540; d) ten thousands digit, 432 706; e) ten thousands digit, 43 247 932; f) billions digit, 6 308 004 975

Comparing numbers that differ by more than one digit. Write "342" and "257" on the board. ASK: Which number has more hundreds? (342) more tens? (257) more ones? (257) Which number do you think is greater—the one with the most hundreds, the most tens, or the most ones? (most hundreds) Why? (hundreds are greater than tens and ones) If some students have trouble seeing that 342 is greater, have them make both numbers with base ten blocks and place the smaller number on top of the larger number to see that there are leftover blocks.

Exercises: Circle the greater number.

a) 731 or 550 b) 642 or 713 c) 519 or 382

Answers: a) 731, b) 713, c) 519

Encourage students who are struggling to compare the number of hundreds more easily by writing the second number below the first number on grid paper or **BLM 1 cm Grid Paper**, if necessary.

$582 = 500 + 82$
$574 = 500 + 74$

Now tell students that you want to compare 582 and 574. Demonstrate how to split the numbers, as shown in the margin. SAY: The hundreds are the same, so you just have to compare 82 and 74.

ASK: Which is greater, 82 or 74? (82) How do you know? (it has more tens) Point out that when comparing any two numbers with the same number of digits, you should start at the left and circle the first digits that are different—those will tell you which number is greater. Have volunteers circle the first, left-most digits that are different in these pairs of numbers:

a) 632 641	b) 873 576	c) 2741 2750	d) 352 418 327 631

Go through the pairs of numbers above one at a time, and have students point their thumbs up or down towards the greater number. (a) 641, b) 873, c) 2750, d) 352 418)

Write “214 567” and “21 456” on the board. SAY: These numbers have the same first digit, the same second digit, and so on. ASK: How can I tell which number is greater? (the number with more digits) Have a volunteer write the numbers one above the other on a grid, lining up the place values. ASK: How does lining up the place values help us see which number is greater? PROMPT: What is the first place value that is different? (hundred thousands) What is the number of hundred thousands in the second number? (zero) Which number is greater? (214 567) ASK: Do we need to look for the first digit that differs when two numbers have different numbers of digits? (no) How can we quickly say which number is greater? (the number with more digits)

Present the numbers in the exercises below on the board, one pair at a time, and have students point their thumbs towards the greater number.

Exercises: Which number is greater?

a) 718 or 523
b) 5234 or 6234
c) 9 843 275 or 9 846 192
d) 874 321 or 874 132
e) 345 089 235 or 76 801 284
f) 789 345 or 789 345 009

Answers: a) 718, b) 6234, c) 9 846 192, d) 874 321, e) 345 089 235, f) 789 345 009

7	1	8
5	2	3

NOTE: Have students who are struggling use grid paper or BLM 1 cm Grid Paper to line up the place values—ones with ones, tens with tens, and so on. (see margin)

Review inequality signs. Remind students that there are shortcut signs to show that one number is greater than the other. These signs are called inequality signs. Write on the board: $3 < 5$. Remind students that “$<$” means “is less than” because the wider end is on the right, so the lesser number is first. Similarly, “$>$” means “is greater than” because the wider end is on the left so the greater number is first.

Exercises: Write “$<$” or “$>$” for the pair of numbers.

a) 32 417 $\square$ 276 329
b) 912 401 $\square$ 91 401
c) 8 322 407 $\square$ 8 322 415
d) 62 713 $\square$ 68 290

Answers: a) 32 417 $<$ 276 329; b) 912 401 $>$ 91 401; c) 8 322 407 $<$ 8 322 415; d) 62 713 $<$ 68 290

Ordering sets of numbers. Write on the board:

3786 2567 3084 678 2193

SAY: I want to order these numbers from least to greatest. Discuss strategies as a class (identifying the smallest number, then next smallest, and so on; comparing the first two numbers and writing the smallest first, then comparing each next number to the numbers already ordered and fitting it where it belongs). Then have volunteers help you order the numbers from least to greatest, recording the answer using the inequality signs. (678 < 2193 < 2567 < 3084 < 3786) Repeat with 67 890, 678 900, 678 678, and 678 678 900, but this time have students order the numbers from greatest to least. (678 678 900 > 678 900 > 678 678 > 67 890)

Exercises: Order the numbers from least to greatest.

a) 3458, 3576, 3479
b) 4987, 6104, 6087
c) 41 387, 25 912, 34 006
d) 3407, 410, 740
e) 678 093, 782 076, 234 987
f) 2 354 000, 798 999, 54 211 235

Answers: a) 3458 < 3479 < 3576; b) 4987 < 6087 < 6104;
c) 25 912 < 34 006 < 41 387; d) 410 < 740 < 3407;
e) 234 987 < 678 093 < 782 076; f) 798 999 < 2 354 000 < 54 211 235

Bonus: Order the numbers from greatest to least.

a) 678 907 342, 6 789 070, 678 907, 678 907 000, 67 890 700

b) 30 409 765, 30 765 409, 76 540 930, 7 654 093, 304 097 650

Answers
a) 678 907 342 > 678 907 000 > 67 890 700 > 6 789 070 > 678 907;
b) 304 097 650 > 76 540 930 > 30 765 409 > 30 409 765 > 7 654 093

Creating numbers from given digits. Write on the board:

3 5 8

Ask students to make all the possible three-digit numbers from these digits. (358, 385, 538, 583, 835, 853) Tell them to try to do it in an organized way. Take suggestions for how to do that. (start with the hundreds digit; write all the numbers that have the hundreds digit 3 first, then work on the numbers with the hundreds digit 5, then 8) ASK: How many numbers are there altogether? (6, 2 starting with each digit) Which of these numbers is greatest? (853) Which is least? (358) Have students reflect: Could they have found the greatest number using these digits without listing all the possible numbers?

SAY: I want to find the greatest number possible using each of the digits 1, 3, 5, and 8 once. ASK: Do I need to find all the numbers I can make from these digits to find the greatest number? (no, you can simply write the numbers in order from greatest to least starting from the left)

Exercises: Create the greatest possible number using each digit once.

a) 2, 9, 7, 8, 4
b) 4, 4, 8, 3, 2
c) 3, 7, 0, 6, 1

Answers: a) 98 742, b) 84 432, c) 76 310

Then ask students to find the least possible number using the same digits. (Note that a number cannot begin with the digit zero, so the least number in part c) is 10 367.) Finally, ask students to use the same digits to find a number in between the greatest and least possible numbers.

Have students use the digits 0 to 9 once each so that
___ ___ ___ ___ ___ > ___ ___ ___ ___ ___.

ASK: Can you place the digits so that ___ ___ ___ ___ >
___ ___ ___ ___ ___ ___ ? (no, a six-digit number is always greater than a four-digit number)

Exercises

1. Use the digits 0, 1, and 2 to create a number that is:
   a) more than 200
   b) between 100 and 200
   c) a multiple of 10
2. Use the digits 4, 6, 7, and 9 to create a number that is:
   a) between 6200 and 6500
   b) larger than 8000
   c) an even number between 9500 and 9700
3. Use the digits 0, 3, 4, 7, and 8 to create a number that is:
   a) a multiple of 10
   b) between 73 900 and 75 000
   c) more than 40 000 and less than 70 000
4. Find the correct missing digit.
   a) 3 2 5 0 < 3 __ 1 0 < 3 3 4 8
   b) 2 5 3 9 1 < 2 5 __ 7 0 < 2 5 5 6 3

ACTIVITIES 1–2 (Optional)

1. Students work with a partner. Partner 1 rolls three dice. Partner 2 makes the greatest possible number with the digits Partner 1 rolled. Partners then switch roles. HARDER: Students work in pairs and compete with each other to make the greatest (or least, or closest to 500) three-digit number. Use three small boxes, bags, or other containers for the hundreds, tens, and ones digit, but choose where to put each roll before doing the next roll. For example, if Partner 1 is trying for the least number and rolls a 2, Partner 2 might choose to place it in the hundreds box. If the next roll is a 1, Partner 2 will want to move the 2 but will not be allowed to.

2. Students use number cards to line up for lunch in order from greatest to least. The number cards should range from one-digit to five-digit numbers. Discuss with students how they can organize themselves into groups. ASK: Are there any groups in which all the numbers in one group will be greater than all the numbers in other groups? If necessary, prompt students with questions such as: Are all the two-digit numbers greater than all the one-digit numbers? How do you know? Should the two-digit numbers go before the one-digit numbers, or after? Who should go right before the two-digit numbers? Who should go first? Before having any group go, ensure that all the groups know which order they go in. ASK: Who belongs in the first group? Have them show their cards. Repeat with the second, third, fourth, and fifth groups.

Extensions

1. How many whole numbers are greater than 4000 but less than 4350? Explain how you know.

   Answer: 349 numbers; any number from 4001 to 4349

2. What is the greatest six-digit number you can create so that:

   a) the number is a multiple of 5

   b) the ten thousands digit is twice the tens digit

   Answers: a) 999 995; b) 989 949

3. Write the number 98 950 on the board and challenge students to find all the greater numbers that use the same digits.

   Answers: 99 058; 99 085; 99 508; 99 580; 99 805; 99 850

4. Write the number 75 095 on the board. Have each student find a number that differs from it in only one digit. Have them ask a partner whether their number is greater or less than 75 095. Whose number is greatest? Have them work in groups of 4, 5, or 6 to order all the numbers they made.

   Sample answers: The greatest number is 95 095, the least number is 15 095, and the closest number is 75 094 or 75 096.

NS6-4 Addition and Subtraction

Pages 28–31

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
algorithm
carry
regrouping

Goals

Students will add and subtract multi-digit numbers, regrouping where necessary.

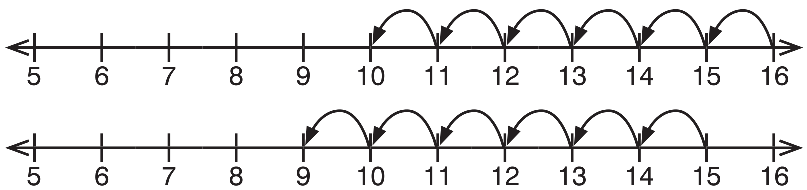
PRIOR KNOWLEDGE REQUIRED

Understands place value
Can add and subtract multi-digit numbers without regrouping
Knows that addition can be used to check subtraction

Mental math minute—number string.

String 1: 16 – 10, 15 – 9, 14 – 8 (6, 6, 6)

Use a number line to explain the strategy: both the starting number and the ending number move 1 step left on a number line, so the number of jumps is the same between the two numbers.



String 2: 84 – 30, 83 – 29, 82 – 28, 81 – 27 (54, 54, 54, 54)

Using expanded form to add three-digit numbers. Show how to add 152 + 273 using the expanded form:

152	___ hundred	+ ___ tens	+ ___ ones
+ 273	___ hundreds	+ ___ tens	+ ___ ones
	___ hundreds	+ ___ tens	+ ___ ones
Regroup:	___ hundreds	+ ___ tens	+ ___ ones

Have students signal the answers for each blank. Remind students that they can regroup 10 tens as 1 hundred, so 12 tens are, in fact, 1 hundred and 2 tens. Have a volunteer demonstrate the regrouping using base ten blocks. Point out that sometimes students will need to regroup a second time if one place value still has a two-digit number. Keep the example above on the board.

Exercises: Write the numbers in expanded form using numerals and words. Then add the numbers. Regroup when necessary.

a) 349 + 229 b) 191 + 440 c) 195 + 246

d) 3186 + 3596 e) 43 159 + 27 242 f) 869 335 + 237 245

Selected answers: a) 578, b) 631, c) 441, d) 6782, e) 70 401, f) 1 106 580

Standard notation for addition with regrouping—three digits. Now demonstrate using the standard algorithm alongside the example on the board of 152 + 273. SAY: 5 tens and 7 tens add to 12 tens, which is 1 hundred and 2 tens. We write the 2 in the bottom row, in the tens column, but we write the 1 hundred above the calculation, in the hundreds column. This way you remember to add the 1 hundred that you carried over from the tens at the same time as the hundreds from the two numbers, so you get 1 + 1 + 2 = 4 hundreds.

	1		
	1	5	2
+	2	7	3
	4	2	5

Have students rewrite any two of the addition statements above, this time using the standard algorithm, and have volunteers demonstrate the recording for all six problems on the board.

Exercises: Add using the standard notation.

a) 358 + 217 b) 475 + 340 c) 643 + 847

d) 978 + 791 e) 1358 + 7217 f) 1235 + 7958

g) 4658 + 8347 h) 94 358 + 18 647 i) 862 595 + 198 857

j) 394 348 + 415 656

Bonus

k) 3875 + 5827 + 2132 l) 15 891 + 23 114 + 36 209

Answers: a) 575, b) 815, c) 1490, d) 1769, e) 8575, f) 9193, g) 13 005, h) 113 005, i) 1 061 452, j) 810 004, Bonus: k) 11 834, l) 75 214

Struggling students can use place value charts alongside the standard algorithm.

Example:

Not correct:

```
  841 765
+ 832 491 780
```

Correct:

```
      841 765
+ 832 491 780
```

SAY: You have to make sure the place values are lined up, the ones with the ones, tens with tens. This can be tricky when the numbers have a different number of digits, but you just have to make sure the ones digits are aligned and the spaces are aligned. The trick to align the place value is to start from the right.

Exercises

1. Add. Regroup when necessary.

   a) 32 405 + 9736 b) 789 104 + 43 896 c) 999 678 + 1322

   d) 94 358 + 8647 e) 652 722 + 798 f) 5973 + 297 588

   Bonus: 17 432 + 946 + 3814 + 568 117

   Answers: a) 42 141, b) 833 000, c) 1 001 000, d) 103 005, e) 653 520, f) 303 561, Bonus: 590 309

2. Add to solve the word problem.

   a) Ron ran 1294 km one year and 1856 km the next. How many kilometres did he run altogether?

b) In an election between three candidates, the candidate who won received 567 802 votes. The other two received 213 435 votes and 342 095 votes. Did the candidate who won get more votes than the other two combined?

Answers: a) 3150 km, b) Yes; the other two combined received only 555 530 votes

Regrouping for subtraction. Write on the board:

46 − 19

SAY: I want to subtract 19 from 46. I would start with subtracting ones, but I see that I do not have enough ones. ASK: What do we do in this case? (regroup 1 ten as 10 ones) Remind students that subtraction is the opposite of addition. When subtracting, you regroup in the opposite direction: to regroup 1 ten as 10 ones, or 1 hundred as 10 tens, and so on. ASK: If I regroup 1 ten in 46 as 10 ones, how many tens and how many ones do I get? (3 tens and 16 ones) Invite a volunteer to write the numbers in a vertical subtraction, lining up place values, and show the regrouping.

Exercises: Regroup and rewrite the subtraction question.

a) 53 − 38 b) 83 − 49 c) 41 − 34 d) 90 − 72

Answers

a)

	4	13
−	3	8

b)

	7	13
−	4	9

c)

	3	11
−	3	4

d)

	8	10
−	7	2

	3	16
	$\cancel{4}$	$\cancel{6}$
−	1	9
	2	7

Finally, demonstrate the standard notation for regrouping, as shown in margin.

Exercises: Subtract using the standard algorithm.

a) 66 − 39 b) 81 − 38 c) 74 − 65 d) 40 − 17

Answers: a) 27, b) 43, c) 9, d) 23

Checking answers for subtraction. Have students perform addition with regrouping to check the answers for the problems they did above. For example, in part a), add 39 + 27. Is your answer 66?

Bonus: What other strategies can you use to check your answer? (For example, in part a), subtract 69 − 39 = 30 and subtract 3 to get 27).

Regrouping when necessary. ASK: How do you know when you need to regroup? (the digit you are subtracting is larger than the digit you are subtracting from) Students can signal the answers to the following exercises.

Exercises: Is regrouping required?

a) 58 − 19 b) 34 − 13 c) 85 − 27 d) 66 − 8

Answers: a) yes, b) no, c) yes, d) yes

	2	16	
	~~3~~	~~8~~	7
−	1	9	2
	1	7	5

Using the standard algorithm to subtract with regrouping. Remind students that when you have more digits, you might regroup not only 1 ten as 10 ones, but also 1 hundred as 10 tens, or 1 thousand as 10 hundreds, and so on. Do the example in the margin together as a class before having students work individually.

Exercises: Subtract, then check by adding.

a) 358 − 129 b) 346 − 183 c) 8625 − 2571

d) 3091 − 1271 e) 2340 − 1528 f) 13 852 − 4680

g) 34 567 − 17 239 h) 987 654 − 293 949

Answers: a) 229, b) 163, c) 6054, d) 1820, e) 812, f) 9172, g) 17 328, h) 693 705

		14	
	7	~~4~~	12
	~~8~~	~~5~~	~~2~~
−	4	5	9
	3	9	3

Do the example in the margin together as a class. Emphasize that you write the second regrouping above the first one, not over the first regrouping, so that you can see each step easily.

Exercises: Subtract, then check by adding.

a) 563 − 175 b) 5415 − 2734

c) 34 422 − 16 358 d) 378 542 − 299 289

Answers: a) 388, b) 2681, c) 18 064, d) 79 253

Bonus: Make up your own subtraction question that requires regrouping twice. Ask a partner to solve your question.

$$\begin{array}{rccc} & \scriptstyle 4 & \scriptstyle 10 & \\ & \cancel{5} & \cancel{0} & 3 \\ - & 1 & 8 & 4 \\ \hline \end{array} \longrightarrow \begin{array}{rccc} & & \scriptstyle 9 & \\ & \scriptstyle 4 & \scriptstyle \cancel{10} & \scriptstyle 13 \\ & \cancel{5} & \cancel{0} & \cancel{3} \\ - & 1 & 8 & 4 \\ \hline \end{array}$$

Borrowing from zero. Present (as vertical subtraction) a case in which the ones need to be regrouped, but the tens digit in the minuend is 0: 503 − 184. ASK: Do I have enough ones to subtract? (no) What do I need to do? (regroup 1 ten as 10 ones) What is my problem? (there are no tens to take from) Explain that, in this case, we need to regroup 1 hundred as 10 tens, then we can easily regroup 1 ten as 10 ones. Show how to record the process (see margin).

Then subtract each place value to get 319. Remind students to line up the place values properly in the next exercises.

Exercises: Subtract using the standard algorithm.

a) 402 − 169 b) 501 − 223 c) 402 − 36

d) 500 − 289 e) 4037 − 2152 f) 90 319 − 6405

g) 145 207 − 1128

Answers: a) 233, b) 278, c) 366, d) 211 e) 1885, f) 83 914, g) 144 079

Have students practise with a variety of different subtraction questions.

Exercises

1. Subtract.

   a) 3695 − 1697 b) 1000 − 854 c) 10 000 − 4356

   d) 45 683 − 1487 e) 33 116 − 13 435 f) 101 363 − 7907

   Answers: a) 1998, b) 146, c) 5644, d) 44 196, e) 19 681, f) 93 456

2. Subtract to solve the word problem.

   a) In 2016, the population of Alberta was 4 067 175 people and the population of Manitoba was 1 278 365 people. How much larger was the population of Alberta in 2016?

   b) In 2006, the population of Saskatchewan was 968 157 people. In 2016, it reached 1 098 352 people. How much did the population of Saskatchewan grow?

   Answers: a) 2 788 810, b) 130 195

Extensions

1. A palindrome is a number whose digits are in the same order when written from right to left as when written from left to right. (747 is a palindrome, 774 is not)

   a) Which numbers are palindromes? 33, 12, 512, 515

   Answers: 33, 515

   b) I am a two-digit palindrome, and 200 more than me is also a palindrome. What number am I?

   Answer: 22, 22 + 200 = 222

   c) A reverse of a number is the number in which the digits are in the opposite order. Example: A reverse of 13 is 31; a reverse of 23 567 is 76 532.

   Write the reverse of each number and add it to the number itself.

   i) 35 ii) 21 iii) 52 iv) 435 v) 1428

   What do you notice?

   Answers: i) 88, ii) 33, ii) 77, iv) 969, v) 9669; the numbers are all palindromes

   d) Find a two-digit number for which you don't get a palindrome by adding it to its reverse. Then add the resulting number to the reverse. Did you get a palindrome? If not, add the resulting number to its reverse. Repeat until you get a palindrome.

Tell students that most numbers will eventually become palindromes, but that mathematicians have not proven whether all numbers will. Over 2 000 000 steps have been tried (using a computer, of course) on the number 196, but mathematicians have still not found a palindrome.

2. Use the digits 1, 2, 3, and 4 once each to write two numbers with the smallest possible difference. Repeat with 1 through 6, 1 through 8, and 0 through 9.

   Solution: The numbers need to be as close as possible on the number line, so the tens digits need to be adjacent: 1 and 2, 2 and 3, or 3 and 4. To make the difference as small as possible, the larger number needs the smallest number of ones possible, and the smaller number needs the largest number of ones possible. This gives three pairs: $23 - 14 = 9$, $31 - 24 = 7$, and $41 - 32 = 9$. The equation $31 - 24 = 7$ is the smallest difference.

   Answers: 1 through 6: $412 - 365 = 47$; 1 through 8: $5123 - 4876 = 247$; 0 through 9: $50\ 123 - 49\ 876 = 247$

3. **A fast method for subtracting from powers of 10 without regrouping.** Do you need to regroup when you subtract from a number whose digits are all 9?

   To subtract a number from a power of 10, subtract 1, then subtract without regrouping. Add 1 back to the answer. Examples:

   $99 - 42 = 57$, so $100 - 42 = 58$
   $999 - 423 = 576$, so $1000 - 423 = 577$

   Use this method to subtract.

   a) $1000 - 768$ b) $10\ 000 - 3892$ c) $100\ 000 - 56\ 381$

   Answers: a) 232, b) 6108, c) 43 619

4. In the number 432, the hundreds digit is 1 more than the tens digit, and the tens digit is 1 more than the ones digit.

   a) Make your own three-digit number with the same properties.

   b) Write the number you wrote in part a) backwards.

   c) Subtract the numbers.

   d) Repeat parts a), b), and c) with several more numbers. What do you notice?

   Advanced students with experience in algebra can try to explain why this happens. Hint: Use expanded form with words and numerals and use *n* for the number of ones in the first number.

   Sample selected answers: a) 543, b) 345, c) $543 - 345 = 198$, d) The difference is always 198. Let *n* be the ones digit of the first number. Write the two numbers in expanded form:

   $(n + 2)$ hundreds $+ (n + 1)$ tens $+ n$ ones
   n hundreds $+ (n + 1)$ tens $+ (n + 2)$ ones

   When subtracting, you get 2 hundreds $-$ 2 ones $= 198$.

NS6-5 Rounding

Pages 32–34

CURRICULUM REQUIREMENT
AB: recommended
BC: required
MB: recommended
ON: required

VOCABULARY
round number
rounding

Goals

Students will round whole numbers to the nearest given digit.

PRIOR KNOWLEDGE REQUIRED

Can determine which multiple of ten, a hundred, or a thousand a number is between
Can find which multiple of ten, a hundred, or a thousand a given number is closest to
Can regroup when adding whole numbers

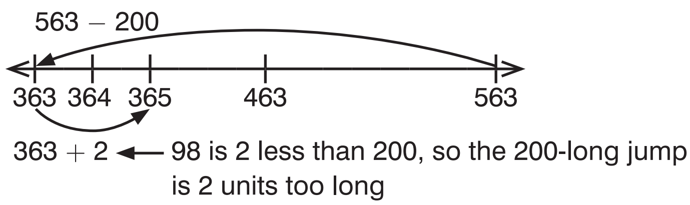
MATERIALS

grid paper or **BLM 1 cm Grid Paper** (p. J-4)

Mental math minute—number string.

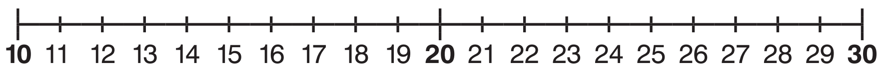
String 1: 563 − 200, 563 − 198 (363, 365)

Present the strategy using open number lines, counting up or down by hundreds, tens, or ones:



String 2: 812 − 200, 812 − 180, 812 − 279 (612, 632, 633)

Rounding two-digit numbers to the nearest ten. Draw a number line from 10 to 30, with 10, 20, and 30 in a different colour than the other numbers.



Circle the numbers 13, 18, 21, and 26, one at a time, and ask volunteers to draw an arrow showing which ten is closest. Tell students that we often want to pretend a number is equal to its closest ten, because multiples of ten are nice round numbers and easier to work with. SAY: That process is called *rounding* to the nearest ten.

Exercises: Round to the nearest ten.

a) 14 b) 19 c) 27 d) 22

Answers: a) 10, b) 20, c) 30, d) 20

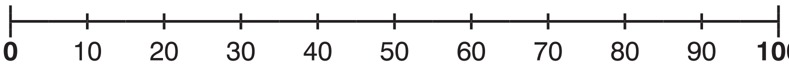
Write "37" on the board. ASK: How many tens are in 37? (3) SAY: The number 37 is between 3 tens and 4 tens. That means it's between 30 and 40. Is 37 closer to 30 or to 40? (40) Repeat with 94. (94 is between 90 and 100, and is closer to 90)

Exercises: Round to the nearest ten.

a) 97 b) 34 c) 546 d) 7 e) 381 f) 569 g) 3 h) 3472

Answers: a) 100, b) 30, c) 550, d) 10, e) 380, f) 570, g) 0, h) 3470

Rounding to the closest hundred. Draw on the board:



ASK: Is 30 closer to zero or to 100? (zero) Have a volunteer draw an arrow showing this. Repeat with 70. ASK: Which multiples of 10 are closer to zero, and which multiples of 10 are closer to 100? (10, 20, 30, 40 are closer to zero; 60, 70, 80, 90 are closer to 100) Which number is a special case? (50) Why is it a special case? (it is the same distance from 0 as it is from 100)

ASK: Where would you place the number 33 on the number line? (between 30 and 40, closer to 30) Have a volunteer show this. ASK: Is 33 closer to zero or to 100? (zero) Repeat with several numbers. Then repeat with a number line from 700 to 800.

Make a table with two headings: "Closer to 300" and "Closer to 400." Name several numbers (342, 356, 312, 385, 352, 331, 327, 390, 309, 351), and ask students to signal whether the number will be closer to 300 (thumbs down) or to 400 (thumbs up). Place the numbers in their correct column as students answer.

ASK: What digit are you looking at to decide? (the tens digit) SAY: When the tens digit is 0, 1, 2, 3, or 4, you round down. When the tens digit is 5, 6, 7, 8, or 9, you round up.

Exercises: What is the nearest hundred?

a) 457 b) 612 c) 908 d) 792 e) 729

Answers: a) 500, b) 600, c) 900, d) 800, e) 700

Tell students that choosing the closest hundred is called rounding to the nearest hundred. Numbers less than 350 are rounded down to 300 and numbers more than 350 are rounded up to 400. ASK: Is 350 closer to 300 or to 400? (neither, it is the same distance from both) SAY: I want to pick 300 or 400 anyway, and I only want to have to look at the tens digit to decide. ASK: Where are all the other numbers with tens digit 5? (in the Closer to 400 column) SAY: When a number is equally close to both hundreds, you round up.

Exercises: Round to the nearest hundred.

a) 250 b) 50 c) 850 d) 950 e) 650

Answers: a) 300, b) 100, c) 900, d) 1000, e) 700

SAY: Do the same thing when rounding to the nearest ten.

Exercises: Round to the nearest ten.

a) 25 b) 45 c) 95 d) 5 e) 55 f) 75 g) 35

Answers: a) 30, b) 50, c) 100, d) 10, e) 60, f) 80, g) 40

Rounding multi-digit numbers to any place value. Show students how numbers can be rounded in a grid. Follow the steps shown below. Example: Round 12 473 to the nearest thousand.

Step 1: Underline the digit you are rounding to.

If the digit to the right of it is 0, 1, 2, 3, or 4, round down: keep the underlined digit the same.

If the digit to the right of it is 5, 6, 7, 8, or 9, round up: add 1 to the underlined digit.

1	$\underline{2}$	4	7	3
	2			

Step 2: Write zeros in all place values to the right of the underlined digit. Copy the digits to the left of the underlined digit.

1	$\underline{2}$	4	7	3
1	2	0	0	0

SAY: So 12 473 to the nearest thousand is 12 000. That makes sense because the number is between 12 000 and 13 000, but is closer to 12 000 than to 13 000.

Exercises: Round to the underlined place value.

a) 3$\underline{5}$ 623 b) 12 $\underline{8}$71 c) 12 9$\underline{4}$3 d) 9$\underline{5}$87

Answers: a) 36 000, b) 12 900, c) 12 940, d) 9600

Rounding with regrouping. Write on the board:

round up

1	7	$\underline{9}$	7	8
		10	0	0

SAY: The 10 hundreds need to be regrouped as 1 thousand. Add it to the 7 thousands to get 8 thousands. Then copy the remaining digits to the left:

1	7	$\underline{9}$	7	8
1	8	0	0	0

SAY: This makes sense because the number is between 179 hundreds and 180 hundreds.

Exercises: Round to the stated place value. Use grid paper.

a) 39 673, thousands b) 12 971, hundreds

c) 12 993, tens d) 9987, hundreds

Answers: a) 40 000, b) 13 000, c) 13 000, d) 10 000

Extensions

1. Teach another way to round a number to the nearest ten: First, add 5. Then, replace the ones digit in the answer with zero. Example:

   $36 + 5 = 41 \longrightarrow 40$ $\qquad$ $32 + 5 = 37 \longrightarrow 30$

   To round to the nearest hundred, add 50 instead of 5. The rounded number will be the answer with the ones and tens digits replaced with 0. Example:

   $842 + 50 = 892 \longrightarrow 800$ $\qquad$ $573 + 50 = 623 \longrightarrow 600$

   You can explain why this works as follows: The number 842 is between 800 and 900. Any number between 800 and 900 will round up to 900 if it is at least halfway to 900. When you add half of 100 to a number that is less than halfway to 900, you get a number still in the 800s. When you add half of 100 to a number that is at least halfway to 900, you get a number that is in the 900s. You can liken this to pouring liquid into a container that is half full. If the amount you are pouring is at least half a container full, you will reach the top, and maybe overflow. If the amount you are pouring is less than half a container full, you will not reach the top. Challenge students to make up a rule for using this method to round to the nearest thousand.

   Use this method to round to the nearest 10, 100, 1000:

   a) 2348 b) 3452 c) 5678 d) 7502 e) 8972

   Answers: a) 2350, 2300, 2000; b) 3450, 3500, 3000; c) 5680, 5700, 6000; d) 7500, 7500, 8000; e) 8970, 9000, 9000

2. Round 365 257 to the nearest ten, hundred, thousand, and ten thousand.

   Answer: nearest ten: 365 260; nearest hundred: 365 300; nearest thousand: 365 000; nearest ten thousand: 370 000

3. Cathy says that 347 rounds to 350 and 350 rounds to 400, so 347 rounds to 400. Is she correct? Explain.

   Answer: No. Cathy needs to think about which place value she is rounding to. If she is rounding to the hundreds, the number 347 rounds to 300 because it is less than 350 and is closer to 300 than to 400. If she is rounding to the tens, 347 rounds to 350 because it is closer to 350 than to 340.

4. Write two numbers that can be rounded to 20 000, 17 000, and 17 400.

   Sample answer: 17 357, 17 432

NS6-6 Estimating in Addition and Subtraction

Pages 35–37

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: recommended

VOCABULARY
approximately equal to sign (≈)
estimate
leading digit
rounding
round number

Goals

Students will estimate sums and differences and choose a method of estimating.

PRIOR KNOWLEDGE REQUIRED

Can round numbers up or down
Can round to the nearest given place value
Can add and subtract multi-digit numbers

MATERIALS

calculator

Mental math minute—number string.

String 1: 16 + 5, 160 + 50, 16 000 + 5 000, 1 600 000 + 500 000 (21, 210, 21 000, 2 100 000)

Use place value to present the following strategy:
16 objects + 5 same objects = 21 objects,
16 tens + 5 tens = 21 tens,
16 thousands + 5 thousands = 21 thousands,
16 hundred thousands + 5 hundred thousands
= 21 hundred thousands

String 2: 14 – 6, 1400 – 600, 140 000 – 60 000, 14 000 000 – 6 000 000 (9, 900, 90 000, 9 000 000)

Review rounding to the leading digit. Use a grid on the board to round 24 758 to the nearest ten thousand, the nearest thousand, and the nearest hundred. (20 000, 25 000, 24 800) Use volunteers to help you. Follow the steps below:

Step 1: Underline the digit you are rounding to.

If the digit to the right of it is 0, 1, 2, 3, or 4, round down: keep the underlined digit the same.

If the digit to the right of it is 5, 6, 7, 8, or 9, round up: add 1 to the underlined digit.

2	<u>4</u>	7	5	8
	5			

Step 2: Write zeros in all place values to the right of the underlined digit. Copy the digits to the left of the underlined digit.

Introduce estimating. Write on the board:

$475 + 321 \qquad 500 + 300$

ASK: Which of these additions is easier? (the second one) Do you think the answers will be close? (yes) Why? (because 500 is close to 475, and 300 is close to 321) Have students calculate both sums to check their prediction. (796 and 800) SAY: Sometimes you don't need an exact answer, just an answer that is close. We call this *estimating*. ASK: In what other context do we use this word? (estimating measurements, such as length) How do we use the word "estimating" with length? (we make an educated guess about the length or distance) Explain that estimating in calculations is similar—we want a number close to the answer but without doing as much work. ASK: Why would you want to estimate answers? (to check answers, to do a quick calculation when we do not need an exact answer but need the answer quickly) Point out that estimation is often used with large numbers, for example, when you need to compare the population of two countries. SAY: It is enough for you to know that the population of Canada is about 37 million people and the population of Mexico is about 130 million people to say that the population of Mexico is larger. You do not need the exact numbers for the populations of both countries. You can also say that the population of Mexico is about 100 million people more than the population of Canada, without doing the exact calculation.

Introduce the approximately equal to sign. Tell students that there is a symbol that looks almost like an equal sign to say that two numbers are almost equal. Write on the board:

$475 + 321 \approx 500 + 300 = 800$

SAY: The symbol that looks like a squiggly equal sign means "almost equal." In mathematics we say *approximately equal*, and we call this sign the "*approximately equal to*" sign.

Estimating sums and differences by rounding. SAY: In the example on the board, we estimated the sum by rounding to the nearest hundred.

Exercises: Estimate by rounding each number to the stated place value.

a) $421 + 159$, tens
b) $4501 - 1511$, hundreds
c) $3652 + 4714$, hundreds
d) $7980 + 1278$, thousands
e) $13\,891 - 11\,990$, thousands
f) $51\,456 - 23\,512$, hundreds

Bonus

g) $8541 + 972 + 37\,218$, thousands

h) $6730 + 9050 - 612$, hundreds

Answers: a) 580, b) 3000, c) 8400, d) 9000, e) 2000, f) 28 000,
Bonus: g) 47 000, h) 15 200

Choosing the digit to round to. Write on the board: $1834 - 1598$. ASK: If I estimate by rounding to the nearest thousand, what do I get? ($2000 - 2000 = 0$) Is zero a reasonable answer? (no) SAY: This estimate shows me that the difference is small, probably far smaller than 1000, but it does not give me more information. ASK: How could I get a better estimate? (round to the nearest hundred) Have students estimate the difference by rounding to the nearest hundred. ($1800 - 1600 = 200$) Then have them find the actual answer. (236) SAY: The first place value in a number is called the *leading digit*. When we estimated to the nearest thousand, we estimated to the leading digit. Estimating to the leading digit is a good strategy to use, but sometimes you need to estimate to a different digit.

Exercises: Estimate the answer by rounding to the leading digit. Then calculate the exact answer.

a) $143 - 127$ b) $789 + 567$ c) $789\,508 + 322$

Answers: a) $100 - 100 = 0$, 16; b) $800 + 600 = 1400$, 1356;
c) $800\,000 + 300 = 800\,300$, 789 830

Discuss the results students got in the above exercises. Would rounding to a different digit work better? In part a), rounding to the leading digit does not give much information; rounding to tens works better. In part b), rounding to the leading digit produces a reasonable result. Rounding to the nearest ten gives a better estimate but does not require less work than adding the actual numbers, so the leading digit is a good digit to round to. In part c), the estimated answer looks rounded to the nearest hundred but is not close to the result you would get rounding the actual answer. When two numbers have a different number of digits, rounding to the leading digit makes no sense for estimating; you need to round to the same place value.

Exercises: Estimate, then calculate the exact answer. Which digit do you round to?

a) $345\,986 + 23\,401$ b) $345\,986 - 23\,401$

c) $345\,986 + 343\,401$ d) $345\,986 - 343\,401$

Sample answers
a) $350\,000 + 20\,000 = 370\,000$, 369 387, ten thousands
b) $350\,000 - 20\,000 = 330\,000$, 322 585, ten thousands
c) $300\,000 + 300\,000 = 600\,000$, or $350\,000 + 340\,000 = 690\,000$, 689 387, hundred thousands or ten thousands
d) $346\,000 - 343\,000 = 3000$, 2585, thousands

Point out that in part c), the calculation when rounding to ten thousands is not much harder than the calculation to hundred thousands, and the estimate is much closer. Though rounding to the leading digit gives a reasonable estimate here, rounding to the next digit is better.

Identifying the digit that the number was rounded to. Write on the board:

48 329 $\approx$ 48 300

ASK: Which digit has this number been rounded to? (hundreds) How do you know? (the last two digits are zero, and the hundreds digit is not zero) Have students round the number to the closest ten, hundred, and thousand to check the answer. (48 330, 48 300, 48 000) Repeat with 48 993 $\approx$ 49 000. Point out that though the hundreds digit is also 0, the number could have been rounded to the nearest hundred, too.

Exercises: What digit was the number rounded to?

a) 39 000 b) 4320 c) 6500

d) 1 345 600 e) 1 230 000 f) 230 010

Answers: a) thousands, hundreds, or tens; b) tens; c) hundreds or tens; d) hundreds or tens; e) ten thousands, thousands, hundreds, or tens; f) tens

SAY: When a number has many zeros, it does not necessarily mean it has been rounded. Present the situations in the exercises that follow, and have students signal thumbs up if the number has been rounded and thumbs down if it has not.

Exercises: Was the number rounded?

a) The Edmonton Oilers won the Stanley Cup in 1990.

b) The population of Canada is 37 000 000.

c) Each package has 40 candies.

d) Jen was born in the year 2000.

e) Scientists discovered that humans arrived in North America at least 25 000 years ago.

Answers: a) no, b) yes, c) no, d) no, e) yes

Investigating how rounding one number up and the other number down affects an estimate. Explain that there are other methods to estimate an addition besides rounding to the closest leading digit. For example, you can round one number up and another number down. Write on the board: $4567 + 2731$. ASK: To round these numbers to the leading digit, would you round up or down? (up and up) Draw a table on the board, as shown below, and have students help you fill it in for this addition. Students should copy the table and leave room for more columns.

Addition	$4567 + 2731$
Regular rounding	$5000 + 3000 = 8000$
Round one up and another down	$5000 + 2000 = 7000$
Actual answer	7298

ASK: Did we get a closer estimate this way? (yes) For the exercises below, have students add a row and columns to the table they started.

Exercises: Use two methods of rounding. Then calculate the actual answer. Which method produces a closer answer?

a) 7234 + 2091

b) 867 + 572

c) 947 832 + 734 603

d) 642 + 348

Answers

Addition	7234 + 2091	867 + 572	947 832 + 734 603	642 + 348
Regular rounding	9000	1500	1 600 000	900
Round one up and another down	10 000	1400	1 700 000	1000
Actual answer	9325	1439	1 682 435	990
Closer method	regular	one up, one down	one up, one down	one up, one down

Write on the board:

	Rounded	How close?
760	800	40 too high
+ 330	300	30 too low
1090	1100	10 too high

SAY: My first estimate was off by 40 and my second estimate was off by 30. ASK: Why is my answer only off by 10? What happened? (one rounded number was too high, the other was too low, and adding them nearly balanced them out)

Examine the results in the table from the previous exercise. Point out that students always rounded to the leading digit. Have students circle the second digit in all the numbers in the first row. Ask students to think why the numbers in the first column gave an answer that was closer to the regular estimate. PROMPT: When you round both these numbers down, what is the combined change? When you round one of these numbers up, how much does the number change? (The combined change down is 325, which is less than the change in each of these numbers when rounded up. Both numbers are very close to the rounded-down estimate, so rounding one of them up produces a bigger error.) ASK: When would it make sense to use the “one up, one down” strategy for estimating addition? (when both numbers are close to the middle of the range; when both second digits are 3, 4, 5, 6, or 7) Emphasize that the round up/round down strategy allows the mistakes in estimation, in other words, the differences between the actual numbers to the rounded numbers, to cancel each other.

Point out that when you estimate a sum of three or more numbers, you might want to use a combination of methods. SAY: For example, when estimating 442 + 543 + 802, it makes sense to round one of the first two addends up and the other down, because the mistakes in estimation will balance each other. It also makes sense to round the third number down, because it is so close to the round hundred. Have students estimate the sum in different ways, (Assign different methods of estimation to different students: rounding all three numbers down, rounding the first number up and the other two down, rounding the first and the third number down and the second number up.) and have students compare the estimates to the actual answer.

Rounding both numbers up when estimating differences.
Conduct a similar discussion for differences. Start with the example of $4567 - 2431 = 2136$. When you estimate the usual way, you get $4567 - 2431 \approx 5000 – 2000 = 3000$. When you round both numbers up, you get $5000 - 3000 = 2000$, which is a closer estimate. Have students fill in a similar table for this subtraction and for the exercises below.

Exercises: Use two methods of rounding. Then calculate the actual answer. Which method produces a closer answer?

a) $7234 - 2991$

b) $867 - 542$

c) $957\ 832 - 734\ 603$

d) $642 - 378$

Answers

Subtraction	$\begin{array}{r} 7234 \\ -\ 2991 \\ \hline \end{array}$	$\begin{array}{r} 867 \\ -\ 542 \\ \hline \end{array}$	$\begin{array}{r} 957\ 832 \\ -\ 734\ 603 \\ \hline \end{array}$	$\begin{array}{r} 642 \\ -\ 378 \\ \hline \end{array}$
Regular rounding	4000	400	300 000	200
Round both numbers up	5000	300	200 000	300
Actual answer	4243	325	223 229	264
Closer method	regular	both up	both up	both up

NOTE: Students can investigate the same subtractions by rounding both numbers down in Extension 1.

Again, discuss which numbers work better with rounding both numbers up. When both numbers are about midway between two ends of the estimation spectrum (numbers with a second digit of 3, 4, 5, 6, 7), rounding both numbers up produces results that are more accurate than regular rounding. Emphasize again that rounding both numbers up in a subtraction allows the errors produced by estimation to cancel each other.

Using estimation to check whether a calculated sum or difference is reasonable. Write on the board:

$273 + 385$

Tell students that Don added these two numbers and got the answer 958. ASK: Does the answer seem reasonable? (no) How can you tell? (the answer will be much less than 900) Point out that even rounding both numbers up will get only 700, so the sum cannot be more than 900.

Exercises: Is the answer reasonable?

a) $1245 + 683 = 1928$

b) $30\,417 + 6685 = 97\,267$

Answers
a) yes, the answer is $1245 + 683 \approx 1200 + 700 = 1900$;
b) no, the answer is $30\,417 + 6685 \approx 30\,000 + 7000 = 37\,000$, not about 100 000

Explain that estimating your answer is particularly important before using a calculator. SAY: When you punch in numbers, you may use a wrong button, such as using addition instead of subtraction, accidentally pressing a button that is one row above or below the button you meant to use, or pressing an extra number. Estimating the answer beforehand helps you identify mistakes.

Exercises: Estimate, then calculate the answer using a calculator. Explain which method of estimation you used.

a) The area of New Brunswick is 72 908 square kilometres, and the area of Nova Scotia is 55 284 square kilometres.

  i) What is the area of both provinces together?

  ii) What is the difference between the areas of the provinces?

b) The population of New Brunswick in 2016 was 747 101, and the population of Nova Scotia in 2016 was 923 598. The population of PEI was 142 907.

  i) What was the population of all three provinces together?

  ii) What is larger, the population of New Brunswick and PEI together, or the population of Nova Scotia? How much larger?

Sample answers
a) i) estimation by rounding to the closest thousand:
$73\,000\text{ km}^2 + 55\,000\text{ km}^2 = 128\,000\text{ km}^2$, exact answer: $128\,192\text{ km}^2$,
ii) estimation by rounding to the closest thousand:
$73\,000\text{ km}^2 - 55\,000\text{ km}^2 = 18\,000\text{ km}^2$, exact answer: $17\,624\text{ km}^2$;
b) i) estimation by rounding to the closest hundred thousand but rounding one of the numbers up (all three numbers rounded down, when two of them have a second digit of 4, will produce a bigger mistake):
$800\,000 + 900\,000 + 100\,000 = 1\,800\,000$, exact answer: 1 813 606,
ii) estimation by rounding to the closest ten thousand:
NB + PEI $\approx 750\,000 + 140\,000 = 890\,000$, which is less than the population of Nova Scotia by about $920\,000 - 890\,000 = 30\,000$,
exact answer: $923\,598 - 890\,008 = 33\,590$

Extensions

1. Round both numbers down to estimate subtraction. Compare the answers to the answers from regular estimation. When does rounding both numbers down work better than regular estimation?

   a) 7234 − 2991 b) 867 − 542

   c) 957 832 − 734 603 d) 642 − 378

   Answers

Subtraction	7234 − 2991	867 − 542	957 832 − 734 603	642 − 378
Regular rounding	4000	400	300 000	200
Round both numbers down	5000	300	200 000	300
Actual answer	4243	325	223 229	264
Closer method	regular	both down	both down	both down

   When both numbers are about midway between two ends of the estimation spectrum (numbers with a second digit of 3, 4, 5, 6, 7), rounding both numbers down produces better results than regular rounding.

2. Use the parts below to show why rounding up both numbers gives a better estimate than one up, one down rounding for $4567 - 2431 = 2136$.

   a) Write the missing numbers.

      i) $4567 = 5000 - \_\_\_\_\_\_$ ii) $2431 = 2000 + \_\_\_\_\_\_$

      iii) $2431 = 3000 - \_\_\_\_\_\_$

   b) The amount of error when estimating the usual way:
      $4567 - 2431$

      $= (5000 - \_\_\_\_\_\_) - (2000 + \_\_\_\_\_\_)$
      $= 5000 - \_\_\_\_\_\_ - 2000 - \_\_\_\_\_\_$
      $= 5000 - 2000 - \_\_\_\_\_\_ - \_\_\_\_\_\_$
      $= (5000 - 2000) - (\_\_\_\_\_\_ + \_\_\_\_\_\_)$
      $= 3000 - \_\_\_\_\_\_$

   c) The amount of error when estimating by rounding up both numbers:
      $4567 - 2431$

      $= (5000 - \_\_\_\_\_\_) - (3000 - \_\_\_\_\_\_)$
      $= 5000 - \_\_\_\_\_\_ - 3000 + \_\_\_\_\_\_$
      $= (5000 - 3000) - \_\_\_\_\_\_ + \_\_\_\_\_\_$
      $= 2000 + \_\_\_\_\_\_$

   d) Which way of estimating reduces the total error of estimation?

Answers: a) i) 433, ii) 431, iii) 569
b) $(5000 - 433) - (2000 + 431)$
$= 5000 - 433 - 2000 - 431$
$= 5000 - 2000 - 433 - 431$
$= (5000 - 2000) - (433 + 431)$
$= 3000 - 864$
c) $(5000 - 433) - (3000 - 569)$
$= 5000 - 433 - 3000 + 569$
$= (5000 - 3000) - 433 + 569$
$= 2000 + 136$
d) Rounding both numbers up reduces the total error of estimation.

3. **Front-end estimation.** In front-end estimation, we ignore all but the first digit of the number, regardless of which ten, hundred, or thousand the number is closest to. For example, by front-end estimation, 37 rounds to 30.

 a) Use both regular rounding and front-end estimation, then calculate the exact answer.

 i) $54 + 29$ ii) $47 + 28$ iii) $31 + 49$

 iv) $57 + 34$ v) $32 + 51$ vi) $35 + 45$

 Do you think front-end estimation gives a better estimate than rounding when adding?

 b) Use both regular rounding and front-end estimation, then calculate the exact answer.

 i) $76 - 57$ ii) $79 - 13$ iii) $58 - 41$

 iv) $56 - 44$ v) $57 - 36$ vi) $41 - 23$

 Do you think front-end estimation gives a better estimate than rounding when subtracting? Explain.

Sample answers

a)

Addition	54 + 29	47 + 28	31 + 49	57 + 34	32 + 51	35 + 45
Regular rounding	80	80	80	90	80	90
Front-end estimation	70	60	70	80	80	70
Actual answer	83	75	80	91	83	80
Which method	regular	regular	regular	regular	same	same

Front-end estimation does not give a better estimate than rounding when adding.

b)

Subtraction	76 − 57	79 − 13	58 − 41	56 − 44	57 − 36	41 − 23
Regular rounding	20	70	20	20	20	20
Front-end estimation	20	60	10	10	20	20
Actual answer	19	66	17	12	21	18
Which method	same	regular	regular	front-end	same	same

Front-end estimation is similar to rounding both numbers down. It produces a better or similar estimate to regular rounding when subtracting if the second digit is close to 5 in both numbers.

4. Write a number that can be rounded to the given numbers.

   a) 1000 and 1400   b) 6000, 5900, and 5870

   Sample answers: a) 1380, b) 5873

5. Can you write a number than rounds to 5800 and to 5870? Explain.

   Sample answer: No, because a number that rounds to 5870 has to be between 5865 and 5874. Any of these numbers rounds to 5900 when rounded to the nearest hundred.

NS6-7 Integers

Pages 38–40

CURRICULUM REQUIREMENT
AB: required
BC: optional
MB: required
ON: optional

VOCABULARY
greater than (>)
integers
less than (<)
negative
positive
whole numbers

Goals

Students will see various real-world contexts in which it makes sense to discuss negative integers.

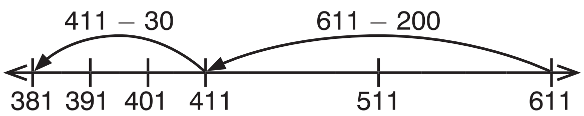
PRIOR KNOWLEDGE REQUIRED

Can compare positive integers using a number line

Mental math minute—number string.

String 1: 611 − 200, 611 − 230 (411, 381)

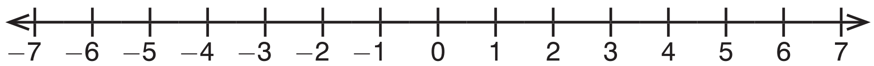
Present the strategy using open number lines, counting up or down by hundreds, tens, or ones:



String 2: 852 − 300, 852 − 320, 852 − 324 (552, 532, 528)

String 3: 914 − 200, 914 − 230, 914 − 228 (714, 684, 686)

Introduce integers. Draw the number line below on the board, leaving out the negative numbers. Point out that if you start at 0, the next whole number up is 1, then 2, and so on. ASK: Have you ever seen numbers with minus signs before? (yes) In what context? (temperature) PROMPT: What can be the temperature in January? SAY: Numbers with a minus sign before them are called negative numbers. They appear to the left of 0 on the number line and have a minus sign in front. Mark −1 and −2 on the number line and have volunteers mark all the remaining numbers to −7.



ASK: How is this number line different from other number lines you've seen before? (some numbers have a minus sign before them) Has anyone ever seen numbers with minus signs before in contexts other than temperature? In what context? Contexts in which students might have seen negative numbers include:

- the +/− rating in basketball or hockey
- time zones
- golf scores that are under (better than) par

Tell students that an *integer* is any number that is a *positive* whole number, zero, or a *negative* whole number.

Ordering integers. Draw students' attention to the number line above. ASK: Which number is greater, 3 or 4? (4) How does the number line show this? (4 is to the right of 3) Which number is greater, 2 or −5? (2) How can you tell? (because 2 appears to the right of -5 on the number line). Repeat with several pairs of integers.

Exercises

1. Use the number line to order the set of numbers from least to greatest.

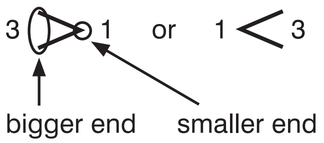
   a) $2, 3, -6$ b) $6, -1, 1, 4, -3$ c) $5, 0, -3, 3, 6, -7$

2. Order the integers and letters from least to greatest. What word do you get?

   M. -1 **E.** 4 **U.** -3 **B.** 0 **N.** -7 **R.** $+5$

   Answers:

-7	-3	-1	0	4	5
N	U	M	B	E	R



Review the signs for "greater than" and "less than." If students have a hard time remembering which is which, remind them that the bigger (wider) end of the sign always goes with the bigger (greater) number. Draw the picture in the margin on the board.

Then point out that "$>$" means "is greater than" and "$<$" means "is less than." Have volunteers read these sentences (inequalities) out loud from left to right: $3 < 5$ (3 is less than 5), $-3 < -2$ (negative 3 is less than negative 2), $1 > -2$ (1 is greater than negative 2). Demonstrate writing the last sentence in words on the board:

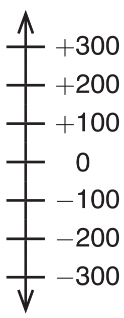
One is greater than negative two.

Exercises: Write the sentence in words.

a) $4 > 0$ b) $-3 < 0$ c) $2 > 1$ d) $-2 < -1$

Answers: a) Four is greater than zero, b) Negative three is less than zero, c) Two is greater than one, d) Negative two is less than negative one.

If students start with the number on the right—for example, a student answers part a) by writing "zero is less than four"—point out that we read from left to right, not right to left.



Sea level. Tell students that we can use integers to describe quantities in opposite directions from a chosen point. For example, we can say that sea level is 0, and distances above sea level are positive while distances below sea level are negative. If a certain type of fish swims 200 m below sea level and a certain type of bird flies 300 m above sea level, we can say that the fish swims at -200 m and the bird flies at $+300$ m. Illustrate this by drawing on the board the vertical number line shown in the margin and keep it on the board for future use.

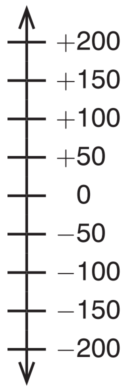
Exercises: Describe the height or depth using integers.

a) 300 m above sea level b) 2000 m below sea level

c) 8400 m above sea level d) 8400 m below sea level

Answers: a) $+300$, b) -2000, c) $+8400$, d) -8400

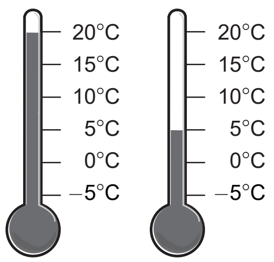
Ordering integers on vertical number lines. Referring to the vertical number line on the board, ASK: How can we tell on this number line which integer is greater? (the integer that is higher up is greater) PROMPT: Which number is greater, $+300$ or $+200$? ($+300$) How does the number line show this? (the number is higher up on the number line)



Exercise: Order the integers from least to greatest using the vertical number line: -50, $+200$, -100, $+150$.

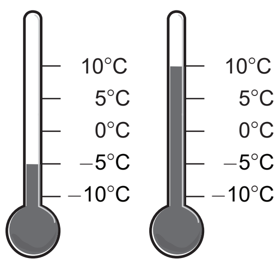
Answer: -100, -50, $+150$, $+200$

What does the greater integer signify in various contexts? Point out that, when comparing height or depth of two objects, the one that is higher up will be represented by the greater integer. SAY: If one integer is greater than another, the position it represents is higher up.



Ask students where they have seen temperature being measured using a vertical number line. (on a liquid-in-glass thermometer) ASK: Are warmer temperatures higher up or lower down? (higher up) Tell students about how liquid-in-glass thermometers work: as the temperature gets warmer, the liquid in the glass bulb expands and rises up the glass tube. Draw the picture shown in the margin on the board. ASK: What does a greater integer mean when talking about temperature? PROMPT: Since 20 is greater than 5, what can you say about the temperature $+20$°C compared to $+5$°C? ($+20$°C is warmer than $+5$°C)

Point out that students can also do the opposite: they can use what they know about temperature to compare integers (instead of using integers to compare temperature). SAY: For example, if you know that $+20$°C is warmer than $+5$°C, you know that $+20$ is greater than $+5$.



Draw the picture shown in the margin on the board. ASK: Which temperature is warmer, 10°C or -5°C? (10°C) How do you know? (because at 10°C you can wear lighter clothes, and it can snow at –5°C) How does the thermometer show that 10°C is warmer than –5°C? (10°C is higher up on the thermometer) Which integer is greater, 10 or -5? Repeat for -10°C or 5°C. SAY: This time, 5°C is warmer, because it is higher up, and so 5 is greater than -10.

Integers are used to describe opposite directions from a given point. Point out the connection between all the uses of integers you have considered so far: they all involve picking an arbitrary point as zero and taking one direction as positive and the other direction as negative. When a vertical position such as sea level is chosen as zero, positions higher up are positive and positions lower down are negative. When a temperature is chosen as zero, warmer temperatures are positive and colder temperatures are negative.

Integers and time zones. Tell students that time zones are also represented using integers, with the time in London, UK, being considered zero. Ask students if they have ever wanted to watch a live event on television but couldn’t because it was on in the middle of the night. Point out that in some parts of the world, it is the middle of the night right now! SAY: If you have friends or relatives in other parts of the world, you have to be careful about what time you call them. Tell students which time zone they are in (e.g., -5, -6, or -7) and explain that the minus sign tells them they are behind London's time, and the number tells them by how many hours.

Exercises: Write integers to describe the time zone.

a) four hours ahead of London

b) two hours behind London

c) three hours behind London

d) one hour ahead of London

Answers: a) $+4$, b) -2, c) -3, d) $+1$

Extensions

1. This extension should be done after students complete **Question 10** on AP Book 6.1 p. 40, since students need to be familiar with the various average temperatures of the planets.

    a) Absolute zero (about -273°C) is the coldest possible temperature. About how much warmer is the coldest planet than absolute zero? (about 60°C; the temperature on Uranus is –216°C and absolute zero is –273°C.)

CONNECTION

Science

    b) The planets are listed in order from closest to the Sun to farthest from the Sun. Some students might be interested in researching why Venus is the hottest planet even though it is only the second closest to the Sun.

CONNECTION

Science

2. Investigate the history of thermometers. Questions students might investigate: How were the scales (Celsius, Fahrenheit, and Kelvin) chosen? How did human body temperature play a role in the Fahrenheit scale? How were 0°C and 100°C chosen? How was 0 Kelvin chosen? How was the value of each degree chosen in each scale? In which order were the three scales invented? Were temperatures warmer than zero ever chosen to be negative?

NS6-8 Opposite Integers

Pages 41–42

CURRICULUM REQUIREMENT
AB: required
BC: optional
MB: required
ON: optional

VOCABULARY
greater than (>)
integers
less than (<)
mirror line
negative
opposite integer
positive

Goals

Students will understand negative integers as being equally far from 0 as their positive counterparts, in the opposite direction.
Students will compare one-digit negative integers.

PRIOR KNOWLEDGE REQUIRED

Can use the greater than (>) and less than (<) signs correctly
Can compare one- and two-digit numbers

MATERIALS

ruler, pencil, and a blank sheet of paper
BLM Number Lines from −6 to 6 (p. C-49)

Mental math minute—number string.

String 1: $100 + 413$, $99 + 414$, $98 + 415$ (513, 513, 513)

Present the strategy using the associative property:

$$\begin{aligned} 100 + 413 &= (99 + 1) + 414 \\ &= 99 + (1 + 414) \\ &= 99 + 415 \end{aligned}$$

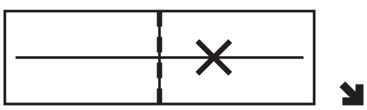
String 2: $800 + 154$, $799 + 155$, $798 + 156$ (954, 954, 954)

Introduce opposite integers by folding a number line through 0.

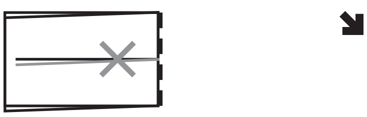
ACTIVITY (Essential)

Give students a ruler, a pencil (not a pen), and a blank sheet of paper for this activity. Give students these instructions:

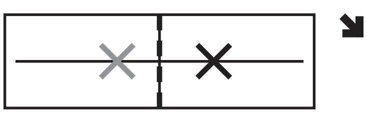
1. Draw a line segment 10 cm long.
2. Draw a mirror line at the midpoint of the line segment: the 5 cm mark.



3. Draw an ✕ to the right of the mirror line. Make the ✕ dark and thick by running your pencil over each line repeatedly.



4. Fold the sheet along the mirror line, with ✕ on the inside.
5. You should be able to see the ✕ through the paper. Rub or scratch the ✕ hard so that it rubs off and makes another mark on the other half of the paper.



6. Unfold the sheet to see where the new ✕ is located. Then check, by measuring, that the new ✕ is the same distance from the mirror line, but in the opposite direction, as the original ✕.

Give students a copy of **BLM Number Lines from −6 to 6.** They can use a different number line for each dot.

Exercises: Draw a dark dot at the given point on a number line, and fold the sheet along the zero mark of the number line. Scratch at the dot to create a new dot on the number line, on the other side of 0. Where is the new point?

a) 5 b) −3 c) 2 d) −4

Answers: a) −5, b) 3, c) −2, d) 4

Tell students that two integers are called *opposite integers* when they are the same distance from zero, but in opposite directions. What students found in the folding activity is the opposite integer to each given integer. ASK: What is the opposite of 5? (−5) What is the opposite of −3? (3) Write 3 and −3 on the board. ASK: What is the same about how we write opposite integers? (they have the same number part) ASK: What is different about how we write them? (whether the "−" sign is there or not)

Exercises: What is the opposite integer?

a) 8 b) −9 c) 10 d) −7 **Bonus:** 23 809

Answers: a) −8, b) 9, c) −10, d) 7, Bonus: −23 809

Explain that, in still water, a bird's reflection looks as far below the surface of the water as the bird is above the surface.

Exercises: Let 0 be the surface of the water. Write an integer showing where the bird's reflection appears to be if the bird is flying:

a) 50 m above the surface b) 85 m above the surface

c) 300 m above the surface d) 420 m above the surface

Bonus: How far below the bird does its reflection appear to be in each part?

Answers: a) −50, b) −85, c) −300, d) −420; Bonus: a) 100 m, b) 170 m, c) 600 m, d) 840 m

The special case of zero. On BLM Number Lines from −6 to 6, have students draw a dark dot at 0, and fold along the mirror line through the 0 mark as before. Have students scratch the dot to make a new mark. Before students unfold the sheet, have them predict where the new mark will be. (in the same place as the old mark) Then have students unfold their sheets to check their prediction. SAY: It doesn't look like there is a new mark. ASK: Do you just need to press harder? What happened? (there isn't a new mark, because the new mark is in the same place as the old mark) Point out that the opposite of zero is still at zero. It is the only number that is the same as its opposite. Remind students that we defined opposite integers by where they are in relation to zero, so it makes sense that zero is a special case.

Comparing two negative integers by comparing their positive opposites first. ASK: By looking at a number line, how can you tell if one integer is greater than another? (if it is to the right, it is greater) Draw on the board a number line from −6 to 6.

Exercises: Use the number line to copy and complete the sentences.

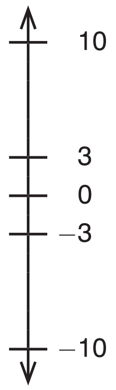
3 is to the __________ of 4, so 3 is __________ than 4.

-3 is to the __________ of -4, so -3 is __________ than -4.

Repeat to compare:

- 5 to 2 and then -5 to -2;
- 2 to 0 and then -2 to 0;
- -1 to -3 and then 1 to 3.

ASK: Does anyone see a pattern? PROMPT: Since you know 3 is less than 5, how does the opposite of 3 compare to the opposite of 5? (the opposite of 3 must be greater than the opposite of 5)



SAY: Another way to see this is on a vertical number line. Draw the number line shown in the margin; SAY: 10 is higher than 3, so -10 will be lower than -3. Students can imagine the folding activity they did earlier: because 3 is closer to 0 before the fold, -3 is closer to 0 after the fold.

Exercises: Write "$<$" or "$>$" in the box.

a) 4 ☐ 7, so -4 ☐ -7.

b) 8 ☐ 6, so -8 ☐ -6.

c) 13 ☐ 9, so -13 ☐ -9.

Bonus: 5000 ☐ 8000, so -8000 ☐ -5000.

Answers: a) $<$, $>$; b) $>$, $<$; c) $>$, $<$; Bonus: $<$, $<$

Finding distances on a number line. Draw a number line on the board from -7 to 7. ASK: Which number is closer to 0: 3 or 4? (3) Repeat the question for these pairs: 3 or -4 (3), -3 or 4 (-3), -3 or -4 (-3), -1 or $+5$ (-1), $+1$ or -5 ($+1$). ASK: How can you tell which number is closer to 0? (choose the number with the smaller number part)

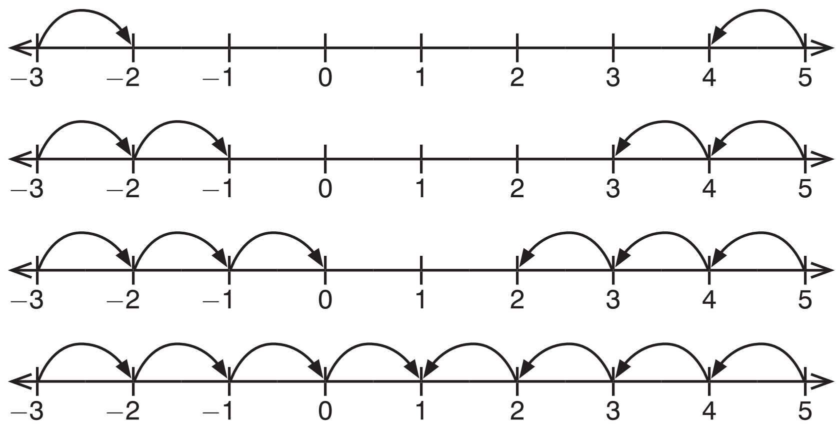
Exercises: Which number is closer to 0?

a) 2 or -3 b) -2 or 3 c) 2 or 3 d) -2 or -3

Bonus: -3000 or $+8000$

Answers: a) 2, b) -2, c) 2, d) -2, Bonus: -3000

Finding the integer halfway between two integers. Draw a number line from -6 to $+6$ on the board. Tell students you want to find the integer that is halfway between -3 and $+5$ on the number line. "Halfway between" two numbers means the same distance from both. Demonstrate pointing at both numbers and moving one step in from each at the same time, until your two fingers meet. The integer you meet at, $+1$, is halfway between -3 and $+5$. For the exercises below, give each student a copy of BLM Number Lines from -6 to 6. If students have trouble keeping track with their fingers, some of them might find it easier to draw hops as shown on the following page.



Exercises: Find the integer halfway between the two given integers.

a) −4 and +2 b) −3 and +1 c) −2 and +6 d) −6 and −2

Answers: a) −1, b) −1, c) +2, d) −4

Zero is halfway between opposite integers. Tell students to look for a pattern in their answers to the following exercises.

Exercises: Write the integer halfway between the two given opposite integers.

a) −3 and +3 b) −4 and +4 c) −1 and +1 d) −7 and +7

Answers: a) 0, b) 0, c) 0, d) 0

When students finish, ASK: What pattern do you notice in your answers? (they are all the same, zero) SAY: zero is always halfway between two opposite integers. That's because opposite integers are by definition the same distance from zero in opposite directions.

Exercises: Finish the sentence: zero is halfway between…

a) −3 and ___. b) −8 and ___. c) −6 and ___. d) −9 and ___.

Bonus: −892 and _____.

NOTE: When you need to pair students up for an activity, you can give each student a card with an integer on it and tell students to find their opposite integer. Use multi-digit integers, and be sure every integer has an opposite. Variation: Include two cards with 0.

Ordering integers in a real-world context. Tell students that the temperature of a freezer for storing and freezing food should be between −30°C and −18°C. ASK: Which of these temperatures are acceptable for a freezer? Students can signal yes (thumbs up) or no (thumbs down) for each one.

a) −27°C b) +3°C c) −12°C d) −19°C

Answer: a) yes, b) no, c) no, d) yes

Extensions

1. How many one-digit negative integers are less than −7?

   Answer: 2 (−8 and −9)

2. a) What number is halfway between 4 and 10? (7)

   b) What number is halfway between −4 and −10? (−7)

   c) How do your answers to parts a) and b) compare?
   (they are opposite)

   d) Repeat parts a) to c) for other examples of positive integers and their negative opposites. How can you find the integer halfway between two negative integers if you know the integer halfway between their positive opposites?

3. What symbol can be put in the box to make the statement true?
   34 207 > $\boxed{}$110 642

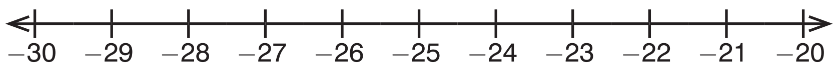
   Answer: a negative sign (−) or a decimal point (.)

4. a) How many six-digit positive integers are greater than 999 985?

   b) How many six-digit negative integers are less than −999 985?

   Answers: a) 999 999 − 999 985 = 14; b) 14, because the six-digit negative integers that are less than −999 985 are exactly the opposites of the six-digit positive integers that are greater than 999 985

5. Use the number line to round each negative number to the nearest ten.



   −29 ≈ _____ −22 ≈ _____ −26 ≈ _____ −24 ≈ _____

   How is rounding negative numbers similar to rounding positive numbers?

   Answers: −29 ≈ −30, −22 ≈ −20, −26 ≈ −30, −24 ≈ −20; Round as though the numbers are positive, then put back the negative sign.

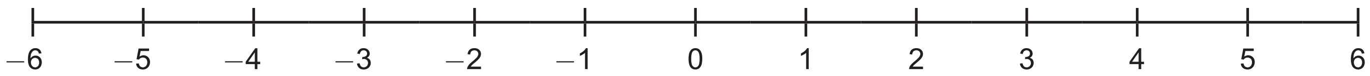
NAME ______________________________ DATE ______________

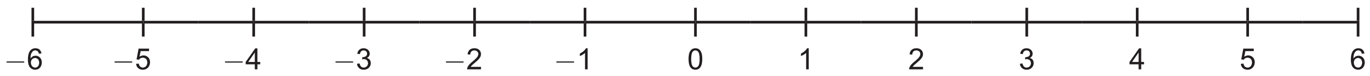
Place Value Cards

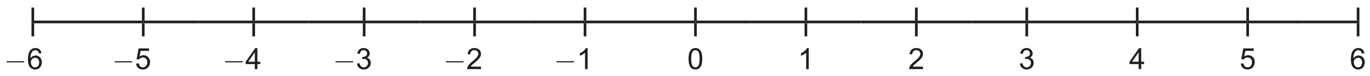
Ten Thousands	Thousands	Hundreds	Tens	Ones
Ten Thousands	Thousands	Hundreds	Tens	Ones

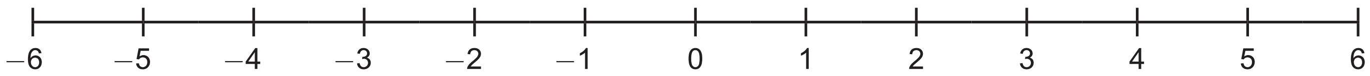
NAME ______________________________ DATE ______________

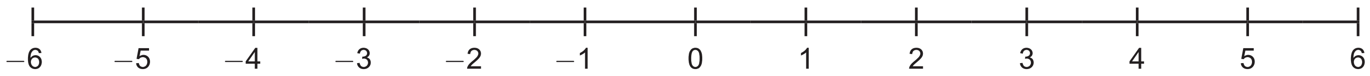
Number Lines from −6 to 6

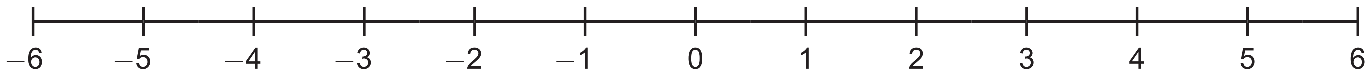


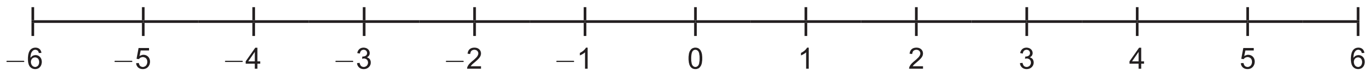


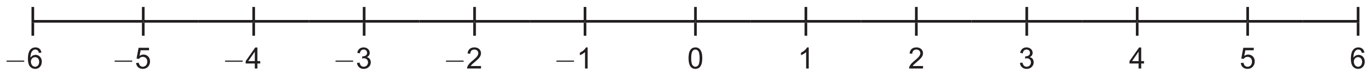












NAME ______________________________ DATE ______________

Hundreds Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

NAME ____________________ DATE ____________

1 cm Grid Paper