Unit 11 Geometry: Transformations

Introduction

This unit will focus on:

- performing, describing, and identifying translations, reflections, and rotations;
- using combinations of transformations to create designs and patterns;
- identifying and plotting points in the first quadrant of a Cartesian coordinate plane; and
- performing and describing transformations of shapes in a Cartesian coordinate plane.

Meeting Your Curriculum

ALBERTA		
Required	G6-13 to 20	including Extension 3 in G6-14 and Extension 2 in G6-15
BRITISH COLUMBIA		
Required	G6-13 to 20	including Extensions 1 and 2 in G6-17
MANITOBA		
Required	G6-13 to 20	
ONTARIO		
Required	G6-13 to 20	

Mental Math Minutes

The mental math minutes in this unit:

- review properties of division with remainders
- use multiplication and division patterns to solve equations by guessing and checking

Generic BLMs

The Generic BLM used in this unit is:

BLM 1 cm Grid Paper (p. T-1)

This BLM can be found in Section T.

Assessment

The lessons covered by a quiz or test are as follows:

	AB	BC	MB	ON
Quiz	G6-13 to 16	G6-13 to 16	G6-13 to 16	G6-13 to 16
Quiz	G6-17 to 20	G6-17 to 20	G6-17 to 20	G6-17 to 20
Test	G6-13 to 20	G6-13 to 20	G6-13 to 20	G6-13 to 20

Additional Information for This Unit

Technology: dynamic geometry software
The Alberta curriculum requires performing transformations using technology. Some of the extensions in this unit use a program called The Geometer's Sketchpad®. If you are not familiar with The Geometer's Sketchpad®, the built-in Help Centre provides explicit instructions for many constructions. Use phrases such as "How to reflect polygons" or "How to construct a line segment of given length" when searching the Index.

G6-13 Translations

Pages 48–50

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
congruent
corresponding
fixed point
image
image under transformation
image under translation
prime symbol (′)
reflection
rotation
transformation
translation
translation arrow

Goals

Students will perform translations on a grid.

PRIOR KNOWLEDGE REQUIRED

Can perform and identify translations
Can measure sides and angles of polygons
Can identify congruent shapes

MATERIALS

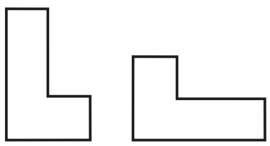
2 identical L-shaped pieces of paper
round counter (e.g., integer tile, round game counter)
rectangular block or a matching paper rectangle
rulers and protractors
paper square (see Extension 1)

Mental math minute—number string.

String 1: Divide. Write your answer with remainder. 24 ÷ 4, 25 ÷ 4, 26 ÷ 4, 27 ÷ 4, 28 ÷ 4, 29 ÷ 4, 30 ÷ 4 (6 R 0, 6 R 1, 6 R 2, 6 R 3, 7 R 0, 7 R 1, 7 R 2)

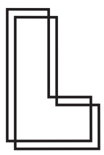
Present the pattern using an array with four dots in a row, adding one dot to the last row, until the row is full. The row that is not full represents the remainder.

String 2: 369 ÷ 3, 370 ÷ 3, 394 ÷ 3 (123, 123 R 1, 131 R 1)
String 3: 400 ÷ 4, 404 ÷ 4, 406 ÷ 4 (100 R 0, 101 R 0, 101 R 2)



Introduce transformations. Show students two copies of an L-shape made of paper that are oriented in different directions, beside each other, as shown in the margin.

Explain that the shapes are identical. Ask students if they remember the correct mathematical term for identical shapes. (congruent shapes)



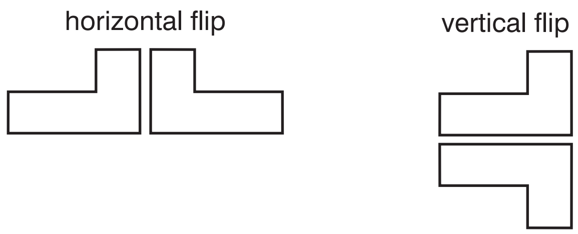
Tell students that you want to move the shapes so that they line up exactly, with one on top of the other, facing the same direction so that one congruent shape completely covers the other. Show moving the shapes, as shown in the margin. Return the shapes to their original position.

Tell students to pretend that the shapes are actually very heavy, very hot sheets of metal, so you need to program a robot to move them. To write the computer program, you have to divide the process of lining up the shapes into very simple steps.

It is always possible to move a figure into any position in space by using some combination of the following three movements:

- Sliding the shape along a straight line without allowing it to turn. This is called *translation*.
- Flipping the shape over. This is called *reflection*.
- Turning the shape around some *fixed point*. This is called *rotation*. A rotation can be a turn around a fixed point that is inside the shape, on the edge of the shape, on its corner, or outside the shape.

Have students tell you, the robot, what steps to perform to position the hot L-shaped sheets of metal one on top of the other. Explain that there are different ways to bring one shape on top of the other, so there are no right or wrong answers. However, some instructions are more efficient than others; in other words, some ways will require fewer steps. When students direct you to rotate or to reflect the shapes, point out that there are very many different ways to reflect or to rotate the shape, so you will need more detail. You need to know the direction of rotation and how much you need to rotate it; for example, is it a quarter turn, or half a turn, or maybe some other turn? Demonstrate reflections using the paper shapes as shown below as you SAY: For a reflection, you also need to know if you flip the shape horizontally or vertically.



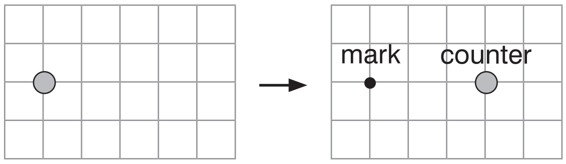
SAY: In this unit you will learn to describe these movements precisely.

Introduce terminology. SAY: These three changes to a figure—translation, reflection, and rotation—are all examples of *transformations*. When a point or a shape is changed by a transformation, the resulting point or shape is called the *image* of the original point or shape. We often add a star (*) or a *prime symbol* (′) to the name of the original point to label the image. For example, the image of point *A* can be labeled as *A*′. We can also use an arrow to show the change from the original to the image. Write on the board:

$A \longrightarrow A'$ or $B \longrightarrow B^*$

SAY: We read these as "*A* is transformed into *A*-prime" and "*B* is transformed into *B*-star." We also say that *A*′ is the *image of A under transformation* and that *B** is the image of *B* under a transformation.

Translating points on a grid. Explain to students that in this lesson they will only perform translations. Use a grid on the board and a round counter to demonstrate sliding a point on a grid; place the counter on a grid intersection and physically slide the point, represented by the counter, on the grid. The diagram below shows a translation of 3 units right. You might want to mark the starting point on the grid.



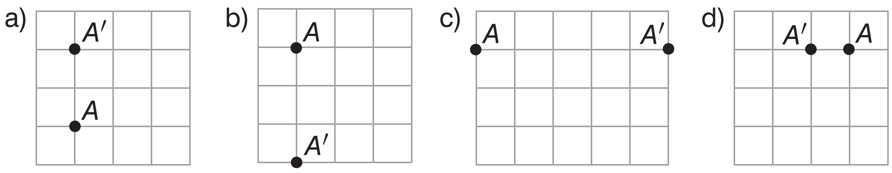
Have students first signal the direction in which the point is translated (right or left, up or down) and then ask them to hold up the number of fingers equal to the number of units the dot is translated. You may wish to draw a large letter L on the left side of the board and a letter R on the right side of the board to help students who have trouble distinguishing between left and right. Demonstrate several new translations with the counter on the grid and ask students to signal the direction and number for each translation.

Invite volunteers to translate a point and have other volunteers describe the translations. Then reverse the task: have volunteers describe the translation and have other volunteers perform them with a counter. Remind students that the result of a transformation is called an image under that transformation. SAY: For example, the result of a translation of a point or shape is called the *image under translation*.

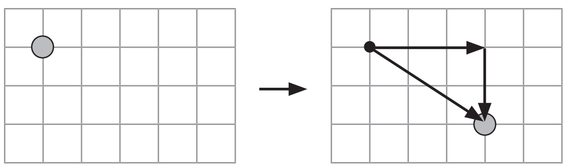
Exercises: Draw a point on a grid and label it *A*. Draw a point *A′* that is the image of *A* under the given translation.

a) 2 units up
b) 3 units down
c) 5 units right
d) 1 unit left

Sample answers



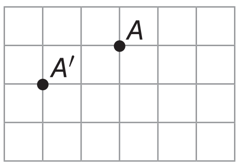
SAY: You can also combine translations. For example, you can move 3 units right and 2 units down. Demonstrate with a counter and draw arrows to show the translations, as shown below:



Exercises: Draw a point on a grid and label it *A*. Draw a point *A′* that is the image of *A* under the given translation.

a) 2 units left, 1 unit down
b) 4 units right, 3 units up
c) 6 units right, 2 units down
d) 3 units left, 4 units up
e) 3 units right, 1 unit up
f) 5 units left, 3 units down

Sample answers

a)


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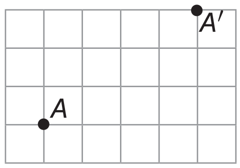

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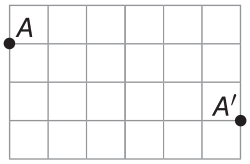

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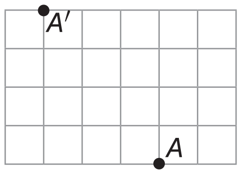

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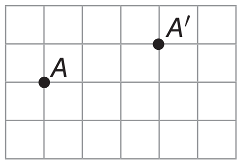

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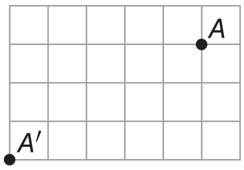
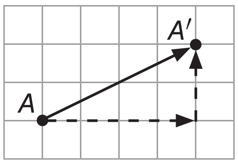


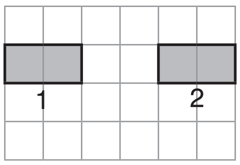
NOTE: Students who are struggling can draw the arrows showing each part of the slide.



Describing translations. SAY: To describe a translation, you need to say how much the point moved and in which direction. Draw the picture in the margin on the board. SAY: You can imagine the arrow from *A* to *A*′ as a combination of two arrows, horizontal and vertical. Trace the dashed arrows with a finger. ASK: How much did point *A* move in the horizontal direction? (4 units) Did it move right or left? (right) Repeat with the vertical arrow. (2 units up) SAY: So the point *A* moved 4 units right and 2 units up.

ACTIVITY (Essential)

Students work in pairs. Partner 1 draws a pair of points on a grid and an arrow from one point to the other. Partner 2 describes the translation. Partner 1 verifies the answer. Partners switch roles.



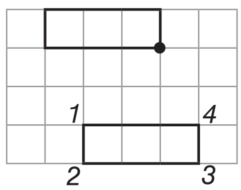
How much did the shape slide? Draw on the board the picture in the margin. ASK: How far did the rectangle slide to the right from Position 1 to Position 2? Accept all answers and record them on the board. Call for a vote if you wish. Students might say the rectangle moved anywhere between 2 and 6 units right. Take a rectangular block or a matching paper rectangle and perform the actual slide, one square at a time, counting the units as a class. The correct answer is 4 units.

Corresponding points. Draw a point at the top right vertex of the rectangle in Position 1. ASK: Can this make it easier to see that the translation was 4 points to the right? (yes) Why? (we know how to translate points) SAY: The vertex I marked and its image are *corresponding* points under a translation. When we talk about transformations, we want to know where each point went to. Invite a volunteer to mark the image of the marked point on the second rectangle. Keep the picture on the board for later use.

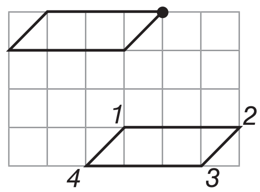
Draw the pictures in the exercises below, one pair of figures at a time, and have students signal the answer by raising the correct number of fingers.

Exercises: Which vertex, 1, 2, 3, or 4, is the image of the vertex marked with a dot under the translation?

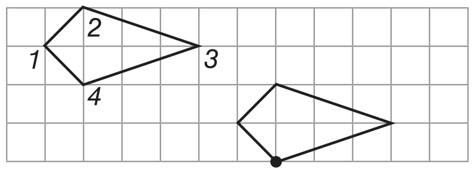
a)



b)



c)



Answers: a) 3, b) 2, c) 4

Under translations, all points on a shape move the same amount in the same direction. Label the vertices of the rectangle in Position 1 from earlier as *A*, *B*, *C*, and *D*. Add the same labels to the vertices of the paper rectangle. Translate the paper rectangle again, from the initial position to the position 4 units to the right and 2 units down. Invite volunteers to label the vertices of the image as $A'B'C'D'$ to show the correspondence. Draw arrows from each vertex to its image. SAY: These arrows are called *translation arrows*. ASK: What do you notice about the translation arrows? (they are all parallel and they are all the same length) Explain that this means that all points on a shape move the same amount in the same direction, so it is enough to draw only one translation arrow to describe a translation. SAY: However, you need to be careful to draw the arrow between a vertex and its image, not any other vertex. Also, the fact that all arrows are the same gives you a way to translate polygons: you can translate each vertex separately and then join the images of the vertices to form the image of the polygon.

Translations preserve length of line segments and size of angles. Give students rulers and protractors. Ask students each to draw a scalene triangle on a grid and measure its sides and angles. Then ask them each to write a translation of their choice. Have students each translate the triangle they drew by using the translation they described and then measure the sides and the angles of the image.

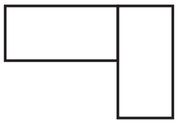
Discuss findings from the translation students just performed. Students should notice that the side lengths of the triangle under translation stayed the same and so did the angle measures. Point out that the result is the same for everyone even though students all drew different triangles and performed different translations.

Translations take polygons to congruent polygons. Ask students to remind you what they know about the sides and angles of congruent polygons. (Congruent polygons have corresponding equal sides and corresponding equal angles; the equal sides and angles come in the same order in both polygons.) ASK: Do translations change the order of vertices? (no) Do they preserve lengths of sides? (yes) Do translations preserve angle sizes? (yes) Do translations take polygons to congruent polygons? (yes) Write on the board:

> If polygon A is an image of polygon B under a translation,
> then polygons A and B are congruent.

ASK: Do you think it works the other way around? Write on the board:

> If polygons A and B are congruent,
> then polygon A is the image of polygon B under a translation.



Explain that if you want to prove a statement is false, you can find just one example that shows that the statement is false. ASK: What would such an example look like for this second statement? (a pair of polygons that are congruent but are not a translation of each other) Ask students to try to draw a pair of polygons like that. (see example in the margin)

Ask students to explain how they know that one polygon is *not* the translation of the other polygon. To prompt students to see the answer, ask them to say from which vertex they would draw a line to another vertex in the shape and show that the line segments joining the vertices are not parallel and not equal in length, so the line segments are not translation arrows. Students should conclude that the line segments that form each shape's sides are equal and the angles are equal, so the two shapes are congruent, but the shape on the left has not been translated to become the shape on the right.

Combining translations. Have students do the following exercises in pairs.

Exercises

a) Draw a quadrilateral that is not symmetrical in any way and label it P.

b) Write a description of a translation of your choice.

c) Translate the polygon P using the description from part b). Label the image P′.

d) Translate the polygon P′ using the description your partner wrote in part b). Label the image P*.

e) Describe the transformation that takes P to P*.

f) Compare your answer in part e) to the answer of your partner. What do you notice?

Selected answer: f) the answers are the same

Discuss the results of the exercises. All students should see that the resulting transformation is a translation. Discuss how the translations combine:

- If both translations have components that move in the same direction, these add. For example, if one translation is 3 units right and another is 2 units right, the total translation is 5 units to the right.

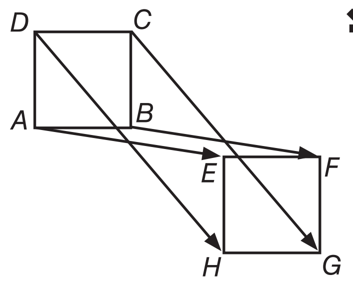
• If one translation has an "up" component and another has a "down" component, they partially cancel out each other. For example, 2 units up followed by 3 units down results in 1 unit down. Have multiple pairs present their answers. Students should also see that the order in which translations are made does not change the overall translation.

Exercises: Emma translated polygon Q 3 units right and 2 units down and then translated the image 4 units left and 5 units down. She labelled the final image Q*. Which translation takes Q to Q*?

Bonus: Which translation takes Q* to Q?

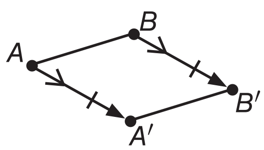
Answers: 1 unit left and 7 units down, Bonus: 1 unit right and 7 units up

Extensions



1. Draw the picture in the margin on the board, first without the arrows. Use a paper square to demonstrate as you SAY: I flipped the square over a horizontal line (demonstrate) and translated it a little to the right. Here is how the vertices changed. Draw the arrows and name each arrow as you draw it: *AE*, *BF*, *CG*, *DH*.

   SAY: The arrows are not all the same length and not all parallel. I think there is no translation that would take the first square, *ABCD*, to the second square, *EFGH*. ASK: Is this correct? (no) Why not? (if you draw the arrows in a different way, you can show a translation with parallel arrows of equal length) How should we draw the arrows to show a translation? (*AH*, *BG*, *CF*, *DE*) Have a volunteer draw the new arrows. Point out that if there is more than one way to draw the correspondence between the vertices, you would need to check all the possible ways to label the vertices of the image. However, if the shapes are not symmetrical, you do not need to worry about that. Have students look at the shapes that they drew before the recent exercises to show shapes that are not translations of each other. Students can change their examples to make the shapes not symmetrical.



2. **Using properties of parallelograms to explain why translations preserve the length of line segments.** Explain that properties of parallelograms give us a way to explain why translations preserve the length of line segments. Draw a line segment and label it *AB*. SAY: If we take a line segment *AB* and translate it, we translate the point *A* and the point *B* in the same way. Draw two identical translation arrows, AA' and BB'. ASK: What do you know about the translation arrows? (they are parallel and equal in length) Label the arrows and join the points A' and B', as shown in the margin.

   ASK: What type of quadrilateral is $A'B'BA$? (a parallelogram) How do you know? (the opposite sides AA' and BB' are parallel and equal) What does this say about *AB* and $A'B'$? (they are parallel and equal) SAY: So, we can see if a line segment has been translated by checking for the same properties as in parallelograms: if line segments AA' and

BB′ are equal and parallel, they show translation. Have students decide if the line segments *AB* and *CD* are translations of each other.

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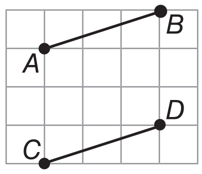

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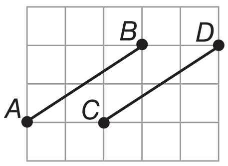

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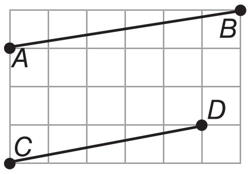


Answers: a) yes, b) yes, c) no

3. a) Draw a rectangle *ABCD* on grid paper, such that $AB = 4$ units and is the horizontal top side and $BC = 5$ units and is the vertical right side.

b) For the points *A* and *B*, draw a broken line (a collection of line segments) that starts at *A* and ends at *B* but does not follow the straight line segment *AB* and does not intersect it. The broken line will look like a "scenic route" or wandering path. Translate the broken line 5 units down so that it starts at *D* and ends at *C*. Erase the old lines *AB* and *CD*.

c) Draw another broken line that starts at *A* and ends at *D*, so that it does not intersect any of the lines you drew in part b) or the sides *AB*, *BC*, or *CD*. It can go along parts of *AD* or intersect it. Translate this broken line 4 units to the right. It should start at *B* and end at *C*. Erase the old lines *BC* and *AD*.

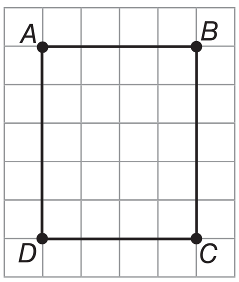
d) You have created a polygon with vertices *ABCD*. Draw a copy of it away from *ABCD*.

e) Translate the polygon you drew in part d) 4 units to the left. Translate the image 4 units to the left again. Repeat several times.

f) Translate the polygon you drew in part d) 5 units down. Translate the image 4 units to the left. Translate the image 4 units to the left again. Repeat several times.

g) A pattern made of congruent shapes that cover the grid without gaps or overlaps is called a tessellation. Does the shape you created produce a tessellation?

Selected sample answers

a)


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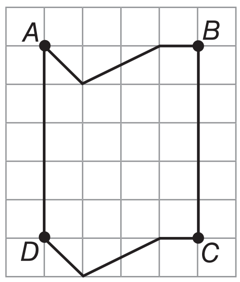

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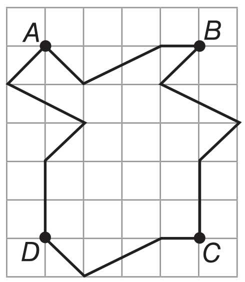
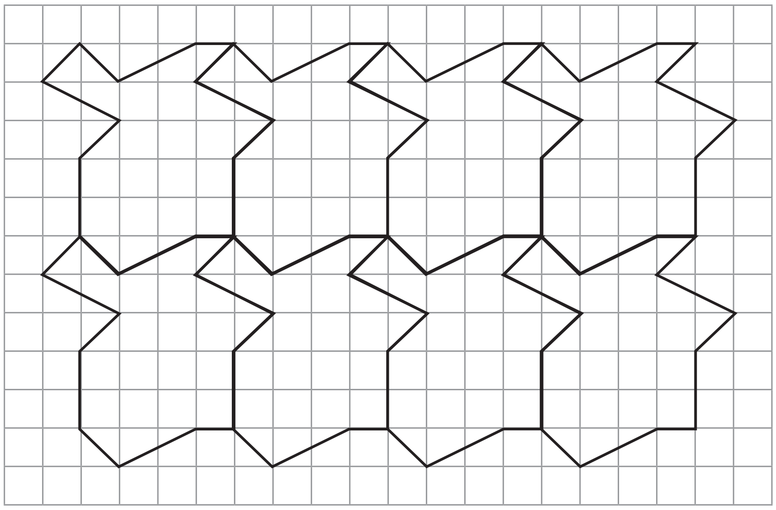


f)



g) yes

G6-14 Reflections

Pages 51–53

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
congruent
corresponding
dividend
divisor
image
image under reflection
midpoint
mirror image
mirror line
orientation
perpendicular
prime symbol (′)
quotient
reflect
reflection
remainder
transformation
translation

Goals

Students will reflect points and shapes and describe reflections.
Students will verify that reflections take polygons to congruent polygons.

PRIOR KNOWLEDGE REQUIRED

Can identify and draw perpendicular lines
Can measure sides and angles of polygons
Can identify congruent shapes
Knows the definition of congruent polygons in terms of sides, angles, and order of elements

MATERIALS

2 identical L-shaped pieces of paper, such as those used in Lesson G6-13
rulers and protractors
coloured chalk or markers
The Geometer's Sketchpad® (see Extension 3)

Mental math minute. Present the following set of problems.

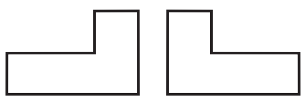
$30 \div 4$ $34 \div 4$ $38 \div 4$ $42 \div 4$ $46 \div 4$

Have students write division with remainder for the first problem. (7 R 2) ASK: What pattern do you see in this set of problems? (the dividends increase by 4 from problem to problem) Draw arrays with four dots in a row to represent the first and the second problems. ASK: How do arrays help us to see the answer to each next problem? (the dividends increase by 4, so each next array just has 1 more full row, and the same row that is not full, so the quotients will grow by 1 and the remainders will stay the same) Record the answers for the rest of the problems. (8 R 2, 9 R 2, 10 R 2, 11 R 2) Have students signal the answer in the exercises below by showing a 0 or 2 with their fingers. **NOTE:** Students will solve harder division problems with a variety of remainders and divisors in the next lesson.

Exercises: Is the remainder 2 or 0?

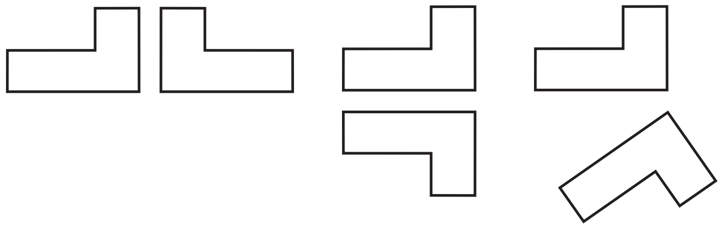
a) $40 \div 4$ b) $440 \div 4$ c) $450 \div 4$ d) $452 \div 4$
e) $800 \div 4$ f) $888 \div 4$ g) $890 \div 4$ h) $8082 \div 4$

Answers: a) 0, b) 0, c) 2, d) 0, e) 0, f) 0, g) 2, h) 2

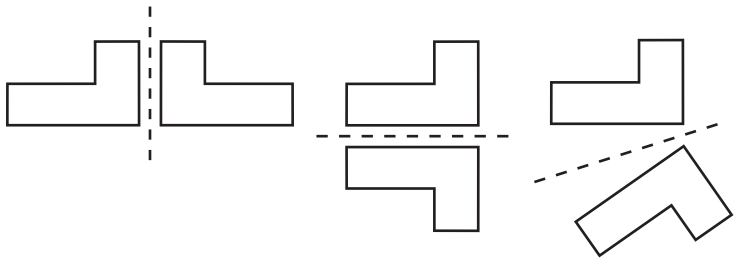


Introduce reflections. Show students two L-shaped pieces of paper. Affix them to the board, as shown in the margin. SAY: I would like to place the shape on the left on top of the shape on the right. ASK: What should I do to the shape on the left? (flip it) Point out that this requires taking the shape off the board. SAY: Another way to make the shape on the left look like the shape on the right is to look at it in a mirror. For that reason, we can say that these two shapes are *mirror images* of each other. Mathematically, we say that the shape on the right is a reflection of the shape on the left. A reflection is another type of transformation.

Remind students that when you “flip” or reflect a shape, there are different ways to do it. Demonstrate several different ways to reflect the L shape on the left, as shown below:



SAY: Suppose there is a mirror between each pair of shapes, so that the original shape is a real one and the other one is the shape in the mirror. ASK: In each case, where would the mirror be? Have a volunteer hold up a sheet of paper as if it were a mirror between the shapes, as shown below (the dashed line represents the mirror):



SAY: If we look at what happens in a plane, the mirror becomes a line. This line is called a *mirror line*. Students might recall that a line of symmetry is also called a mirror line. Point out that the mirror line is indeed a line of symmetry when you consider the original shape and the image as parts of the same picture. Explain that in this unit students will mostly work with horizontal and vertical mirror lines and will work on a grid.

Reflecting points in a line. Draw a vertical line *m* on the board and a point *A* away from the line. SAY: The vertical line is a mirror line, and I am going to reflect point *A* in this line. Demonstrate the steps below and write each step on the board:

Step 1: Draw a line perpendicular to *m* through *A*. Extend the line beyond *m*.

Step 2: Measure the distance from *A* to *m* along the perpendicular line.

Step 3: Mark point *A′* on the other side of *m* so that *A* and *A′* are the same distance from the mirror line *m*.

SAY: The point *A′* is the mirror image of point *A*. Mathematicians say that point *A′* is the *image of A under reflection*.

Exercises

a) Draw a horizontal line *m* and mark a point *A* away from the line.

b) Reflect point *A* in the mirror line *m*.

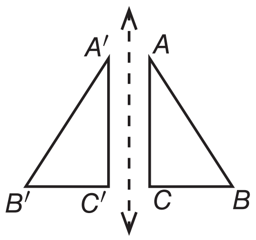
c) Draw a vertical line *n* and mark a point *B* away from the line.

d) Reflect point *B* in the mirror line *n*.

Reflections preserve length of line segments and size of angles. Provide students with rulers and protractors. Ask students each to draw a scalene triangle on a grid, label the vertices, and measure its sides and angles. Then ask them to draw a horizontal or a vertical line of their choice. Encourage some students to draw a horizontal line, some to draw a vertical line, and some to draw a line away from the triangle. Ask at least one student to draw a line that passes through one of the vertices of the triangle and ask another student to draw a line that intersects two of the sides of the triangle. Explain that the line that each student drew will serve as the mirror line for that student's triangle.

Explain that we can reflect the vertices of a triangle and then join the images to reflect the triangle. Have students each reflect the triangle they drew in the mirror line and then measure the sides and the angles of the image. Have them use the prime symbol to label the images of the vertices.

Discuss the findings. Remind students that they showed that translations "preserved" the lengths of the sides and the size of angles. ASK: Does the same happen with reflections? Students should notice that, from their triangle to its image under reflection, the side lengths stayed the same and so did the angle measures. Point out that the result is the same for everyone, although they all had different triangles and different mirror lines, so reflections also preserve lengths of sides and angle sizes.



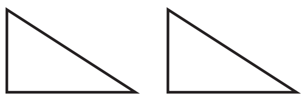
Draw students' attention to the order of vertices in the original triangle and its image. For example, if the original triangle was *ABC* and you needed to go clockwise to get from *A* to *B* to *C*, as shown in the margin, the order in the image triangle is counter-clockwise. Have all students verify that on their triangles.

Reflection and congruence. Ask students what they know about the sides and angles of congruent polygons. (congruent shapes have the same size and shape, so congruent polygons have corresponding sides of equal length, corresponding angles that are equal, and the same order of equal sides and equal angles) Point out that although reflections "flip" the shape, which reverses the order of vertices from clockwise to counter-clockwise, they do not mix up the order of vertices and sides: if two vertices are adjacent to a side of the same length, their images are adjacent to the side of the same length. ASK: Do reflections preserve angle sizes? (yes) Do they preserve lengths of sides? (yes) Do reflections take polygons to congruent polygons? (yes) Write on the board:

If polygon A is a mirror image of polygon B,
then the polygons A and B are congruent.

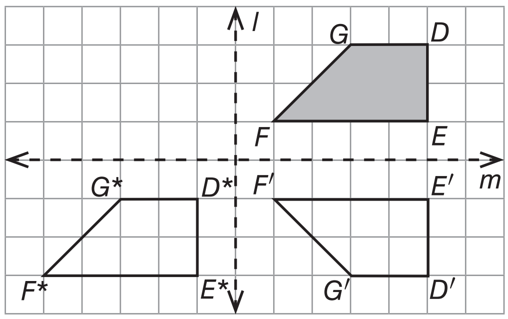
ASK: Do you think it works the other way around? Write on the board:

If polygons A and B are congruent,
then polygon A is the mirror image of polygon B.



Point out that another way to say the second sentence is: “If two polygons are congruent, then they are mirror images of each other.” Give students a few minutes to think about if the statement is correct and then have them find an example showing that this is not true. (see sample example in margin)

Distinguishing between reflections and translations visually. Have students draw a horizontal and a vertical line to act as mirror lines, dividing a sheet of grid paper into four approximately equal parts. Have them pick one part and draw in it a right trapezoid. Have them label the trapezoid *DEFG* so that they read the name clockwise around the trapezoid and shade the shape to identify it clearly as original.



Have students reflect the trapezoid they drew in the horizontal line, labelling the image using ′. Then have them translate the original shape so that it ends in the free region of the page, diagonally from the original shape. Students should use * for labelling the translated polygon.

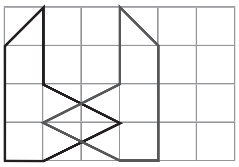
Have students compare the results of reflection and the translation. ASK: How are the images different? Students are likely to say that the image under reflection “points” in a different direction from the original shape, when the image under translation points in the same direction. Draw students’ attention to the order of the letters around the polygon: if you want to start with D and read the letters alphabetically, you need to continue clockwise when reading the letters from the original shape and from the image under translation, but counter-clockwise if you are reading the letters from the mirror images. SAY: When the order of vertices changes to the opposite, say, from clockwise to counter-clockwise, we say that the *orientation* of the shape changes.

Have students reflect the original polygon in the vertical line and use the ″ symbol to label the image. Repeat the discussion. Students will notice that the orientation (in the sense of the order of letters) of both reflected polygons is the same, but they still “point” in different directions, and both polygons “point” in a different direction from the original and from the translation.

Using line segments joining corresponding vertices to distinguish between reflections and translations. Remind students that when they perform a translation, they translate the vertices the same way, so translation arrows are the same length and parallel to each other. Have students draw the translation arrows and verify that.

Ask students to draw the line segments joining the vertices of *DEFG* and the corresponding vertices of one of its mirror images. Have some students use $D'E'F'G'$ and others use $D''E''F''G''$ for this purpose. ASK: Are the line segments parallel? (yes) Are they the same length? (no) Ask multiple students. Point out that students have different trapezoids and reflect in different lines but get the same result.

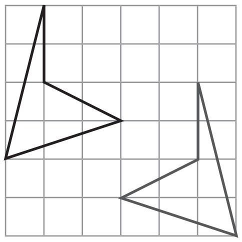
Midpoints of line segments between original and image are on the mirror line. SAY: The point that is exactly halfway between the end points of a line segment is called the *midpoint* of the line segment. ASK: If a line segment is 6 units long, how far from each end point is the midpoint? (3 units) Draw a line segment on the board and invite a volunteer to find the midpoint. Then ask students to find the midpoints of the line segments joining the vertices of *DEFG* and one of its mirror images. ASK: What do you notice about the midpoints? (they are all on the mirror line) What angle does the line segment make with the mirror line? (right angle) Why does this make sense? (To construct the mirror image, you draw a line segment that is perpendicular to the mirror line and mark the image so that the distance from the mirror line is the same to the image and to the original point. This means that the mirror line intersects the line segment at the midpoint.)



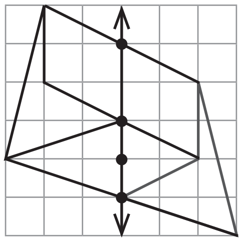
➘ Draw the picture in the margin on the board. Use a different colour for each shape. SAY: These two polygons are reflections of each other. I would like to find the mirror line. How can I use the midpoints of the line segments joining the corresponding vertices to find the mirror line? (Draw line segments between corresponding vertices. Find their midpoints. Join the midpoints to find the mirror line.) Invite a volunteer to demonstrate.

ACTIVITY (Essential)

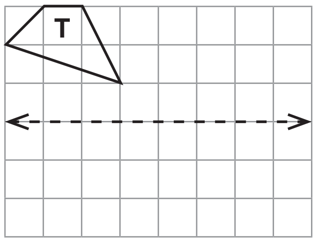
To allow students to practise finding a mirror line, have them draw a non-symmetrical polygon on a grid. Explain that you want them to reflect it in a line of their choice, but instead of drawing the line, they can place a ruler or a pencil to mark the position of the line so that their partners can find the mirror line afterwards. Then have partners exchange notebooks and find the mirror line.



➘ Draw the picture in the margin on the board. ASK: Are these mirror images of each other? Answers may vary; point out that the shapes “point” in opposite directions, so they might be mirror images.

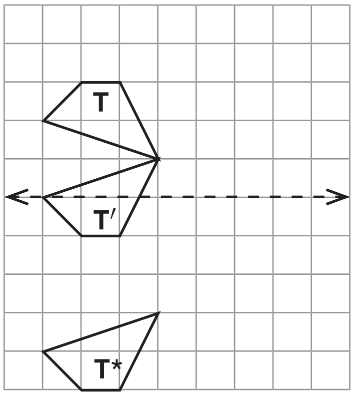


To check, invite volunteers to draw the line segments between the corresponding vertices and find their midpoints. Students will see that the midpoints all fall on the same line (see second image in the margin), but the line segments are not parallel, and there is no line that is perpendicular to all the line segments at the same time. Explain that this means that the shapes are not mirror images of each other. In this case, the shape was first reflected and then translated 2 units down. Invite a volunteer to draw the reflected shape as an intermediate step.

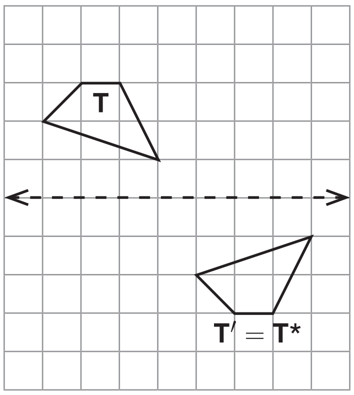


Combining a reflection and a translation. Draw the picture in the margin on the board. Ask students to copy the picture. Then divide students into three groups and have students in each group work in pairs. In each pair, one student will reflect the shape first, then translate the image, and label the final image T′. The other student will reverse the order—translate first and then reflect—and label the final image T*. Each group has a different translation, as shown below. The following shows the original shape, the mirror line, and the results of the transformations.

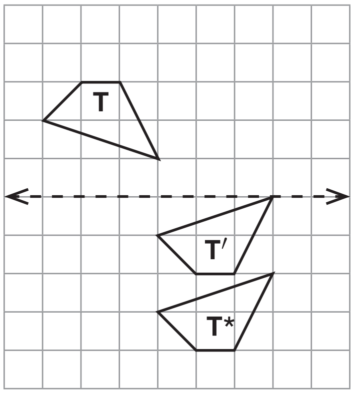
Group 1: 2 units up



Group 2: 4 units right



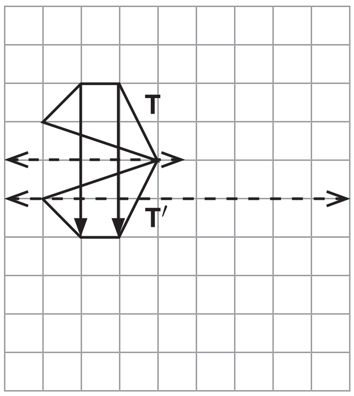
Group 3: 3 units right and 1 unit down

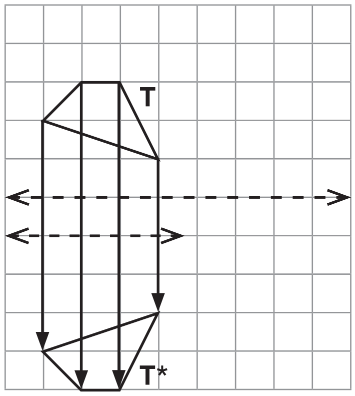


Have students in each pair compare the results (Did you get the same answers? How are the shapes T′ and T* the same, and how do they differ from the original shape T?). Have students discuss the results in their groups and then have groups report on the findings to the class. Students should notice that only Group 2 got the same result for both combinations.

Students should notice that all shapes are congruent, and congruent to the original shape, and that all original shapes “point” the same way, with the lowest vertex on the right, and all final images are oriented the opposite way, with the highest vertex again on the right. Students might also notice that the orientation changed from the original polygon for all the images.

ASK: Could there be a single translation that takes T to T* or to T′? (no) How do you know? (the images “point” in a different direction from T, the orientation changed, and translations do not change orientation) What about a single reflection? (maybe) How can we check? (join the corresponding vertices of the original to the vertices of the image and find the midpoints. If all midpoints are on the same line that is perpendicular to the line segments, that line is the new mirror line) Have students join the vertices, find the midpoints, and report their findings. Only Group 1 will be able to identify a reflection between T and T′ and between T and T*. See the pictures below.





Emphasize that sometimes there is no single transformation that takes one shape onto the other, and two transformations are needed. Moreover, there are multiple ways to take one shape onto the other.

NOTE: Extension 3 is required to cover the Alberta curriculum.

Extensions

1. Triangle *ABC* is reflected in a vertical line *m* to get triangle *A*B*C**. Which transformation will take triangle *A*B*C** to triangle *ABC*?

   Answer: reflection in the same mirror line *m*

2. a) Draw a quadrilateral Q without a line of symmetry on grid paper. Draw a horizontal line *m* away from the quadrilateral.

   b) Reflect the quadrilateral in the line and then translate the image 3 units right and 2 units up. Label the final image Q*.

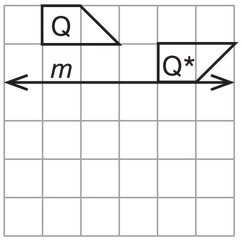
   c) Describe a way to get from Q* to Q. Use translation and then reflection.

   d) Describe a different way to get from Q* to Q. Use reflection and then translation.

e) Did you use the same translation in parts c) and d)?

Sample answers

a–b)



c) Translate Q* 3 units left and 2 units down. Reflect the image in the line *m*.
d) Reflect Q* in the line *m*. Translate the image 3 units left and 2 units up.

Answer: e) no

3. **Investigating reflections using The Geometer's Sketchpad®.** Teach students to reflect polygons in The Geometer's Sketchpad®. Demonstrate how to draw a polygon using the Polygon tool. Tell students that when they want to reflect a shape, they need to create a mirror line and select it using the "Mark Mirror" option in the Transform menu. Then they can select the shape and "Reflect" in the Transform menu. Demonstrate the process. Then have students follow the instructions below.

a) Draw a quadrilateral and label it *ABCD*.

b) Draw a line away from *ABCD*. Label it *m*.

c) Select line *m* and use the "Mark Mirror" option in the Transform menu to label it as the mirror line.

d) Select the quadrilateral *ABCD*. Use the Transform menu to reflect it in mirror line *m*. Label the image quadrilateral $A'B'C'D'$. Does it look congruent to *ABCD*? How is it different from *ABCD*?

e) Move the vertices of *ABCD* to change the shape. Are the quadrilaterals still congruent? Is the image quadrilateral still different from the original in the same way?

f) Move line *m* without turning it. How does the quadrilateral $A'B'C'D'$ change? Does your answer to part e) change?

g) Select one of the points used to create line *m* and move it to turn line *m*. How does the quadrilateral $A'B'C'D'$ change? Does your answer to part e) change?

NOTE: Students have not yet examined rotations, so they are likely to use everyday language to describe the rotation and cannot be expected to be precise.

Selected sample answers: d) the quadrilaterals are congruent, but $A'B'C'D'$ is facing the opposite way from *ABCD*; e) yes, yes; f) the quadrilateral $A'B'C'D'$ is reflected farther away or closer to the quadrilateral *ABCD*, but the quadrilaterals are still congruent, and the image is still facing the opposite way from *ABCD*; g) the quadrilateral $A'B'C'D'$ turns in its position and moves away or closer to the quadrilateral *ABCD*, but the quadrilaterals are still congruent

G6-15 Rotations

Pages 54–56

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
centre of rotation
congruent
corresponding
fixed point
image
mirror line
prime symbol (′)
reflection
rotation
transformation
translation

Goals

Students will rotate points and shapes 90° around a given centre.
Students will verify that rotations take polygons to congruent polygons.

PRIOR KNOWLEDGE REQUIRED

Can identify and draw perpendicular lines
Knows the terms clockwise and counter-clockwise
Knows that the size of an angle is a measure of rotation
Can identify congruent shapes
Knows the definition of congruent shapes in terms of sides, angles, and order of elements

MATERIALS

set squares
rulers
protractors
BLM Rotating a Triangle (p. N-55)
scissors
BLM Rotations Without a Grid (p. N-56) (see Extension 1)
The Geometer's Sketchpad® (see Extension 2)
BLM Find a Flip (pp. N-57–58, see Extension 3)

Mental math minute. Write "450 ÷ 4" on the board. SAY: I can see an easy number that divides by 4 and is smaller than 450. I am separating 450 into 440 and 10. There are other ways to separate 450, such as $400 + 40 + 10$, but I am using 440 and 10. Start recording the solution on the board as shown below. Remind students that when dividing a sum they divide each addend separately. Point out that this is like dividing parts of a very large array. The first array has 110 rows of 4 dots, and the second has 2 rows of 4 dots and 2 dots leftover. ASK: What is the total number of full rows? ($110 + 2 = 112$) What is the total leftover in the division? (2, just the leftover in the second part)

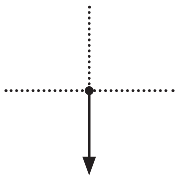
$$\begin{aligned} &450 \div 4 \\ &= (440 + 10) \div 4 \\ &= (440 \div 4) + (10 \div 4) \\ &= 110 + 2 \text{ R } 2 \\ &= 112 \text{ R } 2 \end{aligned}$$

Exercises: Use the method on the board to divide.

a) $58 \div 4$ b) $430 \div 4$ c) $889 \div 4$ d) $1237 \div 4$

Answers: a) 14 R 2, b) 107 R 2, c) 222 R 1, d) 309 R 1

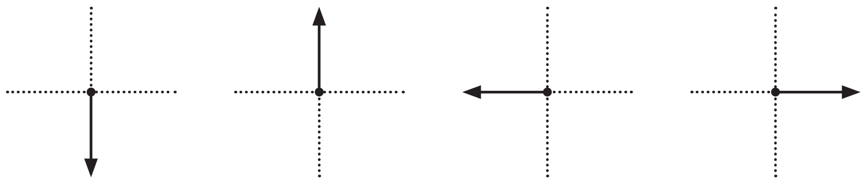
Review clockwise and counter-clockwise and describe turns. Review the meanings of “clockwise” and “counter-clockwise.” Draw several arcs pointing clockwise and counter-clockwise on the board and have students signal thumbs up if the arc points clockwise and thumbs down if it points counter-clockwise.



Draw the picture in the margin on the board. ASK: If this arrow turns 90° clockwise, where will it point? Have students show the direction with their arms and then have a volunteer draw the arrow. (pointing left) Repeat with other starting arrows and other directions; include turns of 180°. Explain that it takes too much time and space to write “clockwise” or “counter-clockwise,” so you will be using short forms: “CW” for clockwise and “CCW” for counter-clockwise.

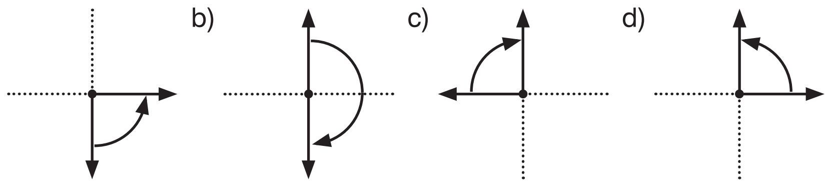
Exercises: From the dark arrow, draw an arc showing the given turn. Draw the arrow after turning.

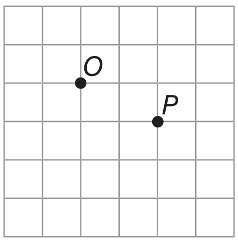
a) 90° CCW b) 180° CW c) 90° CW d) 90° CCW



Answers

a) b) c) d)





Rotating points. Draw the picture in the margin on the board. SAY: I want to rotate point *P* around point *O* 90° clockwise. Demonstrate and start making a list of the steps on the board:

Step 1: Draw the line segment *OP*. Measure its length.

Step 2: Draw an arc clockwise to show the direction of rotation.

Step 3: Place a set square so that:

- the arc points at the diagonal side,
- the right angle is at point *O*, and
- one arm of the right angle aligns with *OP*.

At this point, explain that the line segment *OP* is like a hand on the clock, attached at point *O.* If we turn in the direction of the arc, is it passing through the set square? Trace the turn in the direction of the arc with a finger to check. Turn the set square upside down if needed. Continue demonstrating with the following steps:

Step 4: Draw a ray from point *O* along the side of the square corner. Remove the set square.

Step 5: On the new ray, measure and mark the image point P', so that $OP' = OP$.

For the exercises below, have students always start with points on grid intersections.

Exercises

a) Draw two points on the same horizontal line and label them *S* and *T*. Rotate point *T* around point *S* 90° clockwise.

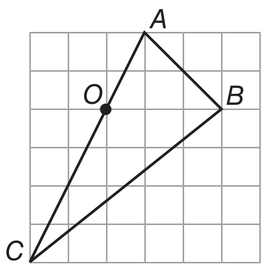
b) Draw point *C* and draw point *D* underneath it, on the same vertical line. Rotate *D* around *C* 90° counter-clockwise.

c) Draw two points *U* and *V* that are not on the same horizontal or vertical line. Rotate point *V* around *U* 90° clockwise.

d) Rotate point *U* around *V* 90° counter-clockwise.

SAY: The point around which you rotate other points is called the *centre of rotation*. You used points that were grid line intersections as centres of rotation. The points you rotated also were grid line intersections. ASK: Were the image points also grid line intersections? (yes)

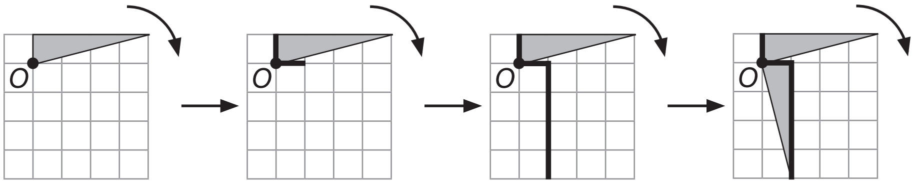
The centre of rotation is a fixed point. SAY: When you perform a transformation, such as reflection, rotation, or translation, some points move, and some points do not. For example, when you make a rotation, the centre of rotation does not move. All other points do. We call points that do not move *fixed points*. A rotation has only one fixed point, the centre. ASK: When you reflect points in a line, are there some points that do not move? (yes) Which points? (the points on the mirror line itself) When you translate shapes or points, are there points that do not move? (no, all points move) SAY: Translations have no fixed points: all points move under translation. Rotations have only one fixed point, the centre of rotation. In any reflection, points on the mirror line never move, so there are infinitely many fixed points in any reflection; there are so many points that you cannot even count them.



Rotating polygons. Draw the picture in the margin on the board. SAY: I want to rotate the triangle 90° clockwise around the centre of rotation *O*. To do that, I need to rotate each vertex and then join the images to form the image of the triangle. Have students draw a similar picture and perform the rotation using a set square, with a volunteer doing the same on the board. Students can use slightly different triangles and different centres of rotation, but for practical purposes, have them use a point on one of the sides of the triangle.

Have students measure the sides and the angles of the original triangle and the image using rulers and protractors. Discuss the results. Students should notice that the side lengths and the angle sizes are preserved in rotation. ASK: Does rotation take polygons to congruent polygons? (yes) If two triangles are congruent, does this mean that one is a rotation of the other? (no) What other transformations can take triangles to congruent triangles? (translations, reflections)

Using the grid to perform rotations of 90°. Draw a right triangle and demonstrate how to rotate the triangle 90° clockwise around the vertex *O*, as shown below. First draw the arc showing the direction of rotation, draw the image of the side adjacent to *O* that aligns with the grid, draw the side perpendicular to the first image side, and then finish the triangle with the third side. (see images below)



Emphasize that you are using the lengths of the short sides of the right triangle and the ones that can be measured by counting squares: the vertical side is 1 unit long, and it rotates into a horizontal side 1 unit long; the horizontal side is 4 units long, and it rotates into a vertical side 4 units long.

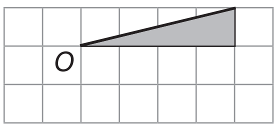
Exercises

a) Draw four right triangles on a grid. Make sure the triangles are not congruent and they are oriented differently (point in different directions) on a page.

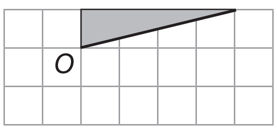
b) On each triangle, label one of the acute angles as *O*. The vertex *O* will be the centre of rotation.

c) Rotate the first two triangles 90° clockwise and the other two triangles 90° counter-clockwise around *O*.

Have students exchange notebooks with partners to check their answers.



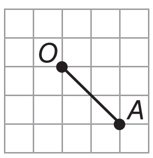
or

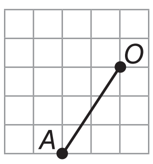


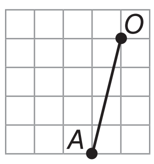
Rotating slanted line segments 90° around an end point. Draw a slanted line segment between two points on a grid and label one of the end points *O*. ASK: How could we use right triangles to rotate this line segment 90° clockwise around the point *O*? Invite volunteers to draw the right triangles that might help with the rotation. The example in the margin shows two possible 1 × 4 triangles, one above and the other below the line segment.

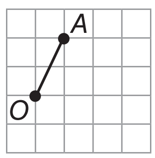
Invite volunteers to perform the rotation. ASK: Does the answer depend on the triangle used? (no)

Exercises: Rotate the line segment 90° clockwise around *O*.

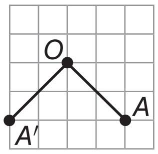
a) 

b) 

c) 

d) 

Answers

a)


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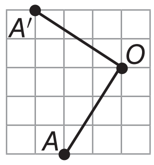

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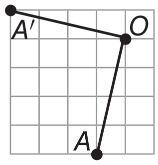
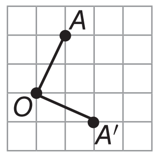
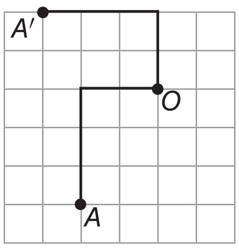


d)


Rotating slanted lines by imagining triangles. Tell students that now you want them to rotate line segments by only imagining the triangles, not drawing them. Emphasize the rule of changing the lengths of the horizontal and the vertical sides: if the original line segment goes 2 units up or down, the line segment rotated 90° will go 2 units right or left depending on the direction of rotation. Have partners draw slanted line segments for each other and label one of the vertices as the centre of rotation. Then have students rotate the line segments 90° clockwise around the marked centre of rotation.

Rotating polygons. Point out that to rotate a point, you can simply draw the line segment joining the point to the centre of rotation and then rotate it following the method students have just used. The grid gives students a shortcut to performing 90° rotations.



Students who are struggling imagining the triangles can draw only the sides of the triangles that follow the grid lines. For example, in question b) above, they can draw the picture in the margin to rotate point *A*.

Exercises

a) Draw a quadrilateral that is not symmetrical in any way. Choose a point *O* away from the quadrilateral. Rotate the quadrilateral 90° counter-clockwise around *O*.

b) Draw a pentagon that is not symmetrical in any way and has a right angle. Rotate the pentagon 90° clockwise around the vertex with the right angle.

ACTIVITY (Essential)

Give students **BLM Rotating a Triangle** and have them cut out the bottom triangle. Have them place the cut-out triangle on top of the triangle at the top of the page to see if the triangles are congruent and if the black dots are in the same places on both triangles. Explain that students will use the black dots as the centres of rotation for the triangle.

When students have placed the cut-out triangle on top of the other triangle and lined up the black dots, have them press the tip of a pencil to the black dot labelled *O* and rotate the top triangle around it clockwise so that the horizontal side becomes the vertical side, coinciding with one of the vertical lines on the grid. Have them hold the cut-out triangle in place, trace it onto the BLM, and label the corresponding vertices using *. Remove the cut-out triangle and compare the results. ASK: How is the rotated image different from the original? (the image turned; it is pointing in a different direction)

Repeat with the second black dot, labelled Q. Label the image triangle $G'H'I'$. ASK: Which transformation takes triangle $G^*H^*I^*$ to triangle $G'H'I'$? (translation 7 units left, 5 units down) If I were to rotate the original triangle 90° clockwise around O and then translate it 7 units left, 5 units down, which triangle would I get? ($G'H'I'$) Have students verify.

Explain that when you want to take a polygon onto a congruent polygon, you can rotate the polygon on a grid to the position of the congruent polygon and then find a translation to bring it to the location you need it to be.

Discuss what students can notice from the activity. Draw students' attention to the order of the letters in the original and the images. In the original triangle, if you want to read the letters in alphabetical order, you go clockwise around the triangle. ASK: In which direction do you go in the image? (also clockwise) Is it true for both images? (yes) Which transformation changes the order? (reflection reverses the order)

Discuss how the position of the centre of rotation influences the image under rotation. When students rotated shapes on a grid, they usually rotated shapes around a point outside the shape, a vertex, or a point on a side of the shape. The images were usually drawn away from the original shape or beside it. ASK: Does this happen when the centre of rotation is inside the shape? (no, the image overlaps the original shape)

Exercises: Fill in the table to summarize. What happens to a polygon that is reflected? Translated? Rotated?

Transformation	Lengths of Sides	Sizes of Angles	Orientation	Position on Page
Reflection				
Translation				
Rotation				

Answers

Transformation	Lengths of Sides	Sizes of Angles	Orientation	Position on Page
Reflection	*same*	*same*	*opposite*	*changed*
Translation	*same*	*same*	*same*	*moved only*
Rotation	*same*	*same*	*same*	*changed*

NOTE: Students will need protractors for **Question 4** on AP Book 6.2 p. 55.

NOTE: Extension 2 is required to cover the Alberta curriculum.

Extensions

1. Have students complete **BLM Rotations Without a Grid**.

   Selected answers: 2. a) 300° counter-clockwise, b) 340° clockwise, c) 210° clockwise, d) 180° counter-clockwise; 3. parallelogram, because the quadrilateral is made from two congruent triangles, with opposite sides the same length

2. **Investigating rotations using The Geometer's Sketchpad®.** Teach students how to perform rotations using The Geometer's Sketchpad®. Explain that before performing a rotation, one needs to mark a centre of rotation and select an angle of rotation. The software always uses the counter-clockwise direction, so that does not need to be specified. One way to select an angle of rotation is to create a new parameter using the Number menu. Remind students to select the "Angle" option for the new parameter. Then have them follow the instructions below to investigate the effects of changing the centre of rotation on the image of the shape.

   a) Use the Polygon tool to draw a triangle.

   b) Use the Number menu to create a new parameter equal to 90°. Use the Transform menu and the "Mark Angle" option to mark the parameter as the angle of rotation.

   c) Draw a point away from the triangle. Label it *A*.

   d) Use the Transform menu and the "Mark Center" option to mark point *A* as the centre of rotation.

   e) Select the triangle, including the interior, the vertices, and the edges. Use the Transform menu to rotate the triangle around point *A* by the angle of 90°.

   f) Move the vertices of the triangle around. Does the image seem to be congruent to the original triangle?

   g) Move the centre of rotation *A* around, including moving it to the sides, the vertices, and the interior of the triangle. How does the image triangle change? Does it change its shape? Do the angles or the lengths of the sides change?

   h) Moving the centre of rotation *A* around is the same as rotating the triangle around the new centre. What type of transformation—reflection, rotation, or translation—moves one image triangle to another image triangle?

   Selected answers: f) yes; g) the image triangle moves around, but its shape, angles, and side lengths do not change; h) translation

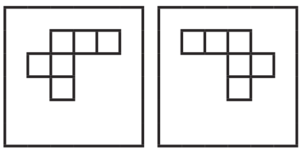
   Emphasize that moving the centre of rotation around is equivalent to rotating the shape around a different centre. So two shapes rotated the same way around different centres are translations of each other.

3. **Find a Flip.** Students can play this game in groups of 2 to 4. To make sure that students identify the transformations correctly, ensure that at least one student can reliably check the answers of the other players in the group.

   Objective: Students create 2 by 2 squares of cards with each card's shape the result of one transformation from the adjacent cards. Students can play cooperatively, working together toward creating eight squares of four cards each (in other words, 2 by 2 squares).

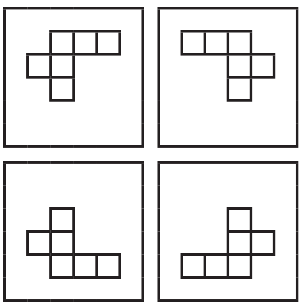
   Materials: **BLM Find a Flip**. There are 32 cards in total: each row on the BLM has four identical shapes, and the next row shows their reflections; thus, there are four sets (or suits) of eight congruent shapes each. The cards do not have a clear top or bottom, so their orientation does not matter.

   Preparation: Provide BLM Find a Flip to each group of students. Students cut out the cards on the BLM. The players shuffle the cards and deal out six cards to each player. Players sort the cards they are given by suit and identify the cards that are reflections of each other.



   Instructions: Students play cooperatively in groups of three to four players so they can see each other's cards, or competitively, in which case, the cards should remain hidden from other players. Players play in turn. Player 1 starts by placing a card at the centre. If Player 2 has a card with a shape that can be obtained from the card already on the table by a single transformation, Player 2 places the card adjacent to the card already in the centre, with the objective of creating a 2 by 2 square of cards. For example, see the pair of cards in the margin: The card on the right is a reflection of the card on the left, and the common vertical side is the mirror line: a student who demonstrates this reflection can flip one card onto the other and look at the cards together against a light source.

   Player 2 says what transformation takes one card to the other, demonstrates the transformation, and picks a new card from the deck. Then Player 3 tries to place another card in the 2 by 2 square of cards. The players continue to take turns and work toward the objective of creating a 2 by 2 square of cards. If any player in turn does not have a card that is a single transformation of any of the cards on the table, that player must pick up all the cards already put down for the square (whether that means one, two, or three cards), and the next player starts a new square but does not need to take another card from the deck. When there are three cards in a square, the fourth card must be placed so that it can be obtained by a transformation from each of the adjacent cards.



   Example: The second picture in the margin shows a completed 2 by 2 square of cards, with all cards showing a reflection of the adjacent cards in the common side. When a 2 by 2 square of cards is completed, all four cards are placed in the common score pile.

G6-16 More Rotations

Pages 57–59

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
angle of rotation
centre of rotation
congruent
corresponding
direction of rotation
fixed point
image
mirror line
prime symbol (′)
reflection
rotation
transformation
translation

Goals

Students will rotate points and shapes around a given centre.
Students will perform combinations of rotations and combine rotations with reflections and translations.
Students will identify and describe rotations.

PRIOR KNOWLEDGE REQUIRED

Knows the terms clockwise and counter-clockwise
Knows that the size of an angle is a measure of rotation
Knows that angle sizes are additive
Can rotate a shape on a grid 90° clockwise or counter-clockwise
Can identify congruent shapes
Knows the definition of congruent shapes in terms of sides, angles, and order of elements

MATERIALS

BLM Rotating a Triangle (p. N-55)
scissors
rulers
BLM 1 cm Grid Paper (p. T-1)
BLM Find a Flip (pp. N-57–58, see Extension 1)

Mental math minute. Remind students that when dividing a sum, they divide each addend separately. Point out that this is like dividing parts of a very large array.

For the following exercises, write the division and the four possible answers on the board. Present one question at a time and have students signal which answer they think is correct by raising the correct number of fingers.

Ask volunteers to explain why some of the answers are not correct. For example, in part a), the remainder of #1 is larger than the divisor, so this cannot be a correct answer.

Exercises: Which answer is correct?

a) 58 ÷ 5

1. 10 R 8 **2.** 11 R 3 **3.** 11 R 2 **4.** 10 R 3

b) 508 ÷ 5

1. 100 R 1 **2.** 110 R 3 **3.** 101 R 3 **4.** 11 R 3

c) 508 ÷ 4

1. 120 R 0 **2.** 127 R 2 **3.** 127 R 0 **4.** 102 R 0

d) 3692 ÷ 3

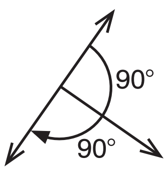
1. 123 R 2 **2.** 1230 R 2 **3.** 1232 R 0 **4.** 1232 R 2

e) 7149 ÷ 7

1. 121 R 2 **2.** 1020 R 9 **3.** 1021 R 2 **4.** 121 R 0

Answers: a) 2, b) 3, c) 3, d) 2, e) 3

Rotations in opposite directions can produce the same result. Ask students to watch how much you are rotating and in which direction. Rotate your entire body, with one arm outstretched, one full turn and ASK: How many degrees did I turn? (360°) In which direction? Repeat, turning in the other direction. Point out that the result of your rotation is the same, regardless of the direction you turned in. Repeat with rotating 90° clockwise and 270° counter-clockwise. SAY: The amount of rotation you perform or the angle that you turn in is called the *angle of rotation*.



Remind students that they actually know that rotations around the same point can be added. Draw the picture in the margin on the board. SAY: If we rotate the top ray clockwise 90°, we get the ray in the middle, and the result is an angle of 90°. We know angle measures can be added. If we rotate the ray in the middle, the image, a further 90° clockwise, we get another angle of 90°, and the total rotation is 180°. When two rotations in the same direction have the same centre of rotation, we can simply add the angles of rotation to get the final angle of rotation.

Ask students again to watch how much you rotate and where you are facing at the beginning and at the end. You can have students count every 90° out loud to keep track. Rotate 180° clockwise and then another 270° (3/4 turn) clockwise. ASK: How much did I turn during the first rotation? (180°) the second rotation? (270°) How many degrees was that in total? (180° + 270° = 450°) What rotation is that equivalent to? (90°) How do you know? (you turned one full turn and another 90°) PROMPT: How many degrees are in a full turn? (360°) Repeat with two consecutive rotations of 270° counter-clockwise to produce a 180° counter-clockwise turn.

On the diagram from earlier, mark *O* on the centre of rotation, point *P* on the upward-pointing ray, and point *P′* as its image on the ray pointing downward to the right. SAY: Now let's look at rotations in opposite directions. Point *P′* is an image of point *P* under a 90° clockwise rotation around *O*. ASK: What rotation in the opposite direction—counter-clockwise—gets you from the point *P* to the point *P′*? (270°) How do you know? (360° − 90° = 270°) Draw an arc to show this rotation and write the subtraction to label it.

Exercises: What turn in the opposite direction would produce the same result?

a) 90° CCW b) 270° CCW c) 180° CW

Answers: a) 270° CW, b) 90° CW, c) 180° CCW

ASK: When we perform a rotation of 180°, does it matter if the rotation is clockwise or counter-clockwise? (no) If we want to rotate a point in a coordinate plane 270° clockwise, what simpler rotation could we do instead and get the same result? (90° counter-clockwise) Have students plot a pair of points on a grid, but not on the same horizontal or vertical grid line, label them *P* and *O*, and rotate *P* 270° clockwise around *O*. Use this opportunity to review rotating points 90° by drawing or imagining a right triangle with the longest, slanted side being *OP*, where *O* is the centre of rotation, as in the previous lesson.

ACTIVITY (Essential)

Give students **BLM Rotating a Triangle** and have them cut out the triangle from the bottom of the page. Have them place the cut-out triangle on top of the triangle at the top of the page and line up the triangles. Have students press the tip of a pencil to the black dot labelled *O* and rotate the top triangle around it 90° clockwise, as they did in the previous lesson, and then rotate the triangle again, around the same point, an additional 90°. ASK: How much did you rotate the triangle in total? (180°) Have them hold the cut-out triangle in place, trace it onto the BLM, and label the vertices using *.

Remove the cut-out triangle and compare the results. ASK: How is the rotated image different from the original? (the image turned upside down) Point out that the side that was horizontal in the original triangle is horizontal in the image triangle as well. Contrast the result with rotation of 90°: when rotating 90° clockwise or counter-clockwise, horizontal sides become vertical and vertical sides become horizontal. Have students also verify that rotating the triangle 180° clockwise and counter-clockwise produces the same result.

Finally, draw students' attention to the order of the letters in the original triangle and the image. Students should see that the triangle is still labelled clockwise after rotation. Emphasize that reflection reverses the order; rotation and translation do not.

Rotating points 180°. Remind students that when rotating a point 90° around another point, they start with drawing a line segment between the points, then use a set square to create the right angle, and then mark the image point the same distance from the centre as the original point. Draw two points, *O* and *P*, on the board and invite a volunteer to draw the line segment between them and measure its length. SAY: I want to rotate point *P* around point *O* clockwise by 180°. How do I draw an angle of 180° with vertex *O* and side *OP*? (extend the line segment *OP* beyond *O* to create a straight angle) Demonstrate the construction on the board following the steps below.

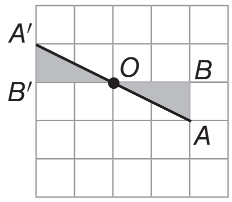
Step 1: Draw line segment *OP*. Measure its length.
Step 2: Extend *OP* beyond point *O*.
Step 3: Mark the point *P′* so that $OP' = OP$.

Exercises:

a) Draw two points, *A* and *B*, on grid line intersections but not on the same horizontal or vertical line.

b) Rotate *B* around *A* 180° clockwise.

c) Rotate *A* around *B* 180° counter-clockwise.

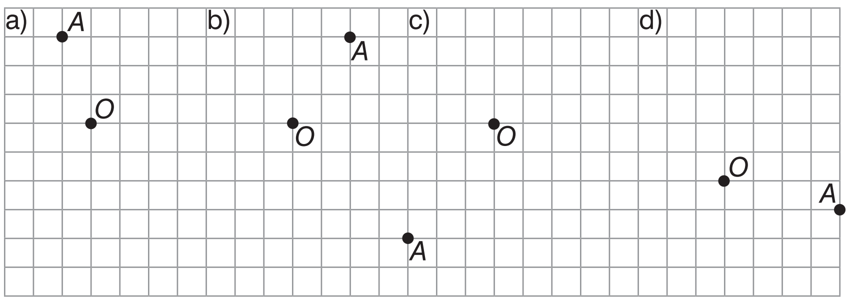


Using a grid to rotate points 180°. Remind students that they used a grid to rotate points 90° clockwise or counter-clockwise. Remind them that they drew or imagined a right triangle and rotated the triangle around one of its vertices. Draw the picture in the margin on the board and explain that it shows rotating point *A* around point *O*. ASK: Is this a 90° rotation around *O*? (no) How much was the triangle turned? (half a turn, 180°, clockwise or counter-clockwise) Discuss how the triangle *AOB* and its image are the same and how they are different. (The triangles are congruent, the horizontal sides are the same length, and the vertical sides are the same length, but if you move from *O* to *A*, you move 2 units right, 1 unit down. When you move from *O* to *A′*, you move 2 units left and 1 unit up, so the directions are opposite.)

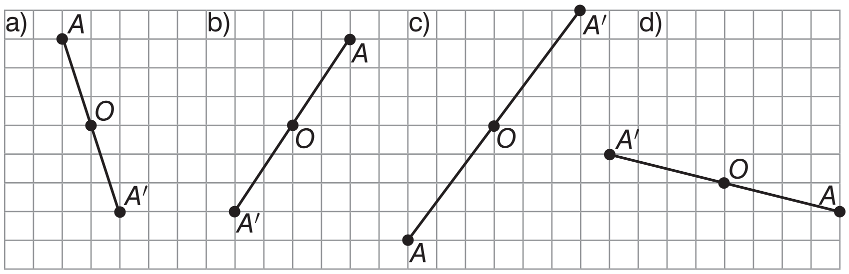
Point out that the picture with triangles *AOB* and *A′OB′* gives us a quick way to rotate points on a grid by 180° similar to what students did with 90° clockwise or counter-clockwise. SAY: All you need to do is to mentally extend the slanted line beyond point *O* as if you were drawing a congruent triangle with horizontal and vertical sides of the same length but in the opposite directions from the centre of rotation. If you go 2 units left and 1 unit up from *A* to *O*, continue another 2 units left and 1 unit up from *O* to *A′*. Demonstrate using the first exercise below: move 1 unit right and 3 units down from *A* to *O* and then move 1 unit right and 3 units down from *O* to *A′*.

Exercises

1. Rotate the point *A* 180° clockwise around point *O*. Draw the line segment *AA′* to check.



Answers

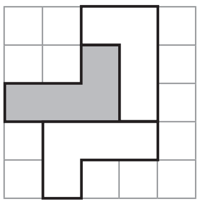


2. a) Draw a right trapezoid.

   b) Rotate the right trapezoid 180° CCW around the vertex with the acute angle.

   c) Draw a pentagon that has no lines of symmetry.

   d) Choose a point away from the pentagon. Rotate the pentagon 180° CW around the chosen point.

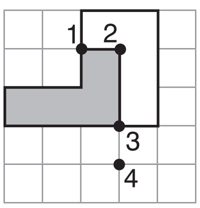


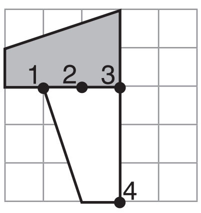
Distinguishing rotations of 90° from rotations of 180°. Draw the picture in the margin on the board. SAY: The grey shape was rotated 90° counter-clockwise to get one of the shapes and 180° (clockwise or counter-clockwise) to get the other shape. ASK: Which shape is produced by a rotation of 90° counter-clockwise and which is produced by a rotation of 180°? (the shape on the top is the image of a 90° CCW rotation; the shape on the bottom is the image of 180° CW or CCW rotation) Have students explain how they know. Answers will vary; make sure students understand that horizontal sides remain horizontal after a rotation of 180°, clockwise or counter-clockwise, and become vertical after a rotation of 90°, clockwise or counter-clockwise.

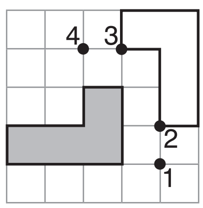
Finding the centre of rotation. Have students copy the grey shape on grid paper and cut it out. (Students can use **BLM 1 cm Grid Paper**.) Have students copy the picture on the board, place the cut-out shape on top of it, and perform both rotations to check. Have them rotate the shape using various points by pressing the tip of a pencil to different points on the outline of the grey shape. After students tried several different points, present the following exercises. Students can signal the answer by raising the number of fingers to show the number of the answer that they think is correct.

Exercises

1. The grey shape was rotated 90° CCW to get the white shape. Which point is the centre of rotation?

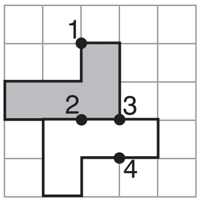
a) 

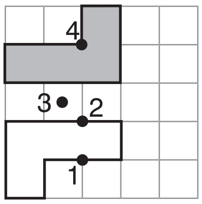
b) 

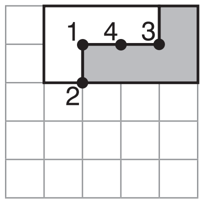
Bonus 

Answers: a) 1, b) 3, Bonus: 4

2. The grey shape was rotated 180° CW to get the white shape. Which point is the centre of rotation?

a) 

b) 

c) 

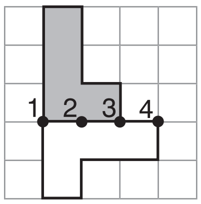
Answers: a) 2, b) 3, c) 4

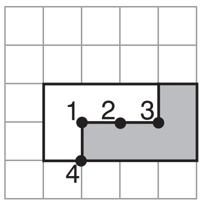
Discuss strategies to find the correct centre of rotation. Point out that when the shape and the image touch each other, it makes sense to look for points that are on the common edge. Moreover, it makes sense to check the vertices first: are there any corresponding vertices that match? The centre of rotation is the only fixed point in rotation, so it should be on the same spot on both the original shape and the image.

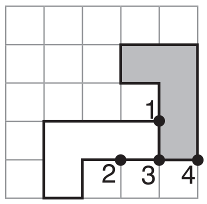
In the case of 180° rotation, there is another way to look for the centre of rotation. If students are familiar with rotational symmetry, have them think of the point that they would rotate the whole picture, original and image, around to make it fall back onto itself, or the visual centre of the whole picture.

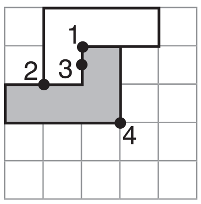
NOTE: Students who are struggling with the exercises below can use the cut-out shape to try to figure out the answers.

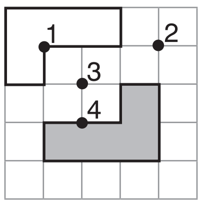
Exercises: The white shape is the image of the grey shape under rotation. Find the centre of rotation and describe the rotation.

a) 

b) 

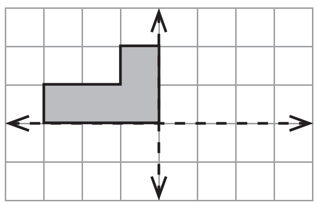
c) 

d) 

Bonus 

Answers: a) 90° CW (or 270° CCW) rotation around point 1, b) 180° CW or CCW rotation around point 2, c) 90° CCW (or 270° CW) rotation around point 3, d) 180° CW or CCW rotation around point 3, Bonus: 180° CW or CCW rotation around point 3

Two reflections in perpendicular mirror lines produce a 180° rotation. Remind students that they can model reflection by flipping a shape over. Flipping it over a horizontal side is like reflection in the mirror line containing this horizontal side. Have students draw a horizontal and a vertical line on grid paper and place the L shape they used as shown in the margin.

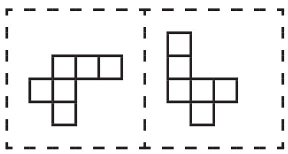


Have students trace the shape, then reflect the shape in the vertical mirror line, and then reflect the image in the horizontal mirror line and trace the image. Repeat with reflecting in the opposite order, in the horizontal line followed by the reflection in the vertical line. ASK: What do you notice? (the result is the same) Is there one transformation that would take the original shape to the image? (yes) Which transformation? (180° rotation around the intersection of the mirror lines)

Repeat by placing the shape differently. Compare answers with the whole class. Did everyone place the shape the same way? (no) Did everyone get the same result: the order of reflections does not matter, and the resulting image is also a 180° rotation around the intersection of the mirror lines? (yes) Point out that the fact that everyone got the same result means that the answer is likely not dependent on the shape and is true for all shapes.

Extensions

1. Have students play Find a Flip (see Extension 3 in Lesson G6-15) with cards from **BLM Find a Flip**. First, have students place the cards so that each card they place is a reflection of the adjacent cards. Then ask the students to use rotations instead so that each card they place is a 90° rotation of the adjacent card. The fourth card will be a 90° rotation of both adjacent cards. Players can even place cards out of order—for example, both sequences are valid: first, second, third, fourth *and* first, second, fourth, third. Players must say around which vertex of the card the rotation was made and in which direction. In the example in the margin, the card on the right appears to be a 90° counter-clockwise rotation of the card on the left around the common vertex on the top.

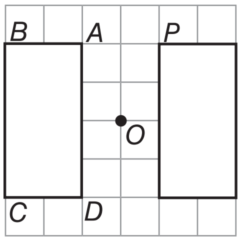


   After students have played the game both ways (reflections and rotations) several times, discuss similarities and differences between the two games. ASK: How many 2 by 2 squares of cards can you complete with cards from the same suit? (2 squares) How many different types of cards are in each suit? (each suit has eight cards of congruent shapes: four identical shapes and four that are their reflections) How are the squares completed in each game different—in other words, how many cards of each type does each square contain? (for rotations, a square has four of the same; for reflections, there are two of one type and two of another type) In which game was it harder to complete the first square? Why? (It is harder to complete the first square in the game with rotations. With rotations, when you place the first card of a suit, there are only three cards you can add to it. With reflections, when you place the first card, there are more card options, so the reflection game is easier.)

Is there a difference in constructing the second 2 by 2 square of cards with the same suit? (no, you have four cards from the same suit left in both cases, and they can always make a square)

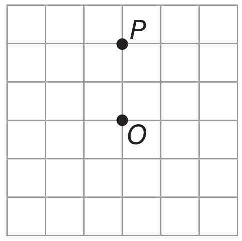
2. Describe the transformation (including the translation arrow, the mirror line, or the amount of rotation around point *O*) that takes the rectangle *ABCD* onto the other rectangle so that …

a) $A \longrightarrow P$ b) $B \longrightarrow P$ c) $D \longrightarrow P$



Answers: a) reflection in the vertical line through *O*, b) translation 4 units right, c) 180° rotation (clockwise or counter-clockwise) around *O*

3. a) Copy the picture.



b) Rotate the point around point *O* as given.

i) $P \longrightarrow P'$: 90° clockwise

ii) $P' \longrightarrow P''$: 180° clockwise

iii) $P'' \longrightarrow P^*$: 270° clockwise

c) Point P^* can be obtained by rotating point *P* around point *O* $90° + 180° + 270° - 360° =$ ______° clockwise.
Explain where each number in the equation comes from.

Selected sample answer: c) 180°; all the rotations in part b) are performed clockwise around point *O*, so the measures are added to get the combined amount of rotation: $90° + 180° + 270° = 540°$. This is more than a full rotation. A full rotation of 360° brings the point *P* to the initial position, so we can subtract 360° to get the rotation after the full turn. $540° - 360° = 180°$, so point P^* is directly down from point *O*.

G6-17 Designs and Transformations

Pages 60–62

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
angle of rotation
centre of rotation
corresponding
direction of rotation
image
mirror line
reflection
rotation
transformation
translation

Goals

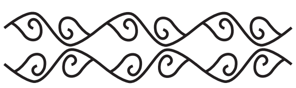
Students will identify transformations used to create patterns and designs.
Students will use transformations and combinations of transformations to create patterns and designs.

PRIOR KNOWLEDGE REQUIRED

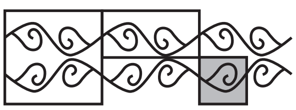
Can identify and perform translations, reflections, and rotations
Can identify congruent shapes
Can extend a repeating pattern

MATERIALS

pictures of designs created from transformations
BLM Find a Flip (pp. N-57–58)
scissors
glue
blank square about 4 cm by 4 cm (see Extension 3)
pattern blocks or **BLM Pattern Blocks** (p. N-59, see Extension 4)

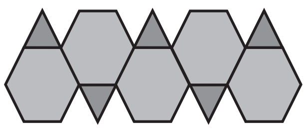


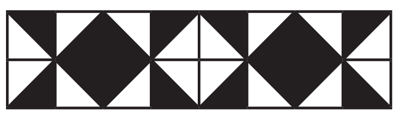
Identifying the smallest part that repeats in the design. Show students several pictures of patterns and designs created by repeatedly using a transformation, such as a frieze pattern (see example in the margin). Explain that by repeatedly using one or several transformations on a simple shape, you can create a beautiful pattern or design. Invite a volunteer to show the part that repeats in the design. For each part that students identify, ask them to explain which transformation is used to repeat this part and create the design.



If students identify the part that repeats, but it is not the smallest, explain that there can be a smaller region that repeats, using more transformations. For example, in the second picture in the margin, students might identify either of the two white rectangles as a repeating part. The largest rectangle is translated to the right to create the repeating pattern. However, the smaller white rectangle can be reflected in the horizontal line to create the larger white rectangle, so the whole pattern can be created using a reflection and translation. Moreover, the grey rectangle can be rotated 180° around the midpoint of one of the vertical sides, as well as reflected in the horizontal line to create the same pattern, so the grey rectangle is the smallest part that was used to create the repeating pattern.

Exercises: What is the smallest part that creates the pattern?

a) 

b) 

Sample answers: a)  b) 

NOTE: Some students might identify half the square shown in the margin as the smallest part creating the pattern. Ask these students to identify all the required mirror lines. This is the correct answer, but it involves reflections in a slant line, which students are not expected to focus on.

Creating designs and patterns by repeated rotation and identifying the rotation.

ACTIVITY 1 (Essential)

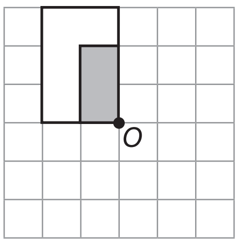
1. Give each student **BLM Find a Flip** and have them cut out eight cards of the same suit (showing the same shape). Assign different suits to different students. Have them create a design that meets the rules below and glue each design to a separate strip of paper. Have students describe the direction, angle, and centre of rotation on the back of the strip of paper.

   a) a 4 by 1 rectangle so that each card is the same 90° CW or CCW rotation of the adjacent cards around a common vertex of the cards

   b) a 4 by 1 rectangle so that each card is a 180° rotation of the adjacent cards around the midpoint of the common side

   Have students swap strips with a partner who used a different suit. Students need to identify the transformations used in the design. They can verify the answer by checking the back of the strip of paper.

Exercises: Copy the picture on grid paper.



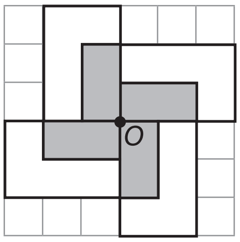
a) Create a design by repeatedly rotating the shape 90° clockwise around point *O*.

b) Create a pattern by repeatedly rotating the shape 90° counter-clockwise around the top-right corner.

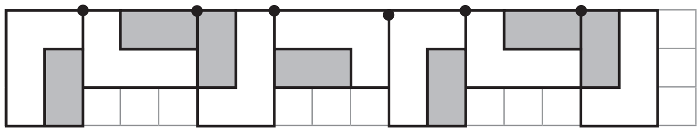
c) Create a pattern by repeatedly rotating the shape 180° clockwise around the middle of the right side.

Answers

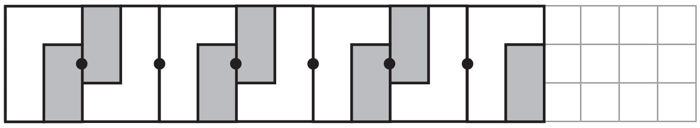
a)



b)



c)



NOTE: Students who are struggling with imagining the image after each rotation can cut out the shape, rotate it by pressing the tip of the pencil to the correct point, and then copy the shape.

Creating designs by repeated reflection and identifying the transformation.

ACTIVITY 2 (Essential)

2. Repeat Activity 1 for the rules below. Have students use cards from a different suit.

   a) a 2 by 2 square so that each card in the square is the same 90° CW or CCW rotation of the adjacent cards around the centre of the 2 by 2 square

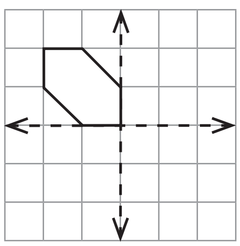
   b) a 2 by 2 square so that each card in the square is a reflection of the adjacent cards in the common side

   c) a 4 by 1 rectangle so that each card is a reflection of the adjacent cards in the common side

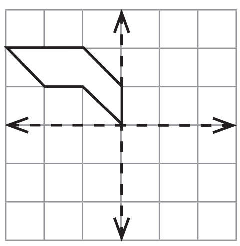
   To make the guessing part harder, have students add one of the strips from Activity 1 as the fourth strip. This will force students to distinguish between reflections and rotations. Have students work with other partners than the ones they worked with in Activity 1.

Exercises: Copy the pictures on grid paper.

A.



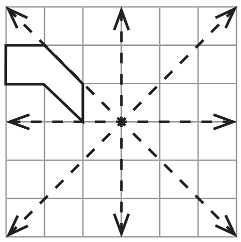
B.



a) Create designs by repeatedly reflecting the shapes in the given mirror lines.

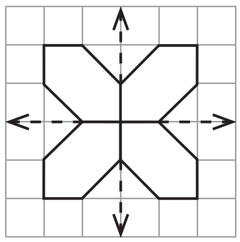
b) Create patterns by repeatedly reflecting the shapes in a vertical line through the right side.

Bonus: Create a design by repeatedly reflecting the polygon in the mirror lines.

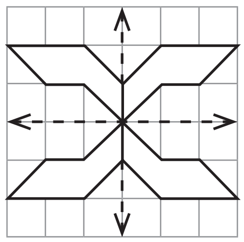


Answers

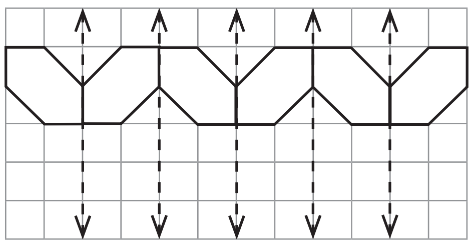
a) A.



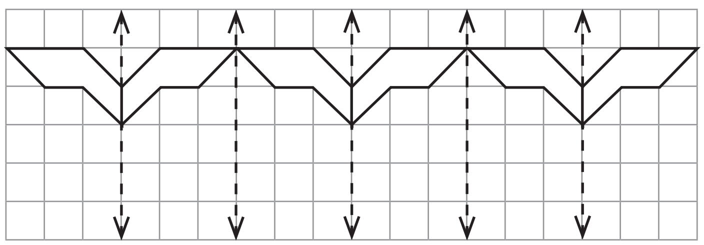
B.



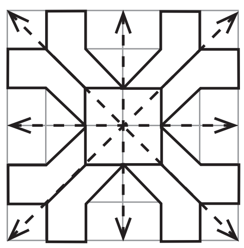
b) A.



B.

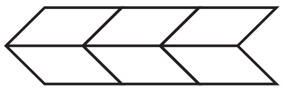


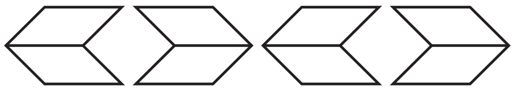
Bonus:

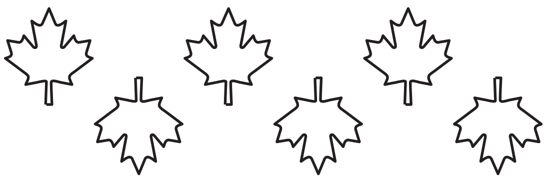


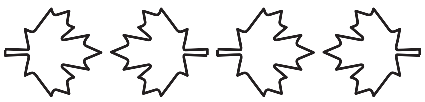
Identifying transformations in patterns. Display the patterns in the following exercises one at a time. Have a volunteer identify the part that repeats. Discuss which transformation or combination of transformations takes the repeated shape to the adjacent shapes. Have students explain how they know which transformation to use. (the shapes are exactly the same and point the same way, so it is a translation; the shapes point in opposite ways, and if I join corresponding vertices, I get parallel line segments of different lengths, so this is a reflection; the shape points in a different direction, and horizontal lines became vertical, so there is a 90° rotation) Have students also identify the mirror lines and the centres of rotation. Encourage multiple answers.

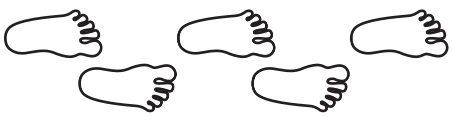
Exercises: Identify the part that repeats.

a) 

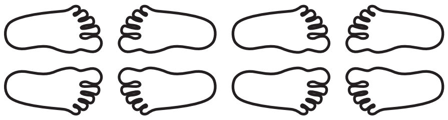
b) 

c) 

d) 

e) 

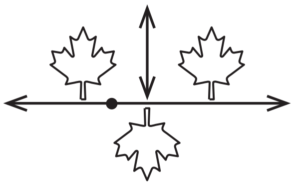
f) 

g) 

Answers

a) reflection in a horizontal line to get the shapes in the bottom row from the shapes in the top row, translation or rotation of 180° around the middle of the common side to get the shapes in the same horizontal row

b) reflection in a horizontal line to get the shapes in the bottom row from the shapes in the top row, reflection in the vertical line to get the shapes in the same horizontal row; translation down and right or rotation of 180° around the point in the centre of the space between each 4 shapes to get the shape situated diagonally



c) reflection in a horizontal line and translation left or right to get from 1 to 2, translation right or reflection in a vertical line to get from 1 to 3; rotation of 180° CW or CCW around the point marked (see margin) to get from 1 to 2 or from 2 to 3; students might also notice that half the leaf can be used to generate the full leaf using a reflection in a vertical line

d) reflection in a vertical line or rotation of 180° around the point midway between the tips of the leaves
e) reflection in a horizontal line and translation to the right
f) reflection in a vertical line
g) reflection in a vertical line and reflection in a horizontal line

NOTE: Students might also replace any rotation of 180° by a combination of reflections, in a horizontal and a vertical line that intersect at the centre of rotation.

Discuss why some patterns allow more descriptions than other patterns. Students should notice that shapes that have some symmetry, such as lines of symmetry or rotational symmetry (students in Ontario should be familiar with the concept from Unit 6), produce more options because reflecting or rotating them results in the same shape as translating.

ACTIVITY 3 (Optional)

3. Find a real-life example of a pattern or design that is made from repeating shapes. Identify the part that is used to create the pattern and describe the transformations used to create the pattern. Students can make a class display of the patterns and designs they found.

NOTE: Extensions 1 and 2 are required in order to cover the British Columbia curriculum.

Extensions

1. Explain that a frieze pattern is a pattern that is created from repeated tiles placed in a row. Several shapes can appear on the same tile, and often the same shape is reflected, rotated, or translated to create the tile. There are seven different types of frieze patterns. Have students follow the instructions below to create a tile for each of the seven types of frieze patterns, starting from the same shape that has no lines of symmetry and no rotational symmetry.

   A. Draw a shape that has no lines of symmetry on grid paper. Your shape is the tile. To create the frieze pattern, translate the shape several times to the right.

   B. Reflect the shape in the horizontal line. Your tile consists of the original and the image.

   C. Reflect the shape in the vertical line. Your tile consists of the original and the image.

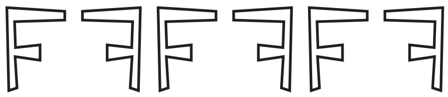
   D. Imagine that your shape is drawn on a rectangle. Rotate the shape 180° CW or CCW around the bottom-right corner of the rectangle. The tile consists of the original and the image.

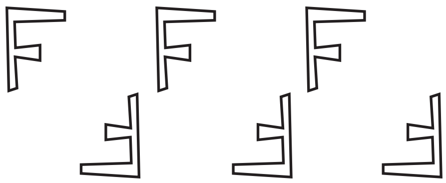
E. Reflect the shape in the horizontal line and then translate it a little to the right. Your tile consists of two shapes, the original and the reflected and translated image.

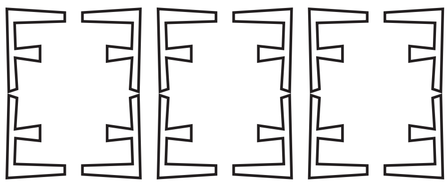
F. Reflect the shape in the horizontal line. Reflect both shapes in the vertical line. Your tile consists of four shapes.

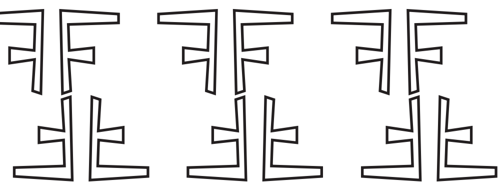
G. Reflect the shape in the vertical line. Reflect both shapes in the horizontal line and translate them to the right. The tile consists of four shapes.

2. Match the frieze pattern to the description in Extension 1.

a) 

b) 

c) 

d) 

Answers: a) C, b) D, c) F, d) G

3. Draw a shape that has no line of symmetry on a square. Use the square with the shape to create a tile pattern.

   a) Rotate the square 90° clockwise around the bottom-right corner repeatedly. Translate the whole row you created 1 unit left and 1 unit down. Repeat, translating the row.

   b) Rotate the square 90° clockwise around the bottom-left corner. Translate the whole column 1 unit right and 1 unit up. Repeat, translating the column.

   c) Did you get the same tiling pattern both ways?

   Selected answer: c) yes

4. Use pattern blocks or cut them out from **BLM Pattern Blocks** to create a shape that has no lines of symmetry. Use rotations to create a design based on the shape you created. Describe the rotations you used.

G6-18 Coordinate Grids

Pages 63–64

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
axis/axes
coordinate grid
coordinates
first, second coordinate
ordered pair
origin
***x*-axis**, ***y*-axis**
***x*-coordinate**, ***y*-coordinate**

Goals

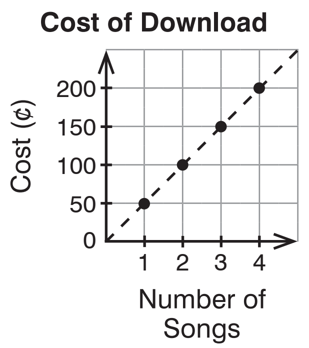
Students will identify and plot points in the first quadrant of a coordinate grid.

PRIOR KNOWLEDGE REQUIRED

Can read line graphs and broken line graphs
Can identify points on a number line made by skip counting

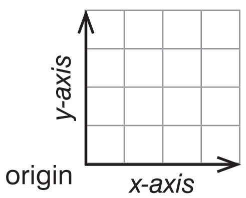
MATERIALS

binders or other dividers
rulers
BLM Grid with Tens (p. N-60)



Review using grids in line graphs. Draw the picture in the margin on the board. Remind students that in line plots they used points on a grid to show the relationship between two quantities. For example, this graph shows the relationship between the number of songs downloaded and the cost in cents. Each dot refers to two numbers. Point at the dots in random order and have students identify what the number of songs and the cost are for each dot. Then draw a dot at (1, 150) and repeat the question. Point out that you could describe any point on a grid using the numbers on the number lines.

SAY: Mathematicians around the world have agreed to give the location of a point on a grid by two numbers in parentheses, or brackets. The first number is always the number above which the point is on the horizontal number line, and the second number is the number to the left, on the vertical number line. This means that the numbers in the pair have a specific order, so we call them an *ordered pair*. Go through all the marked points in the picture on the board and have students identify the ordered pairs. ((1, 50), (2, 100), (3, 150), (4, 200), and the extra point (1, 150))



Introduce the axes and origin. Draw the coordinate grid shown in the margin, but without the labels. SAY: In geometry, we use grids with number lines that always meet at 0. The grid is called the *coordinate grid*. The number lines are called *axes*. Point out the axes and mention that "axes" is the plural of *axis*. SAY: The horizontal number line is called the *x-axis*, and the vertical number line is called the *y-axis*. The point at which the two axes intersect is called the *origin*. Label the axes, the origin, and numbers 0 to 4 on the axes.

Point out that both axes start at 0. Explain that since the origin is the intersection of two number lines, which meet at 0, its ordered pair is (0, 0). It makes sense to draw only one 0 at the place where both number lines meet because in geometry the grids have no squiggly lines or breaks; they are just regular number lines always meeting at 0.

ACTIVITY (Essential)

Students work in pairs and use grid paper. Partners use a divider, such as a binder, to conceal their grids from each other. Each partner draws a coordinate grid and labels it from 0 to 4. Partner 1 marks a point on the grid and tells Partner 2 its ordered pair.
Partner 2 marks the point on her own grid. Partners switch roles a number of times before checking that their grids match.

Introduce the *x*-coordinate and *y*-coordinate. On a grid on the board, mark the point (4, 3) and have students identify the ordered pair. SAY: The ordered pair (4, 3) is called the *coordinates* of the point. Mathematicians say that the point (4, 3) has $x = 4$ or the *x-coordinate* is 4. Trace your finger down from the point and show that it is directly above the number 4 on the *x*-axis.

Then trace with your finger left from the point (4, 3) to the *y*-axis to look at the number on the *y*-axis. SAY: Mathematicians say that this point has $y = 3$ or the *y-coordinate* is 3.

Exercises: Rewrite the coordinates of the point as $x =$ ______, $y =$ ______.

a) (2, 1) b) (1, 3) c) (4, 2) d) (3, 4)

Answers: a) $x = 2$, $y = 1$; b) $x = 1$, $y = 3$; c) $x = 4$, $y = 2$; d) $x = 3$, $y = 4$

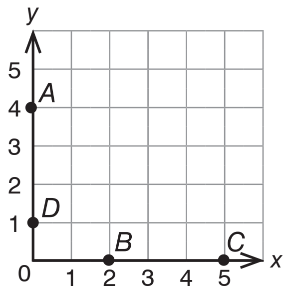
Mark several points on the grid and have students identify the *x* and the *y* for these points. SAY: The *x*-coordinate is often called the *first coordinate*, because it is written first, and the *y*-coordinate is called the *second coordinate*.

Coordinates of points on the axes. Remind students that the axes intersect at 0. Mark the point (3, 0) on the coordinate grid and have students identify the coordinates. Point out that the *y*-coordinate is in fact the height above the horizontal axis, and since the height is 0, the point should be on the horizontal axis itself. Repeat with the point (0, 4), explaining that the *x*-coordinate shows the distance from the point to the vertical axis, measured along the grid line on which the point is situated.

Exercises: Draw a coordinate grid and label it with numbers from 0 to 5. Mark and label the points on the grid.

A (0, 4), *B* (2, 0), *C* (5, 0), *D* (0, 1)

Answers:



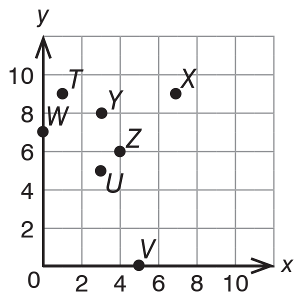
Coordinate grids with scales made by skip counting by 2s. On a grid on the board, draw axes and mark them with intervals of 2. Explain that just as number lines on graphs can be marked with skip counting, number lines on coordinate grids can be marked by skip counting too. Mark several points and ask students to find the coordinates of these points. Start with points that are on the grid lines, such as (2, 4), continue to points that are on only one grid line, such as (2, 3) and (5, 4), and progress to points that are not on grid lines, such as (5, 7).

Exercises

a) Draw a coordinate grid with axes that skip count by 2s from 0 to 10.

b) Mark and label the points on the grid.
Z (4, 6), *Y* (3, 8), *X* (7, 9), *W* (0, 7), *V* (5, 0), *U* (3, 5), *T* (1, 9)

Answers:



Review marking numbers on a number line that only shows tens. Draw a number line from 0 to 40 but label only the tens. Point to a few locations on the line and have students identify the number for each location. Write several numbers and point at locations on the number line. Have students signal with thumbs up and thumbs down to indicate whether the location you are pointing at is the number you wrote.

Coordinate grid with scales made by skip counting by 10s and 5s. Display **BLM Grid with Tens**. Use the point *A* (18, 24) to show students how to determine the coordinates of the point and then invite volunteers to draw lines from other points to the axes. Point to the locations on the grid for each coordinate pair and have students signal thumbs up or thumbs down to indicate whether this point is the one given by the coordinates.

Have students draw a coordinate grid whose axes are marked with intervals of 5 from 0 to 25. Points to plot: *A* (15, 6), *B* (3, 10), *C* (0, 24), *D* (6, 0), *E* (22, 6), *F* (23, 5), *G* (1, 20)

Extensions

1. Draw the points on a coordinate grid.
(2, 6), (4, 4), (5, 7), (7, 8), (5, 2), (3, 4), (2, 1), (0, 0)
Join the points in the order you drew them. Join the first point to the last point. What letter did you make?

Bonus: If you rotate the shape 270° counter-clockwise around any point, what letter will you make?

Answer: N, Bonus: Z

2. Players will need a divider to conceal the coordinate grids they are working on from partners. Player 1 draws a square or a rectangle on the coordinate grid and tells Player 2 the coordinates of the vertices. Player 2 tries to visualize the shape and guess what kind it is before plotting the vertices. Player 2 plots the vertices and checks the answer.

   Advanced: Use other shapes, such as rhombuses, parallelograms, and trapezoids.

3. a) Draw a coordinate grid with axes from 0 to 10.

   b) Plot the pair of points.

      i) (3, 0) and (7, 0) ii) (4, 1) and (9, 1) iii) (5, 2) and (1, 2)

   c) Find the distance between the points in the pair.

   d) Which coordinates are the same in each pair, the first or the second? What is the same about the location of the points?

   e) Which coordinate is not the same? How can you find the distance between the points using the coordinate that is not the same?

   Selected answers: c) i) 4, ii) 5, iii) 4; d) second, the points are on the same horizontal line; e) *x*-coordinates, subtract the smaller coordinate from the larger

4. a) Draw a coordinate grid with axes from 0 to 10.

   b) Plot the pair of points.

      i) (3, 0) and (3, 6) ii) (7, 1) and (7, 9) iii) (0, 2) and (0, 5)

   c) Find the distance between the points in the pair.

   d) Which coordinates are the same in each pair, the first or the second? What is the same about the location of the points?

   e) Which coordinate is not the same? How can you find the distance between the points using the coordinate that is not the same?

   Selected answers: c) i) 6, ii) 8, iii) 3; d) first, the points are on the same vertical line; e) *y*-coordinates, subtract the smaller coordinate from the larger

G6-19 Translations and Reflections on a Coordinate Grid
Page 65–66

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
axis/axes
coordinates
first, second coordinate
grid
image
ordered pair
origin
reflection
translation
x-axis, *y*-axis
x-coordinate, *y*-coordinate

Goals

Students will perform and describe translations and reflections of points and polygons in the first quadrant of a coordinate grid.

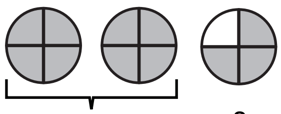
PRIOR KNOWLEDGE REQUIRED

Can identify and perform translations and reflections
Can identify and plot points on a coordinate grid

MATERIALS

BLM Maps (pp. N-61–62, see Extension 3)

Mental math minute. Remind students that they can use division with remainders to convert improper fractions to mixed numbers. For example, 11 ÷ 4 = 2 R 3, so 11/4 can be represented as 2 3/4. Demonstrate using circles divided into fourths, as shown below. Each circle shows a group of four fourths, and the remainder is three fourths, or the fractional part.



$$\frac{11}{4} = \text{2 wholes} + \frac{3}{4} = 2\frac{3}{4}$$

Exercises: Convert the improper fraction to a mixed number.

a) $\frac{13}{4}$ b) $\frac{11}{3}$ c) $\frac{17}{5}$ d) $\frac{51}{8}$ e) $\frac{77}{8}$ f) $\frac{77}{9}$

Answers: a) 3 1/4, b) 3 2/3, c) 3 2/5, d) 6 3/8, e) 9 5/8, f) 8 5/9

Investigate the change in the coordinates under a horizontal or vertical translation. Mark the points (4, 5), (0, 6), and (1, 7) on a grid on the board and ask students to identify the coordinates. Have students draw a coordinate grid (with axes counting by 1s from 0 to 10) and plot the points, joining them into a triangle. Have students translate the vertices of the triangle 3 units to the right and write the original and image coordinates in a table as shown below.

Original	(4, 5)	(0, 6)	(1, 7)
Image	(7, 5)	(3, 6)	(4, 7)

ASK: Which coordinate changed during the translation? (the *x*-coordinate) What about the other coordinate? (it stayed the same) How did the coordinate change—did it increase or decrease? (increase) By how much? (by 3) How do you express it as an operation? (add 3 to the *x*-coordinate)

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Repeat the investigation by translating the points 4 units down. Students will see that the y-coordinate changes, decreasing by 4, and the x-coordinate stays the same.

Generalizing the rule for combined directions. On a new grid, draw the point (4, 5). SAY: When we move this point 3 units right, the x-coordinate increases by 3. ASK: What happens if we move the point 3 units left? (the x-coordinate will decrease by 3) PROMPT: Which coordinate changes? (the x-coordinate) Moving left is the opposite of moving right. Does the x-coordinate increase or decrease? (decrease) Demonstrate both movements with arrows. Write on the board:

3 units right: add 3 to the x-coordinate
3 units left: subtract 3 from the x-coordinate

ASK: So if I wanted to translate a point, say, (7, 7) 3 units left, what point would I get? (4, 7) Invite a volunteer to check. ASK: What if I wanted to translate point (7, 7) 2 units right? (9, 7) Again, invite a volunteer to check.

Repeat the questioning with moving the point (4, 1) 3 units up and 3 units down. Point out that, this time, the y-coordinate changes. Write on the board:

3 units up: add 3 to the y-coordinate
3 units down: subtract 3 from the y-coordinate

Remind students that they can combine translations in different directions. For example, students can translate a shape 3 units right and 4 units down. Ask students to predict the change in the coordinates in the point (5,5) and check.

Exercises

a) Draw a coordinate grid. Plot the points A (4, 5), B (3, 6), C (5, 6), and D (6, 5).

b) How will the coordinates change in each translation?

i) 2 units right, 1 unit up ii) 3 units left, 2 units down

c) Predict the coordinates of the images for each translation.

d) Perform the translations and check your prediction.

Selected answers
b) i) x-coordinate increases by 2, y-coordinate increases by 1;
ii) x-coordinate decreases by 3, y-coordinate decreases by 2
c) i) A' (6, 6), B' (5, 7), C' (7, 7), D' (8, 6); ii) A^* (1, 3), B^* (0, 4), C^* (2, 4), D^* (3, 3)

Investigate the change in the coordinates under a reflection in a vertical line. Mark the points (3, 5), (0, 6), and (1, 2) on a grid on the board and ask students to identify the coordinates. Have students draw a coordinate grid (with axes counting by 1s from 0 to 10) and plot the points, joining them into a triangle. Draw the vertical line through point (4, 0) and have students copy it as well. Have students reflect the vertices of the triangle in the line (see the completed table on the following page).

Original	(3, 5)	(0, 6)	(1, 2)
Image	(5, 5)	(8, 6)	(7, 2)

ASK: Which coordinate changed during the reflection? (the *x*-coordinate) What about the other coordinate? (it stayed the same) Repeat with a reflection in another vertical line through the point (2, 0). (the image points are (1, 5), (4, 6), and (3, 2)) Discuss why the *y*-coordinate did not change. To prompt students to see the answer, have them draw a horizontal line through one of the vertices of the original triangle and look at the *y*-coordinates of different points on the same line. Students will notice that points on the same horizontal line have the same second coordinate. Have them think how they perform a reflection in a vertical line: they start by drawing a line perpendicular to the mirror line, so a horizontal line, and mark a point on the same horizontal line. Naturally, the second coordinate of the image will be the same as the second coordinate of the original—they are on the same horizontal line by construction.

ASK: Are there points on the triangle that had no change of coordinates after the second reflection? (yes) Have students give examples. SAY: The coordinates of the points on the mirror line did not change under reflection because they are fixed points—points on the mirror line stay the same under reflection.

Repeat the investigation by using two horizontal mirror lines, for example, a horizontal line through (0, 3) and another line through (0, 6). Students will see that the *y*-coordinate changes and the *x*-coordinate stays the same because both the original and the image under reflection are on the same vertical line.

Extensions

1. A shape has vertices *A* (2, 2), *B* (3, 5), *C* (5, 5), and *D* (4, 2). Under a translation, vertex *A* moved to *A′* (7, 8). Find the coordinates of the other vertices under the translation. Check by plotting *ABCD* and performing the translation.

   Answer: *B′* (8, 11), *C′* (10, 11), *D′* (9, 8)

2. Marcel translated triangle *ABC* 5 units left and 3 units up. The image triangle has vertices *A** (2, 4), *B** (1, 5), and *C** (2, 3). What were the coordinates of the vertices of triangle *ABC*? Explain.

   Sample answer: Triangle *ABC* was translated 5 units left and 3 units up to get *A*B*C**. This means that translating triangle *A*B*C** 5 units right and 3 units down will get us back to triangle *ABC*. Therefore, the coordinates of the vertices are *A* (7, 1), *B* (6, 2), and *C* (7, 0).

3. Have students complete **BLM Maps**.

Selected answers
1. a) Dubhe; b) (60, 10); c) Alkaid; Bonus: Alioth, (10, 16)
2. b) 10; c) 10; south, 5; d) 5, east, 10, north; e) 10 paces west, 10 paces north
3. b) Red Rock, Tall Fir; c) 10 paces west, 10 paces south
4. a) Bear Cave, Treasure, The Fort; b) (14, 8), (12, 2), (12, 10); c) Clear Spring, Swamp, Bonus: Ear Wood; d) 4 km west; 6 km south; 6 km east, 2 km south; 2 km north, 2 km west; 4 km west, 6 km south; 2 km north, 6 km west, 6 km north

G6-20 Rotations on a Coordinate Grid

Pages 67–69

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
axis/axes
centre of rotation
coordinates
first, second coordinate
image
ordered pair
origin
reflection
rotation
translation
x-axis, *y*-axis
x-coordinate, *y*-coordinate

Goals

Students will perform and describe transformations of points and polygons in the first quadrant of a coordinate grid.

PRIOR KNOWLEDGE REQUIRED

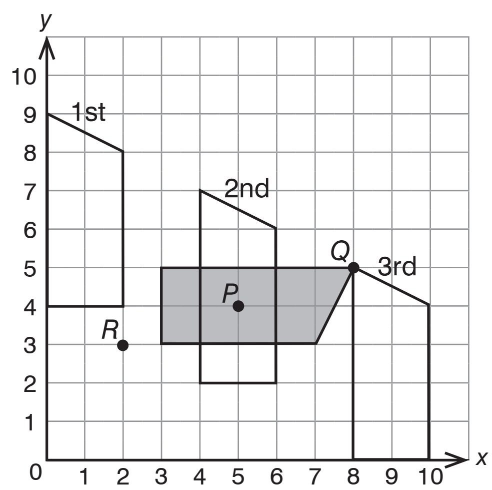
Can identify, describe, and perform translations, reflections, and rotations
Can identify and plot points on a coordinate grid

Mental math minute—number talk. Present this division with remainder: $650 \div 8$. (81 R 2) The following strategies could arise:

use $648 \div 8 = (600 \div 8) + (48 \div 8)$
use $648 \div 8 = (640 \div 8) + (8 \div 8)$
$(640 \div 8) + (10 \div 8) = 80$ R $0 + 1$ R $2 = 81$ R 2

Investigate the change in the coordinates under a 90° rotation. Draw a coordinate grid with axes marked from 0 to 10 on the board and have students draw a similar grid in their notebooks. On the grid, draw a trapezoid with vertices (3, 3), (3, 5), (8, 5), and (7, 3). Ask students to identify the coordinates of the vertices and fill in the first row of the table below for the original points. Have students plot the points on the coordinate grid they drew and join them into the quadrilateral. Have students rotate the quadrilateral 90° CCW around the three points given in the table and fill in the table. (see completed table and grid below)

Original		(3, 3)	(3, 5)	(8, 5)	(7, 3)
Image under 90° CCW rotation around	***R* (2, 3)**	(2, 4)	(0, 4)	(0, 9)	(2, 8)
	***P* (5, 4)**	(6, 2)	(4, 2)	(4, 7)	(6, 6)
	***Q* (8, 5)**	(10, 0)	(8, 0)	(8, 5)	(10, 4)



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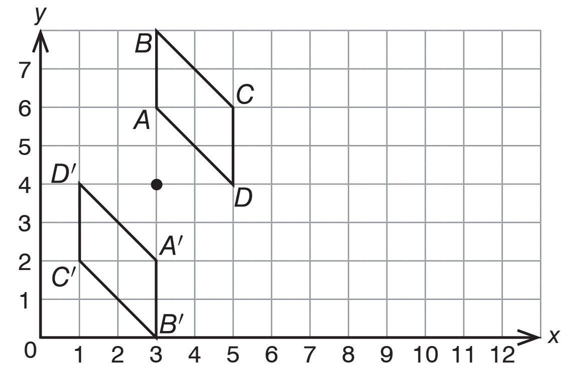
Have students look at each rotation separately. ASK: In the first rotation, is there any vertex that had even one coordinate, *x* or *y*, left unchanged? (no) Repeat with the second and third rotations. In the third rotation, students will notice that the vertex (8, 5) stayed the same. ASK: How is this point special? (it is the centre of rotation) What do we know about fixed points of rotations? (the centre of rotation is the only fixed point of a rotation)

Investigate the change in the coordinates under a 180° rotation. Repeat the previous investigation with quadrilateral *ABCD* with vertices *A* (3, 6), *B* (3, 8), *C* (5, 6), *D* (5, 4), rotating it 180° clockwise around the point (3, 4). Have students use ′ to label the images of the vertices. The resulting vertices are shown below.

Original	*A* (3, 6)	*A* (3, 8)	*A* (5, 6)	*A* (5, 4)
Image	*A*′ (3, 2)	*B*′ (3, 0)	*C*′ (1, 2)	*D*′ (1, 4)



Students will notice that some vertices have the same first or second coordinate before and after the rotation. Discuss why this happens. To prompt students to see the answer, draw their attention to the construction they use to rotate points 180°: the original point, the image, and the centre of rotation are always on the same line, so if the original and the centre of rotation are on the same horizontal line, so is the image, and the *y*-coordinate of all three points is the same. Similarly, the *x*-coordinate of the points on the same vertical line as the centre of rotation does not change when rotating it 180°. Keep the picture on the board for later use.

Rotating polygons with rotational symmetry. Have students rotate the quadrilateral *ABCD* on the board 180° around the point (4, 6). Have them use * to label the image vertices. ASK: What do you notice? (the image coincides with the original; $A^* = C$, $B^* = D$, $C^* = A$, $D^* = B$. Discuss why this happened. (*ABCD* is a parallelogram, which is made of two congruent triangles that are a 180° rotation of each other) If students are familiar with rotational symmetry (see Unit 6, Lesson G6-11), they should recognize that a parallelogram has rotational symmetry of order 2, and students are rotating it around the centre, so it should coincide with itself.

Exercises

a) Draw a coordinate grid.

b) Plot points E (1, 2) and F (1, 6).

c) Find two points G and H, so that $EFGH$ is a square.

d) Find the centre of the square O. What are the coordinates of O?

e) What is the smallest rotation around O that would bring $EFGH$ to a square that occupies the same place?

Bonus: Explain how you know.

Selected answers: c) G (5, 6), H (5, 2); d) O (3, 4); e) 90° clockwise or counter-clockwise; Bonus: A square has rotational symmetry of order 4, so a rotation of 90° should bring it to the same place. OR: If you turn a square 90° around the centre, it does not change.

NOTE: Parts c) and d) have another possible answer, but it involves points in the second quadrant.

Using different transformations to produce the same image. Return to the quadrilaterals $ABCD$ and $A'B'C'D'$ on the board. Have students translate $ABCD$ 2 units left and 4 units down, using $''$ to label the images. Discuss what students notice. ($A'B'C'D'$ and $A''B''C''D''$ occupy the same space, $A'' = C'$, $B'' = D'$, $C'' = A'$, $D'' = B'$) Point out that since a parallelogram does not change when rotated 180° around the centre, rotation and translation produce results that occupy the same space, although they have different image vertices.

Exercises:

a) Draw a coordinate grid.

b) Draw a rectangle on the grid.

c) Reflect or translate the rectangle using a transformation of your choice. Describe the transformation you used.

d) Describe two transformations of a different type that would take the rectangle to the image you produced.

Bonus: Use two different transformations in a row on the rectangle you drew. Then find two different transformations that take the original rectangle to the image rectangle.

NOTE: Encourage students who are familiar with rotational symmetry to use order of rotational symmetry in their explanation in **Question 5.d)** on AP Book 6.2 p. 69.

Extensions

1. Find a sequence of transformations that takes *MNSP* to *M*N*S*P**, keeping the vertices in the same order (*M* to *M**, and so on). Describe the transformations and the change of vertices.

a)

Original	*M* (0, 5)	*N* (1, 7)	*S* (4, 7)	*P* (3, 5)
Image	*M** (10, 3)	*N** (9, 5)	*S** (6, 5)	*P** (7, 3)

b)

Original	*M* (2, 3)	*N* (1, 5)	*S* (2, 7)	*P* (3, 5)
Image	*M** (4, 3)	*N** (6, 4)	*S** (8, 3)	*P** (6, 2)

Sample answers

a)

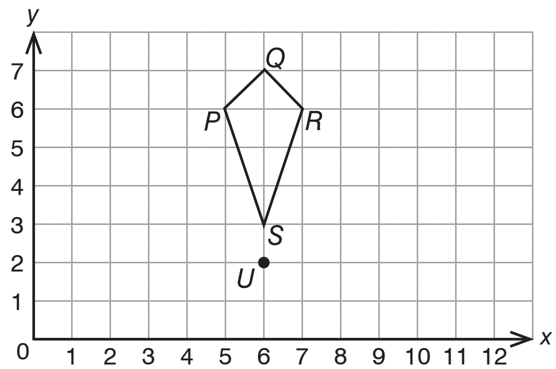
Original	*M* (0, 5)	*N* (1, 7)	*S* (4, 7)	*P* (3, 5)
Reflection in a vertical mirror line though (5, 0)	(10, 5)	(9, 7)	(6, 7)	(7, 5)
Translation 2 units down	*M** (10, 3)	*N** (9, 5)	*S** (6, 5)	*P** (7, 3)

b)

Original	*M* (2, 3)	*N* (1, 5)	*S* (2, 7)	*P* (3, 5)
90° CCW rotation around (2, 3)	(2, 3)	(4, 4)	(6, 3)	(4, 2)
Translation 2 units right	*M** (4, 3)	*N** (6, 4)	*S** (8, 3)	*P** (6, 2)

2. a) Rotate the polygon *PQRS* 90° clockwise around point *U*. What are the coordinates of the vertices of the image?

   b) Describe another transformation or sequence of transformations that would take *PQRS* to occupy the same place, but with *P** being (10, 1).



Answer: a) *P′* (10, 3), *Q′* (11, 2), *R′* (10, 1), *S′* (7, 2)

Sample answer: b) rotate *PQRS* 90° clockwise around point *U*, then reflect the image in the horizontal line through the point *U*

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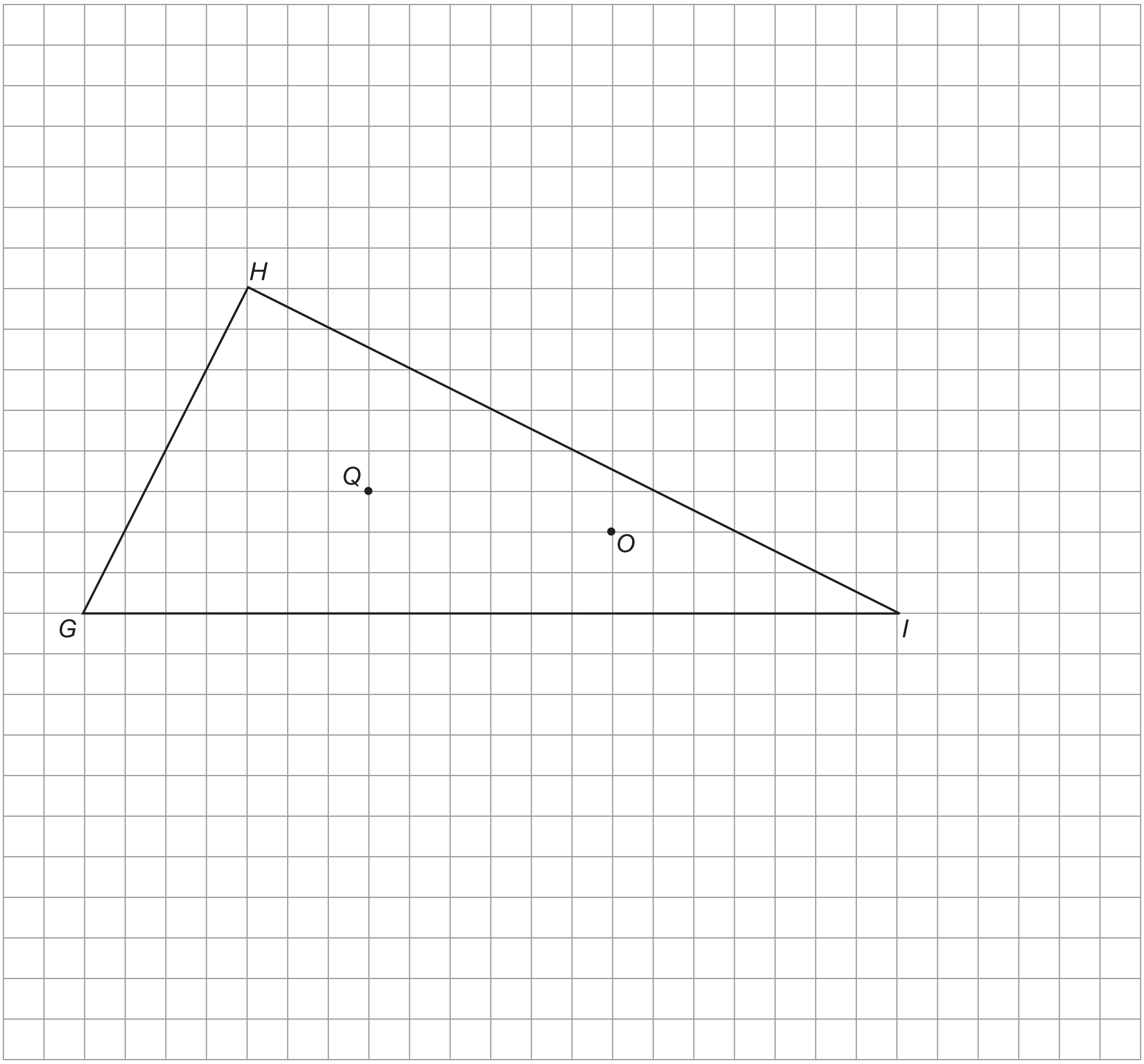
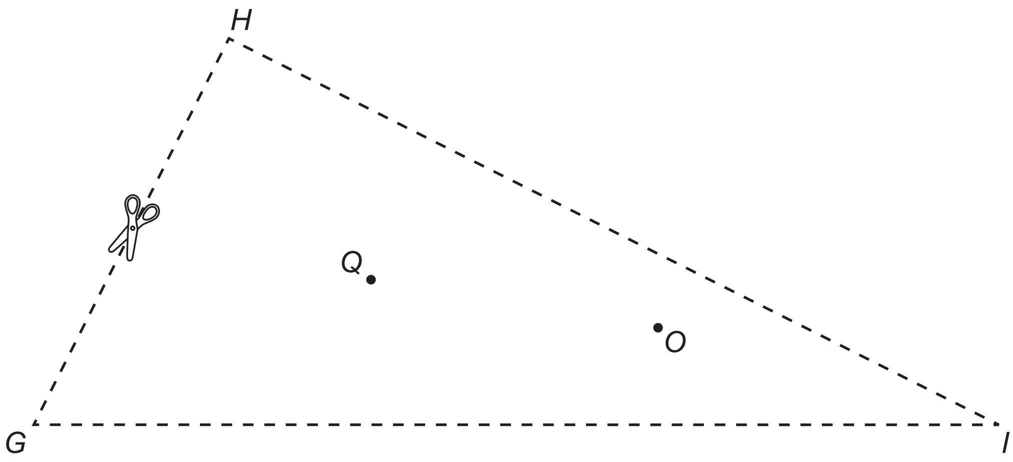
Rotating a Triangle





NAME ______________________________ DATE ______________

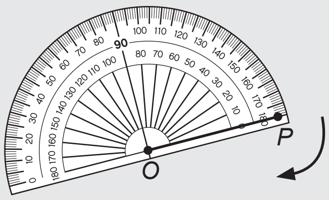
Rotations Without a Grid

To **rotate** point P around point O 60° clockwise:

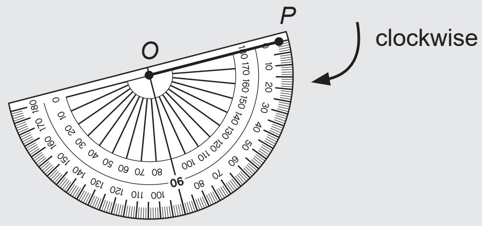
Step 1: Draw line segment OP. Measure its length.

Step 2: Draw an arc clockwise to show the direction of rotation.

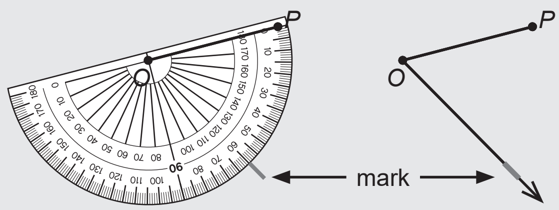
Step 3: Place the protractor so that the origin is at point O and the base line aligns with OP.



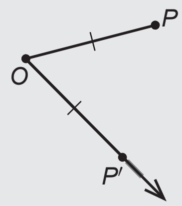
Step 4: Does the scale that counts clockwise have a 0 on the line segment? If not, turn the protractor upside-down.



Step 5: Make a mark at 60° on the scale that counts clockwise. Remove the protractor and draw a ray through the mark, starting at O.



Step 6: On the new ray, measure and mark the image point P' so that $OP' = OP$.

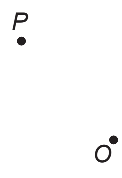


1. Rotate point P around point O by the given angle and direction.

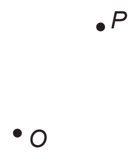
a) 60° CW

b) 20° CCW

c) 150° CCW



d) 180° CW



2. For points O and P in Question 1, what rotation in the opposite direction around point O will take point P to the same image?

a) ______________ b) ______________ c) ______________ d) ______________

BONUS ▶ Use a ruler to draw a triangle ABC. Find the midpoint of side AC and label it M. Rotate ABC 180° clockwise around point M. What type of quadrilateral do ABC and its image make together? Explain.

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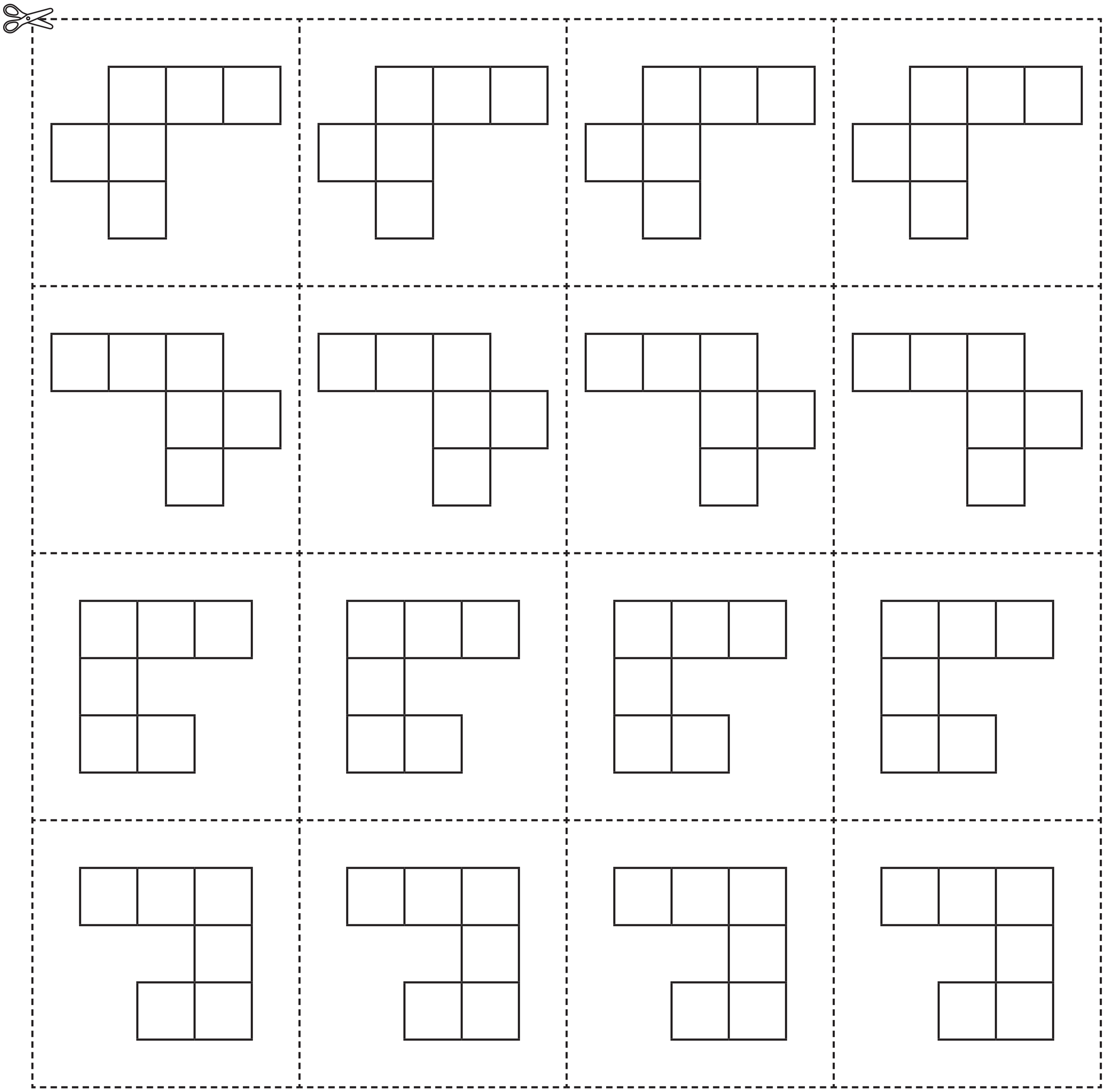
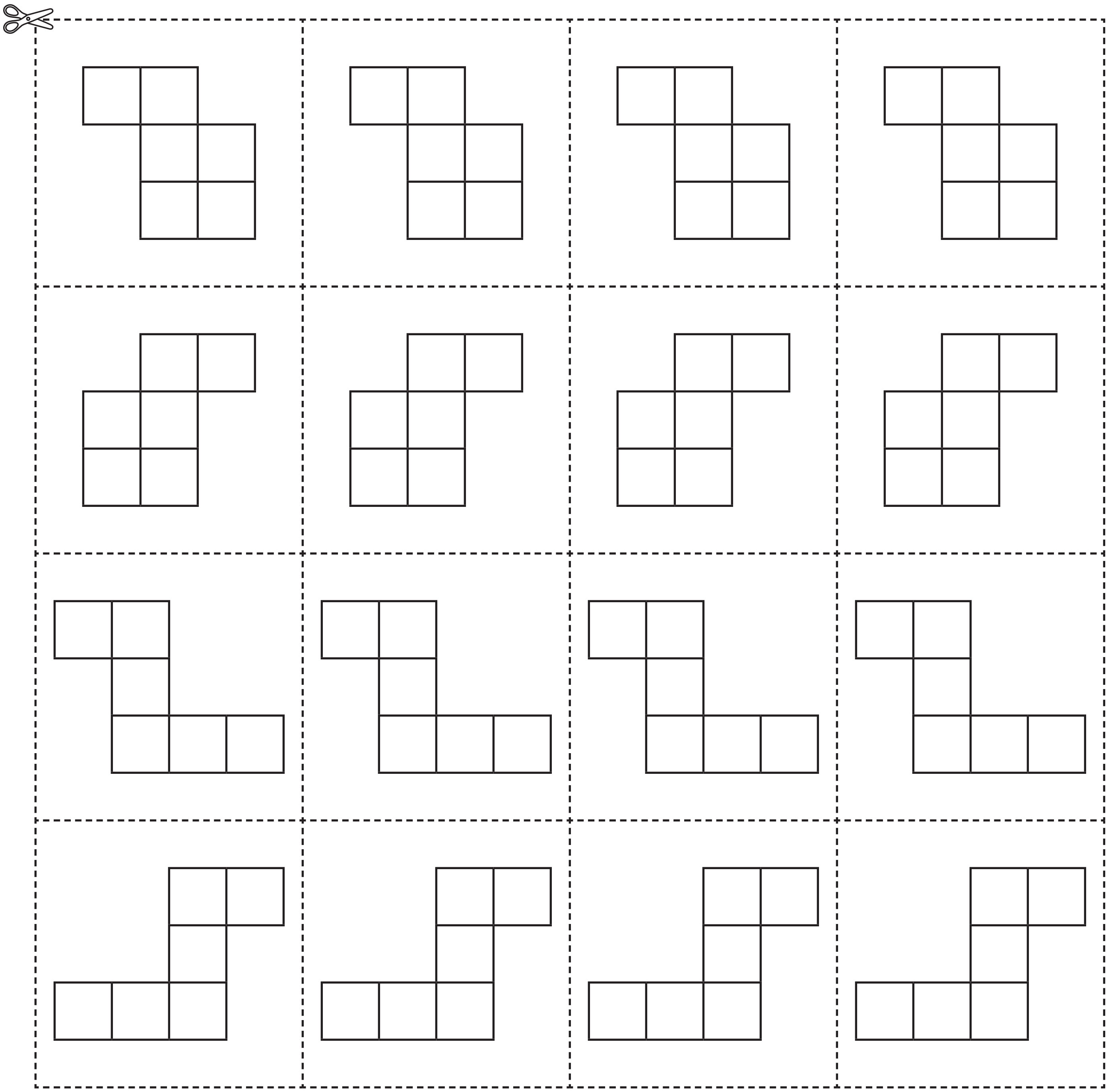
NAME ______________________________ DATE ______________

Find a Flip (1)



NAME ____________________ DATE ____________

Find a Flip (2)



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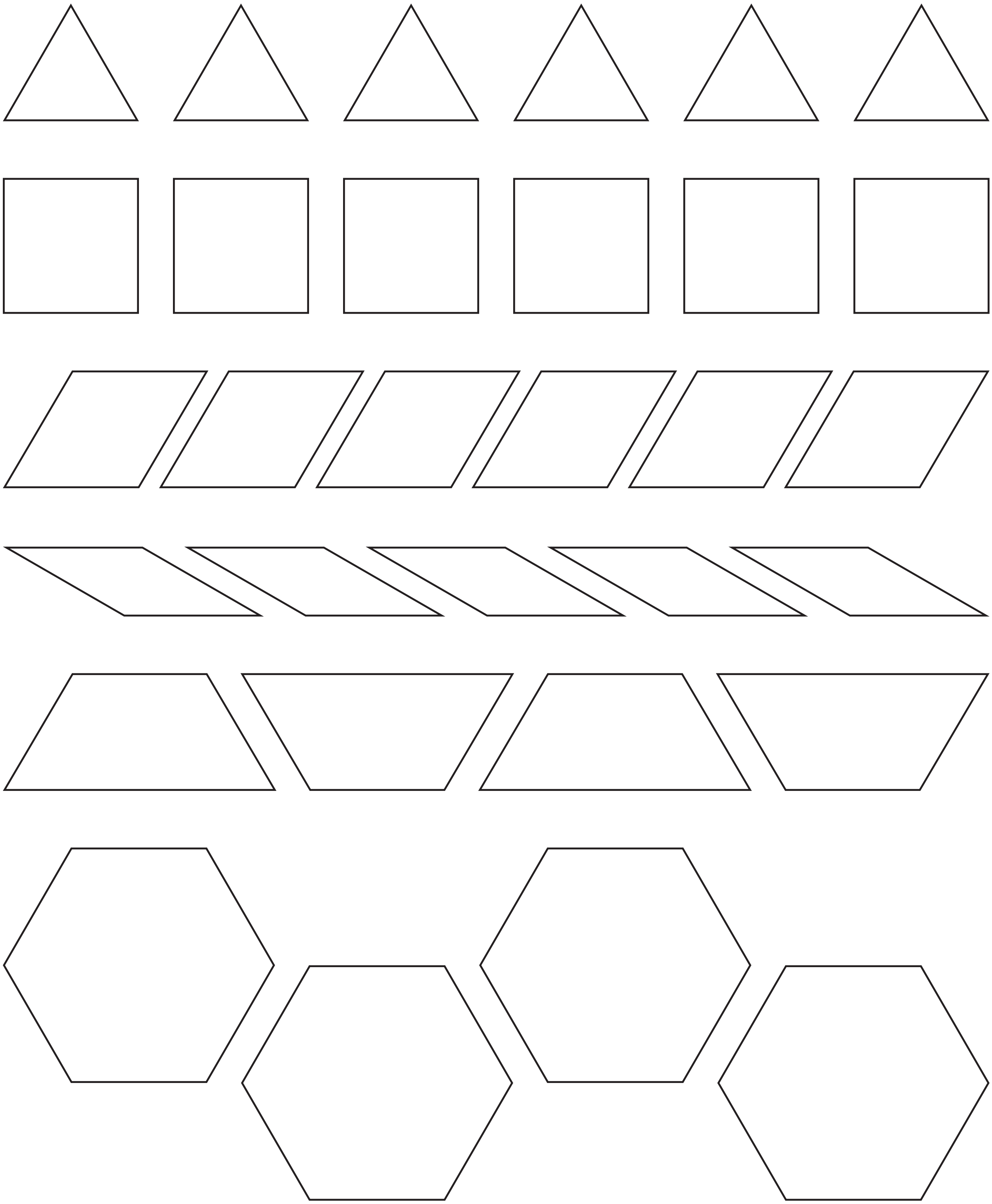
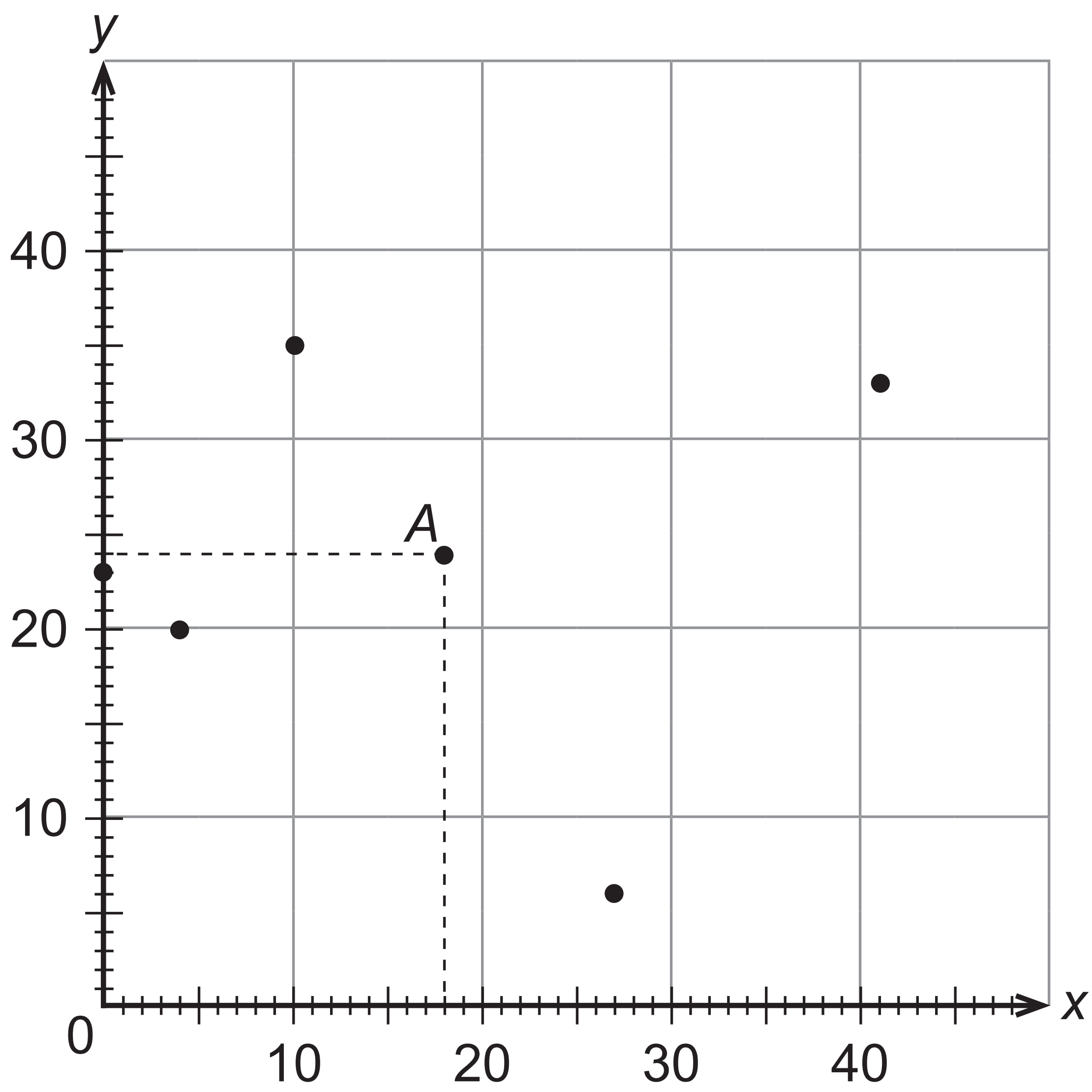
Pattern Blocks



NAME ______________________ DATE ______________

Grid with Tens



B (10, 35) *C* (4, 20) *D* (27, 6) *E* (41, 33) *F* (0, 23)

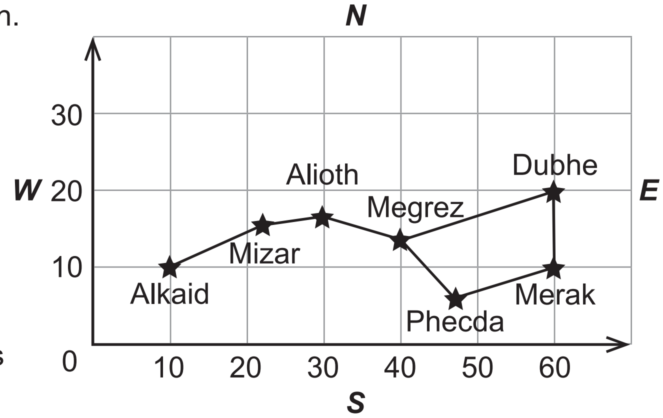
NAME ______________________________ DATE ______________

Maps (1)

1. This is the star map of the Big Dipper, part of the Great Bear constellation.

a) The star at point (60, 20) is the official star of Utah.

What is it called? ______________________



b) What are the coordinates of Merak?

(______, ______)

c) Which star is 50 units west of Merak?

____

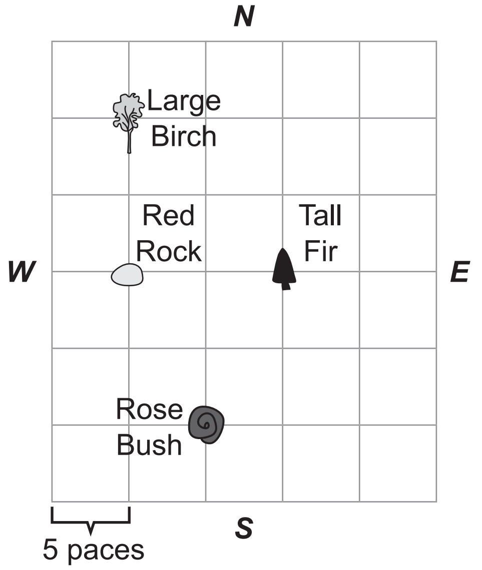
d) Galaxy M81 is located 10 units north and 10 units east from Dubhe. Mark it on the map.

BONUS ▶ What star is at point (30, 16)? ______________________

BONUS ▶ The Pinwheel Galaxy is located 20 units west of Alioth.

Mark it on the map and write its coordinates. (______, ______)

2. This map shows part of Feral Cat Island, where pirates have buried gold, silver, and weapons. Fill in the directions.



a) From the Tall Fir, walk *10* paces (steps) *west* to the Red Rock.

b) From the Red Rock, walk ______ paces north to the Large Birch.

c) From the Red Rock, walk ______ paces ____________

and ______ paces east to the Rose Bush.

d) From the Rose Bush, walk ______ paces ____________

and ______ paces ____________ to the Tall Fir.

e) From the Tall Fir, walk ______________________ and

______________________ to the Large Birch.

3. Mark on the map in Question 2 the point where some treasure is buried.

a) Gold (G): From the Tall Fir, walk 5 paces east and 10 paces north.

Weapons (W): From the Rose Bush, walk 10 paces west and 5 paces south.

Silver (S): From the Large Birch, walk 10 paces south and 5 paces east.

b) What two landmarks is the silver buried between? ______________________________

c) Write directions for walking from Gold to Silver.

____

NAME ______________________________ DATE ______________

Maps (2)

4. This map shows all of Feral Cat Island. Each square on the map has sides 2 km long.

a) Round Lake is at point (4, 8). What is located at each point?

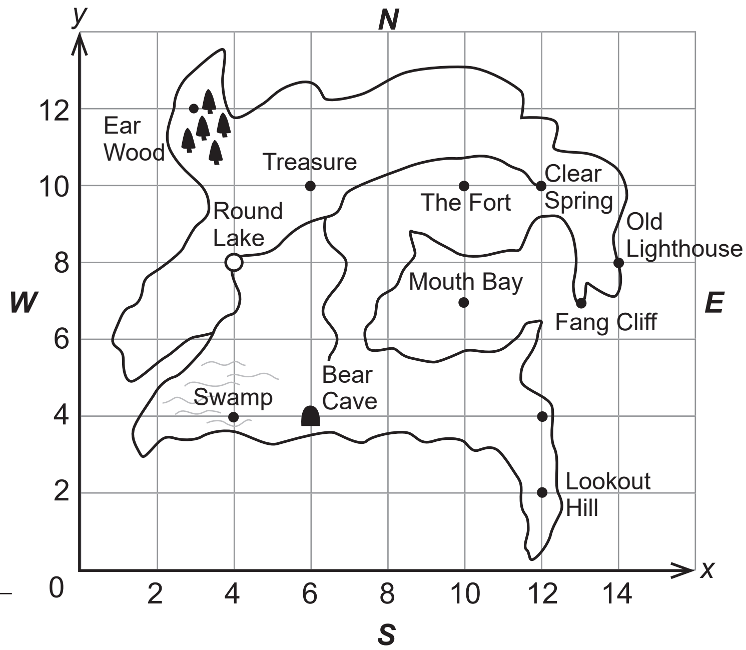
(6, 4) ______________________

(6, 10) ______________________

(10, 10) ______________________

(13, 7) ______________________

Feral Cat Island



b) Give the coordinates for the landmark.

Old Lighthouse ____________

Lookout Hill ____________

Clear Spring ____________

c) Name the landmark located at the point described.

2 km east of the Fort __________________

4 km south of Round Lake _____________

BONUS ▶ 2 km north and 3 km west of the Treasure ______________________

d) Fill in the blanks.

From Round Lake, the Old Lighthouse is ___*10*___ km _____*east*_____.

From the Fort, walk ______ km ____________ to the Treasure.

From the Treasure, the Bear Cave is ______ km ____________.

To walk from the Bear Cave to Lookout Hill, walk ______ km ____________ and ______ km south.

From the Old Lighthouse, walk ______ km ____________ and ______ km ____________ to the Clear Spring.

From the Fort, walk __________________________________ to the Bear Cave.

From Lookout Hill to the Treasure, walk __________________________________.

e) Write your own question that asks for directions and uses the map. Ask your partner to answer it.

____

____

NAME ______________________ DATE ______________

1 cm Grid Paper
