Unit 7 Patterns and Relations: Equations

Introduction

Topics in this unit include:

- distinguishing equations from expressions;
- variable terms and constant terms;
- preservation of equality in models;
- algebraic manipulations of equations;
- testing and verifying solutions to equations;
- solving equations involving integers; and
- solving real-world problems involving equations with one variable.

Meeting Your Curriculum

ALBERTA		
Required	PR7-10, PR7-12 to 18	
Optional	PR7-11	
BRITISH COLUMBIA		
Required	PR7-10 to 16, PR7-18	including Extensions 1 and 2 from PR7-18
Optional	PR7-17	
MANITOBA		
Required	PR7-10, PR7-12 to 18	
Optional	PR7-11	
SASKATCHEWAN		
Required	PR7-10, PR7-12 to 18	
Optional	PR7-11	

Mental Math Minutes

The mental math minutes in this unit:

- practise evaluating expressions using the order of operations
- practise working with integers
- practise working with opposite operations in expressions and equations

Generic BLMs

The Generic BLMs used in this unit are:

BLM 1 cm Grid Paper (p. Q-1)
BLM Strategy Talks (p. Q-2)

These BLMs can be found in Section Q.

Materials

In this unit you will need:

- a double-pan balance or two tables side by side or one table with a divider (such as a ruler or binder)
- paper bags or other opaque, light-weight bags
- two-centimetre connecting cubes or other small, identical objects heavier than a paper bag

Assessment

The lessons covered by a quiz or test are as follows:

	AB	BC	MB	SK
Quiz	PR7-10, 12 to 14	PR7-10 to 14	PR7-10, 12 to 14	PR7-10, 12 to 14
Quiz	PR7-15 to 18	PR7-15, 16, 18	PR7-15 to 18	PR7-15 to 18
Test	PR7-10, 12 to 18	PR7-10 to 16, 18	PR7-10, 12 to 18	PR7-10, 12 to 18

Additional Information for This Unit

The focus of this unit is on linear equations with one variable that have exactly one solution. While equations can have multiple variables and even single-variable equations can have 0, 1, 2, or more solutions, these possibilities need not be emphasized at this grade level.

PR7-10
Analyzing Expressions and Equations
AP Book pp. 1–2

Goals

Students will analyze equations and expressions, identifying variable and constant terms in each. Students will model real-world problems with expressions and equations.

Main Ideas

The value of a variable term depends on the value of the variable while the value of a constant term does not. When representing real-world problems with expressions, unknown or changing quantities are represented with variables. The rate at which a variable changes is represented by the coefficient of the variable. Constant amounts that do not change with the variable are represented by constant terms.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation
Can substitute a value for a variable in an expression or equation
Can distinguish between an expression and equation
Can evaluate a numerical expression using the order of operations
Is familiar with integer addition

Materials

none

Vocabulary

coefficient
constant term
equation
expression
substitute
variable
variable term

Mental Math Minute

Slides 2–3

Remind students that to substitute a value for a variable in an expression means to replace the variable with the value. Explain that sometimes when the numbers are manageable and there aren't many operations, the expressions can be calculated mentally. Display the example in the margin.

$4z + 5, z = 2$
$4(2) + 5 = 8 + 5 = 13$

Exercises

Substitute the value for the variable and evaluate the expression mentally.

a) $5x + 7, x = 2$ b) $5x + 7, x = 3$ c) $5x + 7, x = 10$

d) $12n - 5, n = 3$ e) $12b - 5, b = 5$ f) $12w - 5, w = 8$

Answers: a) 17, b) 22, c) 57, d) 31, e) 55, f) 91

1. Review expressions and equations

Slides 4–11

Key points: An expression represents a numerical value whereas an equation is a statement of equality that might be true or false. An equation is formed by joining two expressions with an equal sign.

Display the word "expression" to the left of the word "equation." Remind students how these two terms are related.

> What does an expression represent? (a numerical value) Are there expressions in an equation? (yes, an equation has two expressions) Does an expression or an equation have an equal sign? (equation)

Display the expressions and equations shown in the margin. Point to each in turn and have students signal left for expression, right for equation, or down for neither.

$54z - 35(9 + x)$
$289 = y$
$476 + 13x = 54z - 35(9 + x)$
$476 + 13x - 54z - 35(9 + x)$
$476 + 13x > 54z - 35(9 + x)$

Explain that a statement comparing two expressions with the symbols $<$ (less than), $\leq$ (less than or equal to), $>$ (greater than), or $\geq$ (greater than or equal to) are all called inequalities. Inequalities are neither expressions nor equations.

Ask students to recall the result of substituting numbers for variables into an expression and simplifying, versus substituting numbers for variables into an equation and simplifying. (a numerical value in the first case vs. a statement of equality that might be true or false in the second) As a class, work through the examples of substitution shown in the margin.

Expression	Equation
$3z + 8$	$3z + 8 = 14$
When $x = 1$,	
$3(1) + 8$	
$= 3 + 8$	
$= 11$	$11 = 14$ is false
When $x = 2$,	
$3(2) + 8$	
$= 6 + 8$	
$= 14$	$14 = 14$ is true

Lead a discussion on the similarities and differences between expressions and equations. Ensure the following points arise:

- both expressions and equations can involve numbers, operations, variables, and brackets
- an equation must contain an equal sign whereas an expression cannot contain an equal sign
- an expression represents a numerical value whereas an equation is a statement of equality that can be either true or false
- you can substitute values for variables in both expressions and equations
- when you substitute values for the variables in an expression and simplify, the result is a number, whereas when you substitute values for the variables in an equation and simplify, the result is a statement of equality that might be true or false

Exercises

1. Write an equation the contains the expression $6h - 7$.

   Sample answer: $6h - 7 = 108$

2. Substitute the given values for the variables and evaluate the expression.

   a) $3x - 9, x = 7$ b) $-10 + 4m, m = 20$ c) $15p - q, p = 3, q = -11$

   Answers: a) 12, b) 70, c) 56

3. Substitute the given value for the variable and simplify. Is the equation true or false?

   a) $4a - 1 = 24, a = 6$ b) $25b + 30 = 120, b = 4$ c) $210 = 90 + 6p, p = 20$

   Answers: a) $23 = 24$ is false, b) $130 = 120$ is false, c) $210 = 210$ is true

4. List two similarities and two differences between expressions and equations.

   Sample answer:

   similarities:

   - both expressions and equations can involve numbers, operations, variables, and brackets
   - you can substitute values for variables in both expressions and equations differences:

   differences:

   - an equation must contain an equal sign whereas an expression cannot contain an equal sign
   - an expression represents a numerical value whereas an equation is a statement of equality that can be true or false

Bonus: Give an example of an expression and an equation. Explain how they are similar and different.

Sample answer: $8s + 17$; $8s + 17 = 3x - 1$. Both the expression and the equation involve numbers, variables, and operations, but only the equation includes an equal sign. The equation joins two expressions with an equal sign.

2. Recognizing constant and variable terms

Key point: In an expression of the form $ax + b$ where a and b are fixed numbers and x is a variable, the value of the variable term "ax" depends on the value of the variable, x, while the value of the constant term "b" does not.

Display the expression shown in the margin. Explain that this expression has two parts, separated by addition. The part with the variable, $3x$, is called the *variable term*, while the part without the variable, 7, is called the *constant term*. Explain that variable means changing, while constant means unchanging. Discuss the reason for these names. Focus on the result of substituting different values for the variable.

$3x + 7$

> What is the value of the variable term if you substitute different values for x, such as 0, 1, and 2? (the value of the variable term changes: the values are 0, 3, and 6) What happens to the constant term if you substitute these different values for x? (the constant term does not change; it is always 7)

Emphasize that the value of the variable term changes, or varies, depending on the value of the variable, while the constant term is always the same, or constant, hence the names.

Display the phrase "variable term" to the left of the words "constant term" and display the expressions and equations in the margin. For each example, point to a variable term or a constant term and have students signal left for variable term or right for constant term.

$21a + (-9)$
$21a - 9$
$-18 + 27m$
$y = -9x - 14$
$4y = -x + 0$
$3f = m$

Ask students to recall what a coefficient is. (the number that is multiplied by a variable) Point out the contrast between the coefficient and the constant term: the coefficient is a number that is multiplied by the variable, whereas the constant term is a number that is added or subtracted to the variable term. Explain that when writing variable terms, coefficients, and constant terms, the negative sign should be included if the term or coefficient is negative. Furthermore, point out that when no number is written for the coefficient of a variable, the coefficient is understood to be one, whereas when there is no constant term written, you can say the value of the constant term is zero.

Refer again to each expression or equation, in turn, and have students identify the variable terms, the variables, the coefficient of each variable, and the constant term.

Exercises

1. Identify the …

   i) variable term. ii) variable. iii) coefficient. iv) constant term.

   a) $25x + 4$ b) $36 - 42h$ c) $-5 + x$ d) $-1x$

   Answers: a) i) $25x$, ii) x, iii) 25, iv) 4; b) i) $-42h$, ii) h, iii) -42, iv) 36; c) i) $1x$ or x, ii) x, iii) 1, iv) -5; d) i) $-1x$, or $-x$ ii) x, iii) -1, iv) 0

2. Write an expression that includes a variable term and a constant term. State the variable term, the variable, the coefficient of the variable, and the constant term.

   Sample answer: expression: $-14p - 23$, variable term: $-14p$, variable: p, coefficient: -14, constant term: -23

3. In the equation, identify the …

   i) coefficient of x. ii) constant term.

   a) $y = 47x - 15$ b) $129 - 53x + 41b = 5c$ c) $4y = -x$

   Answers: a) i) 47, ii) -15; b) i) -53, ii) 129; c) i) -1, ii) 0

Bonus: Identify all the variable terms and constant terms in the given equation.
$-18 + 74x - 6 = -23y + 62 - 58x$

   Answer: variable terms: $74x$, $-23y$, $-58x$; constant terms: -18, -6, 62

3. Working with constant and variable terms in word problems Slides 21–25

Key points: When representing real-world problems with expressions, unknown or changing quantities are represented by variables. The rate at which a variable changes is represented by the coefficient of the variable. Constant amounts that don't change with the variable are represented by constant terms.

Display the sentence in the margin and ask students how many times Sarah's heart will beat during 1 minute of running, 2 minutes of running, and 5 minutes of running. (120, 240, 600) Have students write an expression for the number of beats if Sara runs for m minutes. ($120m$) Discuss what the variable and coefficient represent.

While running, Sara's heart beats at a rate of 120 beats per minute.

> What is the coefficient? (120) What does the coefficient tell us? (how many beats there are each minute while running) What does the variable represent? (the number of minutes Sara runs)

Repeat with the problem in the margin. Ask students to identify the charge for one and two hours, and then to write an expression for the cost of renting a canoe for h hours. (\$25, \$35, $10h + 15$ or $15 + 10h$) Discuss what the variable, coefficient, and constant terms represent.

Abella charges a flat fee of \$15 to rent her canoe plus \$10 per hour.

> What is the coefficient and what does it represent? (10, the hourly rate) What does the variable represent? (the number of hours a person rents the canoe) What is the constant term and what does it represent? (15, the flat fee)

Emphasize that when representing real-world problems with expressions, unknown or changing quantities are represented with variables, a coefficient is multiplied by the variable to represent a quantity that changes when the variable changes, and a constant term is used for a quantity that doesn't change (such as a flat fee).

Exercises

It costs \$25 per hour plus a flat fee of \$55 to rent a boat.

a) Write an expression for the cost of renting the boat for *h* hours.

b) What is the coefficient in your expression? What does it represent?

c) What does the variable represent?

d) What is the constant term in your expression? What does it represent?

e) What is the cost to rent a boat for 4 hours?

Answers: a) $25h + 55$; b) 25, the hourly rate for renting the boat;
c) how many hours the boat is rented; d) 55, the flat fee
Solution: e) $25(4) + 55 = 100 + 55 = \$155$

Extensions

Slides 26–29

1. Explain what a variable is and how it is used in an expression or equation.

   Sample answer: A variable is a symbol, such as a letter, that stands for a number that might be unknown or might change. In an expression or equation, a variable holds a place for a number that can be substituted for the variable. The same rules for operations that apply to numbers in an expression or equation also apply to variables.

2. Explain the difference between variable terms and constant terms in an expression or equation.

   Sample answer: In an expression or equation, a variable term is formed when a variable is multiplied by a number (a coefficient), while a constant term is a number that is not multiplied by a variable. When substituting values for a variable, the value of a variable term varies with the value of the variable, while the constant term remain the same, or constant.

3. On a class trip, students choose how many sandwiches, drinks, fruit bowls, and veggie-stick packets they will order. The teacher buys all the food for the class. Sandwiches cost \$6.50 each, drinks cost \$2.10 each, fruit bowls cost \$3.15 each, and veggie-stick packets cost \$2.25 each. Let *s* be the number of sandwiches, *d* be the number of drinks, *f* be the number of fruit bowls, and *v* be the number of veggie-stick packets the class orders.

   a) Write an expression for the total number of items ordered.

   b) Write an expression for the total cost of all the items ordered.

   c) If the class orders 10 sandwiches, 8 drinks, 5 fruit bowls, and 4 veggie-stick packets, how much does the total order cost?

   Answers
   a) $s + d + f + v$
   b) $\$6.50s + \$2.10d + \$3.15f + \$2.25v$
   c) $\$6.50(10) + \$2.10(8) + \$3.15(5) + \$2.25(4) = \$65 + \$16.80 + \$15.75$
   $+ \$9 = \$106.55$

PR7-11
Linear Relations in Four Quadrants

AP Book pp. 3–5

Goals

Students will graph linear relations in four quadrants. Students will write the formula for a linear relation using its graph or a table of values.

Main Ideas

In a linear relation, when the value of *x* increases by 1, the value of *y* increases by the coefficient of *x* in the formula. The coefficient of *x* and the *y*-intercept can be read from the formula to sketch the graph. Conversely, they can be derived from the graph or a table of values to write the formula.

Summary

Curriculum

AB: optional
BC: required
MB: optional
SK: optional

Prior Knowledge

Can read and plot points in all four quadrants of the cartesian plane

Can distinguish between variable and constant terms in expressions and equations

Can add, subtract, multiply, and divide integers

Can evaluate a numerical expression using the order of operations

Knows that the gap in a linear sequence corresponds to the coefficient of the input variable in the formula for the linear relation

Materials

grid paper or **BLM 1 cm Grid Paper** (p. Q-1)

rulers

BLM Coordinate Grids (p. J-68, optional)

Vocabulary

coefficient
constant term
gap
linear relation
relation
variable term
***y*-intercept**

Additional Information

For this lesson you will need a pre-drawn coordinate grid on the board. If you do not have such a grid, you can photocopy **BLM Large Coordinate Grid** (p. J-67) onto a transparency and project it onto the board. This will allow you to draw and erase lines on the coordinate grid without erasing the grid itself.

Encourage students to use a ruler or other straightedge to draw coordinate grids and graph linear relations. To save time, you can occasionally provide students with **BLM Coordinate Grids.**

Mental Math Minute — Slides 2–3

Review the sign rules for multiplying integers using the examples in the margin.

$(+) \times (+) = +$ $(+3)(+7) = +21$
$(+) \times (-) = -$ $(+3)(-7) = -21$
$(-) \times (+) = -$ $(-3)(+7) = -21$
$(-) \times (-) = +$ $(-3)(-7) = +21$

Exercises

Multiply mentally.

a) $(+4)(+5)$ b) $(-6)(+8)$ c) $(+9)(-7)$ d) $(-8)(-4)$

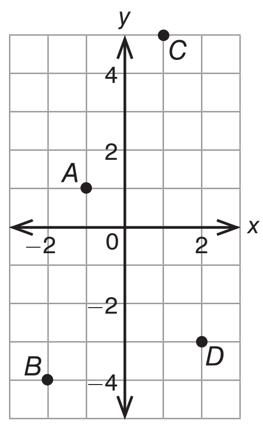
e) $12(-9)$ f) $-11(8)$ g) $-12(-10)$ h) $-12(-12)$

Answers: a) $+20$, b) -48, c) -63, d) $+32$, e) -108, f) -88, g) $+120$, h) $+144$

1. Review reading and plotting points in four quadrants — Slides 4–5

Key points: The *x*-coordinate of a point is negative or positive depending on whether the point is to the left or right of the *y*-axis, respectively. The *y*-coordinate of a point is either negative or positive depending on whether the point is below or above the *x*-axis, respectively.

Display the grid in the margin. Review how to write the coordinates of the points. Emphasize that the *x*-coordinate of a point is negative only if the point lies to the left of the *y*-axis, while the *y*-coordinate of a point is negative only if the point lies below the *x*-axis. Review the quadrant numbering system.



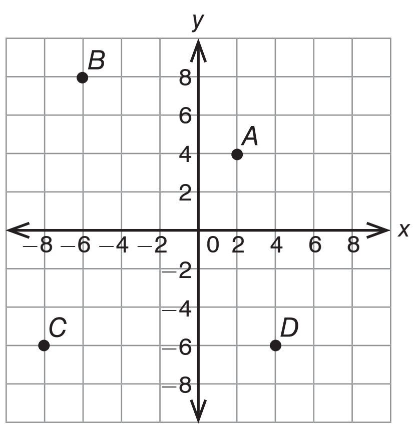
What are the coordinates of the points shown?
(*A* (−1, 1), *B* (−2, −4), *C* (1, 5), *D* (2, −3))
Which labeled point is in each quadrant?
(I. *C*, II. *A*, III. *B*, IV. *D*)

Display the ordered pairs shown. For each pair, point at different locations on the grid while students signal whether you have pointed to the correct location of the ordered pair.

E (3, −2), *F* (−2, 4), *G* (1, 3), *H* (−1, −3)

Exercises

1. Write the coordinates of the points *A*, *B*, *C*, and *D*.



Answers: *A* (2, 4), *B* (−6, 8), *C* (−8, −6), *D* (4, −6)

2. Draw a coordinate grid. Plot and label the points on the grid.

E (−6, −8) *F* (8, 5) *G* (8, −8) *H* (−8, 6)

2. Graphing linear relations in four quadrants

Key points: Linear relations can include negative numbers, so their graphs can be extended through Quadrants I, II, III, or IV. In a linear relation, the value of y when $x = 0$ is called the y-intercept and represents the value on the y-axis where the graph of the relation crosses the y-axis.

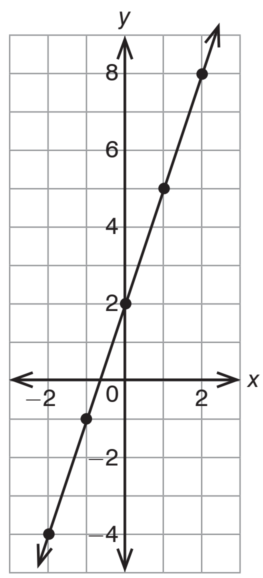
Display the formula and the table of values shown in the margin. Explain that to graph this linear relation, you will start with a table of values that includes negative and positive values for x. Ensure that students know that all four quadrants could be used to graph linear relations. Complete the table as a class.

$y = 3x + 2$

x	y
−2	
−1	
0	
1	
2	

As a class, list the points from the table of values and plot them on a Cartesian plane. Draw a straight line connecting the points.

(−2, −4), (−1, −1), (0, 2), (1, 5), (2, 8)



Explain that the *y-intercept* of a line is the number on the *y-axis* where the line crosses the *y-axis* and discuss how to find it in the table for the formula.

> What is the shape of the graph of this linear relation? (a straight line) What is the number on the y-axis where this line crosses the y-axis? (2) This number is called the y-intercept of the line. What would be the x-coordinate of the point where the line crosses the y-axis? (0) How could you find the y-intercept in the table? (look at the y-value of the point where x is 0)

NOTE: Some sources refer to the y-intercept as the point where the line crosses the y-axis, while other sources refer to the y-intercept as simply the y-coordinate of that point, since the x-coordinate is always zero. We follow the latter approach.

Repeat with the formula $y = -2x + 1$.

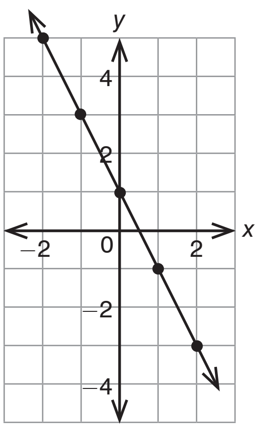
> What is the y-intercept of this line? (1) What is the x-coordinate of the point where the line crosses the y-axis? (0) How could you find the y-intercept in the table? (look at the y-value of the point where x is 0)

$y = -2x + 1$

x	y
−2	5
−1	3
0	1
1	−1
2	−3

(−2, 5), (−1, 3), (0, 1), (1, −1), (2, −3)

Discuss why the y-intercept of the graph of any linear relation will always be the y-coordinate of the point on the line with $x = 0$.

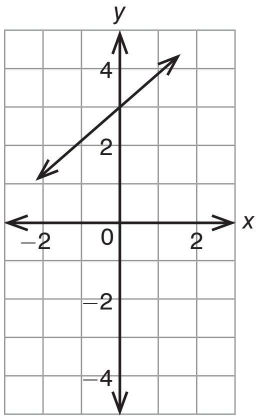


Write any three points on the y-axis. (sample answer: (0, 3), (0, 0), (0, −4)) Why is the x-coordinate always 0? (the horizontal distance from the origin of all points on the y-axis is 0) For the point where a line crosses the y-axis, what will the x-coordinate be? (0) What can you say about the y-coordinate? (it will be the y-intercept of the line)

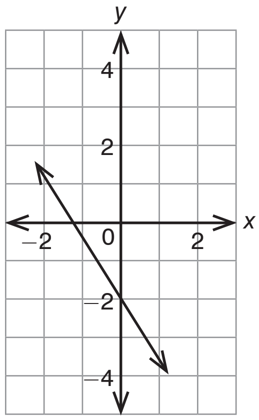
Exercises

1. a) Write the y-intercept of the line.

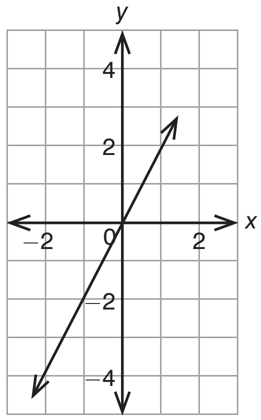
i)



ii)



iii)



b) For each relation in part a), write the coordinates of the point where the line crosses the y-axis.

c) Look at your answers to part b). What can you say about the x-coordinate of each point? What can you say about the y-coordinate?

Bonus: The y-intercept of a linear relation is −356. Write the coordinates of the point where the graph crosses the y-axis.

Answers: a) i) 3, ii) −2, iii) 0; b) i) (0, 3), ii) (0, −2), iii) (0, 0);
c) the x-coordinate is always 0, the y-coordinate is the y-intercept;
Bonus: (0, −356)

2. a) Complete the table of values for the linear relation.

i) $y = 2x - 3$

x	y
−2	
−1	
0	
1	
2	

ii) $y = -3x$

x	y
−2	
−1	
0	
1	
2	

b) Graph the relations from part a). Find the y-intercept on the graphs.

c) How could you find the y-intercepts from the tables in part a)? Do the y-intercepts from the tables match the y-intercepts you found in part b) using the graphs?

Selected answers: a) i) $-7, -5, -3, -1, 1$; ii) $6, 3, 0, -3, -6$; b) i) -3, ii) 0;
c) look at the y-value of the point with $x = 0$; yes

3. Finding the y-intercept from a formula — Slides 15–19

Key points: The y-intercept can be found from the formula for a linear relation by substituting the value 0 for x and simplifying to find the value for y. For a linear relation written in the form $y = ax + b$, the y-intercept is equal to b, the constant term.

Discuss how you could find the y-intercept directly from the formula without making a table or graph but by substituting 0 for x in the formula.

> Where can we find the y-intercept in the table of values? (at $x = 0$) Do we need the other values to find the y-intercept? (no) So how can we find the y-intercept from the formula without making a table of values? (substitute 0 for x and find the value of y)

As a class, substitute 0 for x in each of the formulas shown. Compare the y-intercepts obtained with the constant terms in the formulas and explain why they are equal.

$$\begin{aligned} y &= 14x + 6 \\ &= 14(0) + 6 \\ &= 0 + 6 \\ &= 6 \end{aligned}$$

$$\begin{aligned} y &= -5x - 14 \\ &= -5(0) - 14 \\ &= 0 - 14 \\ &= -14 \end{aligned}$$

$$\begin{aligned} y &= -12x + 0 \\ &= -12(0) + 0 \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

> What is the constant term in each of the three formulas? (6, -14, 0) How is the y-intercept related to the constant term? (they are equal) Why does this make sense? (when you substitute 0 for x, the variable term becomes 0 and you're left with the constant term)

Ask students how they could find the y-intercept just by looking at the formula of a linear relation. (the y-intercept is the constant term)

Exercises

1. a) Substitute 0 for x to find the y-intercept of the linear relation.

   i) $y = 24x - 17$ ii) $y = 30 - 18x$ iii) $y = -20 - 2x$

   b) For each formula in part a), what is the constant term?

   c) How does the constant term compare with the y-intercept?

   Answers: a) i) -17, ii) 30, iii) -20; b) i) -17, ii) 30, iii) -20;
   c) they are equal

2. Identify the y-intercept by simply looking at the formula.

   a) $y = 5x + 3$ b) $y = -\ 32x - 20$ c) $y = -20x - 32$

   Bonus:
   d) $y = 359 - 456x$ e) $17x - 602 = y$ f) $y = -999x$

   Answers: a) 3, b) -20, c) -32; Bonus: d) 359, e) -602, f) 0

4. Deriving a formula from a table of values

Key point: The coefficient of x and the y-intercept can be derived from the table of a linear relation to write a formula.

Display the table of values in the margin and explain to students that their job is to find a formula for the relation using the table.

x	y
0	3
1	-1
2	-5
3	-9

Remind students how to draw gap circles beside the output, or y-coordinate, column. Discuss whether the gap is always the same number and what that says about the relation and the formula for the relation.

> Is the gap always the same number? (yes, -4) What does that tell you about the relation? (the relation is linear) Where will this gap show up in the formula for the linear relation? (the gap is the coefficient of x)

Discuss how to find the y-intercept from the table, and how that can be used together with the coefficient to write the formula for the linear relation.

> Where does the y-intercept appear in the table? (the value of y when x is 0, which is 3) Where will the y-intercept show up in the formula for the relation? (it is the constant term)

Have students use the coefficient and constant term to write the formula. ($y = -4x + 3$) Display the formula for the relation and the labels as shown.

$$y = \overbrace{-4}^{\text{coefficient (gap: change in } y \text{ as } x \text{ increases by 1)}} x \underbrace{+\ 3}_{\text{constant term (}y\text{-intercept)}}$$

Repeat the process of finding a formula from a table of values for the table in the margin.

x	y
-1	-3
0	-1
1	1
2	3

y-intercept: -1
coefficient: $+2$
formula: $y = 2x - 1$

Exercises

a) Find the y-intercept for the linear relation directly from the table.

i)

x	y
-2	1
-1	4
0	7
1	10

ii)

x	y
-1	8
0	3
1	-2
2	-7

iii)

x	y
0	-1
1	-2
2	-3
3	-4

b) Where does the y-intercept show up in the formula for a linear relation?

c) For each relation in part a), find the gap between the y-values from the table.

d) Where does the gap show up in the formula for a linear relation?

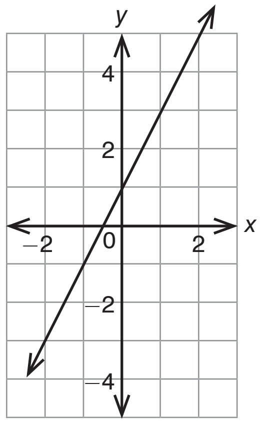
e) Write the formula for each relation in part a).

Answers
a) i) 7, ii) 3, iii) -1
b) the y-intercept equals the constant term
c) i) $+3$, ii) -5, iii) -1; d) the gap equals the coefficient of x
e) i) $y = 3x + 7$, ii) $y = -5x + 3$, iii) $y = -x - 1$

5. Deriving a formula from a graph **Slides 26–29**

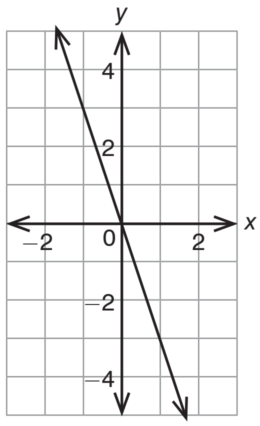
Key point: The coefficient of x and the y-intercept can be derived from the graph of a linear relation and used to write a formula.

Display the graph shown and explain that the formula for a linear relation can be derived from its graph by identifying the constant term (which is the y-intercept) and the coefficient of x (which is the change in y as x increases by 1). As a class, find the values and write the formula.



> What is the y-intercept on the graph? (1) What is the change in the value of y as x increases by 1? ($+2$) Write a formula for the linear relation. ($y = 2x + 1$)

Repeat the process for the graph in the margin. Explain that the "$+0$" constant term (y-intercept) need not be written in the formula.

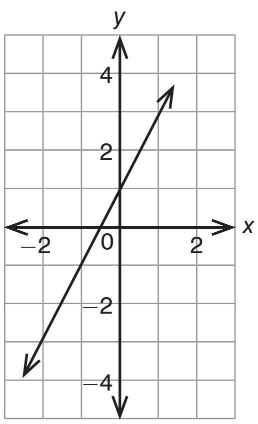


y-intercept: 0
coefficient: -3
formula: $y = -3x + 0$
$y = -3x$

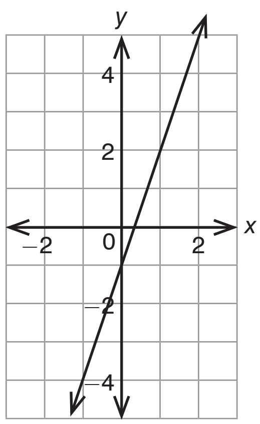
Exercises

a) What is the y-intercept of the linear relation?

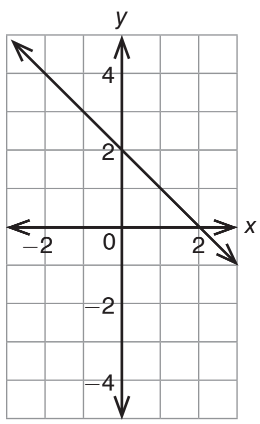
i)



ii)



iii)



b) For the relations in part a), what is the change in y as x increases by 1?

c) Write the formula for each relation.

Answers
a) i) 1, ii) -1, iii) 2
b) i) $+2$, ii) $+3$, iii) -1
c) i) $y = 2x + 1$, ii) $y = 3x - 1$, iii) $y = -x + 2$ or $y = -1x + 2$ or $y = 2 - x$

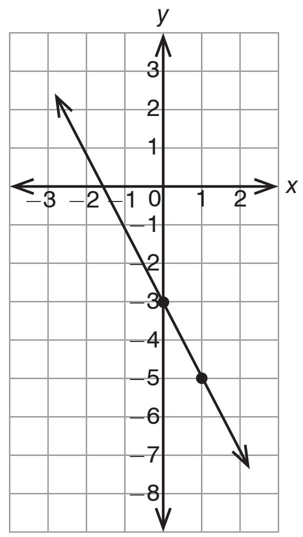
6. Sketching a graph directly from a formula — Slides 30–32

Key point: The coefficient of x and the y-intercept can be derived from the formula of a linear relation and used to sketch its graph.

Display the formula in the margin and explain that the graph of a linear relation can be sketched quickly from the formula once you identify the coefficient and y-intercept.

$y = -2x - 3$

Discuss how to sketch the graph by plotting the y-intercept, using the coefficient to plot a second point, and then connecting the two points to form a line, and then do so as a class.



> Using the formula, what is the y-intercept of the graph? (the constant term, -3) What is the change in y as x increases by 1? (the coefficient, -2) How could we use this information to sketch the graph? (plot a point at $(0, -3)$ for the y-intercept, then plot one more point using the change in y as x increases by 1, e.g., at $(1, -5)$, and then connect the two points to form a straight line)

After completing the graph, review the procedure for sketching a line as a class: first plot a point for the y-intercept, then use the coefficient to find one more point on the line, such as the point where $x = 1$, and then, after having plotted two points, connect the two points with a ruler to form a straight line.

Exercises

Each formula represents a linear relation.

i) $y = 3x - 4$ ii) $y = -2x + 3$ ii) $y = 1 - x$

a) State the coefficient of x for the relation.

b) State the y-intercept for the relation.

c) Find the coordinates of the point for the y-intercept and the point on the line when $x = 1$.

d) Sketch the graph for each relation.

Selected answers
a) i) 3, ii) -2, iii) -1
b) i) -4, ii) 3, iii) 1
c) i) $(0, -4)$, $(1, -1)$; ii) $(0, 3)$, $(1, 1)$; iii) $(0, 1)$, $(1, 0)$

Extensions

1. What is the y-intercept of the linear relation?

   a) $y = 3x + 4 + 9$ b) $y = -7 + x + 18$ c) $y = -23 - 41x - 108$

   Bonus: $y = -30(-4) + 56 - 56x - 23(2) + 230x$

   Answers: a) 13, b) 11, c) -131, Bonus: 130

2. What is the coefficient of x for the linear relation?

   a) $y = 12 + (3 \times 2 + 17)x$ b) $y = (75 \div 5 - 20)x + (14 \times 100)$

   c) $y = (14 \times 38) - (30 \times 2 - 87)x$

   Bonus: $(111 \times 222 + 97) - (54 \div 6 \times 8 + 1)x - (42 \times 43) = y$

   Answers: a) $+23$, b) -5, c) $+27$, Bonus: -73

3. a) On the graph of the relation, as x increases by 1, how does y change?

   i) $y = 3x + 7$ ii) $y = -4x + 111$ iii) $y = 2.5x - 222$ iv) $y = 35 - \frac{5}{3}x$

   b) On the graph of each relation in part a), as x increases by 6, how does y change?

   Answers: a) i) y increases by 3, ii) y decreases by 4, iii) y increases by 2.5, iv) y decreases by 5/3; b) i) y increases by 18, ii) y decreases by 24, iii) y increases by 15, iv) y decreases by 10

PR7-12
Solving Equations by Testing and Revising

AP Book pp. 6–7

Goals

Students will use the method of guessing, checking, and revising to solve simple increasing linear equations.

Main Ideas

The solution to an equation can be found by guessing a value for the variable that would make the equation true, testing by substitution, and then revising the trial value as needed.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation

Can substitute a value for a variable in an expression or equation

Can evaluate a numerical expression using the order of operations

Materials

none

Additional Information

For the sake of simplicity, the examples used in this lesson all involve linear equations where the expression with x on the left side of the equation increases as the value of x increases. For examples involving decreasing expressions, see Extension 4.

Vocabulary

equation
expression
solution
solve
solving
variable

Mental Math Minute Slides 2–3

Ask students to recall different strategies for adding and subtracting two integers mentally. Possible strategies include:

- visualise movement on a number line with the first integer indicating the starting point and the second the direction and magnitude of movement
- if the integers have the same sign, find the sum of the magnitudes and keep the sign
- if the integers have opposite signs, find the difference of the magnitudes and keep the sign of the integer with larger magnitude

Exercises

Add mentally.

a) $(+3) + (+4)$ b) $(-3) + (-4)$ c) $(-3) + (+4)$ d) $(+3) + (-4)$

e) $7 - (+9)$ f) $7 + (-9)$ g) $7 - (-9)$ h) $-7 - 9$

Answers: a) $+7$, b) -7, c) $+1$, d) -1, e) -2, f) -2, g) $+16$, h) -16

1. Identifying solutions to equations Slides 4–14

Key point: If a value for the variable in a single-variable equation makes the equation true, this value is the solution to the equation.

Display the equation in the margin. Ask students what value for the variable would make the equation true. (7) Explain that in any equation with one variable, a value for the variable that makes the equation true is called a *solution* to the equation. In equations that have exactly one solution, we can refer to the solution as *the* solution rather than *a* solution.

$m + 8 = 15$

Explain that you can check whether a value for a variable is the solution by substituting the value and checking if the resulting equation is true. One way to do that is to check the left and right sides of the equation separately. Check the value 7 for m as a class.

Explain that since the expression on the left side is equal to the expression on the right side, the equation is true when $m = 7$. So, 7 is the solution. We can also write "$m = 7$" is the solution.

Solve $m + 8 = 15$.
Try $m = 7$:
LS $= m + 8$ RS $= 15$
$= 7 + 8$
$= 15$
LS = RS, so 7 is the solution
or
LS = RS, so $m = 7$ is the solution

Display the equation in the margin. As a class, test the value of 2 for x. Remind students that the "$\neq$" symbol means "not equal to."

Discuss how evaluating an expression for a value of the variable is similar and different from testing—or verifying—a possible solution to an equation. Focus on the different outcomes: either a numerical value or an equation that is true or false.

$15x + 18 = 78$
Try $x = 2$:
LS $= 15x + 18$ RS $= 78$
$= 15(2) + 18$
$= 30 + 18$
$= 48$
LS $\neq$ RS, so 2 is not the solution

After you substitute a value for a variable into an expression or equation, what is the next step? (simplify by performing the operations) What is the result when you substitute values for the variables in expressions? (a numerical value) What is the result when you test a possible solution in an equation (an equation that is true or false; the value of the variable is the solution only if it makes the equation true)

Emphasize that when substituting a particular value into an equation does not result in a true equation, students are not getting the answer "wrong." Instead, they are successfully ruling out a particular value as the solution.

Exercises

1. Use substitution to verify the given solution to the equation.

   a) $42 + 13t = 55, t = 1$ b) $-55 + 2x = -5, x = 25$ c) $15 - 25m + 3 = 18, m = 0$

   Selected answer
   c) LS $= 15 - 25(0) + 3 = 15 - 0 + 3 = 18$, RS $= 18$, LS $=$ RS,
   so $m = 0$ is the solution

2. Is the given value for the variable the solution to the equation?
   Check by substitution.

   a) $20x + 10 = 130, x = 3$ b) $-15 + b = 20, b = 35$ c) $18 + n = 5n + 10, n = 5$

   Bonus: $400 - 3x + 5(6) = -98 + 15x - (24 \div 2), x = 30$

   Solution: Bonus: $400 - 90 + 30 = 340$ and $-98 + 450 - 12 = 340$, so yes
   Answers: a) no, b) yes, c) no

3. a) Give an example of an equation with one variable. Explain the meaning of "the solution to an equation" using your example to illustrate.

   b) Is the value -4 the solution to the equation $3(14) + T = 38$? Explain how you know.

   Sample answer: a) The solution to an equation with one variable is a value for the variable that makes the equation true. For example, in the equation $3x + 1 = 7$, the solution is 2, since substituting the value 2 for x results in a true equation: $3(2) + 1 = 6 + 1 = 7$.

   Answer: b) Since $3(14) + (-4) = 42 - 4 = 38$, the value -4 for T makes the equation true. So -4 is the solution to the equation.

4. Jagmeet and Liz are assigned two different tasks. Jagmeet substitutes a value for a variable in an expression. Liz tests a possible solution to an equation. Explain how their work is similar yet different.

   Jagmeet:
   $4x + 16$
   when $x = 5$,
   $4(5) + 16$
   $= 20 + 16$
   $= 36$

   Liz:
   $4x + 16 = 40$
   try $x = 5$
   LS $= 4(5) + 16$ RS $= 40$
   $= 20 + 16$
   $= 36$
   LS $\neq$ RS, so 5 is not the solution

Sample answer: Both Jagmeet and Liz substitute a value for a variable and simplify by applying operations. Jagmeet only needs to evaluate an expression, so his result is the numerical value of the expression when the variable, x, has the value 5. Liz is testing a possible solution, 5, in her equation. Since the expressions on the left and right sides of her equation are not equal when x is 5, her result tells her that the value 5 is not the solution to her equation.

2. Guessing and checking using tables — Slides 15–17

Key point: An equation can be solved by trying possible solutions, testing whether they make the equation true, revising the trial value, and repeating if necessary.

Explain that to *solve* an equation with one variable means to find a solution: a value for the variable that makes the equation true. One way to solve an equation is to test values for the variable systematically until you find the correct answer. The guesses can be recorded in a table. Display the equation and table in the margin.

$4x + 27 = 39$

x	4x + 27	True?
0		
1		
2		
3		
4		

Explain that the third column is used to note whether the equation is true, which in this case requires that the expression in the second column equals 39. As a class, fill in the first 4 rows of the table one at a time. Discuss whether the solution has been found after completing each row.

x	4x + 27	True?
0	$4(0) + 27 = 27$	no
1	$4(1) + 27 = 31$	no
2	$4(2) + 27 = 35$	no
3	$4(3) + 27 = 39$	yes
4		

In the row for $x = 3$, is the equation true? (yes)
Have we found the solution to the equation? (yes)
Do we need to continue filling in the table? (no)

Show how to write the solution as $x = 3$, and explain that finding the solution for the variable x is also called *solving for x.*

Exercises

Solve for x by guessing and checking. Use the table.

a) $3x + 14 = 26$

x	3x + 14	True?
1		
2		
3		
4		
5		

b) $14x - 5 = 65$

x		True?
1		
2		
3		
4		
5		

c) $18x - 10 = 44$

x		True?
1		
2		
3		
4		
5		

Answers: a) $x = 4$, b) $x = 5$, c) $x = 3$

3. Guessing and checking using logic to revise test values — Slides 18–22

Key point: When testing values for x in equations of the form $ax + b = c$, where a is positive, the next test value should be higher if the value of the expression $ax + b$ is too low, that is, it is less than c, and lower if the value is too high.

Explain that it's not necessary to try numbers starting from 0 when solving an equation by the testing and revising method. You can choose any first number that seems reasonable and then adjust the next value up or down depending on the result. Display the equation shown and tell students you will start by testing the value 5. Test the value as a class and discuss whether the next value should be higher or lower.

$6m - 7 = 47$

Try $m = 5$:

$$\begin{aligned} LS &= 6m - 7 \qquad\qquad RS = 47 \\ &= 6(5) - 7 \\ &= 30 - 7 \\ &= 23 \end{aligned}$$

> Was the result on the left side of the equation too high or too low? (too low) Should next test value we try be higher or lower than 5? (higher) Why? (a higher test value will make the expression on the left side larger)

Explain that the next test value need not be the next consecutive number, especially if there is a larger gap between the left and right sides of the equation. Test the value 10 as a class and then decide together if the next test value should be larger or smaller. (a bit smaller) Finally, test 9 as a class to conclude that $m = 9$ is the solution to the equation.

Explain that this method for solving an equation has several different names, including "guess and check," "test and revise," and "trial and error."

Exercises

1. a) Substitute 5 for x and say whether 5 is too high or too low.

   i) $4x + 1 = 25$ ii) $15x - 3 = 42$ iii) $20x + 14 = 154$

   b) Use the answers in part a) to try a higher or lower number. Continue until you solve each equation.

   Answers: a) i) $4(5) + 1 = 21$, too low; ii) $15(5) - 3 = 72$, too high; iii) $20(5) + 14 = 114$, too low; b) i) $x = 6$, ii) $x = 3$, iii) $x = 7$

Bonus: Solve the equation $25m + 30 = 530$ by guessing and checking.

Answer: $m = 20$

Extensions

Slides 23–26

1. Consider the equation $4.23x = 12.69$.

   a) Hal says that $x = 0$ is the solution to the equation. Do you agree with Hal? Explain.

   b) Jun says that $x = 1$ is the solution to the equation. Do you agree? Explain.

   Sample answers: a) I disagree with Hal. Substituting 0 gives the equation $0 = 12.69$, which is false; b) I disagree with Jun. Substituting 1 gives the equation $4.23 = 12.69$, which is false.

2. Solve the equation $4.23x = 12.69$ by trial and error.

   Answer: $x = 3$

3. Praveen finds the solution to the equation $5 \times 12 - (154 \times 369 - 263 \times 68)m = 32 + 28$ without doing any complicated calculations. How did Praveen do it? What is the solution?

   Answer: The solution is 0. Praveen could have figured that out by testing out the value 0 for m, or perhaps by noticing that the constant term on the left side, 5×12, which equals 60, is equal to the right side, $32 + 28$.

4. a) Is the linear relation $y = 35 - 2x$ an increasing or decreasing relation? How can you tell?

   b) Test the possible solution $x = 10$ in the equation $35 - 2x = 17$. Is the value of the expression $35 - 2x$ too low or too high?

   c) Should the next test value be larger or smaller than 10? Why?

   d) Solve the equation by trial and error.

   Solutions: a) decreasing, since the coefficient of x is a negative number (-2); b) $35 - 2(10) = 35 - 20 = 15$, so the value of the expression is too low; c) the next value should be smaller than 10, because the expression $35 - 2x$ decreases as x increases, as seen in part a); d) Since $35 - 2(9) = 35 - 18 = 17$, $x = 9$ is the solution.

PR7-13
Solving One-Step Equations Using Models

AP Book pp. 8–9

Goals

Students will use the preservation of equality and models to solve linear equations that require only one step (limited to whole number coefficients and positive solutions).

Main Ideas

An equation is like a balanced scale. If you add, subtract, multiply by, or divide by a number on both sides of an equation, equality is preserved.

Summary

Curriculum

AB: required

BC: required

MB: required

SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation

Can substitute a value for a variable in an expression or equation

Can evaluate a numerical expression using the order of operations

Materials

double-pan balance, paper bags, and two-centimetre connecting cubes for demonstration

40 two-centimetre connecting cubes, 5 paper bags, and ruler per student pair (see Activities 1 and 2)

Vocabulary

coefficient

equation

expression

solution

solve

solving

variable

Mental Math Minute

Slides 2–3

Explain that to check if an equation is true, you need to check that both sides represent the same number. You can use facts about multiplication and division without calculating both sides. For example, doubling both numbers in division does not change the quotient; in multiplication, doubling one factor and halving another factor keeps the product the same.

Exercises

Is the equation true?

a) $84 \div 11 = 42 \div 22$

b) $42 \div 13 = 84 \div 26$

c) $43 \times 20 = 430 \times 2$

d) $5 \times 13 = 10 \times 26$

e) $34 \div 0.5 = 68 \div 1$

f) $34 \div 0.5 = 68 \div 1$

g) $11 \div 2.5 = 44 \div 10$

h) $22 \times 2.5 = 11 \times 5$

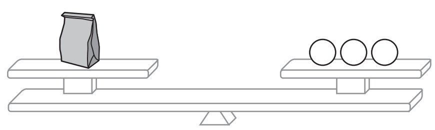
Answers: a) no, b) yes, c) yes, d) no, e) yes, f) no, g) yes, h) yes

1. Modelling addition equations with balances

Slides 4–13

Key points: An addition equation can be represented by a double-pan balance: each side of the balance represents one side of the equation; connecting cubes represent numbers; and a bag of cubes represents a variable. If you add or subtract the same number on each side of the balance (or equation), the balance (or equality) is preserved.

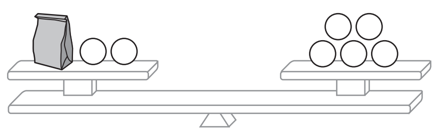
Show students a double-pan balance. Place a paper bag containing three cubes on one pan and three loose cubes on the other pan. The two pans should be balanced. A representation of the set-up is shown in the margin. Explain that all cubes, inside and outside the bag, have the same weight and that the bag (when it's empty) is so light that its weight will not affect the balance. Discuss how it is possible to determine the number of cubes in the bag without looking inside. Reveal the number of cubes in the bag to confirm.



How do you know how many cubes are in the bag without looking inside? (look at the number of cubes on the other side of the balance) How do you know the two sides have the same number of cubes? (because the pans are balanced)

Ask students how they would represent the balanced scale as an equation, where the variable b represents the number of cubes inside the bag. ($b = 3$) Emphasize that each side of the equation represents one side of the balanced scale.

Discuss with students whether the scale will still be balanced after adding two extra cubes to each side. Add two extra cubes to each pan to check. Ask students how they would represent the newly balanced scale as an equation.



$b + 2 \quad = \quad 5$

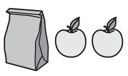
Emphasize that adding the same number of cubes to both sides means the scale will still be balanced. Ask students if subtracting—or removing the same number of cubes—from each side will result in the scale still being balanced. (yes) Test it out by removing first one, and then two cubes, from each side. Have students write the corresponding equations each time. ($b + 1 = 4$, $b = 3$)

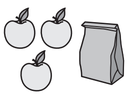
NOTE: Do not change the number of cubes inside the bag. Add or remove only the cubes outside the bag. This preserves the variable (b), represented by the cubes inside the bag.

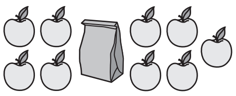
Emphasize that adding or removing the same amount from both sides of a balance maintains the balance or equality, meaning that each side continues to represent the same amount as the other.

Exercises

1. The bag has an unknown number of apples. Let x represent the number of apples in the bag. Write an expression to represent the total number of apples.

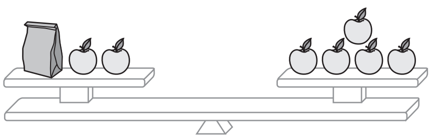
   a) 

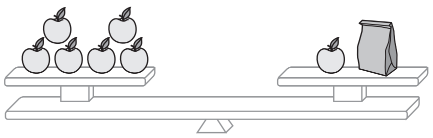
   b) 

   c) 

   Answers: a) $x + 2$, b) $3 + x$ or $x + 3$, c) $4 + x + 5$ or $x + 9$

2. The scales are balanced. Let m represent the number of apples in the bag. Write an equation to represent the total number of apples on each side of the balance.

   a) 

   b) 

   Answers: a) $m + 2 = 5$, b) $6 = 1 + m$ or $6 = m + 1$

3. You start with a balanced scale. If you perform the action, what action must you perform on the other side of the balance to maintain the balance?

   a) add 2 apples to the left side

   b) remove 1 apple from the right side

   c) subtract 3 apples from the left side

   d) add 10 apples to the left side

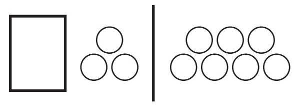
   Bonus: add a bag with x apples on the right side

   Answers: a) add 2 apples to the right side, b) remove 1 apple from the left side, c) subtract 3 apples from the right side, d) add 10 apples to the right side, Bonus: add a bag with x apples on the left side

2. Solving addition equations using balances — Slides 14–20

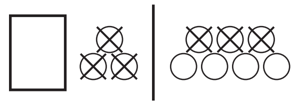
Key point: Solving an addition equation is like removing cubes from both sides of a balance model, to get the variable (bag of cubes) by itself on one side of the balance, or equation.

Explain that you will now use simple pictures to represent a balance scale. Display the image in the margin and explain that the vertical line divides the two sides of the balance, the circles on each side represent connecting cubes, and the rectangle represents the bag containing an unknown number of cubes. Have students write an equation to match the model, where x represents the unknown amount.



$x + 3 \quad = \quad 7$

Discuss with students how they could find the mystery number of cubes in the bag. Accept all answers and acknowledge that students might be able to see the answer immediately ($x = 4$). Students may feel that the approach of getting the bag of cubes all by itself on one side of the balance is not necessary. Explain that the method will be useful when students encounter harder examples. Discuss how the method works using this example. Show the resulting equations and pictures.



$x + 3 - 3 = \quad 7 - 3$

$x \quad = \quad 4$

> How can you get the bag all by itself on one side of the balance? (cross out or remove the extra three circles beside the bag) If you remove three circles from one side of the balance, what must you do to the other side to ensure you maintain the balance? (remove 3 circles) What equation shows removing three circles from each side? ($x + 3 - 3 = 7 - 3$) What equation shows the final balance? ($x = 4$)

Emphasize that adding or subtracting the same amount from both sides of an equation preserves the equality.

Complete part a) of the following exercises as a class before having students complete the rest independently.

Exercises

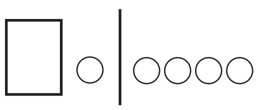
In this model circles represent apples, the rectangle represents a bag with an unknown number of apples, and the line divides two sides of a balance.

i) Add circles so that the model represents the equation.

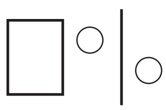
ii) Cross out circles from both sides of the picture to get the bag by itself on one side. Write a new equation to show removing circles from both sides.

iii) Draw a final picture with the crossed-out apples removed. Write a final equation.

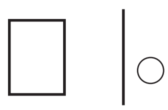
iv) Substitute the value for n in the equation from part i) to verify your answer.

a) 

$n + 2 = 8$

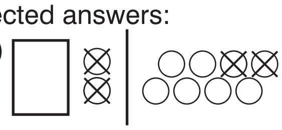
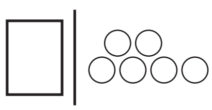
b) 

$n + 1 = 9$

c) 

$n + 4 = 6$

Selected answers:

a) ii)  , $n + 2 - 2 = 8 - 2$; iii)  , $n = 6$

iv) LS $= n + 2 = 6 + 2 = 8$, RS $= 8$, Since LS $=$ RS, $n = 6$ is the solution.

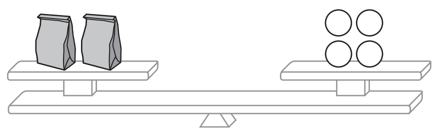
Activity 1: Solving addition equations concretely (Optional) — Slides 21–22

Give each student pair a ruler, paper bag and at least 20 connecting cubes. Partner 2 closes their eyes. Partner 1 places a certain number of cubes in a paper bag and places the bag along with additional cubes outside the bag on one side of the ruler. Partner 1 determines the total number of cubes on the side with the bag and then balances the scale by placing the same total number of cubes on the other side of the ruler. Partner 2 opens their eyes and removes cubes on both sides of the balance to determine how many cubes are in the bag. Partner 2 writes an equation to match each step: the original balance, the subtraction on both sides, and the final balance. Partner 1 opens the bag to reveal whether Partner 2's solution was correct. Partners switch roles and repeat several times.

3. Modelling multiplication equations with balances — Slides 23–29

Key points: A multiplication equation can be represented by a double-pan balance: the number of bags must match the coefficient of the variable. Multiplying or dividing by the same number on both sides of the balance or equation means that the balance—or equality—is preserved.

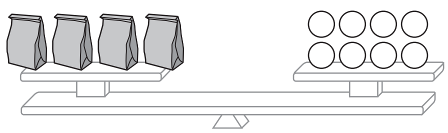
Show students a double-pan balance, this time with two bags hiding two cubes each on one side balanced by four loose cubes on the other side. Explain that the two bags have the same number of cubes inside. Tell students to let x represent the number of cubes in one bag and discuss how to represent the balance with an equation.



> How can you write an expression for the number of cubes on the left side? ($x + x$ or $x \times 2$ or $2x$)
> How can you write an equation to represent the balance? ($2x = 4$)

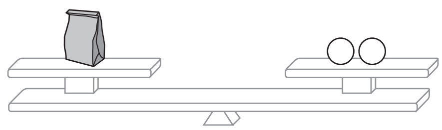
Discuss with students how they could represent the number of cubes on the left side if they do not know the number of cubes in each bag. Emphasize that there needs to be a multiplication: the number of bags by the variable (which is represented by the bag).

Ask students what the balance would look like if they were to multiply both sides by two. (4 bags with the same number of cubes on the left side, and 8 cubes on the right side) Modify the balance to illustrate. Ask students if the two sides are still balanced, and how to represent the balance with an equation. (yes, $4x = 8$)



$4x \quad = \quad 8$

Ask students how they could show division by 4 on both sides of this balance. (divide each side into 4 equal groups and keep only 1 of the 4 equal groups on each side) Have students say how to represent one equal group on each side of the balance. (one bag on the left; two cubes on the right) Modify the balance and have students write the equation for the new balance.

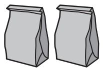


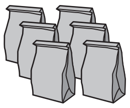
$x \quad = \quad 2$

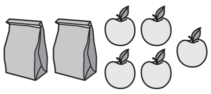
Remind students how easy it is to determine the number of cubes in a bag now that there is only one bag all by itself on one side of the balance. Ask students how many cubes must be in the bag, and then reveal the contents of the bag to confirm. (2 cubes)

Exercises

1. Each bag has the same unknown number of apples. Let y represent the number of apples in one bag. Write an expression to represent the total number of apples.

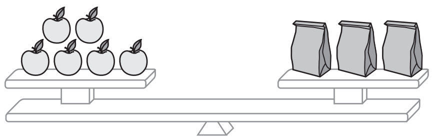
a) 

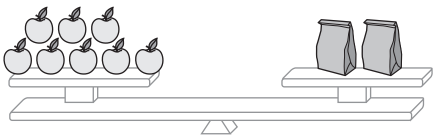
b) 

c) 

Answers: a) $2y$, b) $6y$, c) $2y + 5$

2. The scales are balanced. Let m represent the number of apples in each bag. Write an equation to represent the total number of apples on each side of the balance.

a) 

b) 

Answers: a) $6 = 3m$, b) $8 = 2m$

3. a) If you multiply one side of a balance by a number, what must you do to the other side of the balance to preserve the balance?

b) If you multiply one side of an equation by a number, what must you do to the other side of the equation to preserve the equality?

c) If you divide one side of a balance by a number, what must you do to the other side of the balance to preserve the balance?

d) If you divide one side of an equation by a number, what must you do to the other side of the equation to preserve the equality?

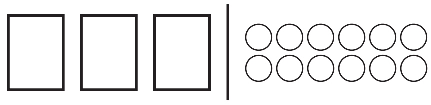
Answers: a) multiply the other side by the same number, b) multiply the other side by the same number, c) divide the other side by the same number, d) divide the other side by the same number

4. Solving multiplication equations using balances

Slides 30–35

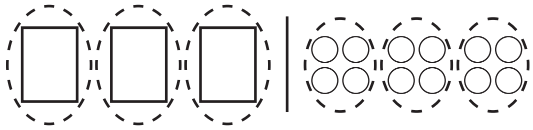
Key point: Solving a multiplication equation is like dividing or removing equal groups (of bags or cubes) from both sides of a balance model, to get the variable (a single bag of cubes) by itself on one side of the balance, or equation.

Display the image shown in the margin and remind students that the vertical line divides the two sides of the balance, the circles represent cubes, and each rectangle represents a bag containing an unknown number of cubes. Explain further that each bag (rectangle) has the same unknown number of cubes. Have students write an equation to match the model, where n represents the unknown amount.

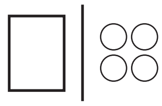


$3n = 12$

Ask students how they could figure out the number of cubes in each bag. Remind students that you are using the approach of getting one bag all by itself on one side of the balance. Discuss how to use division to do this. The solution is shown in the margin.



$3n \div 3 = 12 \div 3$



$n = 4$

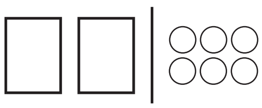
> How can you use division on each side of the balance to get one bag all by itself on the left side? (divide each side by 3) What equation shows dividing both sides by 3? ($3n \div 3 = 12 \div 3$) What equation shows the final balance? ($n = 4$)

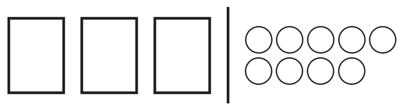
Explain that, just as adding or subtracting the same amount from both sides of an equation preserves equality, so does multiplying or dividing the same amount from both sides of an equation.

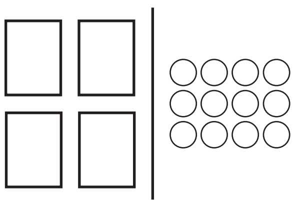
Complete part a) of the following exercises as a class before having students complete the rest independently.

Exercises

The circles represent apples, the rectangles represent bags with the same unknown number of apples, and the line divides two sides of a balance.

a) 

b) 

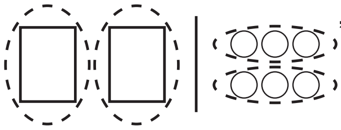
c) 

i) Write an equation to show the number of apples on each side. Use the variable x.

ii) Show how to divide both sides of the picture into equal groups to get the bag by itself on one side. Write a new equation to show dividing on both sides.

iii) Draw a final picture after the division, showing just one of the equal groups on each side. Write a final equation.

iv) Substitute the value for x in the equation from part i) to verify your answer.

Selected answer:
a) i) $2x = 6$
ii)  , $2x \div 2 = 6 \div 2$
iii)  , $x = 3$

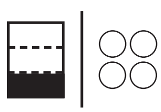
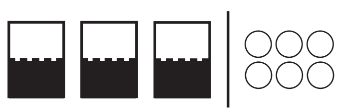
iv) LS $= 2x = 2(3) = 6$, RS $= 6$, Since LS $=$ RS, $x = 3$ is the solution.

Activity 2: Solving multiplication equations concretely (Optional) Slide 36

Students repeat Activity 1, but this time with a focus on division. Partner 1 places the same number of cubes in two to five bags on one side of the ruler and enough loose cubes on the other side to balance the scale. Partner 2 then uses division on both sides of the balance to solve for the unknown number. Partner 2 writes an equation to match each step: the original balance, the division on both sides, and the final balance.

Extensions Slides 37–41

1. Start with the equation $x + 2 = 5$.
   a) Explain how adding the same number to both sides preserves the equality. Use a picture model and equations to support your explanation.
   b) Explain how subtracting the same number to both sides preserves the equality. Use a picture model and equations to support your explanation.
2. Start with the equation $4y = 8$.
   a) Explain how multiplying by the same number (for example, 2) on both sides preserves the equality. Use a picture model and equations to support your explanation.
   b) Explain how dividing by the same number (for example, 2) on both sides preserves the equality. Use a picture model and equations to support your explanation.
3. One way to help solve equations involving division is to draw a rectangle for the variable, x, and then shade fractions of the rectangle to show fractions of x. To multiply on each side, shade the remaining fractions of the rectangle on the left side and add extra groups of circles on the right side.
   a) Write an equation for the picture using the "$\div$" division sign.

      i)  ii)  iii) 

   b) Use the picture models from part a) to solve your equations.

   Answers: a) i) $x \div 2 = 3$, ii) $x \div 3 = 4$, iii) $3(x \div 2) = 6$;
   b) i) $x = 6$, ii) $x = 12$, iii) $x = 4$

PR7-14
Solving One-Step Equations Using Opposite Operations

AP Book pp. 10–11

Goals

Students will use the preservation of equality and reversing operations to solve linear equations that require only one step (limited to whole number coefficients and positive solutions).

Main Ideas

When one side of an equation consists of a variable combined with a number by a single operation, the variable can be isolated by performing the opposite operation on both sides of the equation using the same number. For example, to isolate x in $x + a = b$, subtract a from both sides.

Summary

Curriculum

AB: required

BC: required

MB: required

SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation

Can substitute a value for a variable in an expression or equation

Can evaluate a numerical expression using the order of operations

Materials

none

Vocabulary

isolating the variable

opposite operations

Mental Math Minute **Slide 2**

Exercises

Evaluate the expression mentally.

a) $15 + 10 - 10$ b) $(25 + 7) - 7$ c) $16 - 8 + 8$ d) $(24 - 11) + 11$

e) $4 \times 6 \div 6$ f) $(14 \times 2) \div 2$ g) $54 \div 9 \times 9$ h) $(80 \div 8) \times 8$

Answers: a) 15, b) 25, c) 16, d) 24, e) 4, f) 14, g) 54, h) 80

1. Identifying opposite operations **Slides 3–10**

Key points: Addition and subtraction are opposite operations, just as multiplication and division are opposite operations. Any operation can be undone by applying the opposite operation using the same number (e.g., $4 - 3 = 1$, and $1 + 3 = 4$).

Have students analyze the questions and their answers from the Mental Math Minute exercises. Discuss what patterns they notice. Focus on the result of applying opposite operations. Ask students what operation undoes addition. Explain that two operations that undo each other are called *opposite operations*. Explain that subtraction is the opposite operation for addition and vice versa. Ask what operation undoes multiplication. Explain that division is the opposite operation for multiplication and vice versa.

> What is similar about the answers in the exercises? (you always end up with the quantity you started with) What is similar about all the questions? (each one applies an operation with a number to a starting quantity, and then the opposite operation with the same number)

Display the equations shown in the margin and ask students what number must be filled in to make the equation true. Emphasize that opposite operations are being used to return to the starting quantity. (7, 16, 3, 5)

$45 + 7 - \_\_\_\_ = 45$
$32 \times 16 \div \_\_\_\_ = 32$
$M \times 3 \div \_\_\_\_ = M$
$5n \div \_\_\_\_ = n$

Discuss how opposite operations can be used to return to a starting quantity, no matter what the starting quantity is.

> How do you get back to the starting quantity after…
> - multiplying by 100? (divide by 100)
> - subtracting 37? (add 37)
> - dividing by 25? (multiply by 25)
> - adding 1008? (subtract 1008)

Exercises

1. Write the number that makes the equation true.

a) $12 + 9 - \_\_\_\_ = 12$ b) $47 - 20 + \_\_\_\_ = 47$ c) $20 \times 35 \div \_\_\_\_ = 20$

Bonus: $458 + x - \_\_\_\_ = x$

Answers: a) 9, b) 20, c) 35, Bonus: 458

2. Write the operation that makes the equation true.

   a) $57 + 22 \bigcirc 22 = 57$ b) $99 \div 11 \bigcirc 11 = 99$ c) $h \times 100 \bigcirc 100 = h$

   Answers: a) $-$, b) $\times$, c) $\div$

3. Write the operation and number that make the equation true.

   a) $17 + 3$ ____ $= 17$ b) $20 \div 4$ ____ $= 20$ c) $18 - 25$ ____ $= 18$

   d) $m \times 8$ ____ $= m$ e) $(8 \times m)$ ____ $= m$ f) $8m$ ____ $= m$

   Bonus: $(45h + 15f - 266t) \times m$ ____ $= 45h + 15f - 266t$

   Answers: a) $- 3$, b) $\times 4$, c) $+ 25$, d) $\div 8$, e) $\div 8$, f) $\div 8$, Bonus: $\div m$

4. The operation is applied to a starting number. How could you undo the operation and get back to the number you started with?

   a) add 14 b) multiply by 12 c) subtract x d) divide by K

   Answers: a) subtract 14, b) divide by 12, c) add x, d) multiply by K

Activity: Undoing operations (Optional) Slide 11

Students play in pairs. Player 1 chooses a mystery starting number, writes it down, and then performs one operation on the number. Player 1 tells their partner the operation performed and the result but not the starting number. Player 2 needs to find the mystery starting number by applying opposite operations. Player 1 tells Player 2 if their solution is correct. Players switch roles and play several times.

2. Using opposite operations to solve one-step equations Slides 12–20

Key point: To isolate a variable in an equation where the variable is combined with a number and an operation, perform the opposite (inverse) operation on each side of the equation with the same number.

Display the equation shown in the margin. Explain to students that the goal is to find an equivalent equation where the variable, x, is all by itself on one side of the equation. This process is called *isolating the variable*. Explain how to do so using opposite operations. Work through the steps as a class.

$$x + 3 = 9$$
$$x + 3 - 3 = 9 - 3$$
$$x = 6$$

> How could you use opposite operations on the left side of the equation to reduce it to x? (subtract 3 to cancel out the $+3$) What must be done to the right side of the equation to preserve the equality? (subtract 3)

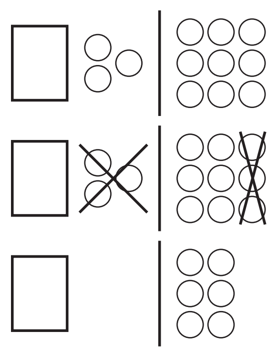
Ask students to recall the models from the previous lesson for representing an equation as a balanced scale. As a class, draw models for the equations.

Emphasize that isolating the variable amounts to solving the equation. Check the solution as a class by replacing x with 6 in the original equation.

$x + 3 = 9$

$x + 3 - 3 = 9 - 3$

$x = 6$



Display the next equation shown in the margin. As a class, write the steps to isolate the variable using opposite operations. Explain that while subtraction equations cannot be modelled in the same way as addition equations, the method of opposite operations still works for isolating the variable. Verify the solution as a class.

$$V - 26 = 45$$
$$V - 26 + 26 = 45 + 26$$
$$V = 71$$

Verify:
LS $= V - 26$ RS $= 45$
$= 71 - 26$
$= 45$ LS $=$ RS, so $V = 71$ is the solution

Repeat with the multiplication and division equations in the margin.

$$5x = 75$$
$$5x \div 5 = 75 \div 5$$
$$x = 15$$

Explain that the variable can be isolated on the left side or right side of the equation. It is more common to have the isolated variable on the left side, but it is always fine to switch the sides of an equation, as in the last example. $42 = h$ means the same thing as $h = 42$, just as $2 + 3 = 5$ means the same thing as $5 = 2 + 3$.

$$6 = h \div 7$$
$$6 \times 7 = h \div 7 \times 7$$
$$42 = h$$
$$h = 42$$

Exercises

1. Isolate the variable by applying the opposite operation. Remember to apply the same operation on both sides of the equation.

   a) $x + 13 = 48$ b) $52 = 4 + x$ c) $y - 28 = 100$

   d) $b \times 8 = 96$ e) $9b = 99$ f) $r \div 12 = 7$

   Selected solution: b) $4 + x = 52$; $4 + x - 4 = 52 - 4$; $x = 48$
   Answers: a) $x = 35$, c) $y = 128$, d) $b = 12$, e) $b = 11$, f) $r = 84$

2. Check your solutions to Exercise 1 by substituting the variable in the original equation.

   Selected answer: d) LS $= b \times 8 = 12 \times 8 = 96$, RS $= 96$, LS $=$ RS, so $b = 12$ is the solution

3. Solving equations with fractional solutions

Slides 21–25

Key points: Division can be written in fraction form. Sometimes the solution to an equation is not a whole number; the solution can be expressed as a fraction or decimal number.

Display the equation in the margin. Remind students that division can also be written in fractional notation. As a class, isolate the variable to solve the equation, using fractional notation for division.

$$4x = 48$$
$$\frac{4x}{4} = \frac{48}{4}$$
$$x = 12$$

Display the next equation from the margin and as a class perform the opposite operation to isolate *x*. Discuss how to simplify the fraction 5/10.

$$10x = 5$$
$$\frac{10x}{10} = \frac{5}{10}$$
$$x = \frac{1}{2} \text{ or } 0.5$$

How can you write 5/10 as a fraction in lowest terms? (divide the numerator and denominator by 5 to get 1/2) How can you write the fraction as a decimal? (divide using long division to get 0.5)

Using the decimal version of the solution (0.5), have students verify that the solution is correct by substituting it into the original equation. Explain that the solution to an equation can be a fraction or a decimal number. When the solution is a fraction, it's not always necessary to write the solution in decimal form. The solution can remain expressed as a fraction.

$$\begin{aligned} LS &= 10x \quad RS = 5 \\ &= 10(0.5) \\ &= 5 \qquad LS = RS\text{, so } x = 0.5 \text{ is the solution} \end{aligned}$$

Display the equation from the margin and guide students to isolate x. Tell students to leave the answer as a fraction.

$$\begin{aligned} 9x &= 5 \\ \frac{9x}{9} &= \frac{5}{9} \\ x &= \frac{5}{9} \end{aligned}$$

Exercises

1. Solve for the variable by applying the opposite operation. Use fraction notation for division.

   a) $8x = 56$ b) $13y = 39$ c) $25q = 175$

   Solutions: a) $(8x)/8 = 56/8$; $x = 7$, b) $(13y)/13 = 39/13$; $y = 3$, c) $(25q)/25 = 175/25$; $q = 7$

2. Solve for the variable by applying the opposite operation. Use fraction notation for division. Leave the solution in fraction notation.

   a) $100m = 50$ b) $12x = 3$ c) $11n = 4$

   Solutions: a) $(100m)/100 = 50/100$; $m = 1/2$; b) $(12x)/12 = 3/12$; $x = 1/4$; c) $(11n)/11 = 4/11$; $n = 4/11$

Extensions

Slides 26–29

1. Solve the equation. Write your answer as a fraction in lowest terms.

   a) $330x + 8 = 550 + 4(2)$ b) $210k = 40 \times 2 + 10$ c) $450q = 13(2) - 16 \div 2$

   Answers: a) $x = 5/3$, b) $k = 3/7$, c) $q = 1/25$

2. Solve the equation. Write your answer as a decimal.

   a) $10y = 7$ b) $55x = 44$ c) $45t = 13(2) - 16 \div 2$

   Answers: a) $y = 0.7$, b) $x = 4/5 = 0.8$, c) $t = 18/45 = 0.4$

3. The equation $10x + 4y = 74$ has two unknowns, x and y.

   a) Find as many whole-number solutions for x and y as you can. Hint: Use organized guessing and checking.

   b) Find solutions that are not whole numbers.

   Answers: a) The whole-number solutions for (x, y) are (7, 1), (5, 6), (3, 11), and (1, 16)

   Sample answers: b) (0, 18.5), (2, 13.5), (4, 8.5), (6, 3.5), (0.5, 17.25)

4. Use organized guessing and checking to find all whole number solutions to the inequality $4n + 8 < 20$.

   Answer: 0, 1, and 2

PR7-15
Undoing Two or More Operations

AP Book pp. 12–13

Goals

Students will use balance models to solve two-step equations involving multiplication and addition. Students will undo two or more operations by using opposite operations in reverse order.

Main Ideas

To isolate a variable in a two-step equation balance model, isolate the variable term first by removing loose cubes and then isolate the variable using division. To undo multiple operations in an expression or equation, opposite operations need to be applied in reverse order.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation
Can substitute a value for a variable in an expression or equation
Can evaluate a numerical expression using the order of operations
Understands that opposite operations undo each other

Materials

double-pan balance, paper bags, and connecting cubes for demonstration
40 two-centimetre connecting cubes, 5 paper bags, and a ruler per student pair (see Activity 1)

Vocabulary

constant term
isolating
opposite operations
variable term

Mental Math Minute — Slide 2

Exercises

Evaluate the expression mentally.

a) $(8 \times 2 + 4) - 4$ b) $(54 \div 9 - 7) + 7$ c) $(64 \div 8 + 19) - 19$

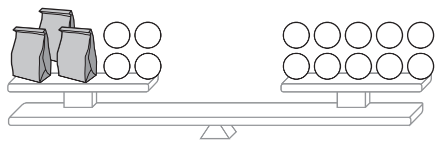
d) $(3 \times 9 + 2 - 2) \div 9$ e) $(5 + 4 \times 8 - 5) \div 4$ f) $(-117 + 9 \times 9 + 117) \div 9$

Answers: a) 16, b) 6, c) 8, d) 3, e) 8, f) 9

1. Modelling two-step equations — Slides 3–11

Key points: In a model for equations using bags and connecting cubes, the total number of bags multiplied by the unknown number of cubes in each bag (i.e., the variable) represents the variable term while the total number of loose cubes on each side represents the constant terms. To isolate a variable using this model, isolate the variable term first by removing loose cubes, and then isolate the variable by division (i.e., by the number of bags).

Display the balanced scale as shown in the margin, using three paper bags, each with two cubes inside, and a double-pan balance. Remind students that each bag holds the same unknown number of cubes. Have students write an equation to match the balanced scale, where *x* represents the unknown number of cubes in each bag. Display the equation, as shown. Have students explain what each part of the equation corresponds to in the balance. Encourage them to use "variable term" and "constant term" in their explanations. Remind students that any term that does not include a variable is a constant term.



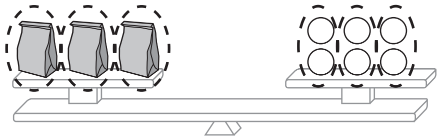
$3x + 4 = 10$

> What does "3*x*," the variable term, correspond to in the balance? (the 3 bags, each with the same unknown number of cubes) What does "4," the constant term on the left side, correspond to? (the 4 loose cubes on the left side) What does "10," the constant term on the right side, correspond to? (the 10 cubes on the right side)

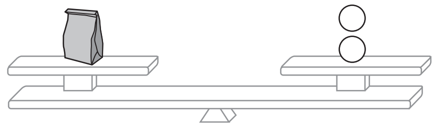
Ask students how they would solve the equation using the balance. Accept all answers but explain that today you will start by isolating the variable term, which corresponds to the three bags. Discuss how to isolate the variable term (the three bags) in the balance.

> Other than the three bags, what else is on the left side of the balance? (4 loose cubes) What must we do to the right side if we remove four cubes from the left side? (remove 4 cubes from the right side to preserve the balance)

Remove four cubes from each side of the balance, and have students write an equation showing this subtraction, as well as the equation showing the balance after the four cubes are removed. ($3x + 4 - 4 = 10 - 4$; $3x = 6$) Point out that students have now reduced the problem to one they already know how to solve. Ask students to recall how to solve the equation and balance showing $3x = 6$ and do so as a class. The steps with the balance and corresponding equations are shown.



$$\frac{3x}{3} = \frac{6}{3}$$



$$x = 2$$

Ask students what the solution is and reveal the contents of the bag to verify. ($x = 2$; there are 2 cubes in each bag)

Ask students to make a model for the equation $2x + 3 = 11$ using pictures. As a class, solve the equation using the model. Finally, verify the solution by substituting the value of the variable into the original equation.

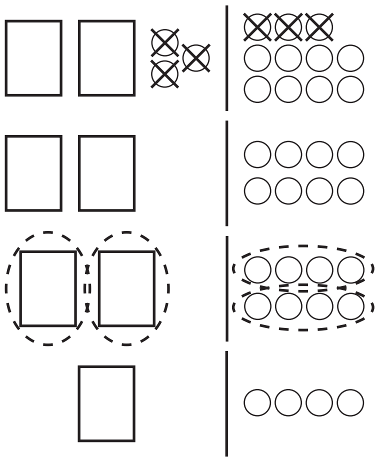
$2x + 3 = 11$

$2x + 3 - 3 = 11 - 3$ subtract 3 from both sides

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ divide both sides by 2

$x = 4$



Check

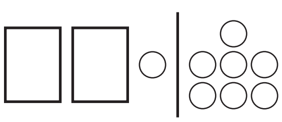
$LS = 2x + 3$ $RS = 11$

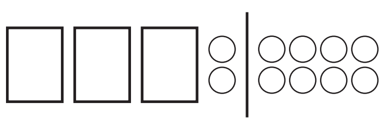
$= 2(4) + 3$

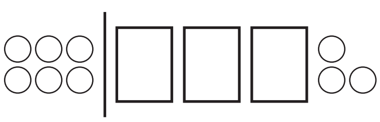
$= 11$ $LS = RS$, so $x = 4$ is the solution

Exercises

1. Each circle represents a cube and each rectangle represents a bag with the same unknown number of cubes. Write the equation shown by the model. Use the variable x for the unknown number of cubes in each bag.

a) 

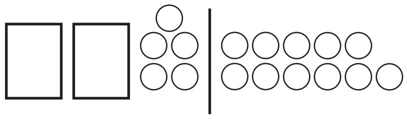
b) 

c) 

Answers: a) $2x + 1 = 7$, b) $3x + 2 = 8$, c) $6 = 3x + 3$

2. Draw a model to represent the equation.

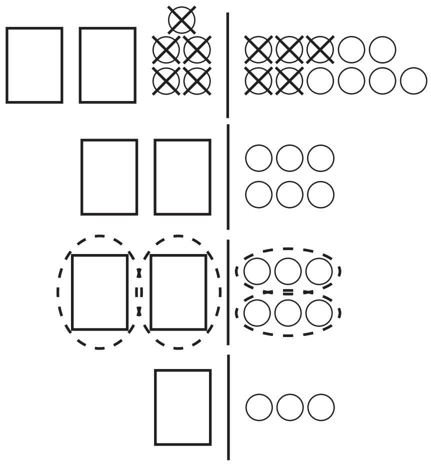
   a) $2x + 5 = 11$ b) $13 = 3x + 4$ c) $4x + 5 = 13$

   Selected answer: a) 

3. Solve the equations in Exercise 2 by using the models you created. Draw a new model and equation for each step of your solution.

   Selected answer:

   a)



$2x + 5 - 5 = 11 - 5$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

Activity 1: Balance the scale (Optional) Slides 12–13

Distribute a ruler, 5 paper bags and 40 connecting cubes to each student pair. Partner 2 closes their eyes. Partner 1 places the same number of cubes in two to five paper bags and places the bags, along with additional cubes outside the bag, on one side of the ruler. Partner 1 determines the total number of cubes on the side with the bags and then balances the scales by placing the same total number of cubes on the other side of the ruler. Partner 2 opens their eyes and performs operations on both sides of the balance to determine how many cubes are in one bag. Partner 2 writes an equation to match each step. Partner 1 opens the bag to reveal whether Partner 2's solution is correct. Partners switch roles and repeat several times.

2. Translating mathematical expressions Slides 14–18

Key point: When translating expressions from words to symbols or symbols to words, the order of operations must be followed.

Display the expression shown in the margin and walk students through the process of writing the actions as steps. Show students how to write the actions in words, in the correct order.

$3x + 7$
Start with x. Multiply by 3. Then add 7.

> What are you starting with? (x) Which two operations are being applied to x? (multiplication by 3 and addition of 7) Which of the two operations happens first? (the multiplication) How do you know? (because of the order of operations)

Ask students how they would need to write the expression if they wanted the 7 to be added to x before multiplying by 3. (using brackets: $3(x+7)$ or $(x+7) \times 3$). As a class, write the steps in words.

$3(x+7)$
Start with x. Add 7. Then multiply by 3.

Complete part a) of the following exercises as a class before having students complete the rest individually.

Exercises

Write the operations in words, in the correct order. Start with the variable.

a) $8 + 9y$ b) $\frac{m}{3} - 6$ c) $14(x - 25)$

Bonus: $(340 + n \times 178) \div 200$

Answers:
a) Start with y. Multiply by 9. Then add 8.
b) Start with m. Divide by 3. Then subtract 6.
c) Start with x. Subtract 25. Then multiply by 14.
Bonus: Start with n. Multiply by 178. Add 340. Then divide by 200.

Complete part a) of the following exercises as a class before having students complete the rest individually.

Exercises

Write an expression to match the description in words. Use brackets only when you need to.

a) Start with b. Multiply by 7. Then add 8.

b) Start with x. Divide by 5. Then subtract 2.

c) Start with p. Subtract 17. Then divide by 10.

Bonus: Start with y. Divide by 4. Subtract 99. Multiply by 32.

Answers: a) $7b + 8$, b) $\frac{x}{5} - 2$ or $x \div 5 - 2$, c) $\frac{p-17}{10}$ or $(p - 17) \div 10$,

Bonus: $32\left(\frac{y}{4} - 99\right)$ or $32 \times (y \div 4 - 99)$

3. Reversing two or more operations Slides 19–26

Key points: Opposite operations undo each other. To undo multiple operations, undo the operations in reverse order.

Explain that you will be solving equations with two operations by undoing the operations, one at a time. Remind students that opposite operations undo each other. To undo two or more operations, you need to work backwards, reversing the last operation first, each time. Compare to the process of putting on and taking off socks and shoes.

In what order do you put on socks and shoes? (socks, then shoes) In what order do you remove socks and shoes? (shoes, then socks)

How is this like undoing two or more operations? (you undo the last operation first and work backwards)

Present the problem shown in the margin. Ask students to tell you which two operations are performed and in what order. (multiply by 4, then add 8) Tell students that you would like to conduct an experiment to find the correct order for reversing the operations. Divide the class into two groups and have the first group undo the addition first, then the multiplication ($48 - 8 = 40$; $40 \div 4 = 10$), while the second group undoes the multiplication first, then the addition. ($48 \div 4 = 12$, $12 - 8 = 4$)

Discuss which order worked to get back to the starting number. Display the diagram shown to illustrate the correct order of reversing the operations.

Start with 10.	10		10	Get back to 10.
Multiply by 4.	40		40	Divide by 4.
Add 8.	48	$\longrightarrow$	48	Subtract 8.

As a class, write the shown series of operations as expressions. Then then find the operations needed to return to x.

Start with x.	x		x	Get back to x.
Multiply by 5.	$5x$		$5x$	Divide by 5.
Add 9.	$5x + 9$	$\longrightarrow$	$5x + 9$	Subtract 9.

Point out that to evaluate an expression, you use the order of operations, whereas to undo operations and return to the starting number or variable, you apply the order of operations in reverse.

Solve Exercise 1.a) of the following exercises as a class before having students complete the rest individually.

Start at 10.	10
Multiply by 4.	40
Add 8.	48

Undo the operations to get back to 10.

Exercises

1. Find the result. Then write the operations in the correct order to get back to the starting number.

   a) Start at 15. Multiply by 3. Add 7.   b) Start at 21. Subtract 6. Divide by 3.

   c) Start at 24. Divide by 6. Add 17.   d) Start at 9. Multiply by 5. Divide by 3.

   Bonus: Start at 16. Add 4. Divide by 2. Multiply by 12. Subtract 135.

   Answers: a) 52, Subtract 7. Divide by 3; b) 5, Multiply by 3. Add 6; c) 21, Subtract 17. Multiply by 6; d) 15, Multiply by 3. Divide by 5; Bonus: -15, Add 135. Divide by 12. Multiply by 2. Subtract 4.

2. Write the result as an expression. Then write the operations in the correct order to get back to the variable.

   a) Start with x. Multiply by 3. Add 7.   b) Start with y. Subtract 17. Divide by 4.

   c) Start with m. Multiply by 2. Divide by 5.   d) Start with n. Multiply by 24. Subtract 75.

   Bonus: Start with x. Multiply by 4. Divide by 7. Subtract 102. Add 13.

   Answers: a) $3x + 7$, Subtract 7. Divide by 3; b) $(y - 17) \div 4$, Multiply by 4. Add 17; c) $2m/5$, Multiply by 5. Divide by 2; d) $24n - 75$, Add 75. Divide by 24; Bonus: $4x/7 - 102 + 13$, Subtract 13. Add 102. Multiply by 7. Divide by 4.

Activity 2: Finding the mystery number (Optional) Slides 27–28

Students play in pairs. Player 1 chooses a mystery starting number, performs an operation on the number, writes down the operation and the result, and then performs a second operation, writing down the second operation and the final result. Player 1 shows their partner the operations they performed and the resulting numbers but not the starting number. Example: Start with 15, multiply by 3, and then subtract 6. Player 1 would write:

Start at ____.	____
Multiply by 3.	45
Subtract 6.	39

Player 2 needs to find the mystery starting number by applying opposite operations. Player 1 reveals the correct answer. Players switch roles and play several times.

Advanced variation: Use 3 or 4 operations.

Extensions Slides 29–34

1. Write the operations you would perform, in order, to isolate x.

   a) Start with x. Add 1. Multiply by 2. Subtract 3. Divide by 4.

   b) $((((x + 9) \times 8) - 7) \div 6)$

   Answers: a) Multiply by 4. Add 3. Divide by 2. Subtract 1; b) Multiply by 6. Add 7. Divide by 8. Subtract 9.

2. Hanna buys her mother a gift of a pair of gloves. She puts the gloves in a gift box, puts the lid on the box, ties a ribbon around the box, and places the box with the ribbon inside a gift bag.

   a) Write steps for Hanna's mother to take the gloves out of the box.

   b) How is this like undoing operations to isolate a variable in an equation?

   Answers: a) Remove the box from the gift bag. Untie and remove the ribbon. Open the box by removing the lid. And, finally, remove the gloves from the box. b) Just as Hanna's mom needed to remove first the gift bag, then the ribbon, and finally the gift box to get to the gloves, you need to undo operations in reverse order to isolate a variable.

3. This "magic trick" will tell you a person's chosen number and their age:

   Step 1: Choose a whole number between 1 and 9.
   Step 2: Multiply the number by 100.
   Step 3: Add the current year (a 4-digit number).
   Step 4: Subtract your year of birth (another 4-digit number).

   The result is a three-digit number. The first digit is the number you chose in step 1, and the next two digits are the age you will be on Dec. 31st of this year.

   a) Does this trick work for you? Try it with different numbers in Step 1.

   b) Try the trick on a friend. Does it work?

c) Write Steps 1 to 4 as an expression, using the variable n for the number chosen in Step 1.

d) Can you figure out why this trick works?

Sample answer: c) If the current year is 2023 and the year of birth is 2010, the expression is $100n + 2023 - 2010$, or $100n + 13$

Solution: d) the current year minus year of birth gives your age as of Dec. 31st of the current year. So, the expression is always $100n +$ age. Since n is a number between 1 and 9, $100n$ will be a three-digit number: the first digit is n and the next two are 0. If your age is less than 100, adding $100n$ to your age will give the result.

PR7-16
Solving Equations with Two or More Operations

AP Book pp. 14–16

Goals

Students will use the preservation of equality and undoing operations in reverse order to solve single-variable linear equations that involve two or more operations.

Main Ideas

To isolate a variable in a linear equation involving two or more operations, undo operations in the reverse order that they were applied (to form the expression with the variable) while always performing the same operations on both sides of the equation to preserve equality.

Summary

Curriculum

AB: required

BC: required

MB: required

SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation

Can substitute a value for a variable in an expression or equation

Can evaluate a numerical expression using the order of operations

Understands that opposite operations undo each other

Can solve one-step equations involving any of the four operations

Materials

BLM Opposite Operations and Two-Step Equations (p. J-69)

double-pan balance, paper bags, and two-centimetre connecting cubes for demonstration

30–40 two-centimetre connecting cubes, 4 paper bags, and a ruler per student

Vocabulary

isolating

opposite operations

Mental Math Minute — Slide 2

Exercises

Evaluate the expression mentally.

a) $\frac{5(12)}{4} \times 4$ b) $\frac{15(2)}{6} \times 6$ c) $\frac{12(4)}{8} \times 8$

d) $\left(\frac{5(12)}{4} + 1 - 1\right) \times 4$ e) $\left(\frac{15(2)}{6} + 7 - 7\right) \times 6$ f) $\left(\frac{12(4)}{8} - 19 + 19\right) \times 8$

Answers: a) 60, b) 30, c) 48, d) 60, e) 30, f) 48

1. Using opposite operations to solve two-step equations — Slides 3–9

Key point: To isolate a variable in a linear equation involving two operations, work backwards by undoing the second operation first.

Display the equation shown in the margin. Discuss how to undo operations to isolate x. Do so as a class, describing each step. Emphasize that each side of the equation must be treated the same way in each step to preserve the equality.

> What operation should we perform on the left side first to isolate the variable term? (subtract 2) What must we do to the right side at the same time to preserve equality? (subtract 2) We have isolated the variable term. What must we now do on each side of the equation to isolate x? (divide by 3)

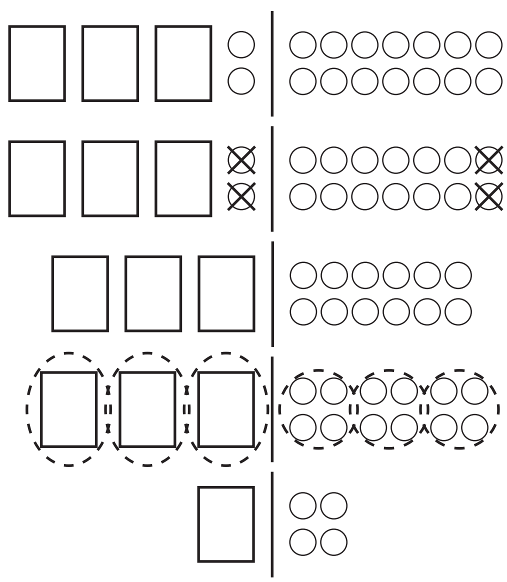
$3x + 2 = 14$	
$3x + 2 - 2 = 14 - 2$	subtract 2 from each side
$3x = 12$	simplify
$\frac{3x}{3} = \frac{12}{3}$	divide each side by 3
$x = 4$	simplify

As a class, verify the solution found by substituting the value for x into the original equation.

LS $= 3x + 2$ RS $= 14$
$= 3(4) + 2$
$= 12 + 2$
$= 14$ LS = RS, so $x = 4$ is the solution

As a class, draw a model to illustrate each step of the solution, as shown.

$3x + 2 = 14$	
$3x + 2 - 2 = 14 - 2$	subtract 2 from each side
$3x = 12$	simplify
$\frac{3x}{3} = \frac{12}{3}$	divide each side by 3
$x = 4$	simplify



Explain that this type of balance model works well for equations involving addition and multiplication, but it is harder to adapt to equations involving division or subtraction. However, even in equations for which this sort of model cannot be drawn, undoing operations in reverse order still works to isolate the variable and solve. Illustrate by solving the example in the margin as a class. Emphasize once more the importance of treating both sides of the equation equally to preserve equality.

$\frac{m}{5} - 12 = 88$	
$\frac{m}{5} - 12 + 12 = 88 + 12$	add 12 to each side
$\frac{m}{5} = 100$	simplify
$\frac{5m}{5} = 5(100)$	multiply each side by 5
$m = 500$	simplify

Exercises

1. Complete **BLM Opposite Operations and Two-Step Equations**.

   Answers
   b) x, $8x$, $8x + 5$, $8x + 5 = 37$; $8x + 5 = 37$, $8x + 5 - 5 = 37 - 5$, $8x = 32$, $8x/8 = 32/8$, $x = 4$; check: 4, 32, 37, yes
   c) x, $x/3$, $x/3 - 23$, $x/3 - 23 = 37$; $x/3 - 23 = 37$, $x/3 - 23 + 23 = 37 + 23$, $x/3 = 60$, $x/3 \times 3 = 60 \times 3$, $x = 180$; check: 180, 60, 37, yes

2. Solve the equation by applying opposite operations. Show each step.

   a) $5x + 6 = 71$ b) $25m - 11 = 214$ c) $\frac{b}{2} + 13 = 52$

   Selected answer: c) $\frac{b}{2} + 13 = 52$, $\frac{b}{2} + 13 - 13 = 52 - 13$, $\frac{b}{2} = 39$
   $\frac{b}{2} \times 2 = 39 \times 2$; $b = 78$

3. Check your solutions to Exercise 2 by substituting the value of b into the original equation.

   Selected answer: c) LS $= \frac{b}{2} + 13 = \frac{78}{2} + 13 = 39 + 13 = 52 =$ RS;

   since LS = RS, $b = 78$ is the solution.

2. Using opposite operations to solve $ax/b = c$ Slides 10–13

Key point: To isolate a variable in an equation of the form $ax/b = c$, first undo the division by b and then undo the multiplication by a.

Display the equation in the margin. Discuss how to undo operations to isolate x and do so as a class, describing each step.

> What can you do first to isolate x? (multiply both sides by 2 to undo the division) What could you do next? (divide both sides by 3 to undo the multiplication)

NOTE: The order of undoing multiplication and division in questions of this form is interchangeable. See Extensions 1 and 2 for more details.

$3x \div 2 = 12$	
$3x \div 2 \times 2 = 12 \times 2$	multiply each side by 2
$3x = 24$	simplify
$\frac{3x}{3} = \frac{24}{3}$	divide each side by 3
$x = 8$	simplify

As a class, solve the equation $4x/5 = 12$ in the same way, where division is written in fraction form.

As a class, verify the solution by substituting the value of x into the original equation.

$$\frac{4x}{5} = 12$$

$\frac{4x}{5} \times 5 = 12 \times 5$	multiply each side by 5
$4x = 60$	simplify
$\frac{4x}{4} = \frac{60}{4}$	divide each side by 4
$x = 15$	simplify

Exercises

1. Solve the equation by applying opposite operations. Show each step.

   a) $5x \div 6 = 10$ b) $\frac{11n}{4} = 22$ c) $\frac{9b}{2} = 54$

   Selected answer: c) $\frac{9b}{2} \times 2 = 54 \times 2$, $9b = 108$, $9b \div 9 = 108 \div 9$, $b = 12$

2. Check your solutions to Exercise 1 by substituting them into the original equation.

   Selected answer: c) LS $= \frac{9b}{2} = \frac{9(12)}{2} = \frac{108}{2} = 54 =$ RS;

   since LS = RS, $b = 12$ is the solution.

3. Using opposite operations to solve $ax/b + c = d$ Slides 14–18

Key point: To isolate a variable in a linear equation involving three (or more) operations, undo operations in the reverse order that they were applied to form the expression.

Display the equation in the margin. Point out that there are three operations on the left side. Ask students to list the operations in the order they are applied, starting with variable x. (start with x, multiply by 6, divide by 5, add 7) Discuss how to undo the operations to isolate x and do so as a class, describing each step.

> What operations would you apply on both sides of the equation, and in what order, to isolate x? (subtract 7, multiply by 5, divide by 6)

As a class, verify the solution by substituting the value of x into the original equation. Discuss the strategy for solving an equation with three or more operations. Compare the process to unwrapping a delivery wrapped in a plastic bag, bubble wrap, and a cardboard box.

> Describe the process for solving an equation with three or more operations. (undo each operation one by one, in reverse order) How is this like unwrapping a parcel wrapped in a plastic bag, bubble wrap, and a cardboard box? (you need to unwrap the outer or last layer first, then the second last layer, and finally the first layer—so cardboard, bubble wrap, and then the plastic bag)

$$\frac{6x}{5} + 7 = 19$$

$\frac{6x}{5} + 7 - 7 = 19 - 7$	subtract 7 from both sides
$\frac{6x}{5} = 12$	simplify
$\frac{6x}{5} \times 5 = 12 \times 5$	multiply both sides by 5
$6x = 60$	simplify
$\frac{6x}{6} = \frac{60}{6}$	divide both sides by 6
$x = 10$	simplify

Exercises

1. Solve the equation by applying opposite operations. Show each step.

   a) $9x \div 6 + 2 = 17$ b) $\frac{11n}{2} - 27 = 6$ c) $\frac{9b}{4} + 10 = 28$

   Selected answer:

   b) $\frac{11n}{2} - 27 + 27 = 6 + 27$; $\frac{11n}{2} = 33$; $\frac{11n}{2} \times 2 = 33 \times 2$;
   $11n = 66$, $\frac{11n}{11} = \frac{66}{11}$; $n = 6$

2. Check your solutions to Exercise 1 by substituting the value for x into the original equation.

   Selected answer:

   b) LS $= \frac{11n}{2} - 27 = \frac{11(6)}{2} - 27 = \frac{66}{2} - 27 = 33 - 27 = 6 =$ RS;
   since LS = RS, $n = 6$ is the solution.

4. Using equations to solve word problems

Slides 19–29

Key point: To represent a real-world problem with an equation, choose what the variable will represent, and then use the information in the problem to write an equation with the variable.

Display the word problem in the margin. Explain that the goal is to represent the problem using an equation with variables, and then solve the equation to solve the problem.

A taxi ride costs $2 per kilometre plus
a flat fee of $3.
Ram paid $11 for a taxi ride.
What distance did Ram travel in the taxi?

Explain that the first step in solving a word problem with an equation is to clearly identify what the variable represents. Discuss what the variable should represent in this problem.

> What is the problem asking you to find? (the distance Ram travelled in the taxi) What should the variable we use represent? (the number of kilometres Ram travelled in the taxi)

Show students how to declare a variable, as shown. Explain to students that they can choose any letter for the variable, but it can be helpful to choose a letter that reminds them of what the variable stands for, such as d to stand for distance.

Let d represent the distance (or number of kilometres) Ram travels in the taxi.

Having clearly defined the variable, discuss how to write an equation to represent the problem using the variable. ($2d + 3 = 11$)

> To begin, ignore the flat fee. What expression tells you how much Ram would need to pay for a taxi ride of d kilometres if there was no flat fee? ($2d$) How could you add the flat fee to the expression? ($2d + 3$) Use this expression and the total fee Ram paid to write an equation. ($2d + 3 = 11$)

As a class, solve the equation and answer the question from the problem. Use a concrete model with paper bags and connecting cubes or pictures of these and write the corresponding equation for each step.

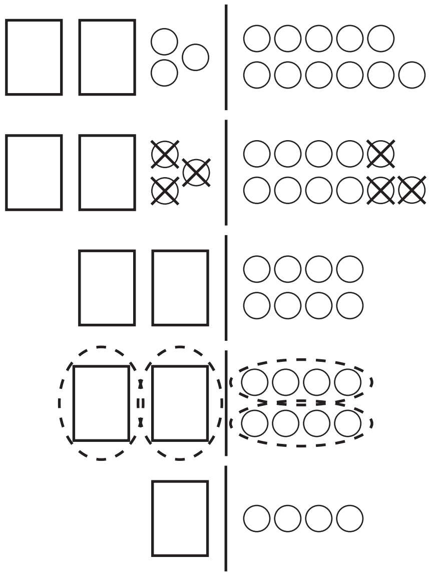
$2d + 3 = 11$

$2d + 3 - 3 = 11 - 3$

$2d = 8$

$2d \div 2 = 8 \div 2$

$d = 4$



Ram travels 4 kilometres in the taxi.

As a class, verify the solution ($d = 4$) in the original equation.

Distribute 4 paper bags and 30–40 connecting cubes to each student for the following exercises.

Exercises

It costs \$4 per hour to rent a bike plus a \$2 flat fee. Ali pays \$14 to rent a bike.

a) The goal is to find out for how long Ali rents the bike.
Complete this statement: Let h be ____________________.

b) Write an expression for the portion of the cost that depends on how long Ali rents the bike.

c) Write an expression for the total cost of renting the bike for h hours, including the flat fee.

d) Use the expression in part c) and Ali's total cost to write an equation.

e) Make a model for the equation using paper bags and cubes. Draw a picture of your model.

f) Solve the equation using your model. Draw a picture and equation for each step.

g) For how long does Ali rent the bike?

Selected answers: a) the number of hours Ali rents the bike, b) $4h$, c) $4h + 2$, d) $4h + 2 = 14$, g) 3 hours

Display the next word problem in the margin.

Discuss what the variable should represent in this problem and write the variable definition statement as a class.

Renting a boat costs \$25 per hour plus a flat fee of \$14.
Jade rents a boat and pays \$164.
For how many hours does Jade rent the boat?

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What is the problem asking you to find? (how many hours Jade rents the boat) What should the variable you use represent? (the number of hours Jade rents the boat)

Let h represent the number of hours for which Jade rents the boat.

Having clearly defined the variable, discuss how to write an equation to represent the problem using the variable.

Write an expression for the portion of the cost that depends on how long Jade rents the boat. ($25 \times h$, or $25h$) Write an expression for the total cost of renting the boat, including the flat fee. ($25h + 14$) Use this expression and the total cost for Jade to write an equation. ($25h + 14 = 164$)

As a class, solve the equation and answer the question from the problem.

$$\begin{aligned} 25h + 14 &= 164 \\ 25h + 14 - 14 &= 164 - 14 \\ 25h &= 150 \\ 25h \div 25 &= 150 \div 25 \\ h &= 6 \end{aligned}$$

Jade rents the boat for 6 hours.

Emphasize that solving a problem with an equation involves:

- defining a variable,
- writing and solving an equation, and
- writing a concluding statement

Exercises

It costs \$10 per hour to use a ski hill and \$45 to rent the skis. Marta rents skis and pays \$105 in total to ski.

a) The goal is to find out for how long Marta went skiing.
Complete the statement: Let h be ____________________.

b) Write an equation with h and solve your equation. Show each step.

c) What does the solution mean? For how long does Marta go skiing?

Answers: a) the number of hours Marta skis; b) $10h + 45 = 10$, $10h + 45 - 45 = 105 - 45$, $10h = 60$, $10h \div 10 = 60 \div 10$, $h = 6$; c) The solution, $h = 6$, tells you the number of hours Marta skis for. Marta goes skiing for 6 hours.

Extensions

Slides 30–34

1. a) Simplify the expressions by adding first and then subtracting.

   i) $5 + 13 - 8$ ii) $16 - 12 + 4$ iii) $x + 1 + 16 - 9$ iv) $x + 7 - 20 + 8$

   b) Simplify the expressions in part a) by subtracting first and then adding. Do you get the same result?

   c) Undo subtraction first and then addition. Do you get back to x?

   i) $x + 13 - 8$ ii) $x - 12 + 4$ iii) $x + 17 - 9$ iv) $x - 13 + 8$

   d) For the expressions in part c), undo addition first and then subtraction. Do you get back to x?

   e) Did the order of doing or undoing addition and subtraction in these expressions change the result?

   Answers: a) i) 10, ii) 8, iii) $x + 8$, iv) $x - 5$; b) yes; c) yes; d) yes; e) no

2. a) Simplify the expressions by multiplying first and then dividing.

   i) $(12 \times 6) \div 3$ ii) $6 \times 4 \div 2$ iii) $(4 \times 8) \div 2$

   b) Simplify the expressions in part a) by dividing first and then multiplying. Do you get the same result?

   i) $12 \times (6 \div 3)$ ii) $6 \times (4 \div 2)$ iii) $4 \times (8 \div 2)$

   c) Undo division first and then multiplication. Do you get back to x?

   i) $(12 \times x) \div 3$ ii) $6x \div 2$ iii) $x \times 8 \div 2$

   d) For the expressions in part c), undo multiplication first, then division. Do you get back to x?

   i) $12 \times (x \div 3)$ ii) $6(x \div 2)$ iii) $8 \times (x \div 2)$

   e) Did the order of doing or undoing multiplication and division in these expressions change the result?

   Answers: a) i) 24, ii) 3, iii) 16; b) yes; c) yes; d) yes; e) no

3. a) Rewrite the equation by turning the multiplication and division into a single fraction on the left side, in lowest terms. Do not change the right side.

   i) $8x \div 12 = 10$ ii) $30z \div 120 + 13 = 16$ iii) $44w \div 55 - 12 = 4$

   b) Solve each equation from part a).

   Answers: a) i) $(2/3)x = 10$, ii) $(1/4)z + 13 = 16$, iii) $(4/5)w - 12 = 4$; b) i) $x = 15$, ii) $z = 12$, iii) $w = 20$

PR7-17
Solving Equations with Integers
AP Book pp. 17–18

Goals

Students will solve one-step linear equations of the form $x + a = b$, where a and b are integers.

Main Ideas

Equations with integers can be solved using the same method of undoing operations and preserving equality, just as in equations with positive numbers. Equations with integers can also be solved by adding opposite integers (with or without models) to isolate the variable.

Summary

Curriculum

AB: required
BC: optional
MB: required
SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation
Can substitute a value for a variable in an expression or equation
Can evaluate a numerical expression using the order of operations
Understands that opposite operations undo each other
Can add and subtract integers
Understands that the sum of opposite integers is 0

Materials

two-centimetre connecting cubes of two different colours
paper bags
rulers

Vocabulary

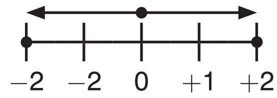
cancel out
magnitude
opposite integers

Mental Math Minute

Slides 2–3

Remind students that opposite integers are the same distance from 0 on a number line but in opposite directions. They have the same magnitude but opposite signs. They add to 0.

Opposite integers:
$+2$ and -2
-5 and 5
x and $-x$



Their sum is 0:
$(+2) + (-2) = 2 - 2 = 0$

Exercises

Add opposite integers to simplify the expression.

a) $5 + (-5)$ b) $(-9) + (+9)$ c) $(-117) + 117$ d) $24 + (-38) + (+38)$

e) $x + 82 + (-82)$ f) $(-18) + y + 18$ g) $n + 42 + (-n)$ h) $(-x) + (-a) + x$

Answers: a) 0, b) 0, c) 0, d) 24, e) x, f) y, g) 42, h) $-a$

1. Using models to solve equations with integers

Slides 4–12

Key points: Connecting cubes of two different colours can be used to represent positive and negative integers in equations. Cubes of opposite colours cancel each other out, just as opposite integers add to 0. The unknown can be isolated by adding cubes of the opposite colour on both sides.

Display the equation in the margin. Explain that an equation with integers can be represented using cubes of two colours for positive and negative numbers, and a paper bag containing an unknown number of positive or negative cubes for a variable. Show students cubes of two different colours and establish which colour will represent positive numbers and which will represent negative numbers: for example, blue for positive and red for negative numbers. Without students seeing, place five positive (blue) cubes into a paper bag. Tell students the bag with an unknown number of positive or negative cubes will represent the variable *b* in the equation. Discuss how to represent the rest of the equation and then do so on a central display table. Use a ruler or other thin object to divide your table into the two sides of the equation.

$b + (-3) = +2$

How can we represent the left side of the equation? (the bag for the variable *b* and 3 negative (red) cubes beside the bag) How can we represent the right side of the equation? (use 2 positive (blue) cubes)

Explain that in the same way that opposite integers add to 0, cubes of opposite colours add to 0 and can be *cancelled out* or removed from the equation. Discuss how to isolate the variable *b* (the paper bag) using the model by adding cubes of the opposite colour.

To isolate the bag of cubes on the left side of the equation, can we subtract three negative cubes from each side? (no) Why not? (because there are no negative cubes on the right side) What can we add to both sides of the equation to cancel out the three negative cubes on the left side? (3 positive cubes)

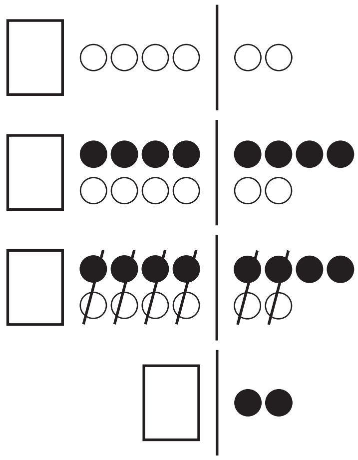
Demonstrate adding three positive cubes to each side of the equation, and then show how the red and blue cubes cancel each other out on the left side of the equation and can therefore be removed. With the bag now isolated on the left side of the equation, ask students how many cubes and of which colour must be in the bag. (5 positive cubes) As a class, write the equations that match these steps.

$$b + (-3) + (+3) = +2 + (+3)$$
$$b + (\cancel{-3}) + (\cancel{+3}) = +2 + (+3)$$
$$b = +5$$

Check the solution by using the model for the original equation. (5 positive cubes combined with 3 negative cubes on the left side results in 2 positive cubes, after cancelling and removing 3 negative cubes by pairing them up with 3 positive cubes)

Work through the equation $x - (+4) = -2$ as a class in the same way, except using pictures instead of concrete materials. Explain that to use the model, a necessary first step is to write any subtraction as an addition, which can always be done using opposite integers. The pictures and equations for each step are shown below: black circles represent positive cubes, white circles represent negative cubes, and the rectangle represents the bag (variable) containing an unknown number of positive or negative cubes.

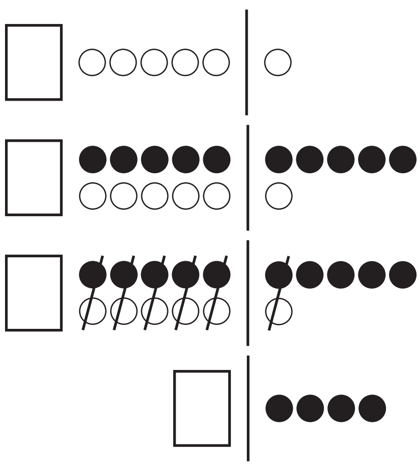
$x - (+4) = -2$ $x + (-4) = -2$	rewrite subtraction as addition
$x + (-4) + (+4) = -2 + (+4)$	add $+4$ to each side
$x + (\cancel{-4}) + (\cancel{+4}) = +2$	cancel and simplify
$x = +2$	simplify



Exercises

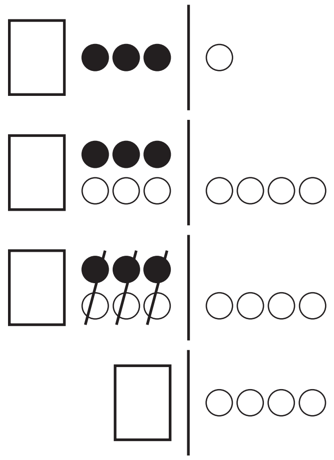
1. Satra solves the equation by using a model. The rectangle represents the variable x, white circles represent negative numbers, and black circles represent positive numbers. Write the equations that match each step of Satra's work.

   a) $x + (-5) = -1$



b) $x - (-3) = -1$

$x + 3 = -1$



Answers: a) $x + (-5) = -1$, $x + (-5) + 5 = -1 + 5$, $x + (\cancel{-5}) + \cancel{5} = -1 + 5$, $x = 4$; b) $x - (-3) = -1$, $x + 3 = -1$, $x + 3 + (-3) = -1 + (-3)$, $x + \cancel{3} + (\cancel{-3}) = -1 + (-3)$, $x = -4$

2. Rani solves the equation without using pictures. Draw pictures that match each step of Rani's work. Use the same kinds of pictures as in Exercise 1.

a) $n + (+3) = -6$
$n + (+3) + (-3) = -6 + (-3)$
$n + (\cancel{+3}) + (\cancel{-3}) = -9$
$n = -9$

b) $w - (+3) = -4$
$w + (-3) = -4$
$w + (-3) + (+3) = -4 + (+3)$
$w + (\cancel{-3}) + (\cancel{+3}) = -1$
$w = -1$

3. Solve the equation $m - (-2) = -1$ using pictures. Write an equation to match each picture in your solution.

Selected answers: $m + (+2) = -1$, $m + (+2) + (-2) = -1 + (-2)$, $m + (\cancel{+2}) + (\cancel{-2}) = -3$, $m = -3$

4. Solve the equation $m + (-2) = -1$ by adding opposite integers to isolate m. Show each step of your work. Draw a picture to match each step of your solution.

Selected answers: $m + (-2) = -1$, $m + (-2) + (+2) = -1 + (+2)$, $m + (\cancel{-2}) + (\cancel{+2}) = +1$, $m = +1$

Activity: Solving integer equations concretely (Optional) — Slides 13–14

Give each student pair a paper bag, a ruler, and at least 20 cubes of two different colours representing positive and negative cubes. Partner 1 places a certain number of cubes in a paper bag while Partner 2 closes their eyes. Note that the cubes in the bag must be all positive or all negative, not a mix. Partner 1 places the bag along with additional cubes (positive or negative, not a mix) outside the bag on one side of the ruler. They determine the integer sum on that side, and then balance the equation by placing the same total number of cubes as the sum (positive or negative) on the other side of the ruler. Partner 2 opens their eyes and isolates the bag by adding opposite integers, being sure to add the same integer to each side of the integer balance. Partner 2 writes an equation to match each step: the original balance, the addition on both sides, and the final balance. Partner 1 opens the bag to reveal whether Partner 2's solution was correct. Partners switch roles and repeat several times.

Key point: Equations with integers can be solved using the method of undoing operations while preserving equality, in the same way that equations with positive numbers are solved.

Display the equation shown in the margin and solve it as a class. Discuss the fact that the solution is a negative number, but the process for solving the equation is the same as before. Verify the solution as a class by substituting the value of x into the original equation.

> What is different about the solution to this equation, compared with the other equations we've been solving up to now? (the solution is a negative number) Did the same method of performing the opposite operation on both sides of the equation work to isolate the variable? (yes)

$x + 15 = 13$
$x + 15 - 15 = 13 - 15$ subtract 15 from both sides
$x = -2$ simplify

Check:
LS $= (-2) + 15$ RS $= 13$
$= 13$
LS $=$ RS, $x = -2$ is the solution.

Repeat the process of solving and checking the solution with the equation $m - 5 = -9$.

$m - 5 = -9$
$m - 5 + 5 = -9 + 5$ add 5 to both sides
$m = -4$ simplify

Check:
LS $= (-4) - 5$ RS $= -9$
$= -9$
LS $=$ RS, $m = -4$ is the solution.

Exercises

1. Solve the equation by applying opposite operations. Show your work.

   a) $n + 7 = 2$ b) $w - 13 = -10$ c) $x + 28 = -56$

   Answers: a) $n + 7 - 7 = 2 - 7$, $n = -5$; b) $w - 13 + 13 = -10 + 13$, $w = 3$; c) $x + 28 - 28 = -56 - 28$, $x = -84$

2. Check your solutions in Exercise 1 by substituting them into the original equation.

   Selected answer: c) LS $= (-84) + 28 = -56 =$ RS, so $x = -84$ is the solution

Use the examples in the margin to review how to simplify expressions with integers.

$(-2) + (+5) = -2 + 5$
$(-2) + (-5) = -2 - 5$
$(-2) - (+5) = -2 - 5$
$(-2) - (-5) = -2 + 5$

Display the equation shown in the margin and solve it as a class. Emphasize that the first step is to use the rules for simplifying integer addition and subtraction.

$w - (-4) = -17$
$w + 4 = -17$ simplify
$w + 4 - 4 = -17 - 4$ subtract 4 from both sides
$w = -21$ simplify

Repeat with the equation $5 + x + (-8) = -37 - (+4)$.

$5 + x + (-8) = -37 - (+4)$
$5 + x - 8 = -37 - 4$ simplify
$x - 3 = -41$ simplify
$x - 3 + 3 = -41 + 3$ add 3 to both sides
$x = -38$ simplify

Exercises

Solve the equation. Show your work.

a) $n + (-16) = -36$ b) $p - (-45) = -15$ c) $-2 + k + (+7) = -51 - (+1)$

Bonus: $(-65) - (-14) + x + (-12) = -198 - (+28) - (-32)$

Answers
a) $n - 16 = -36$, $n - 16 + 16 = -36 + 16$, $n = -20$
b) $p + 45 = -15$, $p + 45 - 45 = -15 - 45$; $p = -60$
c) $-2 + k + 7 = -51 - 1$, $k + 5 = -52$, $k + 5 - 5 = -52 - 5$, $k = -57$
Bonus: $-65 + 14 + x - 12 = -198 - 28 + 32$, $x - 63 = -194$, $x - 63 + 63$
$= -194 + 63$, $x = -131$

3. Solving problems with integer equations **Slides 22–32**

Key point: To represent a real-world problem involving integers with an equation, identify what the variable will represent, and then use the information in the problem to write an equation with the variable.

Display the word problem from the margin. Explain that the goal is to represent the problem using an equation with variables, and then solve the equation to solve the problem.

Ed records the high temperature on Tuesday.
The high temperature on Wednesday is
5 degrees warmer.
If the high temperature on Wednesday is
-3 degrees, what was the high temperature
on Tuesday?

Remind students that the first step in solving a word problem with an equation is to identify the variable. Discuss what the variable should represent in this problem.

> What is the problem asking you to find? (the high temperature on Tuesday) What should the variable we use represent? (the high temperature on Tuesday)

Display the statement in the margin. As a class, use the information in the problem to write an equation.

Let T represent the high temperature on Tuesday.

> Since the high temperature on Wednesday is warmer (higher) than on Tuesday, do we add or subtract to find Wednesday's high temperature from Tuesday's? (add) Write an expression with T for the high temperature on Wednesday. ($T + 5$) Use the expression and the temperature on Wednesday to write an equation. ($T + 5 = -3$)

As a class, solve the equation and answer the question from the problem. Use a concrete model with paper bags and cubes of two colours, or pictures of these, and write the corresponding equation for each step. The equations are shown in the margin.

$$\begin{aligned} T + 5 &= -3 \\ T + 5 + (-5) &= -3 + (-5) \\ T + \cancel{5} + (-\cancel{5}) &= -8 \\ T &= -8 \end{aligned}$$

As a class, solve the equation again, but this time without a model, using only equations and opposite operations. Ask students to compare the two methods and note whether the final answers are the same. Substitute the solution into the original equation to verify.

$$\begin{aligned} T + 5 &= -3 \\ T + 5 - 5 &= -3 - 5 \\ T &= -8 \end{aligned}$$

Check
LS $= (-8) + 5$ RS $= -3$
$= -3$
LS = RS, so $T = -8$ is the solution

Provide students with a ruler, paper bags and two colours of cubes for the following exercises.

Exercises

Olga earns some money on Tuesday. After spending \$7 on Wednesday, she has \$4 less than she did on Tuesday morning. Answer the questions to find how much Olga earned on Tuesday.

a) Complete the statement. Let M be ________________.

b) Write an expression for the total amount Olga gains or loses on Tuesday and Wednesday. Include M in the expression.

c) Use your expression from part b) and the amount of Olga's total loss to write an equation.

d) Solve the equation using a model with paper bags and two colours of cubes. Draw a picture of the model for each step of your solution.

e) Write an equation to match each picture in your solution from part d).

f) Solve your equation from part c) without a model. Use opposite operations. Do you get the same answer?

g) What does your solution in part e) or f) mean? How much does Olga earn on Tuesday?

h) Verify your solution using the original model from part d), and by substituting the value of M into the original equation from part c).

Selected answers: a) Let M be the amount of money Olga earns on Tuesday; b) $M + (-7)$, c) $M + (-7) = -4$, e) $M + (-7) = -4$, $M + (-7) + (+7) = -4 + (+7)$, $M + (\cancel{-7}) + (\cancel{+7}) = +3$, $M = +3$; f) $M + (-7) = -4$, $M - 7 = -4$, $M - 7 + 7 = -4 + 7$, $M = +3$; yes, I got the same answer; g) the solution, $M = +3$, tells me that Olga earned \$3 on Tuesday; h) LS $= (+3) + (-7) = -4 =$ RS; LS $=$ RS, so $M = +3$ is the solution.

Display the next problem in the margin.

Hina wants to know when Pythagoras was born. Hina's teacher tells her that Hypatia died 985 years after Pythagoras was born. If Hypatia died in 415 CE, when was Pythagoras born?

Remind students that the letters CE stand for "common era" and that dates in the common era can be written as a positive number. The letters BCE stand for "before the common era" and dates before the common era can be represented as a negative number.

Remind students that solving a problem using an equation involves:

- defining a variable,
- writing and solving an equation, and
- writing a concluding statement

Let P be the year Pythagoras was born.
$P + 985 = +415$

Solve the equation as a class, using opposite operations. Ask students to explain what the solution means and to write a sentence that answers the question of the problem.

$$P + 985 = +415$$
$$P + 985 - 985 = +415 - 985$$
$$P = -570$$

Pythagoras was born in 570 BCE.

Exercises

The top of a tower is above sea level. The bottom of the tower is 25 metres below sea level. The total height of the tower from the bottom to the top is 97 metres.

a) Write and solve an equation to find how high the tower stands above sea level. Show each step of your work.

b) Check your solution by substituting it into your original equation.

Answers: a) Let t stand for how high the top of the tower is above sea level. $t - (-25) = 97$, $t + 25 = 97$, $t + 25 - 25 = 97 - 25$, $t = 72$. The tower stands 72 metres above sea level. b) LS $= (72) - (-25) = 72 + 25 = 97 =$ RS, LS $=$ RS, so $t = 72$ is the solution.

Extensions

Slides 33–35

1. Solve the equation. Start by simplifying. Combine constant terms together on each side of the equation.

   a) $-6 + (5 - 4)x - 3 = -15 + 8$

   b) $(12 \div 4 - 2)n + 13 + (-21) - 2 = 3 \times 12$

   c) $-13 - 89 = (-8 - 4 + 17 - 4)x + 16$

   d) $13 - 27 = (15 - 18 + 4)w - 18 - (-2)$

   Solutions: a) $-9 + x = -7$, $-9 + x + 9 = -7 + 9$, $x = 2$; b) $n - 10 = 36$, $n - 10 + 10 = 36 + 10$, $n = 46$; c) $-102 = x + 16$, $-102 - 16 = x + 16 - 16$, $-118 = x$; d) $-14 = w - 16$, $-14 + 16 = w - 16 + 16$, $+2 = w$

2. A mystery integer is added to 17. The result is 84 less than double 17. What is the mystery integer?

   Solution: Let x be the mystery integer. $x + 17 = 17 \times 2 - 84$, $x + 17 = 34 - 84$, $x + 17 = -50$, $x + 17 - 17 = -50 - 17$, $x = -67$. The mystery integer is -67.

3. One year, Saskatoon's lowest December temperature was recorded in degrees Celsius. The next month, its lowest temperature was 5 degrees colder. The temperature in January was the opposite integer of 14 more than the product of 7 and 3. What was the lowest temperature in December?

   Solution: Let t be the lowest temperature in December. $t - 5 = -(3 \times 7 + 14)$, $t - 5 = -(21 + 14)$, $t - 5 = -(35)$, $t - 5 + 5 = -35 + 5$, $t = -30$. The lowest temperature in December was -30°C.

PR7-18
Problems and Puzzles: Equations

AP Book pp. 19–20

Goals

Students will translate word problems into equations involving one variable.
Students will apply their knowledge from this unit to solve equations and answer questions related to real-world problems.

Main Ideas

Equalities described in words can be translated into mathematical equations, which can be used to solve word problems.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Understands what a variable is and how it works in an expression or equation
Can substitute a value for a variable in an expression or equation
Can evaluate a numerical expression using the order of operations
Understands that opposite operations undo each other
Can solve two-step equations involving any of the four operations

Materials

BLM Strategy Talks (p. Q-2)
BLM Consecutive Number Problems (p. J-70, see Extension 3)

Vocabulary

algebraic expression
cancel out
coefficient
constant term
expression
numerical expression
variable term

Mental Math Minute Slide 2

Exercises

Substitute the given value for the variable and evaluate the expression mentally.

a) $10x + 7$, $x = 2$ b) $11x + 7$, $x = 3$ c) $5y + 17$, $y = 10$

d) $15n - 6$, $n = 3$ e) $12b - 9$, $b = 9$ f) $12w - 20$, $w = 7$

Answers: a) 27, b) 40, c) 67, d) 39, e) 99, f) 64

1. Translating expressions written in words — Slides 3–8

Key point: In a numerical or algebraic expression written in words, key words give clues as to which operations are being described.

Explain that to solve a word problem, it can be helpful to restate the problem using mathematical expressions. Key words give clues about the operations needed. Display the examples shown below.

Add	Subtract	Multiply	Divide
increased by	decreased by	double	divided by
sum	difference	product	quotient
more than	less than	twice as many	divided into
total	reduced by	times	shared equally

Display the phrases in the margin and explain that the task is to translate the phrases into mathematical expressions. Explain that any variable can be used to represent the unknown number. As a class, begin the process by identifying the key operations.

How do you know that "three divided into a number" is $x \div 3$ instead of $3 \div x$? (because the phrase says "divided into" not "divided by") What are other ways of saying "6 less than a number"? (a number reduced by 6, 6 subtracted from a number)

Explain that a mathematical expression that does not involve any variables is often called a *numerical* expression, whereas an expression that involves variables is called an *algebraic expression*.

7 more than a number
$n + 7$

three divided into a number
$x \div 3$

the product of a number and 5
$w \times 5$, or $5w$

six less than a number
$m - 6$

Exercises

Write an algebraic expression for the description.

a) six more than a number b) a number decreased by seventeen

c) the product of 9 and a number d) the sum of a number and 9

d) a number divided by 12 f) 12 divided into a number

g) four times a number h) double a number

Answers: a) $x + 6$, b) $w - 17$, c) $9y$, d) $y + 9$, e) $x \div 12$, f) $x \div 12$, g) $4r$, h) $2n$

As a class, use the examples in the margin to practise translating words into expressions involving more than one operation.

a number multiplied by 3, then increased by 8
$3n + 8$

4 more than the product of a number and 5
$5w + 4$

six less than half of a number
$(m \div 2) - 6$, or $\frac{m}{2} - 6$

Exercises

Write an algebraic expression for the description.

a) a number multiplied by 4 and then decreased by 7

b) nine more than five times a number

c) a number divided by six then increased by two

Bonus: The product of 13 less than half of a number and 5

Answers: a) $4x - 7$, b) $5n + 9$, c) $m/6 + 2$, Bonus: $(x/2 - 13) \times 5$

2. Translating equations written in words

Slides 9–12

Key points: In an equality described in words, the word "is" means "equal" and is represented by an equal sign. Equalities described in words can be translated into mathematical equations, which can then be used to solve word problems.

Tell students that when translating a written description of a mathematical problem, the word "is" means "equal" and it can be shown by an equal sign. As a class, translate the examples shown in the margin into equations.

$$\underbrace{\text{Five more than a number}}_{x+5}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{eight.}}_{8}$$

$$\underbrace{\text{3 times a number}}_{3n}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{5 less than 32.}}_{32-5}$$

What word in this description of a mathematical problem means "equal?" (is) What two phrases does that word separate? ("five more than a number" and "eight") What expressions could you translate those two phrases into? ($x + 5$ and 8)

Tell students they can now solve these equations to find the mystery number using a method of their choice, such as opposite operations, picture models, or guessing and checking. Solve the second equation as a class using opposite operations.

$$\begin{aligned} 3n &= 32 - 5 \\ 3n &= 27 \\ 3n \div 3 &= 27 \div 3 \\ n &= 9 \end{aligned}$$

Exercises

1. Translate the sentence into an equation.

   a) Ten more than a number is thirteen.

   b) 6 less than a number is 21.

   c) Five times a number is two less than forty-two.

   d) 6 divided into a number is 4 more than 8.

Bonus: Eight less than double a number is four less than ten.

Answers: a) $x + 10 = 13$, b) $y - 6 = 21$, c) $5n = 42 - 2$, d) $m/6 = 8 + 4$, Bonus: $2x - 8 = 10 - 4$

2. Solve the equations in Exercise 1 using any method of your choice.

   Answers: a) $x = 3$, b) $y = 27$, c) $n = 8$, d) $m = 72$, Bonus: $x = 7$

3. Solving word problems using equations — Slides 13–16

Key points: To represent a real-world problem as an equation, identify what the variable will represent, and then use the information in the problem and logical reasoning to write an equation with the variable. Finally, solve the equation to answer the question in the problem.

Display the word problem shown.

Judith joined a walkathon to raise money for charity. She completed the whole walk in 8 hours. Judith takes 15 minutes to walk 1 km. Her breaks during the walk totaled 2 hours. How far did Judith walk?

Ask students how they would start solving the problem. As a class, write a statement to declare the variable. (Let d be the distance Judith walked, in kilometres.) Discuss how to write an equation for the problem. Remind students that speed is equal to the distance travelled divided by the time taken to travel that distance: speed = distance ÷ time.

> How long did Judith walk for? ($8 - 2 = 6$ hours) At what speed does Judith walk? (1 km every 15 minutes) How could we write that speed using kilometres per hour? (there are 60 minutes in every hour, so $60 \div 15 = 4$, so her speed was 4 km per hour) Write an equation for the distance. ($d/(8 - 2) = 60 \div 15$ or $d/6 = 4$)

As a class, solve the equation using opposite operations. Discuss what the solution means and how to answer the question in the word problem.

$$\frac{d}{8-2} = 60 \div 15$$
$$\frac{d}{6} = 4$$
$$\frac{d}{6} \times 6 = 4 \times 6$$
$$d = 24$$

> What does the solution, $d = 24$ mean? (the distance Judith walked, in kilometres) Write a sentence to answer the question in the problem. (Judith walked 24 km in total.)

Exercises

1. Zara bikes from home to her grandma's place. On her way, she stops for 30 minutes. Zara can bike 10 km every half hour. If the trip takes 2.5 hours, what distance did she ride?

   Solution: Let d be the distance Zara rides. The time Zara rides for is 2.5 hours, minus 30 minutes for her stop. Since 30 minutes = 0.5 hours, Zara rides for $2.5 - 0.5 = 2$ hours. Her speed is 10 km every half hour or 20 km/h, so an equation for her speed is $d \div (2.5 - 0.5) = 20$, or $d \div 2 = 20$. Multiplying both sides of the equation by 2 gives $d \div 2 \times 2 = 20 \times 2$, or $d = 40$. So, Zara rides 40 km.

2. Ali is 3 times as old as his daughter. Ali's brother is 35. The sum of Ali's and his brother's ages is 71. How old is Ali's daughter?

   Solution: Let D be the age of Ali's daughter. Ali's age is $3D$. The sum of Ali's and his brother's age is $3D + 35$, and we know that equals 71. This gives the equation $3D + 35 = 71$. $3D + 35 - 35 = 71 - 35$, $3D = 36$, $3D \div 3 = 36 \div 3$, $D = 12$. So, Ali's daughter is 12 years old.

4. Whole-class strategy talk — Slides 17–19

Key point: A variety of strategies can be used to solve problems involving equations and unknowns.

Display the following problem and lead a whole-class strategy talk about how to solve it. (See **BLM Strategy Talks** for detailed guidance on running a whole-class strategy talk.)

Rani's brother is 4 years younger than her. Rani's father is 4 times Rani's age. Rani's mother is 2 years older than Rani's father. The sum of all four ages is 88. How old is Rani's mom?

Possible strategies that students may suggest to obtain equations:

- define a single variable and express all unknowns in terms of that variable
- define a separate variable for each unknown and then express them each in terms of another variable, and finally all in terms of one variable

Possible strategies that students may suggest to solve their equations:

- isolating the variable by applying opposite operations
- testing values and revising

Sample solution: Let R be Rani's age. Her family member's ages are as follows:

- Rani's brother: $R - 4$
- Rani's father: $4R$
- Rani's mother: $4R + 2$

An expression for the sum of their ages: $R + R - 4 + 4R + 4R + 2$, or $10R - 2$ (Since $4R = R + R + R + R$, and $R + R + R = 3R$, etc.) Since the sum is 88, we need to solve the equation $10R - 2 = 88$: $10R - 2 + 2 = 88 + 2$, $10R = 90$, $10R \div 10 = 90 \div 10$, $R = 9$. So, Rani's age is 9, her brother is 5, her father is 36, and her mother is 38 years old.

Extensions — Slides 20–25

1. Invite a local Elder to give a talk on spirit canoe trips with a focus on how variables and equations can help in pre-planning and calculations.

   Don and Alíla7 are planning a canoe trip. They know they can paddle at a pace of 3 km per hour. The distance they need to cover is 18 km.

a) Define a variable for the length of time their journey will take.

b) Write an expression with the variable for how far they can travel over a given length of time.

c) Write an equation and solve it to find out how long their entire journey will take.

Answers: a) Let t be the number of hours the canoe trip will take. b) $3 \times t$, or $3t$, c) $3t = 18$, $3t \div 3 = 18 \div 3$, $t = 6$; The journey will take 6 hours.

NOTE: You may need to help students locate the story and video online for Extension 2.

2. Search online for a video retelling of the story "Small Number and the Big Tree." Watch the video or read the story. Then work with a partner to answer the question.

   a) How tall does Small Number estimate the big tree is?

   b) Suppose that the height of the tree is six times the width of the creek, plus eight metres. If Small number is correct about the height of the tree, how wide is the creek? Use an equation with a variable.

   c) Suppose eight of Small Number's friends hold hands around the Big Tree, and they find that the circumference of the big tree is two decimetres longer than they can reach. If each child has the same arm span (distance from the tip of one out-stretched hand to the other), and the circumference of the tree is 130 decimetres, what is the arm span of each child?

   Answer: a) 50 metres
   Solutions: b) Let w be the width of the creek. $6w + 8 = 50$, $6w + 8 - 8 = 50 - 8$, $6w = 42$, $6w \div 6 = 42 \div 6$, $w = 7$. The width of the creek is 7 m; c) Let a be the arm span of each child. $8a + 2 = 130$, $8a + 2 - 2 = 130 - 2$, $8a = 128$, $8a \div 8 = 128 \div 8$, $a = 16$. The arm span of each child is 16 decimetres, or 160 cm.

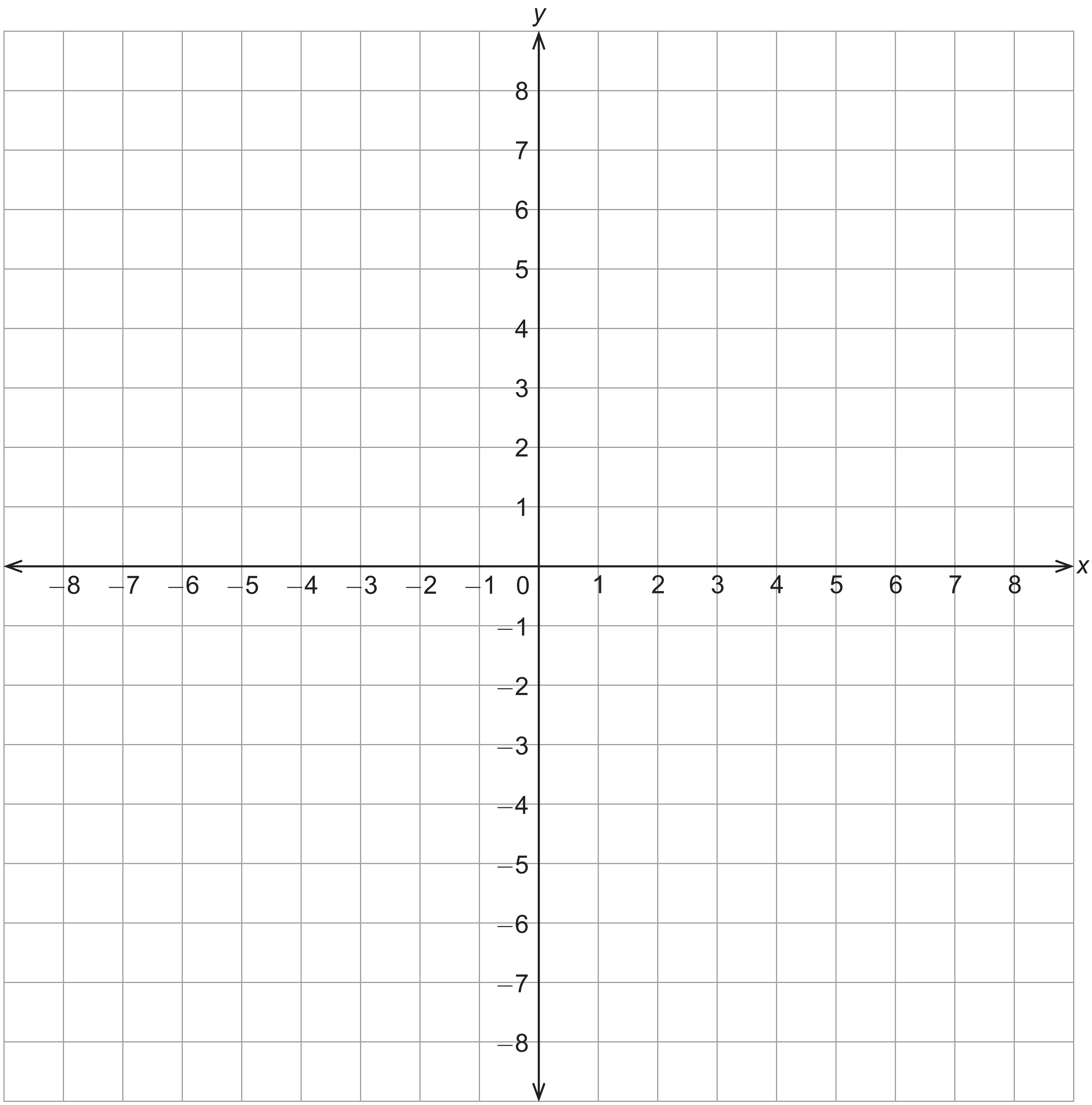
3. When you have two or more variable terms involving the same variable in an expression or equation, you can combine the variable terms into one. For example, $2x + 3x = 5x$, and $16m - 2m + 3m = 17m$.

   Complete **BLM Consecutive Number Problems**.

   Answers
   1. a) 5; b) 8; c) 39; d) $x + 2$; e) 6; f) 9; g) 22; h) 37; i) $x + 2$, $x + 4$; j) $x + 2$, $x + 4$
   2. a) $17 + 18 = 35$; b) i) $x + 1$; ii) $x + x + 1 = 35$; iii) $2x + 1 - 1 = 35 - 1$, $2x = 34$, $x = 17$, so the two numbers are 17 and $17 + 1 = 18$.
   3. a) $x + x + 2 = 32$, $2x + 2 - 2 = 32 - 2$, $2x = 30$, $2x \div 2 = 30 \div 2$, $x = 15$, so the two numbers are 15 and $x + 2 = 17$; b) a) $x + x - 2 = 32$, $2x - 2 + 2 = 32 + 2$, $2x = 34$, $2x \div 2 = 34 \div 2$, $x = 17$, so the two numbers are 17 and $x - 2 = 15$; c) Yes, the two numbers are the same because the equations for the second number were opposite. When x was the smaller number, I added to get the larger number. When x was the larger number, I subtracted to get the smaller number.

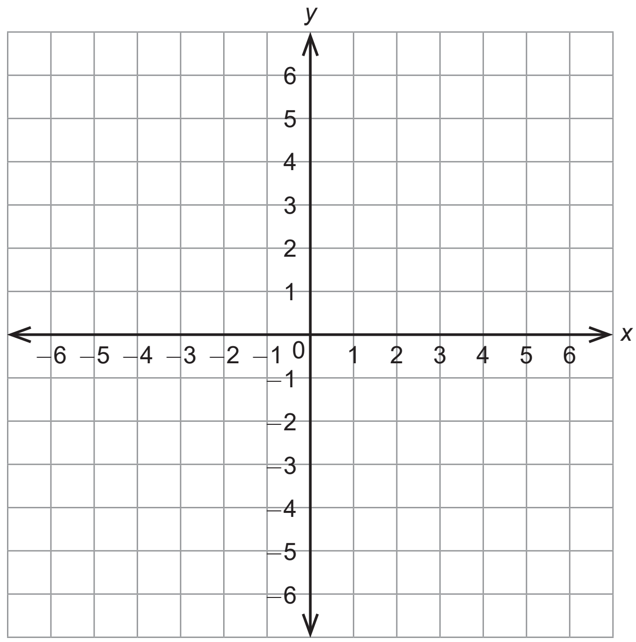
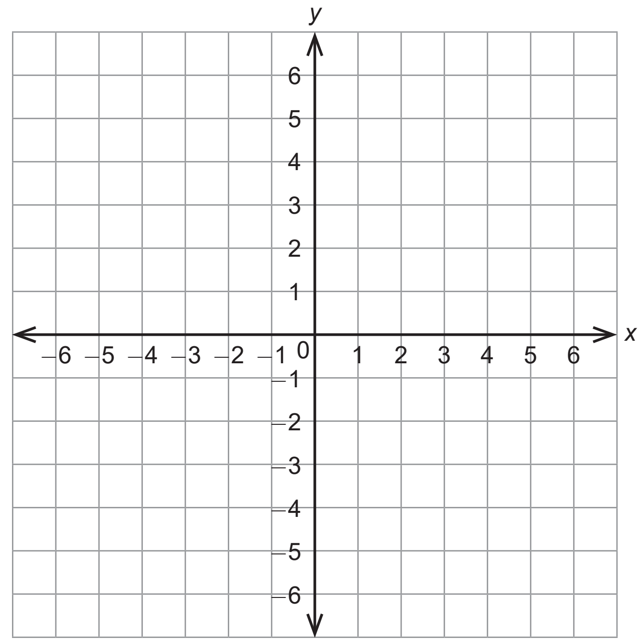
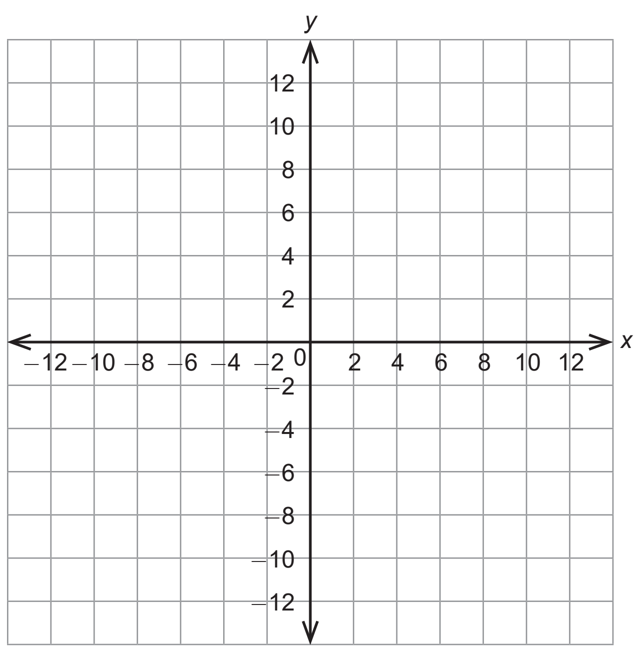
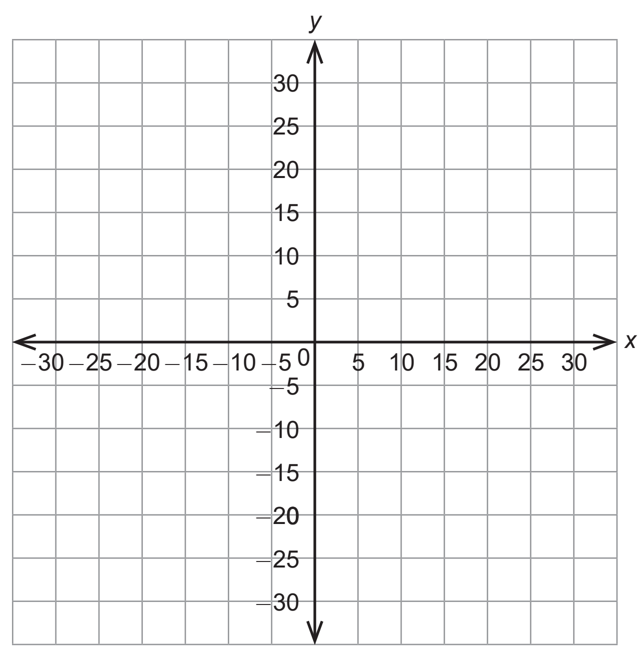
NAME ______________________ DATE ____________

Large Coordinate Grid



NAME ______________________ DATE ______________

Coordinate Grids

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Opposite Operations and Two-Step Equations

1. Jason performs operations starting with secret number x. His result is 37. Write an equation to match how Jason got to 37. Then work backwards to find x.

a) **Jason's operations**

Start with x. x

Multiply by 5. $5x$

Add 7. $5x + 7$

The result is 37. $5x + 7 = 37$

Work backwards to find x

Write the equation again. $5x + 7 = 37$

Undo adding 7 by subtracting 7. $5x + 7 - 7 = 37 - 7$

Write the new equation (simplify). $5x = 30$

Undo multiplying by 5 by dividing by 5. $5x \div 5 = 30 \div 5$

Simplify. You solved for x! $x = 6$

Check your solution by doing the operations in order, the way Jason did them.

Start with your solution: 6 Multiply by 5: 30 Add 7. 37 Did you get 37? yes

b) **Jason's operations**

Start with x. ______

Multiply by 8. ______

Add 5. ______

The result is 37. ______

Work backwards to find x

Write the equation again. ______

Undo adding 5 by subtracting 5. ______

Write the new equation (simplify). ______

Undo multiplying by 8 by dividing by 8. ______

Simplify. You solved for x! ______

Check your solution by doing the operations in order, the way Jason did them.

Start with your solution: ______ Multiply by 8: ______ Add 5. ______ Did you get 37? ______

c) **Jason's operations**

Start with x. ______

Divide by 3. ______

Subtract 23. ______

The result is 37. ______

Work backwards to find x

Write the equation again. ______

Undo subtracting 23 by adding 23. ______

Write the new equation (simplify). ______

Undo dividing by 3 by multiplying by 3. ______

Simplify. You solved for x! ______

Check your solution by doing the operations in order, the way Jason did them.

Start with your solution: ______ Divide by 3: ______ Subtract 23. ______ Did you get 37? ______

NAME ______________________ DATE ______________

Consecutive Number Problems

> Two numbers are **consecutive** if one is the next number after the other.
>
> Examples: 6 and 7 are consecutive integers because 7 is the next integer after 6
> 6 and 8 are consecutive even numbers because 8 is the next even number after 6

1. Fill in the blanks with larger numbers.

a) 4 and ______ are consecutive.

b) 7 and ______ are consecutive.

c) 36, 37, 38, and ______ are consecutive.

d) x, $x + 1$, and __________ are consecutive.

e) 4 and ______ are consecutive even numbers.

f) 7 and ______ are consecutive odd numbers.

g) 16, 18, 20, and ______ are consecutive even numbers.

h) 35, ______, 39, and 41 are consecutive odd numbers.

i) x is even. x, _________, and _________ are consecutive even numbers.

j) x is odd. x, _________, and _________, and are consecutive odd numbers.

2. The sum of two consecutive whole numbers is 35. What are the numbers? Do this problem in two ways.

a) Use a T-table to list all pairs of consecutive whole numbers in order and find the sums. Stop when you reach 35. What are the two numbers?

Two Consecutive Numbers	Their Sum
1, 2	$1 + 2 = 3$
2, 3	$2 + 3 = 5$
3, 4	$3 + 4 = 7$
4, 5	$4 + 5 = 9$
5, 6	$5 + 6 = 11$
⋮	⋮

b) Use algebra.

i) If the smaller of the consecutive numbers is x, write a formula for the other number.
smaller number $= x$
next number $=$ __________

ii) Write an equation using the given information: The sum of the two consecutive numbers is 35.
______________ $= 35$

iii) Solve your equation. What are the two numbers?

3. The sum of two consecutive odd numbers is 32. What are the two numbers? Solve the problem in two ways.

a) Let x be the smaller number.

b) Let x be the larger number.

c) Did you get the same answer both ways? Why?