Unit 3 Patterns and Relations: Linear Relations

Introduction

Topics in this unit include:

- number patterns, linear sequences, and their connection to linear relations;
- linear relations expressed using words, equations, tables, and graphs; and
- analyzing and comparing linear relations expressed in a variety of forms to solve problems.

Meeting Your Curriculum

ALBERTA		
Required	PR7-1 to 4, 6 to 8	
Recommended	PR7-5	review from Grade 6
	PR7-9	review from Units 1 and 3
BRITISH COLUMBIA		
Required	PR7-1 to 4, 6 to 8	including Extension 1 from PR7-2 and Extension 1 from PR7-8
Recommended	PR7-5	review from Grade 6
	PR7-9	review from Units 1 and 3
MANITOBA		
Required	PR7-1 to 4, 6 to 8	
Recommended	PR7-5	review from Grade 6
	PR7-9	review from Units 1 and 3
SASKATCHEWAN		
Required	PR7-1 to 4, 6 to 8	
Recommended	PR7-5	review from Grade 6
	PR7-9	review from Units 1 and 3

Mental Math Minutes

The mental math minutes in this unit:

- practise addition involving two-digit numbers, with and without regrouping
- review Skills 11 to 15 (pp. A-28–29)

Generic BLMs

The Generic BLMs used in this unit are:

BLM 1 cm Grid Paper (p. H-1)
BLM Strategy Talks (p. H-2)

These BLMs can be found in Section H.

Materials

You will need to display coordinate grids (limited to the first quadrant) in many lessons in this unit. One option is to use enlarged copies of the grids from **BLM Small Coordinate Grids** (p. D-67).

BLM Linear Relations Summary (p. D-73–74) summarizes some of the most important concepts, procedures, and rules learned in this unit. It can be used for test preparation after the unit is taught.

Assessment

The lessons covered by a quiz or test are as follows:

	AB	BC	MB	SK
Quiz	PR7-1 to 4	PR7-1 to 4	PR7-1 to 4	PR7-1 to 4
Quiz	PR7-5 to 8	PR7-5 to 8	PR7-5 to 8	PR7-5 to 8
Test	PR7-1 to 4, 6 to 8	PR7-1 to 4, 6 to 8	PR7-1 to 4, 6 to 8	PR7-1 to 4, 6 to 8

Additional Information for This Unit

Students will have an opportunity to practise adding and subtracting integers throughout this unit. Supportive tools such as number lines may benefit some students by reminding them how integer arithmetic works.

The examples and exercises presented in this unit do not require multiplication or division involving negative numbers. While some examples and exercises in the unit can be solved using multiplication involving negative integers, they can also be solved without it. For example, in $250 - 18y$, $y = 3$, while students could calculate $(-18) \times 3$ and add the result (-54) to 250, if they have not learned multiplication involving negative numbers, they can simply calculate 18×3 and subtract the result from 250.

PR7-1
Linear Sequences
AP Book pp. 39–41

Goals

Students will extend sequences by determining the next terms. Students will write a rule to describe how to obtain the terms of a linear sequence.

Main Ideas

A sequence of numbers in which there is a constant difference between successive terms is called a linear sequence. The constant difference in a linear sequence can be a positive or negative number. A unique linear sequence can be described by specifying the starting number and the constant difference.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Can add, subtract, multiply, and divide whole numbers
Can add and subtract integers

Materials

none

Vocabulary

constant difference
linear sequence
non-linear sequence
number pattern
pattern rule
sequence
terms (of a sequence)

Mental Math Minute

Slides 2–3

Skill 11: Deciding if regrouping is required when adding a one-digit number and a two-digit number (p. A-28).

Remind students that when adding a one-digit number to a two-digit number, regrouping is required if the 2 ones digits add to a number greater than 9. Otherwise, no regrouping is required. Work as a class through the examples in the margin, deciding together whether regrouping is required and then performing the addition.

$64 + 3$ (no, 67)
$64 + 5$ (no, 69)
$34 + 6$ (yes, 40)
$34 + 9$ (yes, 43)

Exercises

Do you need to regroup? Add.

a) $53 + 3$ b) $52 + 8$ c) $14 + 9$ d) $68 + 7$

e) $34 + 2$ f) $89 + 2$ g) $25 + 3$ h) $75 + 8$

Answers: a) no, 56; b) yes, 60; c) yes, 23; d) yes, 75; e) no, 36; f) yes, 91; g) no, 28; h) yes, 83

1. Review number patterns

Slides 4–10

Key points: Some number patterns are made by applying the same action to each term to get the next term. Repeating the same action extends the sequence.

Display the sequence in the margin. Use it to remind students that a list of numbers is called a sequence, and that the numbers in the sequence are called the terms of the sequence. Ask students to state the first, fourth, and fifth terms of the sequence.

2, 5, 8, 11, 14, …

Explain that some sequences follow a pattern, so you can determine which numbers come next in the sequence. Such sequences can be called number patterns. Ask students if they see a pattern in the sequence you have been examining.

How do you get from one term to the next in this number pattern? (add 3) What are the next three terms in the sequence? (17, 20, 23)

Ask students what assumption they made to extend the sequence. Point out that they likely assumed that the pattern of adding 3 each time would continue. Emphasize the importance of being aware of any assumptions they make when solving problems.

Discuss what kinds of number patterns students know. Concentrate on different actions that extend patterns in a variety of ways. Have students give examples of different kinds of patterns, and have other students extend them.

Pattern examples

Add 3:	3, 6, 9, 12
Multiply by 2:	3, 6,12, 24
Subtract 10:	108, 98, 88, 78

Then have students present several examples of patterns made by repeating an operation and have other students decide how the number pattern was made.

In the sequence 3, 6, 12, 24, is there a single number you can add or subtract to each term to get the next term? (no) Is there a number you can multiply by or divide by each time to get the next term? (yes, multiply by 2) What are the next three terms in the sequence? (48, 96, 192)

Exercises

1. Use the repeating action given to find the next three terms in the sequence.

   a) add 7 — 13, 20, 27, 34, ___, ___, ___

   b) subtract 9 — 1008, 999, 990, ___, ___, ___

   c) multiply by 5 — 2, 10, 50, ___, ___, ___

   d) divide by 2 — 800, 400, 200, ___, ___, ___

   Bonus: subtract 125 — 285, 160, 35, ___, ___, ___

   Answers: a) 41, 48, 55; b) 981, 972, 963; c) 250, 1250, 6250; d) 100, 50, 25; Bonus: −90, −215, −340

2. The sequence was made by adding or subtracting the same number each time, or by multiplying by or dividing by the same number each time. Determine the operation and the number.

   a) 40, 80, 120, 160, …

   b) 40, 80, 160, …

   c) 1620, 540, 180, …

   d) 840, 680, 520, …

   Bonus: −15, −45, −75, −105

   Answers: a) add 40; b) multiply by 2; c) divide by 3; d) subtract 160; Bonus: subtract 30

2. Exploring constant differences in linear patterns — Slides 11–17

Key points: Some number patterns have a constant difference between successive terms, while others do not. Only sequences made by adding or subtracting the same number have constant differences, and only these are called linear sequences.

As a class, compare the two sequences shown. Concentrate on the actions that create the sequences. Then have students write two more terms in each sequence.

5, 10, 15, 20, …
5, 10, 20, …

What is the same about these two sequences? (they both start with 5, 10) What is different? (the third terms are not the same) What action do you repeat for each sequence? (for the first sequence, add 5; for the second sequence, multiply by 2)

+5 +5 +5 +5 +5
5, 10, 15, 20, 25, 30

+5 +10 +20 +40
5, 10, 20, 40, 80

Display empty circles hovering between terms in both sequences shown. Explain that the number inside each circle is called the gap between terms. It indicates how much you need to add or subtract to each term to get the next term. Emphasize that the gap includes a sign, which determines whether the number is added or subtracted. Have students signal the signs and numbers that belong in the circles.

Ask students which of the two sequences has a constant difference between terms. Explain that sequences with gaps that are always the same are called *linear sequences*.

Repeat with two sequences made by subtraction and division. In each case, identify the signs and numbers that belong in the circles. Explore the action that repeats.

-8 -8 -8 -8 -8
400 , 392 , 384 , 376 , 368 , 360

-200 -100 -50 -25
400 , 200 , 100 , 50 , 25

> Which of the four sequences have the same gap between successive terms? (the first and third) Which of the sequences are linear sequences? (the ones made by addition or subtraction)

Emphasize that only sequences made by adding or subtracting the same number have constant differences between terms, and only these sequences are called linear sequences. Sequences that do not have a constant difference between terms are called *non-linear sequences*.

Exercises

1. Write the number that is added (with a plus sign) or the number that is subtracted (with a minus sign) to get from each term to the next. Is the sequence linear or non-linear?

   a) 7 , 15 , 23 , 31 , 39

   b) 850 , 775 , 700 , 625

   c) 25 , 50 , 100 , 200

   Answers: a) $+8$, $+8$, $+8$, $+8$; linear; b) -75, -75, -75; linear; c) $+25$, $+50$, $+100$; non-linear

2. Is the sequence linear? Explain why or why not.

   a) 3000, 300, 30, 3

   b) 3000, 2000, 1000, 0

   c) 50, 150, 250, 350

   Bonus: 7, 14, 21, 28, 35, 42, 50

   Answers: a) no, because the difference between successive terms is not constant; for example, the first gap is -2700, while the second gap is -270; b) yes, because there is a constant difference of -1000 between terms; 1000 is subtracted each time; c) yes, because the difference between terms is constant: $+100$; Bonus: no, because between the sixth and seventh term the gap is $+8$, while between other terms the gap is $+7$

Key points: Subtracting a number is the same as adding its opposite. The gap in a linear sequence can be described as a positive or negative number. In an increasing linear sequence, the gap is positive, while in a decreasing linear sequence, the gap is negative.

Ask students to think of two ways they could describe how to obtain the next term in the linear sequence shown. In linear sequences, the gap between terms can be thought of as a positive or negative number. Students can also think of the action as addition: when the gap is negative, simply add the negative number.

27, 21, 15, 9
subtract ___ each time
OR
add ___ each time

Display the linear sequences shown and have students identify whether the sequences are increasing or decreasing.

27, 21, 15, 9
11, 24, 37, 50

> Do the numbers get larger or smaller? (smaller; larger) What is the gap? (-6, $+13$) Which sequence is increasing and which is decreasing? (the sequence with the positive gap is increasing; the sequence with the negative gap is decreasing)

Exercises

1. a) What is the gap of a linear sequence?

   b) Why can you always think of the gap as a number that is added?

   Answers: a) the constant difference between the terms; b) since the gap can be a positive or negative number, for decreasing linear sequences, you simply add the negative gap each time

2. Explain how you can use the sign of the gap to determine whether a linear sequence is increasing or decreasing.

   Answer: if the gap is positive, the linear sequence is increasing; if the gap is negative, the linear sequence is decreasing

3. Write the gap as a positive or negative number. Is the sequence increasing or decreasing?

   a) 202, 199, 196, 193

   b) 188, 197, 206, 215

   c) -38, -31, -23, -15

   d) -1, -2, -3, -4, -5

   Answers: a) -3, decreasing; b) $+9$, increasing; c) $+7$, increasing; d) -1, decreasing

4. Describing a number pattern with a rule

Key point: You can describe a unique linear sequence by specifying the starting number and the constant difference.

Use the examples shown to discuss what is needed to fully describe a linear sequence. Lead students to an understanding that both the gap and the starting number are required.

Can you come up with two different sequences that both start with 11 and have a gap of +5? (no) Is the starting number and gap enough to generate a unique linear sequence? (yes)

10, 15, 20, 25, …
10, 5, 0, −5, −10, …
10, 13, 16, 19, …
15, 20, 25, 30, …
2, 7, 12, 17, …

Explain that stating the starting number and how to obtain each term from the previous term is called a rule for a number pattern. Have students write rules for the five patterns above. Some solutions are shown.

10, 15, 20, 25, … (Start at 10 and add 5 each time.)
10, 5, 0, −5, −10, … (Start at 10 and add −5 each time, Start at 10 and subtract 5 each time.)

Exercises

1. Use the pattern rule to write the first five terms of the sequence.

   a) Start at 5 and add 13 each time.

   b) Start at 13 and add 5 each time.

   c) Start at 100 and add −7 each time.

   Bonus: Start at −50 and subtract −10 each time.

   Answers: a) 5, 18, 31, 44, 57; b) 13, 18, 23, 28, 33; c) 100, 93, 86, 79, 72; Bonus: −50, −40, −30, −20, −10

2. State the starting number and gap for the sequence. Write a rule for the linear sequence.

   a) 150, 171, 192, 113

   b) 299, 309, 319, 329

   Answers: a) 150, +21, start at 150 and add 21 each time; b) 299, +10, start at 299 and add 10 each time

3. Write the starting number and the gap for the sequence. Write the rule for the linear sequence in two ways using addition and subtraction.

   a) 55, 50, 45, 40, 35

   b) 405, 386, 367, 348

   Answers: a) 55, −5, start at 55 and add −5 each time, start at 55 and subtract 5 each time; b) 405, −19, start at 405 and add −19 each time, start at 405 and subtract 19 each time

Bonus: Write the first 5 terms of two different sequences that have the first term 30 and the second term 60. Write the pattern rule for each.

Sample answer: 30, 60, 90, 120, 150 has this rule: start at 30 and add 30 each time; 30, 60, 120, 240, 480 has this rule: start at 30 and multiply by 2 each time

1. a) The following sequence is made by alternating two rules: adding 3 and multiplying by 2. Extend the sequence to find the ninth term.

   4, 7, 14, 17, 34

   b) The following sequences were made by alternating two actions. In each case, identify the actions and extend the sequence to find the ninth term.

   i) 5, 15, 11, 21, 17, 27, 23

   ii) 1600, 800, 1200, 600, 1000, 500

   iii) 2, 10, 30, 38, 114, 122

   iv) −125, −175, −170, −220, −215, −265

   Answers: a) 154, b) i) add 10, subtract 4, 29; ii) divide by 2, add 400, 850; iii) add 8, multiply by 3, 1122; iv) subtract 50, add 5, −305

2. When the gaps between terms of a sequence keep changing, the gaps sometimes form their own number pattern. You can look at the gaps between the gaps to predict the next gap and the next term of the original sequence.

   a) Complete the gap circles to find the sixth term of the number pattern. Complete the table.

   +1 +1 +1 ( )

   +5 +6 +7 +8 ( )

   5 , 10 , 16 , 23 , 31 , ___

Sequence Terms	Gaps	Gaps Between the Gaps
5	+5	+1
10	+6	+1
16	+7	+1
23	+8	
31		

   b) The sequences follow a pattern. Look for a pattern in the Gaps Between the Gaps column of the table to extend the original sequence. Complete the table.

i)

Sequence Terms	Gaps	Gaps Between the Gaps
10		
25		
45		
70		
100		

ii)

Sequence Terms	Gaps	Gaps Between the Gaps
−100		
−102		
−103		
−103		
−102		
−99		

Answers: a) +1, +9, 40

b) i)

Sequence Terms	Gaps	Gaps Between the Gaps
10	+15	+5
25	+20	+5
45	+25	+5
70	+30	+5
100	+35	+5
135	+40	
175		

ii)

Sequence Terms	Gaps	Gaps Between the Gaps
−100	−2	+1
−102	−1	+1
−103	0	+1
−103	+1	+1
−102	+2	+1
−99	+3	
−96		

PR7-2
Variables and Expressions
AP Book pp. 42–43

Goals

Students will learn how variables are used in expressions. Students will substitute a value (limited to integers) for each unknown in an expression and then evaluate the expression.

Main Ideas

A variable is a symbol that represents a number that may be unknown or may change. The value of an algebraic expression depends on the numbers substituted for its variables.

Summary

Mental Math Minute	D-12
1. Writing expressions with variables	D-12
2. Evaluating expressions	D-13
3. Contrasting equations and expressions	D-14
Extensions	D-15

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Can apply the order of operations to evaluate numerical expressions (accounting for brackets, the four operations over whole numbers, and addition and subtraction over integers)

Materials

none

Additional Information

Extension 1 is required in order to cover the British Columbia curriculum.

Vocabulary

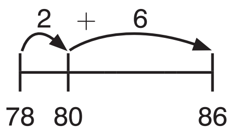
equation
expression
substitute
variable

Mental Math Minute

Skill 12: Adding a one-digit number using the nearest multiple of 10 (pp. A-28–29).

Remind students of the technique of using the nearest multiple of 10 when adding a one-digit number to a two-digit number where regrouping is needed. Use the examples shown.

$78 + 8$:



$78 + 8$: 2 from 8 to 80,
6 more from 80 to 86,
so $78 + 8 = 78 + 2 + 6$
$= 80 + 6$
$= 86$

Exercises

1. What number do you need to add to get to the next multiple of 10? What is the next multiple of 10?

   a) 27 b) 16 c) 62 d) 91 e) 84 f) 33

   Answers: a) 3, 30; b) 4, 20; c) 8, 70; d) 9, 100; e) 6, 90; f) 7, 40

2. Add.

   a) $25 + 6$ b) $43 + 8$ c) $84 + 9$ d) $78 + 9$

   e) $99 + 5$ f) $57 + 9$ g) $33 + 8$ h) $87 + 6$

   Answers: a) 31, b) 51, c) 93, d) 87, e) 104, f) 66, g) 41, h) 93

1. Writing expressions with variables

Key points: A variable is a symbol that represents a number that may be unknown or may change. All rules that apply to numbers in expressions also apply to variables.

Display the example expressions shown. Point out the variables and ask students to explain what a variable is. Remind students that a variable is a symbol such as a letter that represents a number. The number might be unknown or might change.

18	y
$17 + 801$	$(26 - h) \div 7$
$(13 \times 5) + (x - 12)$	$6 \times t$

Explain that all six examples shown are expressions: combinations of numbers and variables with operations and sometimes brackets. An expression can be as simple as a single number or variable, or it can be highly complex. Remind students that a variable is treated like a number in an expression—all rules that apply to a number in an expression also apply to a variable.

Ask students how they would write an expression for the money earned by a worker given the worker's hourly rate and the number of hours they work. Examples and answers are shown.

Hourly Rate	Time	Expression for Money Earned
\$15 per hour	4 hours	15×4
\$12 per hour	t hours	$12 \times t$
\$20 per hour	N hours	$20 \times N$

> How much money do you earn if you get paid \$20 per hour and you work for N hours? ($20 \times N$)

Remind students that multiplication can be shown using brackets, or by writing a number beside a variable. Ask students to rewrite the expressions in the table using these alternative notations. ($15(4)$, $12t$, $20N$)

Exercises

Write an expression for the cost of renting a canoe for the given amount of time.
Write the multiplication two different ways.

a) hourly rate: \$10 per hour
time: 6 hours

b) hourly rate: \$15 per hour
time: x hours

c) hourly rate: \$13.50 per hour
time: m hours

Bonus

d) hourly rate: \$$M$ per hour
time: 3 hours

e) hourly rate: \$$R$ per hour
time: t hours

Sample answers: a) \$10 $\times$ 6, 10(6); b) \$15 $\times$ x, 15x; c) \$13.50 $\times$ m, \$13.50m;
Bonus: d) $M \times 3$, $3M$; e) Rt, tR

2. Evaluating expressions **Slides 9–11**

Key points: Every expression represents a numerical value. If the expression has variables, its value depends on the values of the variables it contains. An expression's value can be determined by substituting the variables with numbers.

Explain that every expression has a numerical value. If the expression has no variables, its value is fixed. If the expression includes variables, then the value of the expression depends on the values of the variables. To find the value of an expression with variables, the first step is to replace, or *substitute*, the variables with numbers. The next step is to perform the operations, following the order of operations.

Display the expressions shown along with the given values of the variables. Evaluate the expressions as a class, showing each step. Remind students about the importance of using brackets when substituting a value for a variable.

$4 + h, \quad h = 9$
$= 4 + (9)$
$= 13$

$15k - 8, k = 4$
$= 15(4) - 8$
$= 60 - 8$
$= 52$

$20 - y, y = -9$
$= 20 - (-9)$
$= 20 + 9$
$= 29$

> How do you find the value of an expression with variables? (substitute the values of the variables in the expression and then perform the operations) In which of these examples was it especially important to use brackets? (the second and the third)

Exercises

Substitute the given value for the variable and evaluate the expression.

a) $7n, n = 100$

b) $3 - h, h = 297$

c) $2x + 8b, x = 14, b = 2$

d) $250 - 18y, y = 3$

e) $x + 24(5), x = -9$

f) $75(2) - q, \ q = -53$

Bonus: $40m + 50n - p, m = 20, n = 2, p = -5$

Answers: a) 700, b) -294, c) 44, d) 196, e) 111, f) 203, Bonus: 905

3. Contrasting equations and expressions

Slides 12–20

Key points: An equation consists of two expressions joined together with an equal sign. It represents a statement of equality that will be either true or false, depending on what numbers are substituted for variables.

Remind students that an equation is two expressions joined together with an equal sign. An expression cannot contain an equal sign; it can contain only numbers, variables, operations, and brackets. An expression represents a numerical value, whereas an equation represents a statement of equality. An equation will be either true or false, depending on which numbers are substituted for the variables.

Work through the examples shown as a class. Highlight the similarity between the words "equal" and "equation" as an aid for students in remembering which of the words "expression" and "equation" refers to the statement involving an equal sign.

Expression	**Equation**
$4x + 7$	$4x + 7 = 15$
When $x = 1$,	
$4(1) + 7$	
$= 4 + 7$	
$= 11$	$11 = 15$ is false
When $x = 2$,	
$4(2) + 7$	
$= 8 + 7$	
$= 15$	$15 = 15$ is true

What is the difference between "$4x + 7$" and "$4x + 7 = 15$"? (the first is an expression that represents a numerical value, the second is an equation: a statement of equality which will be either true or false depending on which number you substitute for x) What happens when you substitute the value of 1 for x? (the expression evaluates to 11, and the equation says $11 = 15$, which is false) What happens when you substitute the value of 2 for x? (the expression evaluates to 15, and the equation says $15 = 15$, which is true)

Exercises

1. Fill in the blank(s) using "equation" or "expression."

   a) An _______ contains an equal sign.

   b) An _______ represents a numerical value.

   c) An _______ represents a statement of equality that might be either true or false.

   d) An _______ is formed by joining two _______s with an equal sign.

   e) When you substitute numbers for variables into an _______ and perform the operations, you end up with a number.

   f) When you substitute numbers for variables into an _______ and perform the operations, you end up with a statement of equality which will be either true or false.

   Answers: a) equation, b) expression, c) equation, d) equation, expression, e) expression, f) equation

2. a) Substitute numbers for the variables. What is the value of the expression?

   i) $5x - 9, x = 7$ ii) $100 - 4m, m = 20$ iii) $25p - q, p = 2, q = -10$

   b) Substitute the values of the expressions you calculated in part a). Is the equation true or false?

   i) $5x - 9 = 24, x = 7$ ii) $100 - 4m = 20, m = 20$ iii) $25p - q = 40, p = 2, q = -10$

   Answers: a) i) 26, ii) 20, iii) 60; b) i) $26 = 24$ is false, ii) $20 = 20$ is true, iii) $60 = 40$ is false

Solve the first part of the following exercises as a class before having students complete the remaining parts individually.

Exercises

Substitute numbers for the variables on both sides of the equation. Is the equation true or false? Use LS for left side of the equation and RS for right side of the equation.

a) $14 + 3T = 11T - 2, T = 2$ b) $42x + 5 = 90 - x, x = 2$

c) $5M + 75 = 12M + 15, M = 10$ d) $80 - y = 160 + y, y = -40$

Selected solution:

a)
$$\begin{aligned} \text{LS} &= 14 + 3T & \text{RS} &= 11T - 2 \\ &= 14 + 3(2) & &= 11(2) - 2 \\ &= 14 + 6 & &= 22 - 2 \\ &= 20 & &= 20 \end{aligned}$$

Since LS = RS, the equation is true when $T = 2$.

Answers: b) false, c) false, d) true

NOTE: You may need to help students locate the story and video for Extension 1 online.

Extensions

Slides 21–27

1. Search online for a video retelling of the story “Small Number and the Old Canoe.” Watch the video or read the story. Then work with a partner to answer the question.

   a) Let the variable x represent the number of brothers Small Number’s great-grandpa had. What clues in the story provide information about x?

   b) Since the father of Small Number’s grandpa built the canoe with two of his brothers, what can you say about x?

   c) Since Small Number’s grandpa says the three totem poles were each built by one of his uncles, does that mean x is at least 3? Explain.

   d) Is there an upper limit for x given in the story?

   Selected answers: b) x is at least 2 ($x \geq 2$), c) not necessarily, since one of Grandpa’s uncles could have built more than one of the totem poles, or Grandpa could be including maternal uncles; d) no

2. Variables can be used in statements that involve the phrases "less than," "less than or equal to," "great than," or "greater than or equal to." Remember that these words can also be shown with symbols.

Symbol	Meaning in Words
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to

a) Use a variable and symbols to translate each sentence into a mathematical statement.

i) Sara is under 15 years of age.

ii) The temperature today was more than 20°C.

iii) Boris needs to save at least $100 in the summer.

iv) At most, 5 people can fit in Satra's car.

b) The variable x represents an integer. Use the information given to find all the possible values for x.

i) $x \geq -2$, and $x < 3$.

ii) $x > 1$, $x \leq 7$, and x is an even number.

iii) $x < 3$, $x > -4$, and x is not a positive number.

iv) $x > 10$, $x \leq 45$, x is a multiple of 9, and x is odd.

Sample answers: a) i) Let S represent Sara's age. $S < 15$, ii) Let t represent today's temperature. $t > 20$°C, iii) Let B represent the amount of money Boris needs to save. $B \geq \$100$, iv) Let n represent the number of people that can fit in Satra's car. $n \leq 5$.

Answers: b) i) -2, -1, 0, 1, or 2; ii) 2, 4, or 6; iii) -3, -2, -1, or 0; iv) 27 or 45

3. Evaluate the expression for the given values of the variables.

a) $1430a - (560 - 20b) + 40 \div y$ $\quad a = 2, b = 30, y = 8$

b) $-15 + 3(m - 4) - Y + z \div 12$ $\quad m = 13, Y = -21, z = 108$

c) $1200 \div s - q + 160 \div ((10 - r) \times 8)$ $\quad q = -13, r = 5, s = 600$

Answers: a) 2905, b) 42, c) 19

4. Ross babysits once a week. Sometimes he works for 3 hours, sometimes 4 hours, and sometimes 5 hours. Sometimes he gets paid $12 an hour, sometimes $15 an hour, and sometimes $18 an hour.

a) Assign two variables for the changing quantities.

b) Write an expression for the total amount that Ross makes in a week.

c) How much money does he make if he works for 6 hours and gets paid $18 an hour?

Sample answers: a) let h be the number of hours Ross babysits in a week and d be the number of dollars per hour he makes that week; b) Ross makes $h \times d$ or hd dollars in a week; c) $108

PR7-3
Linear Relations
AP Book pp. 44–45

Goals

Students will evaluate formulas that relate term numbers to term values in linear sequences. Students will determine if a pattern is linear by recognizing if the sequence it generates is linear.

Main Ideas

A relation can be represented in many ways, including as a rule written in words, a formula relating the two quantities, or a table. A linear sequence produces a linear relation between the term numbers and term values.

Summary

Curriculum

AB: required

BC: required

MB: required

SK: required

Prior Knowledge

Can substitute values for variables into expressions and equations

Can apply the order of operations to evaluate numerical expressions (accounting for brackets, the four operations over whole numbers, and addition and subtraction over integers)

Can find the gap between terms of a sequence

Materials

none

Vocabulary

equation
expression
formula
input variable
output variable
relation
substitute
term
term number
term value
variable

Mental Math Minute

Slides 2–3

Skill 13: Adding tens and ones separately to add two-digit numbers without regrouping (p. A-29).

Remind students that when regrouping is not needed, it is easy to add two 2-digit numbers by simply adding the tens and ones separately. Practise as a class with the example shown.

$34 + 52$
Add the tens: $30 + 50 = 80$
Add the ones: $4 + 2 = 6$
Add the totals: $80 + 6 = 86$

Exercises

Add the tens, add the ones, and then add the totals.

a) $46 + 32$ b) $54 + 41$ c) $61 + 36$ d) $82 + 17$

e) $35 + 44$ f) $27 + 62$ g) $18 + 51$ Bonus: $23 + 34 + 41$

Answers: a) 78, b) 95, c) 97, d) 99, e) 79, f) 89, g) 69, Bonus: 98

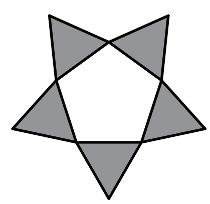
1. Finding formulas for number patterns

Slides 4–14

Key points: A rule for extending a number pattern is different from the pattern's formula. The rule tells you the starting number and the way to obtain the next term from any given term. The formula tells you how to find the value of any term from its term number.

NOTE: A rule for a sequence that tells you the starting number and how to find a term from the previous term is also called a recursive rule. A rule for a sequence that uses a formula to calculate each term from the term number is called an explicit rule. However, you need not teach this terminology to students at this grade level.

Display the shape shown in the margin. Ask students to identify the number of triangles and pentagons used for one figure, and to determine how many of each type of shape would be needed for two, three, or four such figures. Show students how to keep track of the number of pentagons and triangles using a table. A sample is shown. Ask students if they can describe a rule for finding the number of triangles given the number of pentagons.



Number of Pentagons (*p*)	Number of Triangles (*t*)
1	5
2	10
3	15
4	20

How can we write the rule for finding the number of triangles (t) from the number of pentagons (p) as an equation? ($p \times 5 = t$, or $t = 5p$)

Remind students that a *formula* is an equation involving two or more variables that allows you to calculate the value of one variable if you know the value of the others. The relationship between the number of pentagons and triangles in the example can be described in words (multiply the number of pentagons by 5 to find the number of triangles), or as a formula ($t = 5p$) using variables to represent the quantities. In this example, if we are trying to find the number of triangles (t) from the number of pentagons (p), then p is called the input or *input variable* and t is called the output or *output variable*. In a formula, we normally write the output variable all by itself on the left side of the equal sign. Draw students' attention to the difference with the tables, in which the output is on the right, not the left.

Exercises

A shape is made using triangles and squares.

a) Use the table to find a formula to calculate the number of triangles from the number of squares.

i)

Number of Squares (s)	Number of Triangles (*t*)
1	4
2	8
3	12

ii)

Number of Squares (*s*)	Number of Triangles (*t*)
1	12
2	24
3	36

iii)

Number of Squares (*s*)	Number of Triangles (*t*)
7	63
9	81
12	108

b) Use the formula from part a) to find the number of triangles needed for 10 squares.

Bonus: Use the formula from part a) to find the number of triangles needed for 53 squares.

Answers: a) i) $t = 4s$, ii) $t = 12s$, iii) $t = 9s$; b) i) $t = 4(10) = 40$, ii) $t = 12(10) = 120$, iii) $t = 9(10) = 90$; Bonus: i) $t = 4(53) = 212$, ii) $t = 12(53) = 636$, iii) $t = 9(53) = 477$

Display the sequence shown and explain that the sequence is made by applying the same operation repeatedly. Ask students to describe a rule for the sequence by stating the starting number and how to obtain the next term from any given term. Show students how to record the terms of the sequence using a table.

5, 10, 15, 20, 25
Start at 5 and add 5 each time.

Point out that, in a sequence, the term number tells the position of a term in the sequence, while the *term value* tells you what the value is for a particular term. Note that the "term value" is often simply called the "term," but using the phrase "term value" helps to clearly distinguish it from the "term number." For each term number in the example shown, ask students for the corresponding term value.

Term Number (*n*)	Term Value (*v*)
1	5
2	10
3	15
4	20
5	25

Compare the two different ways of describing this sequence: a rule for extending the sequence, which tells you how to find a term using the previous term; and a formula, which tells you how to find a term directly from the term number. In this case, the formula is $v = 5n$.

What is the 6th term in the sequence? (30) Did you use the rule for extending the sequence or the formula to find the 6th term? Why? (sample answer: the rule for extending the sequence, since we already know the 5th term, which makes it easy) What is the 20th term? (100) Did you use the rule for extending the sequence or the formula? Why? (the formula, since extending to the 20th term would take too long)

Repeat this process for the sequence 8, 9, 10, 11, 12. (Start at 8, add 1 each time; $v = n + 7$; 6^{th} term: 13; 20^{th} term: $v = 20 + 7 = 27$)

Exercises

a) Use the rule for extending the sequence to write the first four terms. Fill in the table for the sequence.

i) Start at 4 and add 4 each time.

Term Number (*n*)	Term Value (*v*)
1	
2	
3	
4	

ii) Start at 8 and add 8 each time.

Term Number (*n*)	Term Value (*v*)
1	
2	
3	
4	

iii) Start at 12 and add 1 each time.

Term Number (*n*)	Term Value (*v*)
1	
2	
3	
4	

b) Use the table to find a formula for calculating the term value (v) from the term number (n).

c) Find the 5^{th} term in the sequence and the 25^{th} term in the sequence.

d) In part c), did you use the rule for extending the sequence or the formula? Explain your choice.

Answers: a) i) 4, 8, 12, 16; ii) 8, 16, 24, 32; iii) 12, 13, 14, 15; b) i) $v = 4n$, ii) $v = 8n$, iii) $v = n + 11$; c) i) 5^{th} term: 20, 25^{th} term: $v = 4(25) = 100$; ii) 5^{th} term: 40, 25^{th} term: $v = 8(25) = 200$; iii) 5^{th} term: 16, 25^{th} term: $v = 25 + 11 = 36$

Sample answer: d) I used the rule for extending to find the 5^{th} term in the sequences since I already knew the 4^{th} term. I used the formula for the 25^{th} term in each sequence since it was much faster than extending all sequences to term 25.

2. Introduce relations

Slides 15–23

Key points: A relation—a relationship between two sets of quantities, or variables—can be represented in many ways, including in words, with an equation, and with a table. A (linear) sequence generates a (linear) relation between the term values and term numbers.

Explain that a *relation* is a relationship between corresponding values of two quantities, or variables. Provide some examples (see margin) of two quantities that could be related and ask students to come up with more examples. Ask students for examples of different ways of expressing the relationship between two sets of quantities. (an equation with variables representing the two sets of quantities, a statement of the relationship in words, a table with corresponding values for the two sets of quantities)

- number of pentagons and triangles in shapes
- the term numbers and term values in a sequence
- the number of hours you rent a canoe and the cost
- the number of hours you travel in a car at a constant speed and the distance travelled

Display the sequence shown and as a class fill in a table for the sequence. Explain that any table with corresponding values represents a relation. Moreover, any sequence can be thought of as a relation between the term numbers and term values.

3, 7, 11, 15, 19

Display circles for gaps between terms in the right column of the table, as shown. Explain that circles beside a table can be used to record the gap or difference between terms, just like circles between terms of a sequence can be used to keep track of the gaps. Point out that the term numbers in this table are in counting order, so the gap between the term numbers is always +1. Fill in the gaps in the right column as a class. (+4 in each)

Term Number (*n*)	Term Value (*v*)
1	3
2	7
3	11
4	15
5	19

> What is the gap between the numbers in the right column? (+4, +4, +4, +4) Is the gap always the same number? (yes, +4) Is the relation a linear relation? (yes)

Explain that, as with sequences, when the gap between the term values is always the same, the relation is linear.

Repeat the above process for the sequence 3, 6, 12, 24, 48. Students should see that the relation generated is not a linear relation.

Before assigning the next set of exercises, remind students that for any relation where the first variable can be used to calculate the second, we call the first variable the input variable and the second the output variable. In the case of a sequence, we can call the term number the input and the term value the output.

Exercises

Fill in the gaps for each relation. Is the relation linear?

a)

Input (*A*)	Output (*B*)
1	7
2	10
3	13
4	16

b)

Input (*A*)	Output (*B*)
1	7
2	15
3	23
4	30

c)

Input (*A*)	Output (*B*)
1	80
2	40
3	20
4	10

d)

Input (*A*)	Output (*B*)
1	80
2	60
3	40
4	20

Answers: a) gaps: +3, +3, +3; yes, linear; b) gaps: +8, +8, +7; no, non-linear; c) gaps: −40, −20, −10; no, non-linear; d) gaps: −20, −20, −20; yes, linear

Display the equation shown. As a class fill in a table for the values of A (the input variable) from 1 to 5. With students, complete the table as shown.

> How do you find the values of the output variable, B, for each value of the input variable, A? (substitute each value of A in the expression $10 - 2A$) What are the gaps between the values of B? (-2, -2, -2, -2) Is the relation linear? (yes)

$B = 10 - 2A$

Input (A)	Output (B)	Gap
1	$10 - 2(1) = 10 - 2 = 8$	
		-2
2	$10 - 2(2) = 10 - 4 = 6$	
		-2
3	$10 - 2(3) = 10 - 6 = 4$	
		-2
4	$10 - 2(4) = 10 - 8 = 2$	
		-2
5	$10 - 2(5) = 10 - 10 = 0$	

Exercises

1. Make a table showing the output for the values 1 to 4 of the input variable. Find the gaps between the values of the output variable. Is the relation linear?

   a) $B = 20 - A$

   b) $y = 3n + 2$

   Answers

   a) yes, linear

Input (A)	Output (B)	Gap
1	19	
		-1
2	18	
		-1
3	17	
		-1
4	16	

   b) yes, linear

Input (n)	Output (y)	Gap
1	5	
		$+3$
2	8	
		$+3$
3	11	
		$+3$
4	14	

2. Make a table showing the term values when the term number is 1 to 4. Find the gaps between the term values. Is the relation linear?

   a) Start at 4 and multiply by 4 each time.

   b) Multiply the term number by 4 to get each term.

   Answers

   a) no, non-linear

Term Number	Term Value	Gap
1	4	
		$+12$
2	16	
		$+48$
3	64	
		$+192$
4	256	

   b) yes, linear

Term Number	Term Value	Gap
1	4	
		$+4$
2	8	
		$+4$
3	12	
		$+4$
4	16	

Bonus: Does the equation $B = A \times A$ represent a linear relation?
Hint: Make a table for the values 1 to 4 of the input variable.

Answer: no

Extensions

Slides 24–27

NOTE: Extension 1 is a prerequisite for Extension 2.

1. A family is having a party. This is the formula for the number of chairs they will need for the party: $g + 4 = c$.

   The variable *g* represents the number of guests and *c* is the total number of chairs needed.

   a) How many people are in the family?

   b) How many chairs do they need if the number of guests is …

   i) 5? ii) 9? iii) 0? iv) 1? v) 21?

   c) The family has 15 chairs. How many guests could they have at the party?

   Answers: a) 4, b) i) 9, ii) 13, iii) 4, iv) 5, v) 25; c) 11

2. A family with a baby is having a party. This is the formula for the number of chairs they will need for the party: $g + f - 1 = c$.

   The variable *g* is the number of guests, *f* represents the number of family members, and "$-$ 1" represents the baby, who does not need a chair.

   a) If there are 10 guests and 5 family members, how many chairs will they need?

   b) A different family with a baby has 10 chairs, how many guests and how many family members could be at this party?

   Answers: a) $10 + 5 - 1 = 15 - 1 = 14$; b) Since $g + f - 1 = 10$, that means $g + f = 11$ in this case, so the possible values are:

g	0	1	2	3	4	5	6	7	8	9	10
f	11	10	9	8	7	6	5	4	3	2	1

3. The relation between the variables *x* and *y* is given by the equation. Is the relation linear?

   Hint: Make a table and find the value of *y* when *x* is 1, 2, 3, or 4. Once you substitute the value for *x* in the equation, you can guess and check to find the value of *y*.

   a) $5 + x = 10 - y$ b) $x + y = 10$ c) $x \times y = 60$

   Answers: a) yes, b) yes, c) no

PR7-4
Formulas for Linear Relations

AP Book pp. 46–49

Goals

Students will represent linear sequences using formulas for finding the term value from the term number. Students will analyze whether oral and written patterns are linear in nature and will use linear relations to solve problems.

Main Ideas

For all linear sequences, term *n* of the sequence is equal to the product of the gap and *n*, combined by addition or subtraction with an offset. The offset can be found by comparing the gap and the first term. Linear relations can be used to solve real-world problems.

Summary

Curriculum

AB: required

BC: required

MB: required

SK: required

Prior Knowledge

Can substitute values for variables into expressions and equations

Can apply the order of operations to evaluate numerical expressions (accounting for brackets, the four operations over whole numbers, and addition and subtraction over integers)

Can find the gap between terms of a sequence

Can find the formula for a simple linear relation involving only multiplication

Materials

none

Vocabulary

coefficient

equation

expression

input variable

magnitude

opposite integer

output variable

substitute

term number

term or term value

variable

Mental Math Minute

Slides 2–3

Skill 14: Adding tens and ones separately to add two-digit numbers with regrouping (p. A-29).

Remind students that even when regrouping is needed, one way to add two 2-digit numbers is by adding the tens and ones separately. Practise as a class with the example shown.

$38 + 56$
Add the tens: $30 + 50 = 80$
Add the ones: $8 + 6 = 14$
Add the totals: $80 + 14 = 94$

Exercises

Add the tens. Add the ones. Add the totals.

a) $54 + 27$ b) $33 + 37$ c) $24 + 66$ d) $29 + 38$

e) $35 + 45$ f) $15 + 79$ g) $45 + 81$ h) $86 + 28$

Answers: a) 81, b) 70, c) 90, d) 67, e) 80, f) 94, g) 126, h) 114

1. Finding formulas for simple linear sequences

Slides 4–12

Key point: For linear sequences in which the gap is the same as the starting number, the term value can be found by multiplying the gap by the term number.

Display the sequence rule shown. As a class, fill in the table for the first 5 terms of the sequence and find the gap between terms. (always $+4$) Ask students how they could have used the sequence rule to predict the gap will always be $+4$. (the rule says to add 4 each time) Point out that this is a special sequence since the starting number is equal to the gap, which makes it especially easy to find a formula for the sequence. Have students find the formula. If necessary, remind them that n and v should be in their formula. ($v = 4n$)

Start at 4 and add 4 each time.

Term Number (n)	Term Value (v)
1	4
2	8
3	12
4	16
5	20

Add a column in the table and show students how to express the terms in this sequence using repeated addition by applying the sequence rule. Then add a fourth column to write the repeated additions as multiplications.

Term Number (n)	Term Value (v)	Repeated Addition	Multiplication
1	4	4	4×1
2	8	$4 + 4$	4×2
3	12	$4 + 4 + 4$	4×3
4	16	$4 + 4 + 4 + 4$	4×4
5	20	$4 + 4 + 4 + 4 + 4$	4×5

Since the second term of the sequence is $4 + 4$, how do you write the third term as repeated addition? ($4 + 4 + 4$) How can you use multiplication to write the 5^{th} term? (4×5) How can you use multiplication to write the n^{th} term? ($4n$)

Display the formula in two ways, as shown. Explain to students that the table shows a systematic way to find a formula for

term value $= 4 \times$ *term number*
$v = 4n$

calculating the term value from the term number, and a similar approach can be used even for more complicated sequences.

Ask students if this logic only works when the starting number and gap are $+4$, or if it would work for finding the formula for any linear sequence that has the same starting number as the gap. (it works for any linear sequence that has the same starting number as the gap)

Explain that in a formula, the number that is multiplied by a variable is called the *coefficient* of that variable. Ask students what the coefficient of n is in this formula. ($+4$, that is, the gap)

Have students predict what the formula will be for the sequences shown without making a table. The answers are displayed as well.

Start at 5 and add 5 each time.
($v = 5n$)

Start at 9 and add 9 each time.
($v = 9n$)

Start at 127 and add 127 each time
($v = 127n$)

Exercises

1. a) Complete the table for the sequence with this rule: Start at 3 and add 3 each time.

Term Number (n)	Term Value (v)	Repeated Addition	Multiplication
1		3	3×1
2		$3 + 3$	3×2
3			
4			
5			

b) Write an expression for the given term of the sequence using multiplication.

i) 4 ii) 10 iii) 126 iv) n

c) Write a formula to find the term value (v) from the term number (n).

d) Use the sequence rule, repeated addition, and multiplication to explain why the coefficient of n in the formula is equal to the gap of the sequence.

Selected answers: b) i) 3×4, ii) 3×10, iii) 3×126, iv) $3 \times n$ or $3n$; c) $v = 3n$

Sample answer: d) The sequence rule says to add 3 each time to get the next term. So this gap of 3 is added repeatedly as many times as the term number, or multiplied by the term number, to get each term.

2. Write a formula to find the term value (v) from the term number (n) just by looking at the sequence rule, without creating a table.

a) Start at 2 and add 2 each time.

b) Start at 8 and add 8 each time.

c) Start at 1000 and add 1000 each time.

Bonus: Start at G and add G each time.

Answers: a) $v = 2n$, b) $v = 8n$, c) $v = 1000n$, Bonus: $v = Gn$

Key points: For increasing linear sequences, the gap equals the coefficient of the term number (input variable). Term n of the sequence is equal to the product of the gap and n plus (or minus) an offset. The offset can be found by comparing the gap and the first term (or the product "gap $\times$ term number" to the term value for any known term).

Display the sequence rule shown below and ask students how the rule is similar but different from the previous rule: "Start at 4 and add 4 each time." (same gap, but the starting number is now 1 higher) Display an empty version of the table shown. As a class, fill in the first two columns for the rule shown.

Rule: Start at 5 and add 4 each time.

Term Number (n)	Term Value (v)	Repeated Addition	Multiplication
1	5	$4 + 1$	$(4 \times 1) + 1$
2	9	$(4 + 4) + 1$	$(4 \times 2) + 1$
3	13	$(4 + 4 + 4) + 1$	$(4 \times 3) + 1$
4	17	$(4+ 4 + 4 + 4) + 1$	$(4 \times 4) + 1$
5	21	$(4 + 4+ 4 + 4 + 4) + 1$	$(4 \times 5) + 1$

Formula: $v = 4n + 1$

Then show students how to write the first term as the gap plus another number, in this case, 1. As a class, fill in the third column of the table, reminding students that they need to add the gap to each term to get the next term.

Complete the fourth column of the table as a class, reminding students that repeated addition can be written as multiplication. As you fill in the fourth column, place brackets around the repeated addition in the third column to make the product more visible. Have students write the formula for the sequence. ($v = 4n + 1$)

> What is the gap in this sequence? (+4) How can you write the starting number, 5, as the gap plus some number? ($5 = 4 + 1$) Can you use the table to find a formula for this sequence? ($v = 4n + 1$)

Now consider with students a similar sequence, this one with the rule "Start at 3 and add 5 each time." Start with an empty version of the table shown on the next page and fill in the first two columns. Ask students how they could express the first term (3) as the gap (5) minus another number. ($3 = 5 - 2$) As a class, use this to fill in the rest of the table. Have students write the formula for the sequence.

Rule: Start at 3 and add 5 each time.

Term number (n)	Term value (v)	Repeated Addition	Multiplication
1	3	$5 - 2$	$(5 \times 1) - 2$
2	8	$(5 + 5) - 2$	$(5 \times 2) - 2$
3	13	$(5 + 5 + 5) - 2$	$(5 \times 3) - 2$
4	18	$(5 + 5 + 5 + 5) - 2$	$(5 \times 4) - 2$
5	23	$(5 + 5 + 5 + 5 + 5) - 2$	$(5 \times 5) - 2$

Formula: $v = 5n - 2$

Remind students that in any formula, the number that is multiplied by a variable is called the *coefficient* of that variable. Ask students to identify the coefficients in the two formulas you have explored. (4, 5)

Ask students what they notice about the gaps (4, 5) and the coefficients (4, 5) in each of these sequences. (they are always equal) Discuss why the gap equals the coefficient. Emphasize that the gap is the number being added each time, and so it is added as many times as the term number, and this repeated addition amounts to multiplication by the term number.

Using the fourth column of the tables, ask students if the number that needs to be added or subtracted to the product "gap $\times$ term number" is always the same, and how this number could be found. (yes; by subtracting "gap $\times$ term number" from the term value)

Exercises

Consider the sequence with this rule: Start at 2 and add 8 each time.

a) Write the first five terms of the sequence.

b) What is the gap for this linear sequence?

c) Write the first term value as the gap minus another number.

d) Complete the table for the sequence.

Term number (n)	Term value (v)	Repeated Addition	Multiplication
1	2	$8 - 6$	
2		$(8 + 8) - 6$	$(8 \times 2) - 6$
3			
4			
5			

e) Write an expression for the given term of the sequence using multiplication.

i) 5 ii) 6 iii) 100 iv) n

f) Write a formula to find the term value (v) from the term number (n).

Selected answers: a) 2, 10, 18, 26, 34; b) +8, c) $2 = 8 - 6$, e) i) $(8 \times 5) - 6$, ii) $(8 \times 6) - 6$, iii) $(8 \times 100) - 6$, iv) $8n - 6$; f) $v = 8n - 6$

Complete the first part of the exercise below as a class before having students complete the rest individually.

Exercises

Complete the table by finding the gap of the linear sequence first, and then calculating gap $\times$ n. Then write the number that must be added to (or subtracted from) "gap $\times$ n" to get the term value (v). Fill in the blanks to describe in words how to get the term value from the term number, and then write a formula for the sequence.

a)

Term Number (n)	**Gap $\times$ n**	**Term Value (v)**
1		5
2		8
3		11

Add ___

Multiply by ___ and then add ___.

Formula: ___________.

b)

Term Number (n)	**Gap $\times$ n**	**Term Value (v)**
1		9
2		15
3		21

Add ___

Multiply by ___ and then add ___.

Formula: ___________.

c)

Term Number (n)	**Gap $\times$ n**	**Term Value (v)**
1		3
2		11
3		19

Subtract ___

Multiply by ___ and then Subtract ___.

Formula: ___________.

d)

Term Number (n)	**Gap $\times$ n**	**Term Value (v)**
1		1
2		3
3		5

Subtract ___

Multiply by ___ and then subtract ___.

Formula: ___________.

Answers

a) gap: $+3$; gap $\times$ n: 3, 6, 9; add 2; multiply by 3 and then add 2; $v = 3n + 2$

b) gap: $+6$; gap $\times$ n: 6, 12, 18; add 3; multiply by 6 and then add 3; $v = 6n + 3$

c) gap: $+8$; gap $\times$ n: 8, 16, 24; subtract 5; multiply by 8 and then subtract 5; $v = 8n - 5$

d) gap: $+2$; gap $\times$ n: 2, 4, 6; subtract 1; multiply by 2 and then subtract 1; $v = 2n - 1$

3. Finding formulas for decreasing linear sequences **Slides 27–31**

Key points: For all linear sequences, the gap equals the coefficient of the term number (input variable). In decreasing linear sequences, term n is equal to the product of the magnitude of the gap and n subtracted from an offset.

Explain that you are now going to consider a decreasing linear sequence, where the same amount is subtracted each time. Display an empty version of the table shown on the following page. As a class, fill it in one column at a time and find the formula for the sequence. In the column for repeated addition, finding the first repeated addition might not be obvious. You have to think about how to get to the term value by subtracting 2.

Explain that subtracting 2 each time means the number being subtracted increases by 2 each time. Have students find a formula for the sequence.

Start at 20 and subtract 2 each time.

Term Number (n)	Term Value (v)	Repeated Addition	Multiplication
1	20	$22 - 2$	$22 - (2 \times 1)$
2	18	$22 - (2 + 2)$	$22 - (2 \times 2)$
3	16	$22 - (2 + 2 + 2)$	$22 - (2 \times 3)$
4	14	$22 - (2 + 2 + 2 + 2)$	$22 - (2 \times 4)$
5	12	$22 - (2 + 2 + 2 + 2 + 2)$	$22 - (2 \times 5)$

$v = 22 - 2n$

> What is the gap in this sequence? (-2) How can you write the starting number, 20, as some number minus 2? ($20 = 22 - 2$) Use the table to find a formula for this sequence. ($v = 22 - 2n$)

Point out that the gap is -2, but the formula could be found by multiplying n by 2 (the magnitude of -2), and then subtracting the result from 22. Discuss the similarities and differences between this formula and the formula for an increasing linear sequence. (similarities: the coefficient of n is equal to the gap in both cases; differences: in a decreasing linear sequence (like this one), you subtract the product of the magnitude of the gap and n from a number, whereas in an increasing sequence you add to or subtract from this product)

Exercises

Consider the sequence given by this rule: Start at 40 and subtract 5 each time.

a) How can you write the starting number as some number minus 5?

$40 = \_\_\_ - 5$

b) Complete the table for the sequence.

Term Number (n)	Term Value (v)	Repeated Addition	Multiplication
1	*40*	$\_\_\_ - 5$	$\_\_\_ - (5 \times 1)$
2		$\_\_\_ - (5 + 5)$	$\_\_\_ - (5 \times 2)$
3			
4			
5			

c) Write a formula to find the term value (v) from the term number (n). $v = \_\_\_\_\_\_\_\_$

Selected answers: a) $40 = 45 - 5$, c) $v = 45 - 5n$

Key point: A (linear) written or oral pattern can be represented by a (linear) relation between the term numbers and corresponding term values to solve problems.

Display the problem shown. Challenge students to write the first two sentences as a pattern rule. (Start at 8 and add 3 each time.) Discuss how expressing the problem with this rule makes it clear that the pattern is a linear pattern.

A plant is 8 cm tall at the end of one week.
It grows 3 cm more each week.
How tall will it be at the end of four weeks?

Ask students what approach they would use to answer the question. Explain that many approaches are possible, but that you will start by creating a table. Display an empty version of the table shown and fill in the rows as a class.

Week Number (n)	Plant Height (h)
1	8 cm
2	11 cm
3	14 cm
4	17 cm

> How could you use the table to answer the question? (look at the height in the row after 4 weeks, which is 17 cm)

Display the follow-up problem shown in the margin and ask students if they would extend the table to answer the question. (no, it would take too long) Ask students how they could find a formula to calculate the plant height, h, in terms of the week number, n. Have students write the formula for the sequence, and then find the height of the plant at the end of 40 weeks. ($h = 3n + 5$; when $n = 40$, $h = 3(40) + 5 = 120 + 5 = 125$ cm)

Assume the plant continues to grow 3 cm each week.
How tall would it be at the end of 40 weeks?

Discuss why the sentence "Assume the plant continues to grow 3 cm each week" is important in this problem. For example, the growth of the plant might follow a linear pattern for some time but then it might change; after some time the plant might start growing faster or slower or it might stop growing. Emphasize once more the importance of being aware of any assumptions made when solving problems.

Display the problem shown in the margin as well as an empty version of the table. As a class, fill in the table. Ask students how they could use the table to answer the question. (look at the number of poles in the row for 3 tipis). Ask students if the relation shown in the table is linear and have them explain why. (yes, because the gap between terms is always $+14$)

Each tipi uses 14 poles.
How many poles do you need for 3 tipis?

Number of Tipis	Number of Poles
1	14
2	28
3	42
4	56

Ask students if they would extend the table or use a formula to find how many poles are needed for 20 tipis and have them explain why. (a formula, because extending the table would take too long) Challenge students to find the formula to calculate the number of poles needed (p) from the number of tipis (t), and to use the formula to find the number of poles for 20 tipis. Remind students that this is a special linear sequence where the starting number and gap are the same. Display the formula and the calculation. ($p = 14t$; $p = 14(20) = 280$)

Exercises

1. Fill in a table for the first four terms of the pattern described. Is the pattern linear?

   a) The population of a new town is 100 people. The population doubles every year.

Year (y)	Number of People (N)
1	
2	
3	
4	

   b) The first house on a street has address number 129. The address numbers decrease by 2 each time.

House Position (p)	Address Number (A)
1	
2	
3	
4	

   c) Haru has $40 saved at the end of Week 1. They save $15 more each week.

Week Number (n)	Money Saved (M)
1	
2	
3	
4	

   d) The volume of water in a large tank is 1000L. Each day half of the water is used up.

Day Number (d)	Volume of Water (V)
1	
2	
3	
4	

   Answers: a) 100, 200, 400, 800; no; b) 129, 127, 125, 123; yes; c) $40, $55, $70, $85; yes; d) 1000L, 500L, 250L, 125L; no

2. For the linear patterns in Exercise 1, write a formula for calculating the output variable from the input variable.

   Answers: b) $A = 131 - 2n$, c) $M = 15n + 25$

3. A bike rental store charges $32 to rent a bike for one day and $10 for each additional day.

   a) Complete the table.

Number of Days (n)	Cost (C)
1	
2	
3	
4	

   b) Is the relation linear?

   c) Write a formula to calculate the cost (C) of renting the bike for the number of days (n).

   d) How much would it cost to rent the bike for …

   i) 10 days? ii) 2 weeks? iii) the month of June?

   Answers: a) $32, $42, $52, $62; b) yes, c) $C = 10n + 22$, d) i) $122, ii) $162, iii) $322

1. a) Complete the table.

Term Number (n)	Term (t)
1	$2^2 - 1^2 =$ ___
2	$3^2 - 2^2 =$ ___
3	$4^2 - 3^2 =$ ___
4	$5^2 - 4^2 =$ ___
5	$6^2 - 5^2 =$ ___
6	$7^2 - 6^2 =$ ___

   b) Use the table in part a) to write a formula for the number in the n^{th} row of the table.

   c) Will $23^2 - 22^2$ be in the 22^{nd} row or the 23^{rd} row? How do you know?

   d) Use your formula to find $23^2 - 22^2$.

   e) Check your answer using a calculator or long multiplication.

   Answers: a) 3, 5, 7, 9, 11, 13; b) $2n + 1$; c) 22^{nd} row, since the second base in the subtraction always matches the term number; d) $23^2 - 22^2 = 2(22) + 1 = 44 + 1 = 45$; e) $23^2 - 22^2 = 529 - 484 = 45$

2. The Fibonacci sequence can be generated using the following rules:
   - The first term is 1.
   - The second term is also 1.
   - To get the third term and all higher terms, add the two previous terms.

   So, the 3^{rd} term is $1 + 1 = 2$, the 4^{th} term is $1 + 2 = 3$, and so on.

   a) Write the first 10 terms of the Fibonacci sequence.

   b) Will the number 80 be a term in the Fibonacci sequence? Explain.

   c) Is the Fibonacci sequence linear?

   d) What do you notice about the gaps in the Fibonacci sequence?

   Answers: a) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55; b) no, the 11^{th} term is $55 + 34 = 89$, which is already higher than 80, and all terms beyond the 11th term will only be larger; c) no; d) if you leave off the first gap (0) they also form a Fibonacci sequence

NOTE: Extensions 3 and 4 should be done in order.

3. A sequence of numbers starts at 1. The gaps in the sequence form a linear pattern: they start at 3 and increase by 2 each time.

   a) Write the first five terms of this sequence.

   b) Find a formula to calculate the term (t) from the term number (n).

   Answers: a) 1, 4, 9, 16, 25; b) $t = n \times n$, or $t = n^2$

4. Consider the sequence 3, 6, 11, 18, 27.

   a) Write the gaps as a sequence. Is there a pattern in the gaps?

   b) Assume this sequence continues with the same pattern in the gaps. Write a rule for the sequence by specifying the starting number and how to obtain the next term from a given term.

   c) Find a formula to calculate the term (t) from the term number (n).

   Answers: a) 3, 5, 7, 9; the gaps start at 3 and increase by 2 each time; b) start at 3 and add a new gap each time; to find the gaps, start at 3 (the gap between the first and the second term), and add 2 to the gap each time; c) $t = n \times n + 2$, or $t = n^2 + 2$

PR7-5
Introduction to Coordinate Grids
AP Book pp. 50–51

Goals

Students will review ordered pairs and plotting ordered pairs as points in the first quadrant of a coordinate grid. Students will write points from a grid as ordered pairs and identify the *x*- and *y*-coordinates.

Main Ideas

An ordered pair can be represented as a point on a grid. The first and second coordinates of an ordered pair correspond to the distances from the origin along the horizontal and vertical axes, respectively. Changing the order of the coordinates changes the location of the point.

Summary

Curriculum

AB: recommended
BC: recommended
MB: recommended
SK: recommended

Prior Knowledge

Can use letters as variables to represent numerical values

Can plot numbers on number lines including number lines with labels marked by skip counting

Materials

BLM Small Coordinate Grids (p. D-67, for display)

binders or other dividers

rulers

BLM Small Coordinate Grids (p. D-67)

BLM Grid with Tens (p. D-68)

grid paper or **BLM 1 cm Grid Paper** (p. H-1, see Extension 2)

Vocabulary

axes
axis
coordinate grid
coordinates
first coordinate
grid
ordered pair
origin
scale
second coordinate
***x*-axis**
***x*-coordinate**
***y*-axis**
***y*-coordinate**

Skill 15: Adding two-digit numbers with and without regrouping (p. A-29).

Review the method of adding tens and ones separately to add two-digit numbers with and without regrouping. Use the examples shown.

$46 + 21$
Add the tens: $40 + 20 = 60$
Add the ones: $6 + 1 = 7$
Add the totals: $60 + 7 = 67$

$68 + 56$
Add the tens: $60 + 50 = 110$
Add the ones: $8 + 6 = 14$
Add the totals: $110 + 14 = 124$

Exercises

Add the tens. Add the ones. Add the totals.

a) $63 + 18$ b) $33 + 42$ c) $24 + 64$ d) $49 + 37$

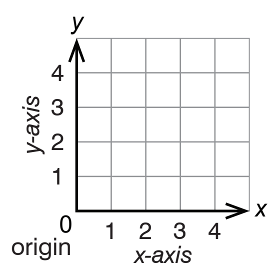
e) $35 + 43$ f) $25 + 59$ g) $55 + 91$ h) $96 + 86$

Answers: a) 81, b) 75, c) 88, d) 86, e) 78, f) 84, g) 146, h) 182

1. Identifying points on a coordinate grid Slides 4–7

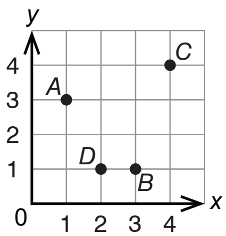
Key points: A coordinate grid includes a horizontal number line (called the horizontal axis, or *x*-axis) and a vertical number line (called the vertical axis, or *y*-axis). Points on the grid-line intersections can be located using an ordered pair. Changing the order of the numbers in an ordered pair changes the location of the point.

Display the coordinate grid shown, but without the labels. Remind students that a grid is a series of evenly spaced horizontal and vertical lines. Geometry and other areas of mathematics use a specific type of grid called a coordinate grid. It has a horizontal number line called the *x-axis* and a vertical number line called the *y-axis*. Point out the axes and mention that *axes* is the plural of *axis*. The point at which the two axes intersect is called the *origin*. Label the axes, the origin, and numbers 0 to 4 on both axes.



Emphasize that both axes start at 0. Explain that, since the origin is the intersection of two number lines, which meet at 0, the origin is often shown with only one 0 at the place where both number lines meet.

Explain that mathematicians around the world have agreed to give the location or address of a point on a grid with two numbers in parentheses, or brackets. The first number tells you how far away a point is from the vertical axis; look for it on the horizontal line, directly below the point. The second number tells you how far away a point is from the horizontal axis; look for it on the vertical line, directly to the left of the point. This means that the numbers in the pair have a specific order, so we call them an ordered pair. Mark the point (2, 4) on the grid and guide students to find the location and label the ordered pair in the correct order. Repeat with the point (4, 2) and use the two examples to emphasize the importance of order in an ordered pair.



Exercise

Write the coordinates of the points shown.

Answer: *A* (1, 3), *B* (3, 1), *C* (4, 4), *D* (2, 1)

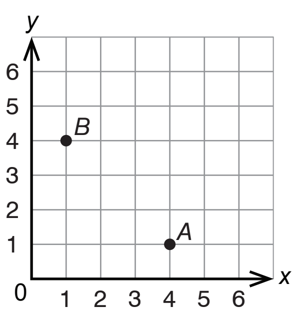
Activity 1: Plotting points in the first quadrant (Essential) Slide 8

Students work in pairs. Each student will require a sheet of grid paper. Ask all students to draw a coordinate grid, labelling it from 0 to 4. Partners use a divider, such as a binder, to conceal their grids from each other. Partner 1 marks a point on the grid and tells Partner 2 its ordered pair. Partner 2 marks the point on their own grid. Partners switch roles several times before checking that their grids match.

2. Working with the *x*- and *y*-coordinates of points Slides 9–13

Key points: The first and second coordinates of a point are called the *x*- and *y*-coordinates of the point, respectively. The *x*-coordinate of an ordered pair corresponds to the distance from the origin along a horizontal line, while the *y*-coordinate corresponds to the distance along a vertical line.

Draw a coordinate grid and label it with numbers 0 to 6 on both axes. Mark the point (4, 1) and have students identify the ordered pair before you display it. Explain that the numbers 4 and 1 in the ordered pair are called the *coordinates* of the point. The first coordinate is also called the *x-coordinate*, so the point (4, 1) has *x*-coordinate 4, which can be written as $x = 4$. Demonstrate that the point is directly above the number 4 on the *x*-axis. Repeat similarly with the *y-coordinate,* or second coordinate, of the point.



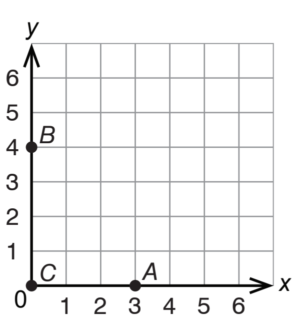
Exercises

Rewrite the coordinates of the point as $x =$ ___, $y =$ ___.

a) (3, 1) b) (2, 3) c) (3, 2) Bonus: (198, 306)

Answers: a) $x = 3$, $y = 1$; b) $x = 2$, $y = 3$; c) $x = 3$, $y = 2$;
Bonus: $x = 198$, $y = 306$

Remind students that the axes intersect at 0. Mark the point (3, 0) on the coordinate grid and have students identify the coordinates. Point out that the *y*-coordinate tells the distance from the horizontal axis, and since the distance is 0, the point must be on the horizontal axis itself. Repeat with the point (0, 4). Then mark the origin and have students identify its coordinates.

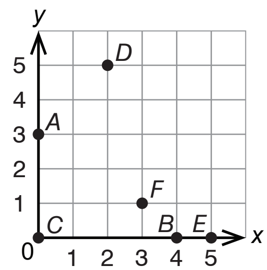


Exercise

Draw a coordinate grid and label it with numbers 0 to 5. Mark and label the points on the grid.

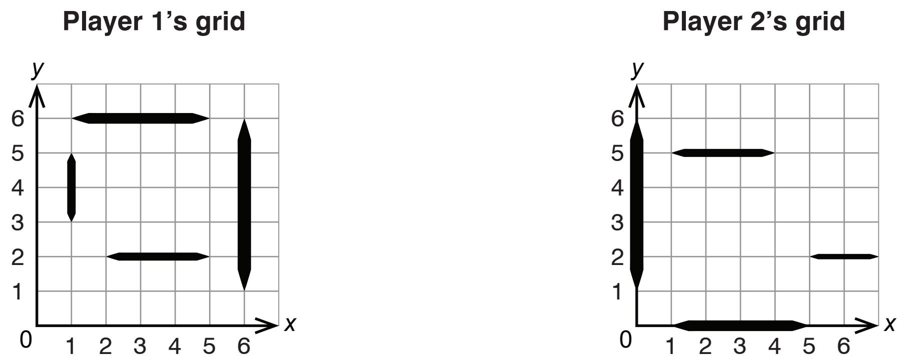
A (0, 3) *B* (4, 0) *C* (0, 0) *D* (2, 5) *E* (5, 0) *F* (3, 1)

Answer:



Activity 2: Ships in the first quadrant (Optional) Slide 14

Pairs of students will play a game of strategy with ships on a grid. Distribute **BLM Small Coordinate Grids** to each student. Students will use the first two grids on the BLM. Without letting their partners see, students draw four non-overlapping horizontal and vertical line segments on the first grid. The four line segments must run along grid lines with endpoints on grid corners, and they must be of the following lengths: 2, 3, 4, and 5 units (see example).

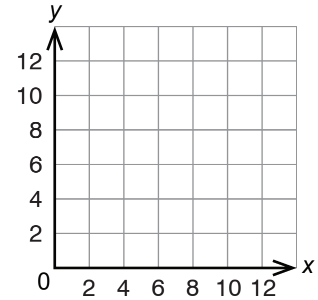


To begin the game, Player 1 guesses a point on their partner's grid by stating its ordered pair. If this point is part of a line segment on Player 2's grid, Player 2 replies "hit"; otherwise, it is a "miss." Player 1 records the guess on the second grid, marking a hit with an X and a miss with an O. If it is a hit, Player 2 also marks an X on that point. Player 2 then has a turn guessing. When all grid points of a player's line segment are crossed out, that player says "one down." (Note that the line segment of length 2 contains 3 grid points, the segment of length 3 contains 4 grid points, and so on.) Play continues until all line segments or "ships" have been found.

3. Using scaled coordinate grids Slides 15–20

Key points: The axes on coordinate grids can have scales made with any numbers. Using larger scales allows for showing larger numbers in a small space but makes it harder to distinguish between numbers that are close together.

Display the grid shown. Explain that just as number lines on graphs can be marked with skip counting, number lines on coordinate grids can be marked by skip counting too. Mark several points and ask students to find the coordinates of these points. Start with points that are on the grid lines, such as (8, 4), continue to points that are on only one grid line, such as (4, 3) and (5, 6), and progress to points that are not on grid lines, such as (9, 7).



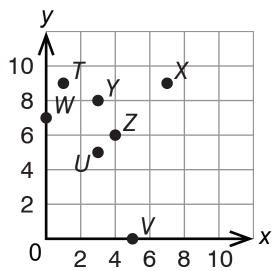
Exercises

a) Draw a coordinate grid with axes that skip count by 2s from 0 to 10.

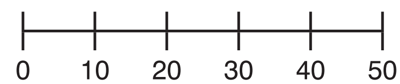
b) Mark and label the points on the grid.

Z (4, 6), *Y* (3, 8), *X* (7, 9), *W* (0, 7), *V* (5, 0), *U* (3, 5), *T* (1, 9)

Answers: a–b)



Display a number line from 0 to 50 but label only the tens, as shown. Point to a few locations on the line and have students identify the number for each location. Write several numbers and point at their locations on the number line. Have students signal with thumbs up or thumbs down to indicate whether the location you are pointing at is correct for the number you wrote.



Display **BLM Grid with Tens**. Use the point *A* (18, 24) to show students how to determine the coordinates of the point by drawing horizontal or vertical lines. Invite volunteers to draw lines from other points to the axes and label it with the coordinates. Point to the locations on the grid for each coordinate pair (for points *B* to *F*) and have students signal thumbs up or thumbs down to indicate whether this point is the one given by the coordinates.

Explain that the number you skip count by on a number line is called the scale. Discuss the advantages and disadvantages of using larger versus smaller scales for grids. By the end of the discussion, emphasize that using larger scales allows for the display of large numbers in a small space but makes it harder to distinguish between numbers that are close together.

If you want to show large numbers in a small space, would a large scale or small scale work better? (large scale) Would a large scale or small scale work better if you need to distinguish clearly between numbers that are close together? (small scale)

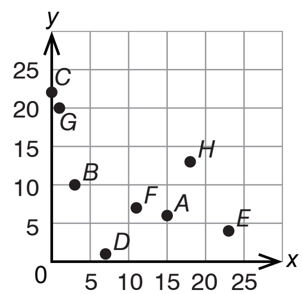
Exercise

Draw a coordinate grid whose axes are marked with intervals of 5 from 0 to 25. Plot these points:

A (15, 6) *B* (3, 10) *C* (0, 23) *D* (7, 1)

E (23, 4) *F* (11, 7) *G* (1, 20) *H* (18, 13)

Answer:



Activity 3: Ships on scaled coordinate grids (Optional) Slide 21

Students repeat Activity 2 in pairs but this time use one of the scaled grids from BLM Small Coordinate Grids. Each partner will need two copies of the BLM.

Extensions Slides 22–27

1. Plot the given points on the scaled coordinate grid marked from 0 to 16, from "Small Coordinate Grids."

   A (6, 4) *B* (5, 1) *C* (3, 1) *D* (1, 12) *E* (3, 12) *F* (4, 6) *G* (5, 9)

   H (7, 9) *I* (8, 6) *J* (9, 12) *K* (11, 12) *L* (9, 1) *M* (7, 1)

   Join the points in the order. Join the first point to the last point. What letter did you make?

   Answer: W

2. Have students play this game in pairs. Pairs will need grids from BLM Small Coordinate Grids, or students can draw their own grids using grid paper or **BLM 1 cm Grid Paper**. They will also need a divider to conceal their coordinate grids from their partners. Students draw a square or a rectangle on the coordinate grid and tell their partner the coordinates of the vertices. Their partner tries to visualize the shape and guess what kind it is before plotting the vertices. Then their partner plots the vertices and checks the answer. Have students switch roles and do it again.

   Advanced variation: Use other shapes, such as rhombuses, parallelograms, and trapezoids.

3. a) Draw a coordinate grid with axes from 0 to 10.

   b) Plot the pair of points.

      i) (2, 0) and (8, 0) ii) (4, 1) and (9, 1) iii) (7, 4) and (0, 4)

   c) Find the distance between the points in the pair.

   d) Which coordinates are the same in each pair, the first or the second? What is the same about the location of the points?

e) Which coordinate is not the same? How can you find the distance between the points using the coordinate that is not the same?

Selected answers: c) i) 6, ii) 5, iii) 7; d) the second coordinates are the same, the points are on the same horizontal line; e) the *x*-coordinates are different, subtract the smaller coordinate from the larger one

4. a) Draw a coordinate grid with axes from 0 to 10.

   b) Plot the pair of points on the coordinate grid.

   i) (3, 1) and (3, 6) ii) (8, 1) and (8, 9) iii) (0, 2) and (0, 8)

   c) Find the distance between the points in the pair.

   d) Which coordinates are the same in each pair, the first or the second? What is the same about the location of the points?

   e) Which coordinate is not the same? How can you find the distance between the points using the coordinate that is not the same?

Selected answers: c) i) 5, ii) 8, iii) 6; d) the first coordinates are the same, the points are on the same vertical line; e) the *y*-coordinates are different, subtract the smaller coordinate from the larger one

PR7-6
Tables and Graphs
AP Book pp. 52–54

Goals

Students will create a table of values for a linear relation and create a graph from the table of values. Students will describe patterns found in a graph to draw conclusions.

Main Ideas

The corresponding values in a table of values form ordered pairs that can be graphed as points. The table represents a linear relation when the graph of the table forms a straight line.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Can use letters as variables to represent numerical values

Can plot ordered pairs as points on a coordinate grid (limited to positive integer coordinates)

Can evaluate an expression with a variable when given the value of the variable

Can determine whether a relation given by a sequence or formula is linear

Materials

playing cards
BLM Small Coordinate Grids (p. D-67, for display)
BLM Plotting Points from Tables (p. D-69)
grid paper or **BLM 1 cm Grid Paper** (p. H-1)
BLM Determining Linearity from Tables and Graphs (p. D-70)
BLM Increasing and Decreasing Relations (p. D-71)
ruler

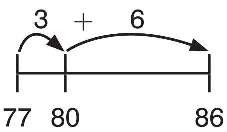
Vocabulary

axis
coordinate grid
coordinates
graph
grid
ordered pair
table of values
x-axis
x-coordinate
y-axis
y-coordinate

Skill 12: Adding a one-digit number using the nearest multiple of 10 (pp. A-28–29).

Remind students of the technique of using the nearest multiple of 10 when adding a one-digit number to a two-digit number where regrouping is needed. Use the examples shown to illustrate this.

$77 + 9$



$77 + 9$: 3 from 77 to 80,
6 more from 80 to 86,
so $77 + 9 = 77 + 3 + 6$
$= 80 + 6$
$= 86$

Use a deck of playing cards to generate random exercises for students to add a two-digit number and a one-digit number. Remove the face cards, tens, and aces from the deck, keeping only cards numbered 2 through 9. Shuffle the remaining cards and draw one card, showing it to students. Students write this number as the tens digit of the two-digit number. Repeat for the ones digit of the two-digit number, and finally for the one-digit number. Students perform the addition, keeping in mind that it might or might not involve regrouping. Repeat 5 to 10 times.

1. Graphing points from a table of values

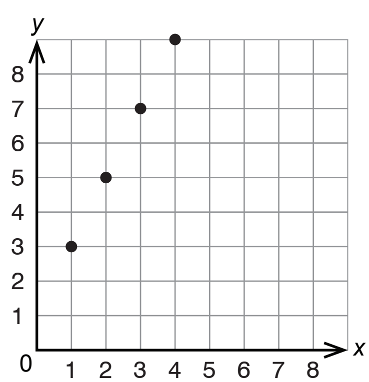
Slides 4–9

Key points: A table of values for a relation shows several values of one quantity and the corresponding values for another quantity. The corresponding values form ordered pairs that can be graphed as points.

Display the table shown without the ordered pair column. Ask students what real world situations this relation might represent. (sample answer: cost of an overseas phone call (in dollars) for a given number of minutes) Explain that, for any relation, the corresponding values can form ordered pairs, with the input being the *x*-coordinate and the output being the *y*-coordinate. Add the ordered pair column and, as a class, fill in the ordered pair for each row. Tell students that a table for a relation is called a *table of values*.

Input (*x*)	Output (*y*)	Ordered Pair
1	3	(1, 3)
2	5	(2, 5)
3	7	(3, 7)
4	9	(4, 9)

Explain that the list of ordered pairs can be plotted as points on a coordinate grid. Display a coordinate grid and plot the points as a class.



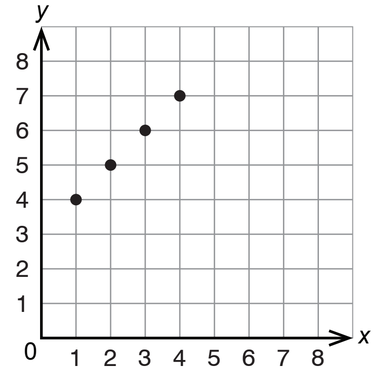
Explain that any relation can be represented with points on a coordinate grid. The collection of points for the relation is called the *graph* of the relation. If the relation is given as a sequence or with a formula, the first step in drawing the graph is to form a list of ordered pairs, which can be done using a table.

Display the sequence rule and the table headings as shown on the next page. In this case, the input is the term number and the output is the term value.

Fill in the table as a class. Have students determine the term values for the first four terms. Then, have students form ordered pairs, and as a class plot the points on a graph. The solutions are shown.

Start at 4 and add 1 each time.

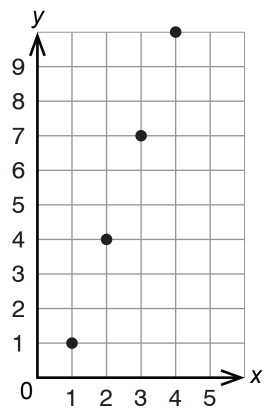
Input (x)	**Output (y)**	**Ordered Pair**
1	4	(1, 4)
2	5	(2, 5)
3	6	(3, 6)
4	7	(4, 7)



As a class, repeat the process of forming a table of values and plotting the points on a coordinate grid for the relation given by the formula $y = 3x - 2$. The solution is shown.

Explain that having the input and output values is the same as having the x- and y-coordinates of the points, so it is not necessary to explicitly list the ordered pairs—if students have a table of coordinate values, they can go directly to graphing. Distribute **BLM Plotting Points from Tables** for the following exercises.

Input (x)	**Output (y)**
1	$3(1) - 2$ $= 3 - 2$ $= 1$
2	$3(2) - 2$ $= 6 - 2$ $= 4$
3	$3(3) - 2$ $= 9 - 2$ $= 7$
4	$3(4) - 2$ $= 12 - 2$ $= 10$



Exercise

Complete BLM Plotting Points from Tables.

Selected Answers: 1. (1, 9), (2, 7), (3, 5), (4, 3); 2. a) 0, 3, 6, 9; b) 10, 8, 6, 4; 3. a) (1, 3), (2, 5), (3, 7), (4, 9); b) 6, 7, 8, 9; c) 7, 5, 3, 1

2. Determining linearity from graphs — Slides 10–14

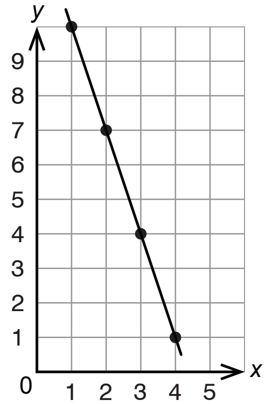
Key point: A table of values represents a linear relation when the graph of the table forms a straight line.

Display the table shown and have students determine if the relation is linear.

In a table of values where the gap between the input values is always the same, how do you check whether the relation is linear? (check if the gap between the output values is always the same) Is the gap between the output values always the same in this relation? (yes, -3)

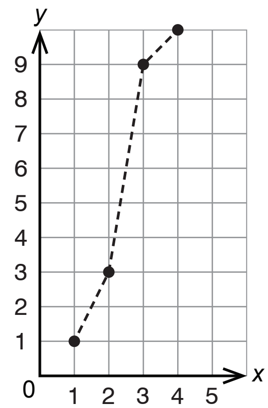
Input (x)	**Output (y)**
1	10
2	7
3	4
4	1

As a class, plot the points for the relation on a coordinate grid. Ask students whether they think the plotted points lie on a single straight line. After taking votes, use a ruler to draw a straight line connecting the four points. The graph is shown.



Repeat the process with the table of values shown. The completed graph is also shown.

Input (*x*)	Output (*y*)
1	1
2	3
3	9
4	10



Is the relation linear? (no) Does the graph of the relation form a single straight line? (no)

Explain that the graph of a linear relation will always form a straight line, while the graph of a non-linear relation will not form a straight line. Point out the connection between the words line and linear.

Distribute grid paper or **BLM 1 cm Grid Paper** and **BLM Determining Linearity from Tables and Graphs** for the following exercises.

Exercise

Complete BLM Determining Linearity from Tables and Graphs.

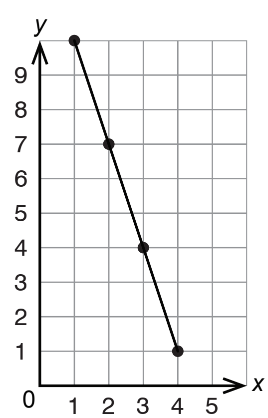
Answers: 1. a) 2, 7, 12, 17; linear; b) 2, 4, 8, 16; not linear; c) 5, 8, 11, 14; linear; d) 16, 12, 8, 4; linear; 2. a) forms a straight line; b) does not form a straight line; c) forms a straight line; d) forms a straight line; 3. A and D, because each of these graphs is a straight line. B and C are not straight lines and so they do not show linear relations.

3. Distinguishing increasing graphs from decreasing graphs **Slides 15–20**

Key point: In the graph of a linear relation, the line moves up from left to right if the gap is a positive number, whereas the line moves down from left to right if the gap is a negative number.

Revisit the linear relation given by the table and graph shown. Lead students to recognize that the graph of a decreasing linear sequence is a line that moves down as it goes from left to right.

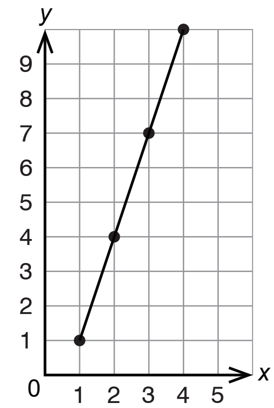
Input (*x*)	Output (*y*)
1	10
2	7
3	4
4	1



Are the output values in the table increasing or decreasing? (decreasing) What are two ways of knowing that the values will decrease? (the output values are getting smaller, the gap is negative) How do the *y*-values on the graph change as you move from left to right? (they also decrease)

Repeat this process with the linear relation given by the table and graph shown.

Input (x)	Output (y)
1	1
2	4
3	7
4	10



How can you tell if a linear relation is increasing or decreasing by using the gap? (a positive gap means increasing while a negative gap means decreasing)
How can you tell if a linear relation is increasing or decreasing by using the graph? (if the line goes up from left to right, the relation is increasing; if the line goes down from left to right, the relation is decreasing)

Provide students with grid paper for Question 3 on **BLM Increasing and Decreasing Relations** in the exercises below.

Exercise

Complete Questions 1 to 3 from BLM Increasing and Decreasing Relations.

Answers: 1. a) decreasing, since the line moves down from left to right; b) increasing, since the line moves up from left to right; c) increasing, since the line moves up from left to right; d) decreasing, since the line moves down from left to right; 2. a) 5, 9, 13, 17; the gap is positive and the relation is increasing; b) 15, 12, 9, 6; the gap is negative and the relation is decreasing; 3. a) straight line, going up from left to right (increasing); b) straight line, going down from left to right (decreasing)

Do the first part of the following exercises as a class before having students complete the rest independently. Show students how to move between points and the corresponding values on each axis to answer the questions.

Exercise

Complete Question 4 from BLM Increasing and Decreasing Relations.

Answers: a) 6, b) 5, c) 2, d) 5.5

1. A relation is given by the following table.

Input (x)	Output (y)
100	200
200	400
300	600
400	801

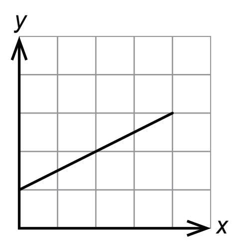
a) Is the relation linear? Explain how you know.

b) Marco makes a graph of the relation on a coordinate grid with a scale of 100 on both axes. Why might it be difficult to tell from this graph whether the relation is linear or not?

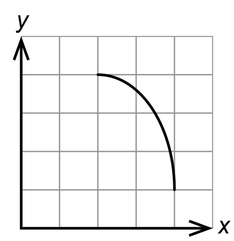
Answers: a) No, because although the input values increase by the same amount, not all output values increase by the same amount. The first two gaps are 200 while the third gap is 201.; b) because the point (400, 801) is very close to (400, 800)

2. The following graphs are missing the labels on the axes to show the scale. Can you still tell which ones are linear? Explain how you know.

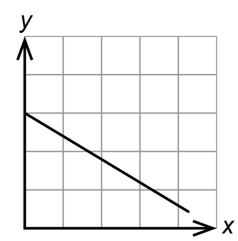
A.



B.



C.



Sample answer: Graphs A and C are linear, while B is non-linear. Regardless of the scales on axes, Graphs A and C are both straight lines, so as x increases by the same amount, y changes by the same amount. This is not true in Graph B, which is not a straight line.

3. a) On grid paper, graph the relation given by the equation $y = x^2 + 2$.

Hint: Make a table of values using the values 0, 1, 2, 3, and 4 for x. Remember, x^2 means $x \times x$.

b) Will the points all lie on a straight line? Is the relation linear?

c) Connect the points with a smooth curve. The curve you drew is called a parabola.

Selected answer: b) no, no

4. Is the sequence 4, 4, 4, 4, … a linear sequence? Explain your answer.

Sample answer: yes, because there is a constant gap of 0 between terms; or, yes, because the graph of the sequence forms a straight (horizontal) line

PR7-7
Analyzing Relations and Graphs
AP Book pp. 55–57

Goals

Students will match a set of linear relations to a set of graphs, and vice versa, and explain their reasoning. Students will describe, using everyday language, the relationship shown on a graph to help them solve problems.

Main Ideas

The gap between successive terms in a linear relation can be seen on its graph as the distance between the *y*-values of successive points. The starting value and the gap can be found on the graph of a linear relation and used to find a formula for the relation.

Summary

Curriculum

AB: required
BC: required
MB: required
SK: required

Prior Knowledge

Can use letters as variables to represent numerical values

Can plot ordered pairs as points on a coordinate grid (limited to positive integer coordinates)

Can evaluate an expression with a variable given the value of the variable

Can determine whether a relation given by a sequence, formula, or graph is linear

Materials

BLM Small Coordinate Grids (p. D-67, for display)

rulers

grid paper, **BLM 1 cm Grid Paper** (p. H-1), or **BLM Small Coordinate Grids** (p. D-67)

BLM Solving Problems with Graphs of Linear Relations (p. D-72)

dice (two per student, see Extension 3)

Vocabulary

axis
coefficient
coordinate grid
coordinates
graph
grid
scale
table of values
x-axis
x-coordinate
y-axis
y-coordinate

Mental Math Minute

Slides 2–3

Skill 13: Adding tens and ones separately to add two-digit numbers without regrouping (p. A-29).

Remind students that, when regrouping is not needed, it is easy to add two 2-digit numbers by simply adding the tens and ones separately. Practise as a class with the example shown.

$24 + 55$
Add the tens: $20 + 50 = 70$
Add the ones: $4 + 5 = 9$
Add the totals: $70 + 9 = 79$

Exercises

Add the tens. Add the ones. Add the totals.

a) $16 + 32$ b) $24 + 41$ c) $61 + 28$ d) $52 + 15$

e) $55 + 44$ f) $37 + 22$ g) $28 + 61$ Bonus: $43 + 32 + 13$

Answers: a) 48, b) 65, c) 89, d) 67, e) 99, f) 59, g) 89, Bonus: 88

1. Finding a coefficient from a graph

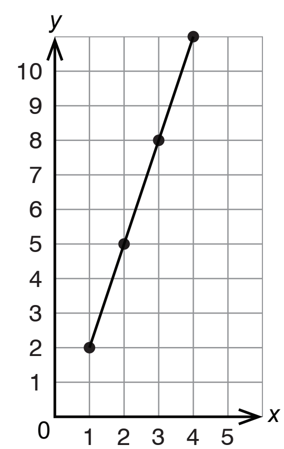
Slides 4–10

Key point: In the graph of a linear relation, if you increase the *x*-coordinate by one, the change in the *y*-value corresponds to the coefficient of *x* in the formula for the relation (which is the gap in a linear sequence).

Display the formula shown and ask students what steps they could take to sketch the graph of this linear relation. Explain that one way is to make a table of values first. Guide students to complete a table of values, plot the points on a coordinate grid, and draw a line with a ruler to connect the points. The solution is shown.

$y = 3x - 1$

Input (x)	Output (y)
1	2
2	5
3	8
4	11



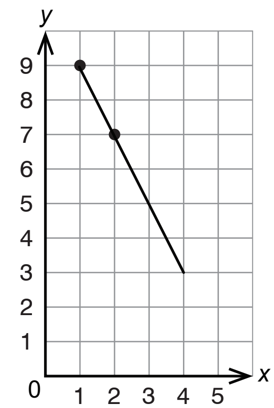
Lead students to notice that the change in *y* as *x* increases by one corresponds to the gap in the table, which is equal to the coefficient of *x* in the formula.

On the graph, how much does *y* increase each time *x* increases by one? (+3) Where do you see this number in the table? (the change in the *y*-value corresponds to the gap) Why would these values be the same? (the values of *y* on the graph are the same as the output values in the table)

Explain that there is a way to sketch the graph of a linear relation using the formula without the need to make a table of values first. All that is needed is the coefficient of *x* and one point on the graph. Sketch the graph of the linear relation once again, but this time start with the first point (where $x = 1$ and $y = 2$) and show students how to plot the next point (where $x = 2$) by changing the *y*-value by adding or subtracting the coefficient of *x*. Once the second point (2, 5) has been plotted, use a ruler to connect the two points with a straight line that extends to match the graph drawn previously. Ask students if this method produced the same graph. (yes)

Display the formula shown and as a class sketch a graph directly from the formula. Involve students in finding the point at $x = 1$ (1, 9), the change in y as x increases to 2 (-2), and the second point on the line (2, 7). Demonstrate again using a ruler to draw a line by carefully connecting the two points.

$y = 11 - 2x$



Provide Students with grid paper, **BLM Grid Paper**, or **BLM Small Coordinate Grids** to complete Exercises 2 and 3 in the following set of exercises.

Exercises

1. Identify the coefficient of x in the formula.

   a) $y = 8x - 2$ b) $y = 3 + 4x$ c) $y = 13 - 7x$

   Bonus

   d) $y = -x + 10$ e) $x + 5 = y$ f) $y = 99 - x + 1$

   Answers: a) $+8$, b) $+4$, c) -7, Bonus: d) -1, e) $+1$, f) -1

2. Consider the linear relation given by the formula $y = 2x + 1$.

   a) What is the coefficient of x in the formula?

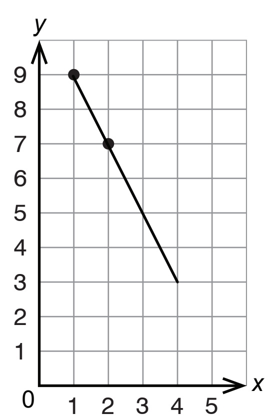
   b) What will be the y-coordinate of the point on the graph with $x = 1$?

   c) How will the value of y change in the graph when x increases by 1?

   d) Sketch the graph of the linear relation.

   Answers: a) 2, b) 3, c) It will increase by 2.

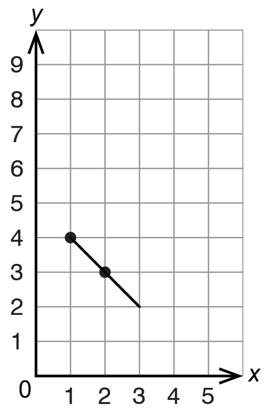
   d)

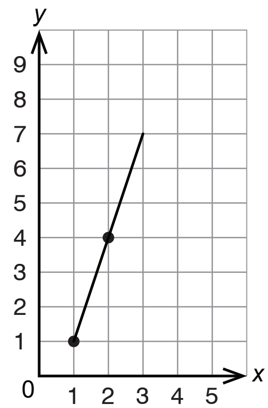


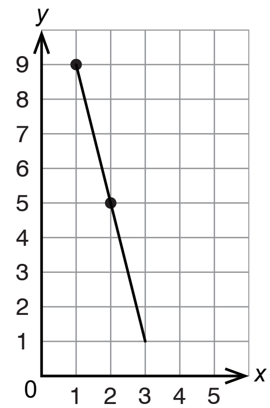
3. Sketch the graph of the linear relation. Start by finding one point on the graph, and then use the coefficient of x to find another point. Use a ruler to draw a straight line connecting the two points.

a) $y = 5 - x$ b) $y = 3x - 2$ c) $y = 13 - 4x$

Answers

a)


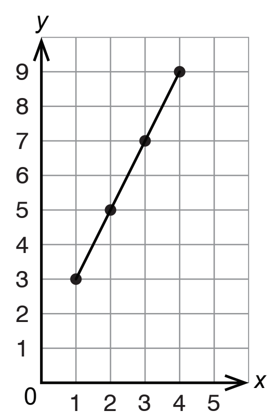
b)


c)


2. Using a graph to find a formula **Slides 11–17**

Key point: Information from the graph of a linear relation (such as the change in the y-value as the x-value increases by 1, and the value of y when x is 1) can be used to find a formula for the relation.

Display the graph shown and as a class make a table of values using the graph. Fill in the table with the values 1 to 4 in the column for x and ask students for the corresponding values of y. Leave the middle column blank for now. Show students how to read the values of y for the given values of x from the graph. The completed table of values is shown in the margin.



Input (x)	Gap $\times$ x	Output (y)
1		3
2		5
3		7
4		9

Ask students to recall how to obtain a formula for a linear relation by using a table of values. Remind students to find the gap between the output values first. (+2) Students should then multiply the gap by the input. Fill in the middle column as a class and have students determine how to use addition or subtraction to obtain the output. (multiply the gap and x, and then add 1) The completed table is shown. Ask students to write a formula for the linear relation. ($y = 2x + 1$)

Input (x)	Gap $\times$ x	Output (y)
1	$2(1) = 2$	3
2	$2(2) = 4$	5
3	$2(3) = 6$	7
4	$2(4) = 8$	9

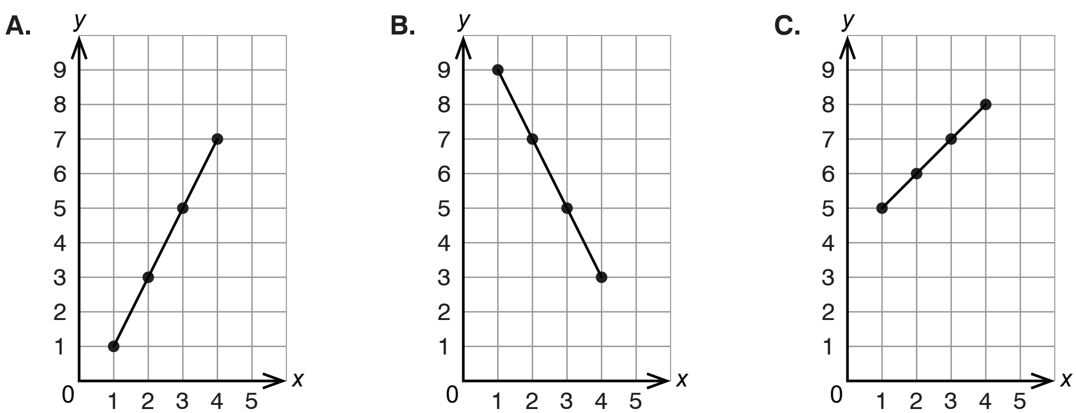
Formula: $y = 2x + 1$

Explain that there is a way to get the formula of a linear relation from its graph without using a table of values. Ask students to recall how to find the coefficient of the formula on the graph. (the change in the value of y as x increases by 1) Demonstrate using the graph displayed. (+2) Ask students to read from the graph what the value of y is when x is 1, and how to obtain that value using addition or subtraction with the coefficient. ($2 + 1 = 3$) Finally, ask students to use this information to write the formula for the linear relation. ($y = 2x + 1$)

Show students how to obtain a linear sequence rule from the graph of this relation of the form "Start at ___ and add/subtract ___ each time." Ask students how to obtain the starting number from the graph. (find the value of y when x is 1, which is 3 in this case) Ask students how to obtain the gap. (find the change in the value of y when x increases by 1, which is +2 in this case) Have students complete the sequence rule. (Start at 3 and add 2 each time.)

Exercise

The graphs of three linear relations are shown.



a) The graphs can represent linear sequences. Use the graphs to find the gap of each linear sequence.

Hint: Look at the change in the value of y as x increases by 1.

b) Use the graphs to find the starting value of each linear sequence.

Hint: Look at the value of y when $x = 1$.

c) Write a sequence rule for each linear sequence of the form "Start at __ and add/subtract __ each time."

d) Combine the gap with another number using addition or subtraction to get the value of y when $x = 1$.

e) Write a formula for each linear relation.

f) Use each graph to make a table of values. Use your table to check your answers to parts a) to e).

Selected answers: a) A. +2, B. −2, C. +1; b) A. 1, B. 9, C. 5; c) A. Start at 1 and add 2 each time; B. Start at 9 and subtract 2 each time; C. Start at 5 and add 1 each time; d) A. $2 - 1 = 1$, B. $11 - 2 = 9$, C. $1 + 4 = 5$; e) A. $y = 2x - 1$, B. $y = 9 - 2x$, C. $y = x + 4$

Key points: Several properties of relations can be used to match relations and graphs, including whether the relation is increasing or decreasing, whether the relation is linear or non-linear, the coefficient of the input variable, and the value of the output variable when the input variable is one.

Display the three tables and three graphs shown below. Explain that each relation shown in the tables is also shown in one of the graphs, and that students need to be detectives to figure out which table matches which graph. Suggest to students to begin by identifying the gaps between the output values. Ask students whether the relation shown in Table I is linear. (yes, because the gap is the same number, $+3$) Then ask students whether Table I shows an increasing or decreasing relation, and to explain how they know. (increasing, since the values go up, or since the gap is a positive number) Students should now be able to identify which graph corresponds to Table I, since only one of the graphs is both increasing and linear.

I.

Input (*A*)	Output (*B*)
1	2
2	5
3	8

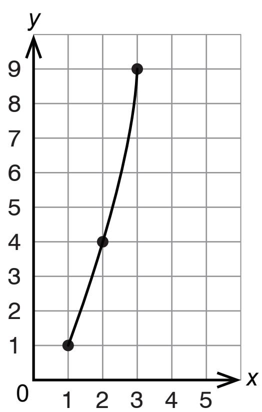
II.

Input (*n*)	Output (*y*)
1	1
2	4
3	9

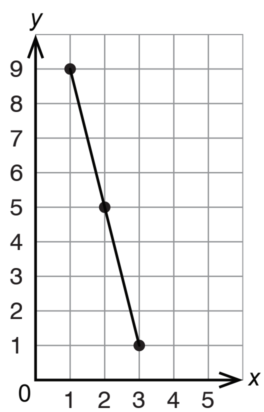
III.

Input (*A*)	Output (*y*)
1	9
2	5
3	1

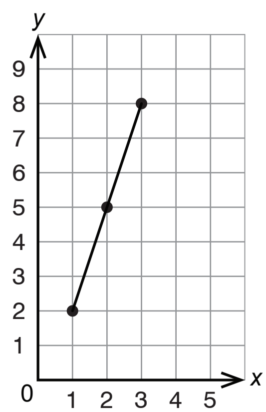
A.



B.



C.



Repeat with Tables II and III. (Table I is linear and increasing and corresponds to Graph C; Table II is non-linear and increasing and corresponds to Graph A; Table III is linear and decreasing and corresponds to Graph B)

Point out that it was possible to identify the correct graphs by simply considering the shapes of the graphs.

Exercises

a) Write the gaps between the output values in the tables.

i)

Input (A)	Output (B)
1	3
2	6
3	9

ii)

Input (n)	Output (y)
1	9
2	4
3	1

iii)

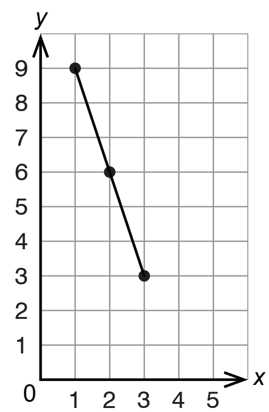
Input (A)	Output (y)
1	9
2	6
3	3

b) Which relations from part a) are increasing?

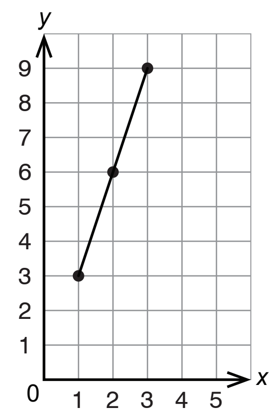
c) Which relations from part a) are linear?

d) Match each relation from part a) to its graph.

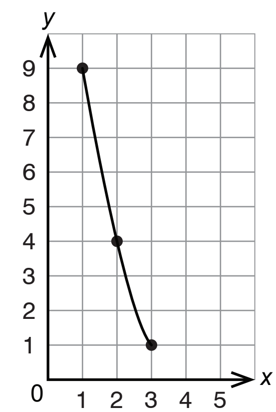
A.



B.



C.



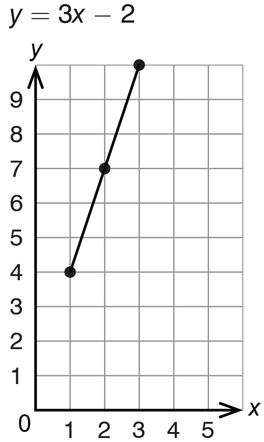
e) Explain how you did the matching in part d).

Answers: a) i) +3, +3; ii) −5, −3; iii) −3, −3; b) i only; c) i and iii; d) i) B, ii) C, iii) A, e) Table i and Graph B show the only increasing relation; Table ii and Graph C show the only non-linear relation; Table iii and Graph A show the only relation that is both linear and decreasing

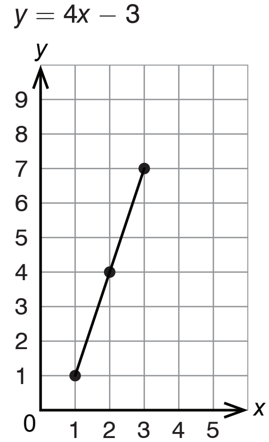
Display the formulas and graphs shown below and explain that this time students will match three formulas to their corresponding graphs. Explain that since all the relations are increasing and linear, students need to pay attention to the coefficients and starting points. Guide students in how to find these values in the formulas and on the graphs.

I. $y = 3x - 2$ **II.** $y = 4x - 3$ **III.** $y = 3x + 1$

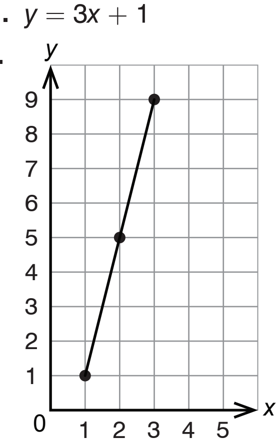
A.



B.



C.



What are the coefficients of x in the formulas? (I. $+3$, II. $+4$, III. $+3$) Which graph shows the coefficient $+4$? How do you know? (Graph C, since the value of y increases by 4 when x increases by 1) How can you tell the other two graphs apart? (by the starting numbers)

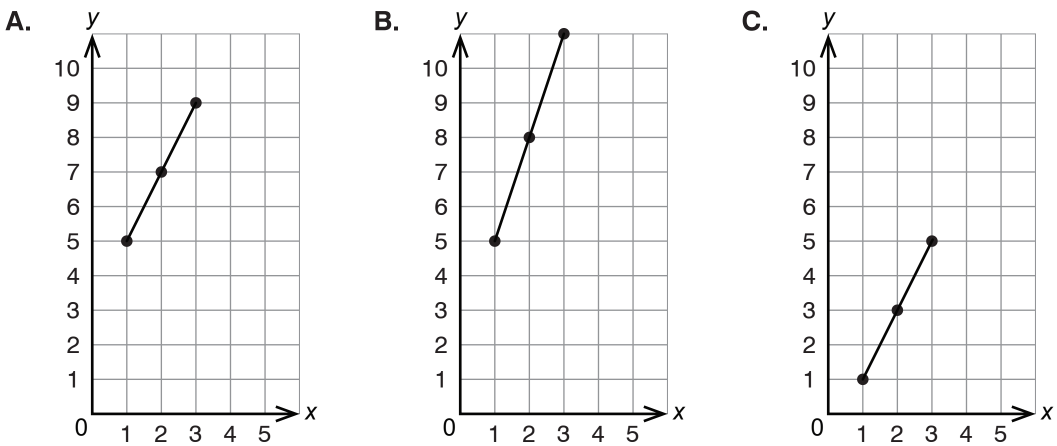
Exercises

a) The formula represents a linear relation. Find the value of y when $x = 1$ for the formula.

i) $y = 2x - 1$ ii) $y = 2x + 3$ iii) $y = 3x + 2$

b) Write the coefficient of x for each formula from part a).

c) Each graph represents a linear relation. Find the value of y when $x = 1$ for each graph.



d) Find the change in y as x increases by 1 in each graph from part d).

e) Each graph in part c) represents a linear sequence that is also represented by a formula in part a). Match the formulas and graphs.

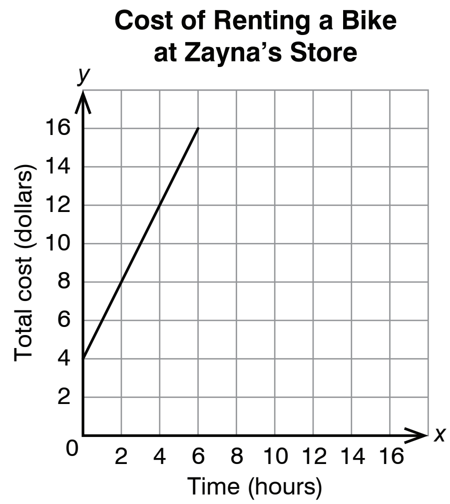
f) Explain how you did the matching in part e).

Answers: a) i)1, ii) 5, iii) 5; b) i) $+2$, ii) $+2$, iii) $+3$; c) A. 5, B. 5, C. 1; d) A. $+2$, B. $+3$, C. $+2$; e) i. C, ii. A, iii. B; f) Formula i and Graph C show the only linear sequence with starting number 1 and gap 2; Formula ii and Graph A show the only linear sequence with starting number 5 and gap 2; Formula iii and Graph B show the only linear sequence with starting number 5 and gap 3

4. Linear relations in the real world

Slides 29–31

Key points: The graph of a linear relation can be extended to find other points that lie on the line. Graphs of linear relations can be used to represent and solve real-world problems.



Display the graph shown and ask students to identify what the graph shows using the title. Have students identify the scale on each axis and what quantity is represented on each axis using the labels. Ask students to describe the graph in words. (the graph is a straight line that goes up from left to right) Explain that linear relations, and their graphs, can represent real-world situations, like in the graph shown.

In **BLM Solving Problems with Graphs of Linear Relations** for the following set of exercises, solve Question 1 as a class before having students solve Question 2 independently. Show students how to extend the line to answer Question 1, part c).

Exercise

Complete BLM Solving Problems with Graphs of Linear Relations.

Answers: 1. a) i. \$8, ii. \$12, iii. \$10, iv. \$14, b) 6 hours, c) \$18; 2. a) 40 m, b) 60 m, c) Amo: 30 s; Rena: 25 s; d) 40 m, because when the race starts at 0 seconds, Amo is 40 m away from the starting line; e) 20 seconds after the start of the race; Bonus: $140 - 110 = 30$ m.

Extensions

Slides 32–37

NOTE: Extension 1 is a prerequisite for Extension 2.

1. Extending the graph of a linear relation so that it touches the y-axis can make it easier to find a formula for the relation.

   a) Graph the relation $y = 3x + 2$. Include the point with x-coordinate 0 on your graph.

   b) What is the value of y when x is 0? Use the formula.

   c) On the graph, what is the value of y when the line touches the y-axis?

   d) The value of y when a line touches the y-axis is called the y-intercept of the line. Where do you see the y-intercept in the formula for the linear relation?

   e) How much does the value of y increase as x increases by 1? Where do you see this number in the formula?

   f) Complete the description of the graph: "The value of y increases by ___ as x increases by 1, starting from the y-intercept of ___."

   Selected answers: b) 2, c) 2, d) the number added to "$3x$," e) by 3; the coefficient of x; f) 3, 2

2. a) Write the y-intercept for each linear relation just by looking at the formula.

   i) $y = 2x + 3$ ii) $y = 4x + 1$ iii) $y = 3x$ iv) $y = 10 - 2x$

   b) Use the coefficient of x and the y-intercept to complete the description of the graph for each relation from part a): The value of y ___ (increases/decreases) by ___ as x increases by 1, starting from the y-intercept of ___.

   c) Sketch the graph of each relation by using your descriptions in part b).

   d) Check your answers in part c) by making a table of values.

   Selected answers: a) i) 3, ii) 1, iii) 0, iv) 10; b) ii) The value of y increases by 4 as x increases by 1, starting from the y-intercept of 1.; iv) The value of y decreases by 2 as x increases by 1, starting from the y-intercept of 10.

3. Students will each need grid paper or BLM 1 cm Grid Paper and two dice. Students will roll the dice five times and record the pairs of numbers they get. For each pair of numbers, students will call the larger number l and the smaller number s. They will write one of two formulas, $l \times n + s$ or $l \times n - s$, on a separate sheet of paper. Tell them that, if the dice show the same number, then l and s will be equal, which is fine. All formulas should be different, so if the same pair of numbers appears a second time, students should use $l \times n - s$ if they used $l \times n + s$ previously, and vice versa. If the same pair of numbers appears a third time, students need to roll again. Students will then graph the sequences given by these formulas on graph paper but should record the formula separately and not in the same order as the graphs. Next, students can swap their formulas and graphs with a partner and match the formulas produced by their partners to their corresponding graphs.

PR7-8
Problems and Puzzles: Linear Relations

AP Book pp. 58–60

Goals

Students will solve problems and puzzles that relate to the concepts developed throughout this unit about linear relations.

Main Ideas

Linear sequences can be represented as linear relations between the term numbers and term values. Linear relations can be represented by rules, formulas, tables, and graphs.

Summary

Curriculum

AB: required

BC: required

MB: required

SK: required

Prior Knowledge

Can use letters as variables to represent numerical values

Can plot ordered pairs as points on a coordinate grid (limited to positive integer coordinates)

Can evaluate an expression with a variable given the value of the variable

Can determine whether a relation given by a sequence or formula is linear

Can use properties of linear relations to match formulas or tables with graphs

Materials

BLM Small Coordinate Grids (p. D-67, for display)

ruler

BLM Strategy Talks (p. H-2)

Additional Information

Extension 1 is required in order to cover the British Columbia curriculum.

Vocabulary

axes

graph

linear

non-linear

number pattern

sequence

term number

term value

Mental Math Minute — Slides 2–3

Skill 14: Adding tens and ones separately to add two-digit numbers with regrouping. (p. A-29)

Remind students that, even when regrouping is needed, one way to add two 2-digit numbers is by adding the tens and ones separately. Practise as a class with the example in the margin.

$48 + 26$
Add the tens: $40 + 20 = 60$
Add the ones: $8 + 6 = 14$
Add the totals: $60 + 14 = 74$

Exercises

Add the tens. Add the ones. Add the totals.

a) $34 + 26$ b) $23 + 38$ c) $14 + 76$ d) $27 + 58$

e) $55 + 29$ f) $15 + 79$ g) $55 + 92$ h) $96 + 68$

Answers: a) 60, b) 61, c) 90, d) 85, e) 84, f) 94, g) 147, h) 164

1. Review finding the formula for a linear sequence — Slides 4–14

Key points: The n^{th} term of a linear sequence is equal to the product of the gap and n, combined by addition or subtraction with an offset. The offset can be found by comparing the product "gap $\times$ n" to the n^{th} term for any known term.

Display the sequence shown above the table. Review the process for finding a formula for the sequence by working with students to complete a table with the three columns headings shown. Fill in the table and gap circles together as a class.

8, 14, 20, 26, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1	$6 \times 1 = 6$	8
2	$6 \times 2 = 12$	14
3	$6 \times 3 = 18$	20

+6
+6

Add 2

> Comparing the second and third columns, what must you add to get the term value (v) from the product of the gap and the term number (n)? (add 2 to gap $\times$ n to get v)

Discuss how to obtain a formula for the sequence from the table, and how to use the formula to calculate the 10^{th} term in the sequence.

> Look across one row of the table. "Which operations could take you from the term number to the term value?" (multiply by 6 and add 2) What is the formula for finding the term value, v, from the term number, n? ($v = 6n + 2$) Using the formula, what is the 10^{th} term in the sequence? ($v = 6(10) + 2 = 60 + 2 = 62$)

Discuss what assumptions were made about the sequence in the problem. Emphasize that to determine the 10^{th} term in the sequence, it was important to assume the sequence would continue the same pattern of increasing by 6 each time.

Repeat this process for the two sequences 2, 9, 16, 23 and 90, 88, 86, 84. Remind students that sometimes, to get from the second column in the table to the third column, you might need to subtract a number or subtract from a number. In the decreasing sequence, where the gap is negative, explain to students that they don't need to multiply n by a negative number, since they can instead multiply by the magnitude of the gap, and then subtract the product from a number.

Exercises

1. Find a formula for the sequence. Then find the value of the 10^{th} term in the sequence.

a) 11, 15, 19, 23, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1		
2		
3		

Add ___

Formula: ___________

10^{th} term: ___________

b) 57, 54, 51, 48, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1		
2		
3		

Subtract from ___

Formula: ___________

10^{th} term: ___________

c) 1, 5, 9, 13, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1		
2		
3		

Subtract ___

Formula: ___________

10^{th} term: ___________

d) 190, 180, 170, 160, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1		
2		
3		

Subtract from ___

Formula: ___________

10^{th} term: ___________

Answers
a) $v = 4n + 7$, 10^{th} term: $4(10) + 7 = 40 + 7 = 47$
b) $v = 60 - 3n$, 10^{th} term: $60 - 3(10) = 60 - 30 = 30$
c) $v = 4n - 3$, 10^{th} term: $4(10) - 3 = 40 - 3 = 37$
d) $v = 200 - 10n$, 10^{th} term: $200 - 10(10) = 200 - 100 = 100$

2. What assumption did you need to make in Exercise 1 about the sequences? Explain.

Sample answer: I needed to assume that the sequences continue with the same gap so that I could write a formula and determine the 10^{th} terms of the sequences.

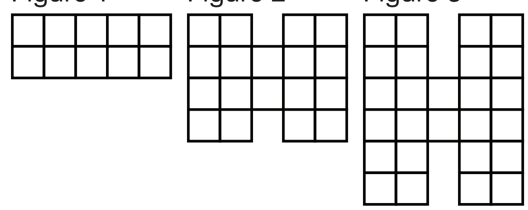
2. Solving problems with linear patterns

Key point: Puzzles and problems that involve linear number patterns can be solved by finding the gaps in the number patterns, which in turn can be used to develop formulas.

Explain to students that the work they have done with linear patterns can help them to solve a wide range of puzzles and real-world problems: How many square patio tiles will there be in the 10^{th} figure of a growing pattern of shapes? How much does it cost to rent a bicycle for a given number of hours? How much does a long-distance phone call cost for a given number of minutes? All of these examples may involve linear patterns. Identifying number patterns is the key—many problems involving linear number patterns can be solved using the techniques students have learned already.

Display the pattern shown, and help students notice that the number of blocks in the figures follows a linear pattern. Explore how each figure can be built from the previous figure. Shade the extra squares added each time to highlight the change.

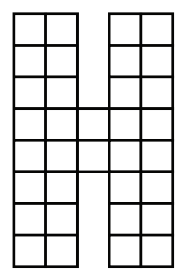
Figure 1 Figure 2 Figure 3



> How can you get Figure 2 from Figure 1? (add 4 blocks above the top row and 4 blocks below the bottom row) How can you get Figure 3 from Figure 2? (add 4 blocks above the top row and 4 blocks below the bottom row) What would Figure 4 look like? (like Figure 3 but with an extra 4 blocks above the top row and 4 blocks below the bottom row)

Have students draw what they think Figure 4 would look like, and after discussing students' proposed answers, display the solution shown. Have students write down the number of blocks for each figure and discuss the number pattern formed.

Figure 4



> How many blocks are used in each figure? (10, 18, 26, 34) How can you describe this number pattern using a pattern rule? (Start at 10 and add 8 each time.)

As a class, find an expression for the number of blocks in the n^{th} figure and use it to calculate the number of blocks in the 12^{th} figure of the pattern without drawing it. The solution is shown in the margin.

Number of blocks in n^{th} figure:
$8n + 2$
Number of blocks in 12^{th} figure
$8(12) + 2 = 96 + 2$
$= 98$

Emphasize that number patterns come up in many different problems. Identifying possible number patterns and looking at the gap in number sequences can help to determine whether a number pattern is linear or not. For linear patterns, the gap can be used to find a formula or an expression for the n^{th} term in the pattern.

Exercises

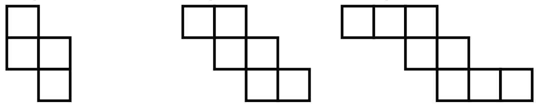
The number of blocks in the sequence of figures forms a linear number pattern.

i) Write the number of blocks in each figure as a sequence.

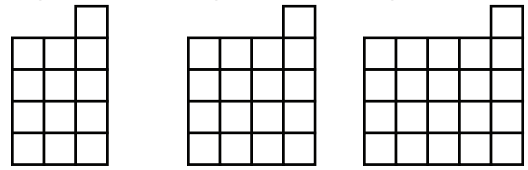
ii) Write an expression for the number of blocks in the n^{th} figure.

iii) Find the number of blocks in the 15^{th} figure.

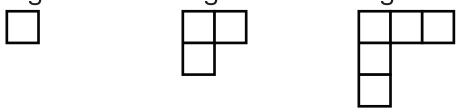
a) Figure 1 Figure 2 Figure 3



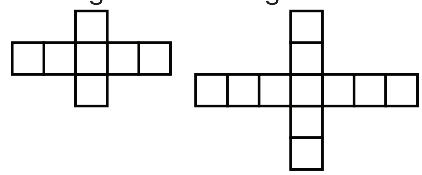
b) Figure 1 Figure 2 Figure 3



c) Figure 1 Figure 2 Figure 3



d) Figure 1 Figure 2 Figure 3



Answers
a) i) 4, 6, 8, …; ii) $2n + 2$; iii) $2(15) + 2 = 30 + 2 = 32$
b) i) 13, 17, 21, …; ii) $4n + 9$; iii) $4(15) + 9 = 60 + 9 = 69$
c) i) 1, 3, 5, …; ii) $2n - 1$; iii) $2(15) - 1 = 30 - 1 = 29$
d) i) 3, 7, 11, …; ii) $4n - 1$; iii) $4(15) - 1 = 60 - 1 = 59$

3. Whole-class strategy talk

Slides 22–23

Key points: A variety of strategies can be used to solve problems involving number patterns.

Display the following problem and lead a whole-class strategy talk about how to solve the problem. (See **BLM Strategy Talks** for detailed guidance on running a whole-class strategy talk.)

Carla and Marta are each saving money to buy a \$50 scooter. The money that they have at the end of each week is shown in the table. If both patterns continue, who will save enough money for the scooter first? Explain your thinking.

Week	Carla	Marta
1	\$1	\$5
2	\$2	\$10
3	\$4	\$15
4	\$8	\$20

Possible strategies that students may suggest:

- graphing each sequence and extending the graphs
- analyzing the gaps between successive terms to compare the sequences, and make predictions based on that comparison
- finding sequence rules for the sequences
- finding formulas for the sequences

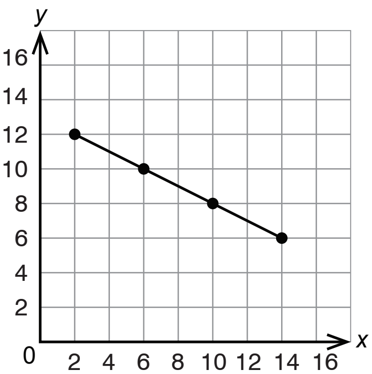
Sample solution: Marta's savings follow a linear pattern, with a gap of $+5$. The n^{th} term of Marta's sequence is given by the expression $5n$. Since $5(10) = 50$, Marta's savings will not pass \$50 until after the 10^{th} week. The gaps in the number pattern for Carla's savings are 1, 2, 4. The gap keeps increasing. If both patterns continue, the gaps in Carla's savings will keep doubling every week. Extending Carla's sequence, we get 1, 2, 4, 8, 16, 32, 64 … After the 4^{th} week, the gap is already larger than 5, and by the 7^{th} week, Carla has already saved more than \$50. So Carla will be able to buy the scooter first.

Key points: The features of a graph (such as linear versus non-linear and increasing versus decreasing) can be used to predict what relation a graph might represent.

Display the graph and problem shown. Discuss which relation the graph might be showing and how students came to their conclusions. Focus on whether the graph shows a linear or non-linear, increasing or decreasing relation.

> Does the graph show a linear or non-linear relation? (linear) Does the graph show an increasing or decreasing relation? (decreasing) Which of the situations described represents a decreasing linear relation? (the third one)

Discuss what other situations the graph could represent. (for example, the number of episodes left to watch if you watch two episodes every four days) Encourage students to suggest various ways to label the two axes based on different scenarios, including units.



Which of these relations could the graph represent?

1. The cost (in $) of renting a bicycle for *n* hours.
2. The water left in a tank after *n* minutes if half the remaining amount drains every minute.
3. The distance from home of someone driving home at a constant speed.

Exercises

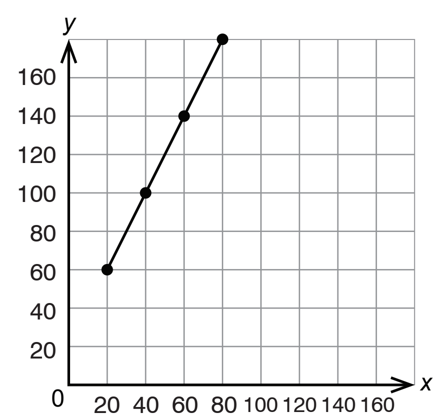
a) Which of these relations could the graph represent?

A. The population of a town each year, given that the population triples every 20 years

B. The cost of a taxi ride (in dollars) for a given number of minutes

C. The money (in dollars) left in your bank account after the given number of days if you spend the same amount every 20 days

b) What else could this graph represent? Write your own title and labels for the graph.



Selected answer: a) Relation B is the only relation the graph could represent, since B would be both linear and increasing. Relation A is not linear and Relation C is decreasing.

NOTE: You may need to help students locate the story and video for Extension 1 online.

1. Search online for a video retelling of the story “Small Number Counts to 100.” Watch the video or read the story. Then work with a partner to answer the following questions.

   a) Write the first 4 numbers that Small Number would say each time he reaches his own tipi.

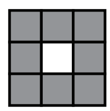
   b) Write a pattern rule for the sequence in part a). Will the pattern continue? Explain how you know.

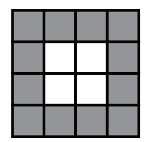
   c) Is Small Number’s answer correct? Use what you know about divisibility and linear patterns to explain why or why not.

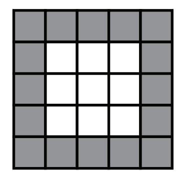
   Answer: a) 1, 8, 15, 22

   Sample answers: b) Start at 1 and add 7 each time. The pattern will continue in the same way because Small Number is repeating the same action: counting 7 objects positioned in a circle. c) The number 98 is a multiple of 7, since $7 \times 14 = 98$. So Small number would be at the last tipi in the circle when he says the number 98. He needs to go two tipis further to reach 100, so he would reach the second tipi in the circle, or the tipi one south of his own. So Small Number’s answer is correct.

2. How many shaded squares and how many unshaded squares would be in Figure 7?

   
   Figure 1

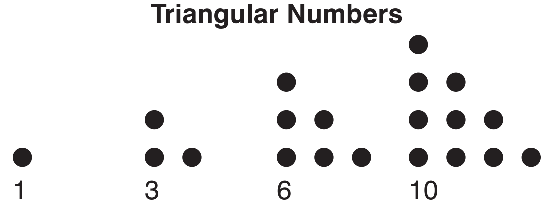
   
   Figure 2

   
   Figure 3

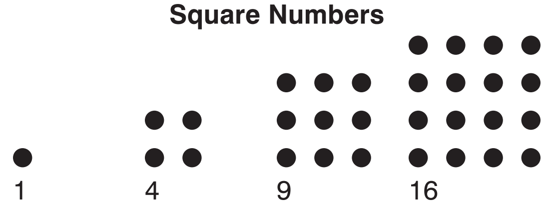
   Solution: There are n^2 unshaded squares in the n^{th} figure and $4n + 4$ shaded squares. So, in Figure 7, there will be $4(7) + 4 = 28 + 4 = 32$ shaded squares and $7^2 = 7 \times 7 = 49$ unshaded squares.

3. The ancient Greeks investigated numbers that could be arranged in geometric shapes.

   Triangular Numbers

   Square Numbers

   a) The first four triangular numbers are shown.

      i) Find the 5th and 6th triangular numbers by drawing pictures.

      ii) Describe the pattern in the triangular numbers.

      How does the gap change?

      iii) Find the 8th triangular number by extending the pattern you found in part ii).

   b) Repeat parts i) to iii) with the square numbers.

   Answers: a) i) 15, 21; ii) Start at 1 and add the gap each time. The gap starts at 2 and increases by 1 each time. OR: Start at 1 and add the term number each time. iii) $21 + 7 = 28$, $28 + 8 = 36$; the 8th triangular number is 36; b) i) 25, 36; ii) Start at 1 and add the gap each time. The gap starts at 3 and increases by 2 each time. iv) $36 + 13 = 49$, $49 + 15 = 64$; the 8th square number is 64

PR7-9
Cumulative Review: Units 1 and 3
AP Book pp. 60–64

Goals

Students will review concepts developed in this unit and Unit 1 by solving problems.

Main Ideas

Divisibility properties for numbers can be used to factor whole numbers. Integers can be ordered, added, and subtracted. Linear relations can be represented by rules, formulas, tables, and graphs.

Summary

Mental Math Minute	D-65
Extensions	D-65

Prior Knowledge

Can determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10

Can find factors of a number

Understands why a number cannot be divided by 0

Can add and subtract integers

Can use letters as variables to represent numerical values

Can plot ordered pairs as points on a coordinate grid (limited to positive integer coordinates)

Can evaluate an expression with a variable given the value of the variable

Can determine whether a relation given by a sequence or formula is linear

Curriculum

AB: recommended

BC: recommended

MB: recommended

SK: recommended

Materials

playing cards

Vocabulary

none

Additional Information

This lesson is an opportunity for students to review concepts in Units 1 and 3 by working on the problems in the AP pages.

Mental Math Minute

Slides 2–3

Skill 15: Adding two-digit numbers with and without regrouping (p. A-29).

Review the method of adding tens and ones separately to add two-digit numbers with and without regrouping. Use the examples in the margin.

Use a deck of playing cards to generate random exercises for students to add 2 two-digit numbers. Remove the face cards, tens, and aces from the deck, keeping only cards numbered 2 through 9. Shuffle the remaining cards and draw one card, showing it to students. Students write this number as the tens digit of the first two-digit number. Repeat for the ones digit of the first two-digit number, and then for each digit of the second two-digit number. Students perform the addition, keeping in mind that it might or might not involve regrouping. Repeat 5 to 10 times.

$56 + 23$
Add the tens: $50 + 20 = 70$
Add the ones: $6 + 3 = 9$
Add the totals: $70 + 9 = 79$

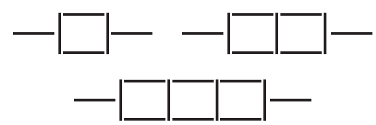
$78 + 65$
Add the tens: $70 + 60 = 130$
Add the ones: $8 + 5 = 13$
Add the totals: $130 + 13 = 143$

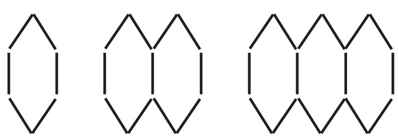
Extensions

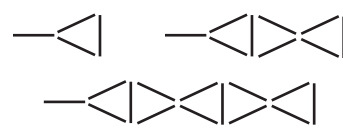
Slides 4–8

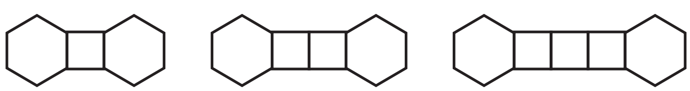
1. Find the number of toothpicks in the 20th figure of the sequence.

a) 

b) 

c) 

d) 

Bonus: 

Selected solutions: a) Since there are 4 toothpicks in the first figure and 3 more in each subsequent figure, the 20th figure would use $3(20) + 1 = 61$ toothpicks; Bonus: Since there are 14 toothpicks in the first figure and 3 more in each subsequent figure, the 20th figure would use $3(20) + 11 = 71$ toothpicks

Answers: b) $3(20) + 3 = 63$, c) $5(20) + 1 = 101$, d) $5(20) - 1 = 99$

2. a) Figure out the formula for the sequence 4, 10, 16, 22. Then, using your understanding of divisibility rules and factors, explain why 216 cannot be a member of the sequence.

   b) Use the formula for the sequence and make an equation to check whether 216 can be a member of the sequences.

   i) 8, 12, 16, 20 ii) 4, 7, 10, 13

Answer: a) The formula for the sequence is $v = 6n - 2$. $6n$ is always divisible by 6, but if we subtract 2, the number is never divisible by 6. Since 216 is divisible by 6, it cannot be a term in the sequence.

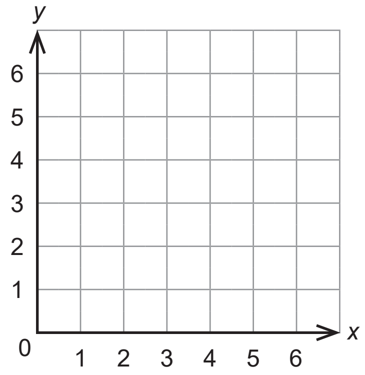
Solutions

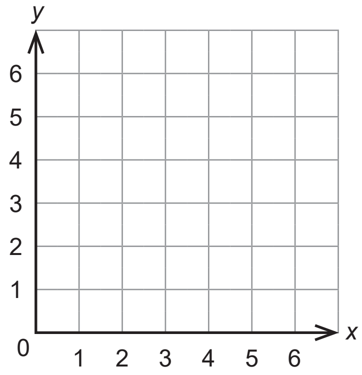
b) i) The formula for the sequence is $v = 4n + 4$. We'll know that 216 is a term in the sequence if we can find a positive integer n for which $216 = 4n + 4$. By guessing and checking values for n, we find that when $n = 53$, $4(53) + 4 = 212 + 4 = 216$. So, 216 is the 53rd term in the sequence. Another way to reason through the problem is to notice that the sequence consists of all multiples of 4 greater than or equal to 8. So, if 216 is divisible by 4, it must be somewhere in the sequence. Indeed, $216 = 200 + 16$, and since 200 and 16 are both divisible by 4, 216 is also divisible by 4 and is a term in the sequence.

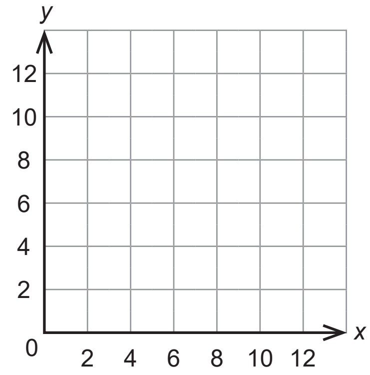
ii) The formula for the sequence is $v = 3n + 1$. If we assume that 216 is a term in the sequence, then 215 would have to be divisible by 3. However, we know the divisibility rule for 3: the digits must add up to a multiple of 3. We know that $2 + 1 + 5 = 8$ is not divisible by 3, so 215 is not divisible by 3 either. This means 216 is not a term in the sequence. Another approach is to notice that $3n + 1$ is always one more than a multiple of 3, and since 216 is a multiple of 3 ($2 + 1 + 6 = 9$), we know that 216 cannot be a term in the sequence.

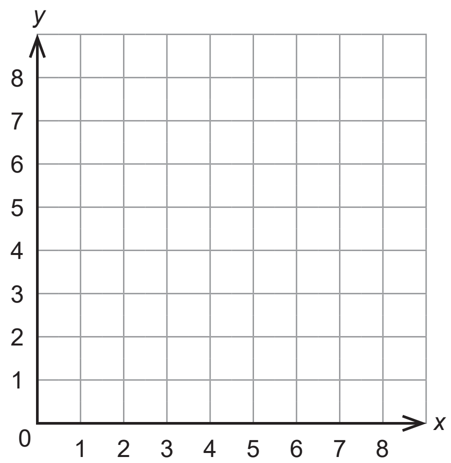
NAME ______________________________ DATE ______________

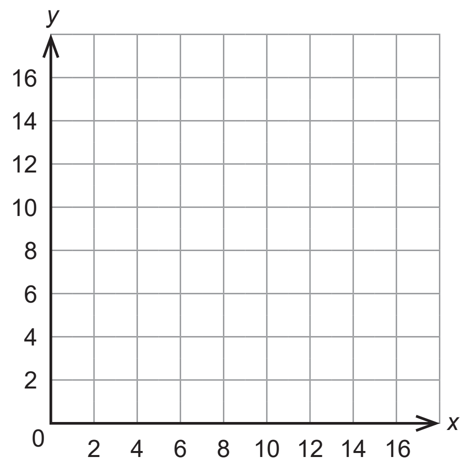
Small Coordinate Grids

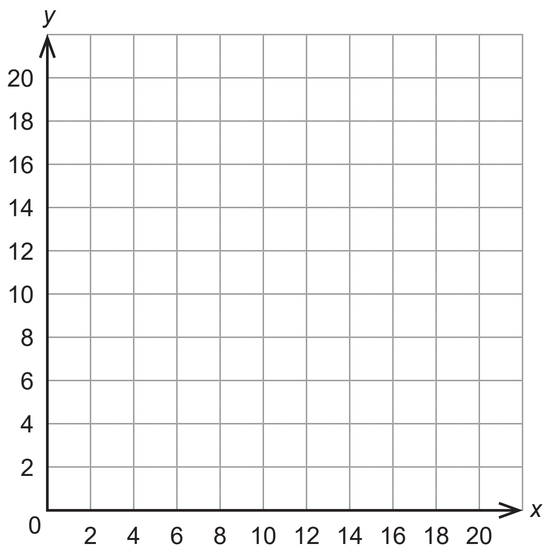


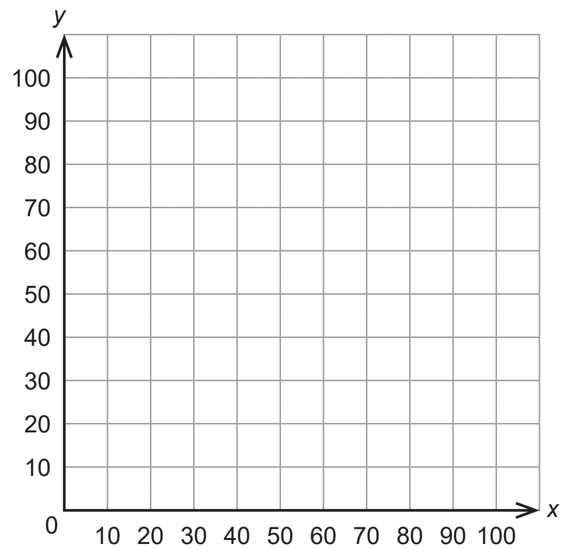






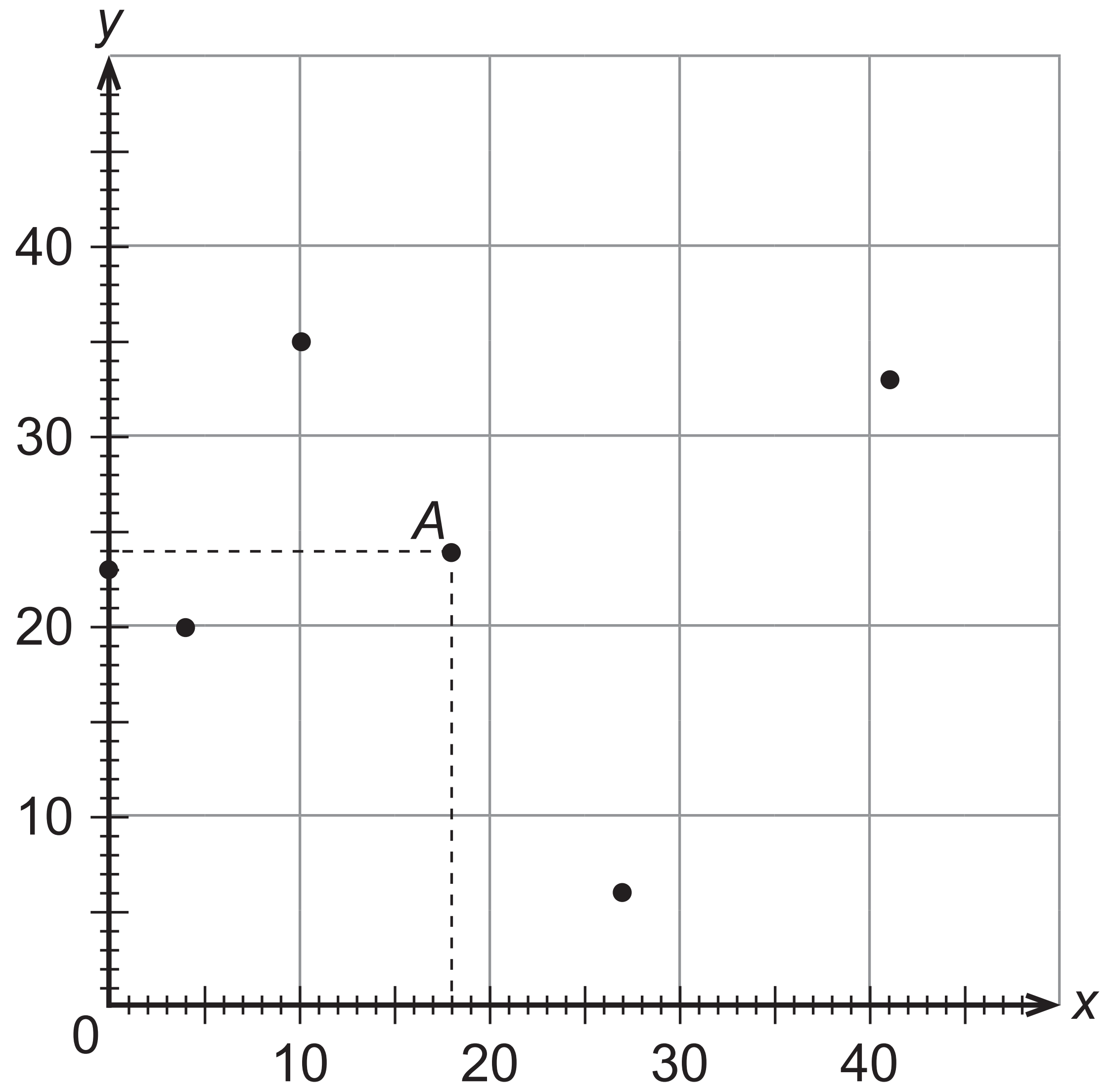






NAME ______________________ DATE ____________

Grid with Tens



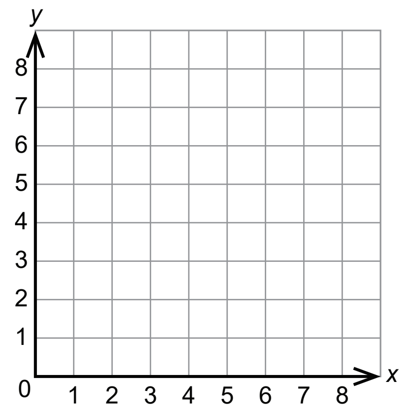
B (10, 35) *C* (4, 20) *D* (27, 6) *E* (41, 33) *F* (0, 23)

NAME ______________________ DATE ____________

Plotting Points from Tables

1. Write the ordered pairs for the relation.
Then plot the points on the grid.

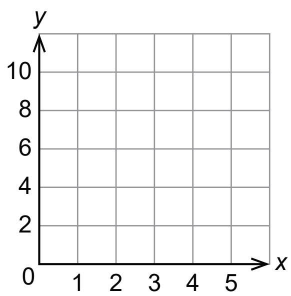
Input (x)	Output (y)	Ordered Pair
1	9	
2	7	
3	5	
4	3	



2. Fill in the table for the first four terms of the sequence. Then plot the points on the grid.

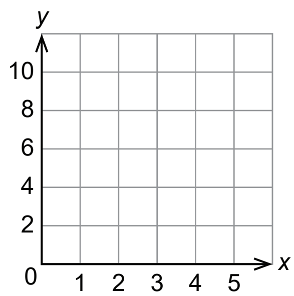
a) Start at 0 and add 3 each time.

Input (x)	Output (y)
1	
2	
3	
4	



b) 10, 8, 6, 4

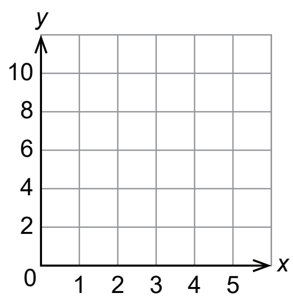
Input (x)	Output (y)
1	
2	
3	
4	



3. Use the formula for the relation to fill in the table. Then plot the points on the grid.

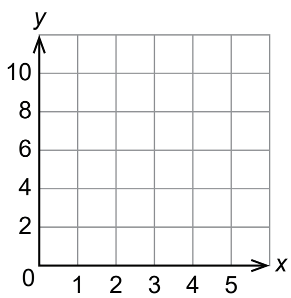
a) $y = 2x + 1$

Input (x)	Output (y)	Ordered Pair
1	*$2(1) + 1 = 3$*	*(1, 3)*
2		
3		
4		



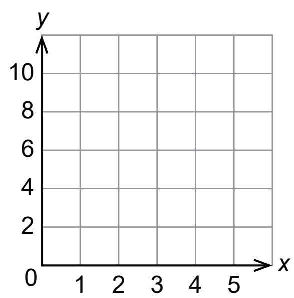
b) $y = x + 5$

Input (x)	Output (y)
1	
2	
3	
4	



c) $y = 9 - 2x$

Input (x)	Output (y)
1	
2	
3	
4	



NAME ______________________________ DATE ______________

Determining Linearity from Tables and Graphs

1. Complete the table for the relation and find the gaps between the output values. Is the relation linear?

a) Start at 2 and add 5 each time.

Input (x)	Output (y)
1	
2	
3	
4	

____

b) Start at 2 and multiply by 2 each time.

Input (x)	Output (y)
1	
2	
3	
4	

____

c) $y = 3x + 2$

Input (x)	Output (y)
1	
2	
3	
4	

____

d) $y = 20 - 4x$

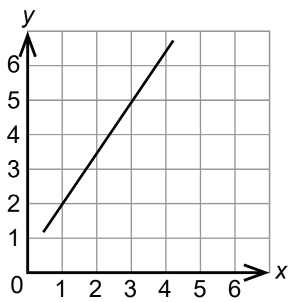
Input (x)	Output (y)
1	
2	
3	
4	

____

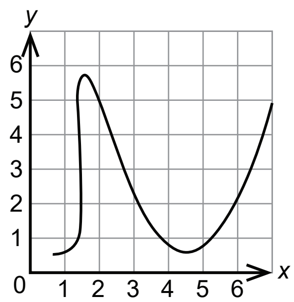
2. Draw one coordinate grid where the y-values go up to 20. Graph each relation from Exercise 1 on this grid and label the graphs a) to d). State whether each graph forms a straight line.

3. Which graphs show a linear relation? Explain.

A.



B.



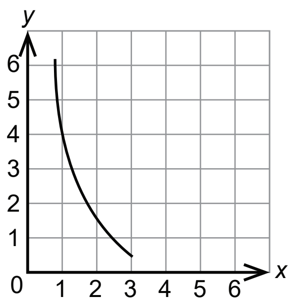
____

____

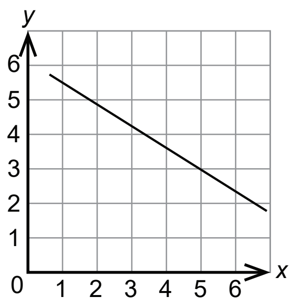
____

____

C.



D.

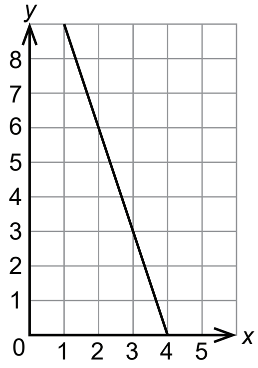


NAME ______________________________ DATE ______________

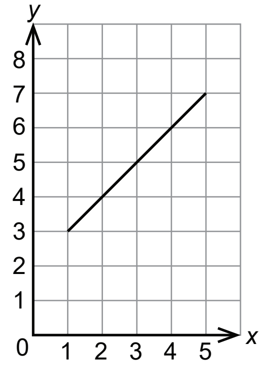
Increasing and Decreasing Relations

1. Is the linear relation increasing or decreasing? How do you know?

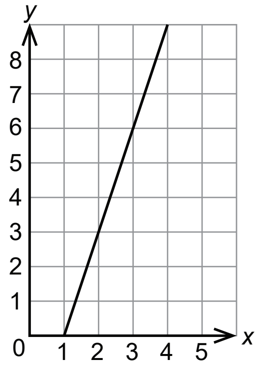
a)



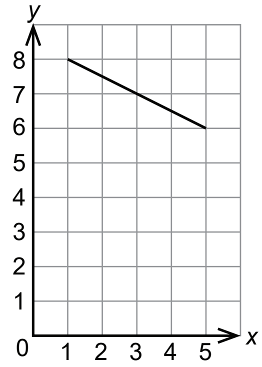
b)



c)



d)



2. Complete the table. Is the gap between the output values a positive or negative number? Is the relation increasing or decreasing?

a) $y = 4x + 1$

Input (x)	Output (y)
1	
2	
3	
4	

b) $y = 18 - 3x$

Input (x)	Output (y)
1	
2	
3	
4	

3. For each part of Question 2, describe what you think the graph will look like. Then graph the relation to check.

4. a) In Question 1, part a), what is the value of y when x is 2? ________

b) In Question 1, part b), what is the value of x when y is 7? ________

c) In Question 1, part c), what is the value of x when y is 3? ________

Bonus ▶ In Question 1, part d), what is the value of y when x is 6? ________

NAME ______________________________ DATE ______________

Solving Problems with Graphs of Linear Relations

1. a) How much do you have to pay to rent a bike from Zayna's store for …

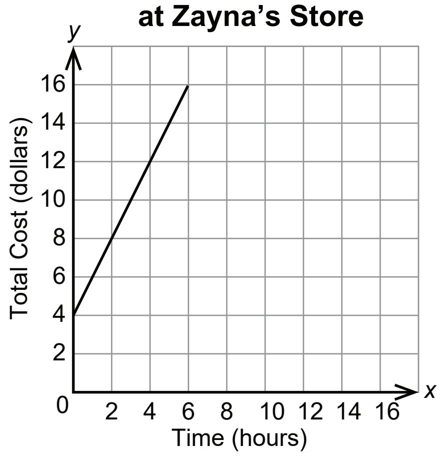
i) 2 hours? ii) 4 hours?

iii) 3 hours? iv) 5 hours?

b) Abdul rented a bike from Zayna's store and paid $16. How many hours did he rent it for?

c) How much would it cost to rent a bike from Zayna's store for 7 hours?

Cost of Renting a Bike at Zayna's Store



2. a) How far from the start is Rena after 10 seconds?

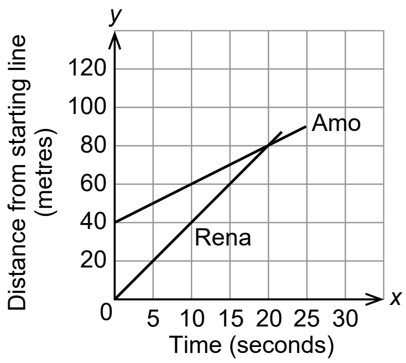
b) How far from the start is Amo after 10 seconds?

c) How long does it take Amo and Rena to finish the race? Extend the lines on the graph to find out.

d) How much of a head start does Amo have? Explain how you know.

e) When does Rena overtake Amo?

Rena and Amo's Progress in a 100 m Race



Bonus ▶ If Rena and Amo each keep running at the same speed, how far ahead of Amo would Rena be after 35 seconds from the start of the race?

NAME ______________________________ DATE ______________

Linear Relations Summary (1)

Variables, expressions, equations, and formulas

An **expression** is formed by combining one or more numbers or **variables** with operations and possibly brackets. An expression can be as simple as a single number or variable.

An **equation** is formed by joining two expressions with an equal sign.

Expressions: $5z - 8$ $14y$ 81 Equations: $5z - 8 = 14y$ $14y = 81$

An expression represents a numerical value, whereas an equation represents a statement of equality that might be true or false depending on what numbers are substituted for variables. To remember what “equation” means, look at the common letters:

equation equal sign

A **formula** can be an equation that shows how to calculate an **output variable** from an **input variable**. In a formula, the **coefficient** of a variable is the number multiplied by that variable.

In $v = 5n$, the coefficient of n is 5. In $y = 32x$, the coefficient of x is 32.

Linear sequences and relations

A **relation** is when you have corresponding values between two quantities. A sequence forms a relation between the **term numbers** and **term values**.

Linear sequences are made by adding the same number each time. This is the **gap** between term values, and it can be a positive or negative number.

A relation is **linear** if its graph forms a straight line.

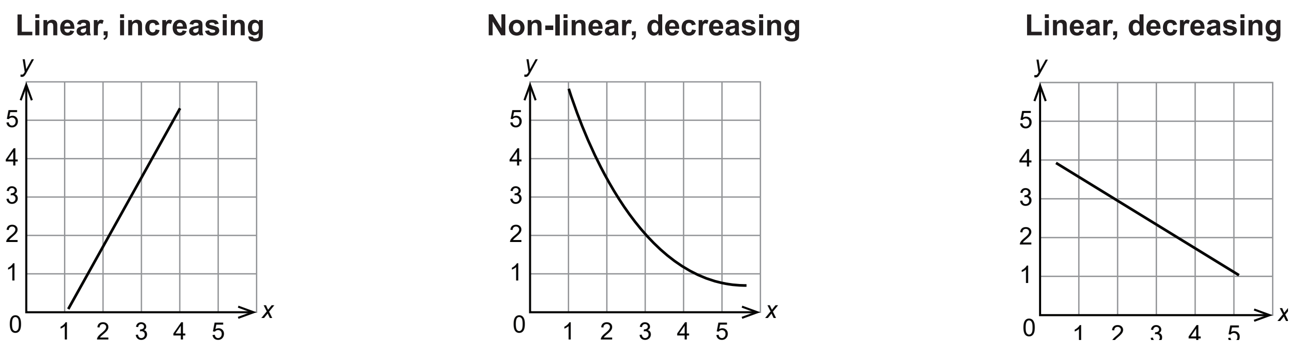
If the graph does not form a single straight line, the relation is **non-linear**.

A sequence in which the terms get larger each time is called **increasing**.

A sequence in which the terms get smaller each time is called **decreasing**.

If the graph of a relation goes up from left to right, it is increasing.

If the graph of a relation goes down from left to right, it is decreasing.



NAME ______________________ DATE ____________

Linear Relations Summary (2)

Matching graphs, tables, rules and formulas for linear relations

Since the values of y on the graph of a linear relation are the same as the output values, the change in the value of y as x increases by 1 is equal to the gap between the output values.

Remember, this gap is equal to the coefficient in the formula.

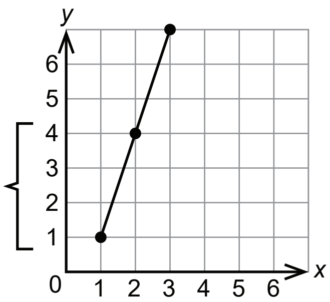
Input (x)	Output (y)
1	$3(1) - 2 = 1$
2	$3(2) - 2 = 4$
3	$3(3) - 2 = 7$

$+3$

$+3$

$y = \mathbf{3}x - 2$

y increases by **3** as x increases by 1



Finding formulas for linear sequences

For **increasing linear sequences**, multiply the gap by the term number, and find the number you must add or subtract to get the term value.

For **decreasing linear sequences**, multiply the magnitude of the gap by the term number, and find the number you must subtract from to get the term value.

Examples:

Increasing linear sequence where the gap is larger than the starting number

a) 1, 5, 9, 23, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1	4×1	1
2	4×2	5
3	4×3	9

$+4$

$+4$

Subtract __3__

Formula: $v = 4n - 7$

10th term: $4(10) - 3 = 40 - 3 = 37$

Decreasing linear sequence

b) 190, 180, 170, 160, …

Term Number (n)	Gap $\times$ n	Term Value (v)
1	$-(10 \times 1)$	190
2	$-(10 \times 2)$	180
3	$-(10 \times 3)$	170

-10

-10

Subtract from __200__

Formula: $v = 200 - 10n$

10th term: $200 - 10(10) = 200 - 100 = 100$