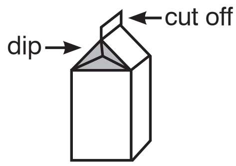
Unit 6 Measurement

In this unit, students will distinguish between right and skew prisms and construct nets of 3-D shapes. They will also determine the relationships between the height, the area of the base, and the volume of right prisms and cylinders and connect volume to capacity. They will develop and use the formulas for finding the volume of right prisms and cylinders. Students will also determine the surface area of right prisms and cylinders and solve problems involving volume and surface area of prisms and cylinders.

ONLINE GUIDE

Tips for working with modelling clay



Materials
Students will benefit from seeing a variety of 3-D shapes, both prisms (right and skew) and not prisms. Such shapes are required in some lessons. If you do not have a commercial set of 3-D shapes, you can either make some from modelling clay (you will find tips for working with modelling clay on our website or create shapes using nets provided on **BLM Nets of 3-D Shapes** (pp R-48, U-1–U-24). You can also find examples of different 3-D shapes among boxes. For example, some chocolate boxes (e.g., Toblerone) are triangular or hexagonal prisms, and you can turn a standard milk carton into a pentagonal prism by cutting off the strip on the top and covering the dips in the bases with paper.

You will also need a variety of boxes and cylindrical cans of different dimensions. Ahead of time, ask students to bring from home various boxes and cans, such as empty medicine packages, soup cans, tea boxes, cereal boxes, and so on. Boxes in the shape of prisms that are not rectangular prisms will be very useful as well.

Capacity vs. Volume
Volume is the amount of space taken up by a three-dimensional object and capacity is defined as how much a container can hold. One way to distinguish volume from capacity (at least at this level) is to look at the units in which they are measured: volume is measured in linear units cubed—centimetres cubed (cm^3), metres cubed (m^3), kilometres cubed (km^3)—while capacity is measured in millilitres or litres (mL or L).

Meeting Your Curriculum
Lessons ME8-9 through ME8-12 and ME8-16 deal with nets, volume, and the surface area of prisms, which students in Ontario learned in Grade 7. However, to be able to solve problems involving volume and surface area of both prisms and cylinders, as required by the Ontario curriculum, students need a good review of this material. Ontario teachers are encouraged to go through these lessons at least briefly.

For students following the WNCP curriculum, lesson ME8-9 contains essential review material, and the rest of the unit is core curriculum.

ME8-9 Right Prisms

Pages 146–147

CURRICULUM EXPECTATIONS
Ontario: 7m34, 7m49; **8m5, 8m7, review**
WNCP: 6SS3; **essential for 8SS2, [C, CN, V]**

VOCABULARY
right prism
skew prism
face
edge
vertex
skeleton
volume
length
width
height

PROCESS ASSESSMENT

8m7, [C]
Workbook Question 3b)

Goals

Students will identify and sketch right prisms.

PRIOR KNOWLEDGE REQUIRED

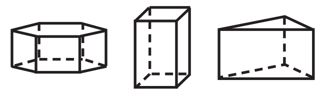
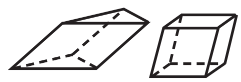
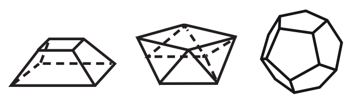
Can identify right angles
Can identify right prisms
Can find the volume of a right rectangular prism
Can multiply or divide decimals
Is familiar with cubic units of measurement

MATERIALS

a variety of right and skew prisms (see Introduction)
BLM Nets of 3-D Shapes (pp R-48, U-1–U-24)
modelling clay and toothpicks (to make a skeleton)
grid paper

Review prisms. *Prisms* have two identical (congruent) polygonal faces called bases and side faces that are parallelograms. Students might be familiar only with right prisms, whose side faces are rectangles. Present several 3-D shapes (do not include skew prisms for now) one at a time and have students tell whether each shape is a prism or not. To assess students at a glance, you can ask them to answer "yes" and "no" in ASL (shake your fist up and down for "yes," touch the thumb with the pointer and the middle finger together for "no"). Then ask volunteers to place all the prisms base down.

Right prisms and skew prisms. A *skeleton* of a prism is a model that has only edges and vertices, no faces. Show students how you can make a skeleton of a prism using modelling clay and toothpicks: make two copies of a base, add vertical edges to one of the bases, and attach the other base on top. Place two copies of a skeleton on the table, base down, and shift the top base of one of them so that the whole prism is tilted to the side. Explain that the prism you have created is called a *skew prism*. The original prism is called a *right prism*. Display several prisms (as below; you can find the nets for them on **BLM Nets of 3-D Shapes**) and have students sort them into right prisms and skew prisms as a class. Add several shapes that are not prisms and have students explain why these are not prisms. Note that answers will vary for shapes that are not prisms (side faces are not parallelograms, bases are not the same size, bases are congruent but are not translations of each other, etc.)

Type of shape	Properties	BLM Nets of 3-D Shapes
Right prisms 	The top face is directly above the bottom face. Side edges are vertical.	Shapes 1–11 (3, 4, 5 shown here)
Skew prisms 	The top face is shifted from the bottom face. Side edges are not vertical. Side faces are parallelograms.	Shapes 12–15 (12 and 15 shown here)
Not prisms 	The side faces are not parallelograms, bases are not the same size, bases are facing different ways, etc.	Shapes 16–24 (20, 21, 22 shown here)

NOTE: For prisms that have all faces in the shape of a parallelogram, any pair of faces can be taken as bases. If four of the faces are rectangles, you can take the remaining two faces to be bases and conclude that the prism is a right prism. The following shapes on **BLM Nets of 3-D Shapes** are composed only of parallelograms, should you want to focus on them and have students determine which are right prisms and which are not.

- 3—a right rectangular prism (any pair of opposite faces can be placed directly above each other)
- 8—a right prism with a parallelogram base (only the non-rectangular faces—the parallelograms—can be placed directly above each other)
- 12—a skew prism with three different pairs of identical parallelogram faces (no pair of faces can be placed directly above each other)
- 13—a skew rectangular prism (two faces are rectangles, but no pair of faces can be placed directly above each other)

The angle between the base and the side faces. Show students a book that is open partway. Explain that just as the space between two rays with the same endpoint forms the angle between two lines, the space between two faces joined at an edge forms the angle between the faces. This angle can be acute, right, or obtuse (show each with the pages of the book).

Sketch two lines, one vertical and the other horizontal. **ASK:** What is the angle between the lines? (90°, right angle) Place a tall box (such as a large cereal box) on your desk. **SAY:** The sides of the box are vertical and the desk is horizontal. What is the angle between the sides of the box and the desk? (90°, right angle) Hold up prisms one at a time and invite volunteers to stand them on their bases next to the cereal box. **ASK:** Can you place the prism so that it touches the side of the box with a whole side face? Can you do it for all the side faces? Have volunteers check all the side faces. As a class, sort the prisms into those that have at least one side face that doesn't sit flush against the box and those for which any side face can be placed flush against the box. If the prism has all faces that are parallelograms, you can try different faces as bases. Look for a pair of bases such that all side faces can be placed flush against the box (for example, the prism on **BLM Nets of 3-D Shapes (8)** has one pair of bases, the parallelogram faces, that allow all side faces to be placed flush against the box, so it should be placed in the first group). **ASK:** How would you describe the shapes in the two groups? (right prisms and skew prisms) What is the angle between the side faces and the base for right prisms? (a right angle)

PROCESS EXPECTATION

Connecting

Hidden lines. Draw a picture of a cube using dashed lines for the hidden edges. Explain to students that the edges on the back of the shape are often drawn using dashed lines to indicate that we can't see them. The dashed lines and the solid lines might intersect in the picture, but if the point of intersection is not a vertex, there is no real intersection between the edges there. The lines intersect in the picture because the picture is flat, but the shape itself is 3-dimensional.

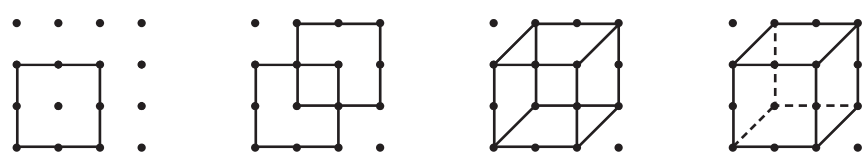
Sketching cubes. Show your students how they can draw a picture of a cube.

Step 1: Draw a square that will become the front face.

Step 2: Draw another square of the same size, so that the centre of the first square is a vertex of the second square.

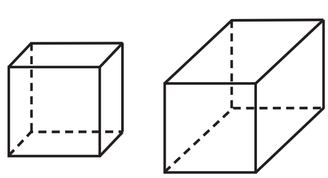
Step 3: Join the corresponding vertices with lines as shown.

Step 4: Erase parts of the lines that represent hidden edges, to make them dashed lines.



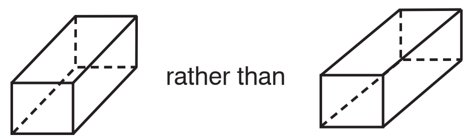
Step 1 Step 2 Step 3 Step 4

Struggling students will find it helpful to draw cubes on dot paper or grid paper.

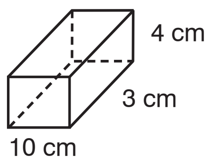


Sketching right rectangular prisms. Sketch the two diagrams at left on the board and draw students' attention to the difference between these shapes and the previous cube. Both shapes at left are not cubes—they are rectangular prisms of different lengths, with the front and back faces being squares. The side faces, the top, and the bottom look different because the prisms have different lengths. **ASK:** How was Step 2 performed differently in each drawing? (the bottom left vertex of the back face is not at the centre of the front face—it is closer to the bottom left vertex of the front face in the thinner shape and farther from that vertex in the longer shape)

ASK: What would you do in Step 2 to draw a very long rectangular prism? Point out that the corner of the back face should not sit on the diagonal, so that the edges do not overlap:



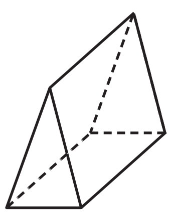
Have students draw a long rectangular prism.



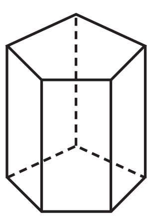
Add dimensions to one of the prisms you sketched, as shown. **ASK:** Could these be the dimensions of this prism? What is wrong? Have a student rearrange the dimensions to better match the picture.

Remind students that to find the volume of a right rectangular prism, they multiply the length, the width, and the height. Have students find the volume of the prism above.

Ask students to sketch a rectangular prism, add some dimensions, and swap their sketch with a partner. Have students find the volume of their partner's prism. Then ask students to sketch a rectangular prism that has a volume of 300 cm^3 and compare their sketches with a partner. Did partners draw the same prism? Can they draw a different prism with the same volume?



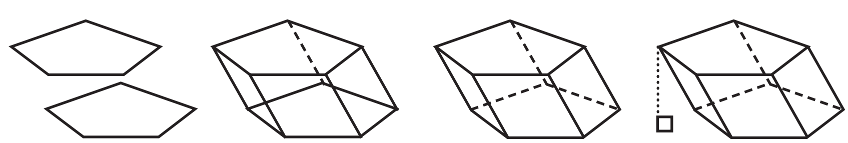
Sketching other right prisms using the same method. Have students draw other prisms, such as triangular or pentagonal prisms, the same way they drew rectangular prisms. Point out that the rectangular faces in these sketches are distorted—they look like parallelograms. The bases (the front and back faces) are not distorted.



Sketching a prism standing on a base rather than on one of the rectangular faces. Point out to students that in this position, the front face is a rectangle and is not distorted, while the bases, which are now at the top and the bottom, are distorted. To see the distortion of the base in this position, suggest that students hold a pattern block or a paper polygon horizontally, slightly below eye level. They should see that the polygon in the base appears to be squashed vertically (shorter and wider) compared to the polygon viewed head on. To draw a prism standing on a base, draw the base shorter and wider than it is when viewed head on, then draw the second (top) base directly above the bottom base and join the vertices to produce the side faces.

Extension

Sketch a skew prism. Sketch the first (bottom) base as when drawing a right prism, but sketch the second (top) base so that it is not directly above the first base. Also, add a line perpendicular to the top base to show that there is an angle. **EXAMPLE:**



ME8-10 Nets of Right Prisms

Pages 148–150

CURRICULUM EXPECTATIONS
Ontario: 6m41, 7m41; **8m2, 8m6, 8m7**
WNCP: **8SS2, 8SS3, [C, R, V]**

VOCABULARY
net
face
right prism
parallelogram
triangle
triangular prism
rectangular prism

Goals

Students will identify nets of 3-D shapes, identify shapes from nets, construct nets of right prisms, and construct shapes from given nets.

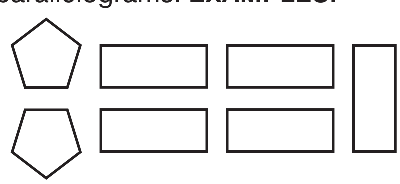
PRIOR KNOWLEDGE REQUIRED

Can identify right prisms as prisms
Can identify the base of a prism

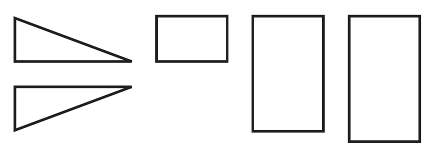
MATERIALS

BLM Nets of 3-D Shapes (pp R-48, U-1–U-24)
BLM A Net or Not a Net? (p R-49)
small rectangular prisms (1 per student)
boxes of different dimensions
BLM Is It a Net? (p R-50)

Identify 3-D shapes from their faces. On the board, draw several shapes (that together are the faces of a right prism) and ask your students which 3-D shape they make. If students have trouble identifying the shapes, ask them to circle the base(s) first, by looking for shapes that are not rectangles or parallelograms. **EXAMPLES:**

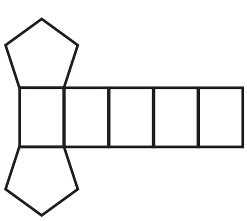


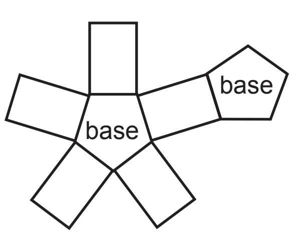
(pentagonal prism)



(triangular prism)

Introduce nets. Hold up a net for a cube, e.g., **BLM Nets of 3-D Shapes (1)**. Ask students to identify the shapes the net is made of. (squares) **ASK:** What 3-D shape that you know has all its faces that are squares? (cube) Fold the net, to show that it indeed folds into a cube. Explain that a *net* of a 3-D shape shows all the faces of the shape attached together and can be folded into the 3-D shape.



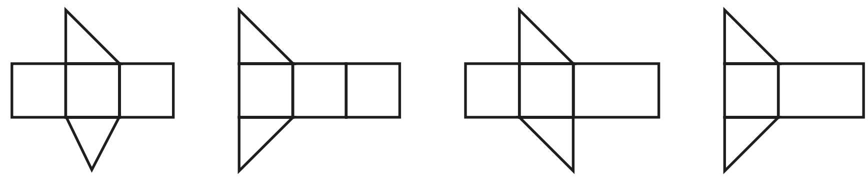


Making nets. Hold up a triangular or pentagonal prism (you can make one using **BLM Nets of 3-D Shapes (2, 6)**). **ASK:** Which shapes are the faces of this prism? How many bases does it have? What is the shape of the side faces? How many side faces does it have? If you wanted to make a net for this prism, the easiest way would be draw the band of rectangles for side faces (created by rolling a prism and tracing the side faces in turn) and then add the bases. Illustrate this on the board.

Explain that another way to make a net for this prism is to start with a base (draw it on the board and write “base” on it), add a side face along each edge of the base (draw one side face and ask volunteers to draw the rest), and finish with the second base. Model this method on the board as well.

Different nets for the same prism. This would be a good time to do Activity 1, below.

A net or not a net? After students finish Activity 1, draw several incorrect examples of nets for a triangular prism on the board and ask volunteers to explain why these drawings cannot serve as nets for prisms:



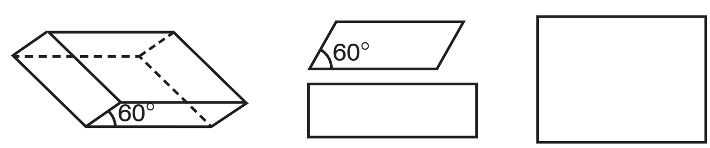
The bases are not the same	*The middle face is too short*	*The base at the bottom is flipped*	*A side face is missing*

PROCESS ASSESSMENT
8m6, [V]
Workbook Question 5

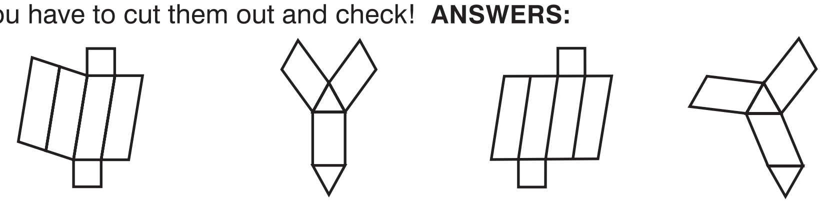
Have students work in pairs, with one student drawing a picture that will not work as a prism net and the other student explaining why the drawing cannot be a prism net. Have students switch roles several times. Then have each pair choose two of their favourite "nets" and swap them with another pair of students. Afterwards, each group of four can present one of their drawings to the class and explain why it cannot be a prism net. For a more challenging task, students can draw pictures that might or might not work as nets, and the class can predict if these are prism nets (students can use thumbs up and thumbs down to show their answer). After each vote, ask students to sketch the net on a sheet of paper. When the voting is done, have different students redraw the nets to scale, cut them out, and fold them, to check the prediction.

EXTRA PRACTICE:

1. How many of each type of face would you need to make this prism?



2. Have students complete **BLM A Net or Not a Net?** Students can use tape to join the faces. This exercise emphasizes the fact that it's very hard to correctly identify nets for skew prisms just by looking at them. You have to cut them out and check! **ANSWERS:**



skew square prism	*skew triangular prism*	*(no)*	*(no)*

Nets and dimensions. Give each student a small rectangular box with different length, width, and height (e.g., small empty pill package). Have students number the faces of their prism. Ask students to measure the sides of the prism and to mark the dimensions of each side on a drawing of the side. Have them number the sides in their drawings as they numbered the sides on the box. How many sides should they have drawn? (6) Ask students to sketch a net for the same prism, and mark the numbers on the faces of the net as well. Then ask students to mark the dimensions on each face of the net. Finally, ask them to place the prism on their desks and to identify each face of the prism, and then each face of the net, as top, bottom, right, left, front and back faces.

Have pairs swap boxes and construct nets for each other's boxes. This time they need to draw the sides of the boxes to scale. Students should mark the dimensions on the nets. Do the nets they drew to scale look similar to the sketches their partners made beforehand? Have students cut out their nets, fold and tape them, and check that the folded net is the same size as the prism they started with.

ACTIVITIES 1–2

1. Let students explore various ways of creating nets for the same solid rather than memorizing a single net. They will need various prisms with faces that are not regular polygons. You can ask students to make such prisms using modelling clay (they can use plastic knives to cut the faces flat) or nets from **BLM Nets of 3-D Shapes (3, 5, 6, 7, 8, 9)**. Each student should work with at least two different prisms.

   Have students trace the faces of their prisms on a piece of paper in order to create a net. **ASK:** them to cut out the nets they have drawn. Let them cut off faces of the net (one at a time) and re-attach the faces at different places. Will the new net fold into the same prism? Which edges are places where you would want to re-attach faces and which are not?

PROCESS EXPECTATION

Making and investigating conjectures

2. Give each student nets for three different shapes: a right prism, a skew prism, and a non-prism. (You can use nets from **BLM Nets for 3-D Shapes**.) Have students identify which net belongs to a right prism and which belongs to a skew prism, and then guess which shapes these are. Students can also try to describe what the non-prism will look like. Students should then construct the shapes from the nets to check their predictions. Pairs can compare the shapes they produced and explain to each other why their non-prisms are not prisms.

PROCESS EXPECTATION

Visualizing

Extension

Give students **BLM Is It a Net**? Ask students to predict whether each drawing is the net of a 3-D shape. How are the nets the same? (They have the same overall shape, and both have two square faces.) How are they different? (net A has 4 parallelogram faces, net B has 8 triangular faces) Have students cut out the nets, fold them, and check their predictions. (Net A does not fold into a 3-D shape, but net B does—it folds into a shape called an *antiprism*.) How does breaking the parallelograms into triangles help the picture become a net? (When the net is folded, the triangles have an angle between them, allowing the side faces to fit around both squares.)

ME8-11 Volume of Rectangular Prisms

Pages 151–154

CURRICULUM EXPECTATIONS
Ontario: 7m17; **8m5, 8m7, review**
WNCP: 6SS3; **8SS4, [C, CN]**

VOCABULARY
length
width
height
area
volume
cubic units
rectangular prism

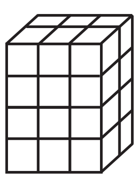
Goals

Students will find the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED

Is familiar with area and perimeter
Can find the area of a rectangle
Knows the formula for the area of a rectangle
Can multiply or divide decimals
Is familiar with linear and square units of measurement

Units of length, area, and volume. Review with students the various units used to measure length and area. Point out that 1-dimensional objects, like strings and line segments, have only 1 dimension, length, which we measure in centimetres, metres, kilometres, and so on. Objects that have area are 2-dimensional; they have length and width, and we measure their area with square units, such as m^2 (where the raised 2 reminds us that they are 2-dimensional). Remind students how the square metres show up in the calculation of area: $1\text{ m} \times 1\text{ m} = 1\text{ m}^2$. Explain that objects that have length, width, and height are 3-dimensional, and we measure their volume in *cubic units*, such as cm^3. Show students a centimetre cube and point out that its sides are all 1 cm long. Ask students what other measurement units for volume they know. How large are these units?



Remind students that the third dimension in 3-D figures is called *height*. Identify the length, width, and height in the prism at left. Students can review the formula for the volume of a rectangular prism using Questions 1 and 2 on Workbook page 151.

Use the terms length, width, and height to label the multiplication statement that gives the volume of the prism at left:

$$\underset{\textit{length}}{3\text{ cm}} \times \underset{\textit{width}}{2\text{ cm}} \times \underset{\textit{height}}{4\text{ cm}} = 24\text{ cm}^3$$

PROCESS ASSESSMENT

8m5, [CN]

ASK: What does the raised 3 mean? (three 1 cm sides were multiplied to get one cubic cm)

Draw several prisms on the board, mark the height, width, and length (you can use different units for different prisms), and ask students to find the volume. **EXAMPLES:**

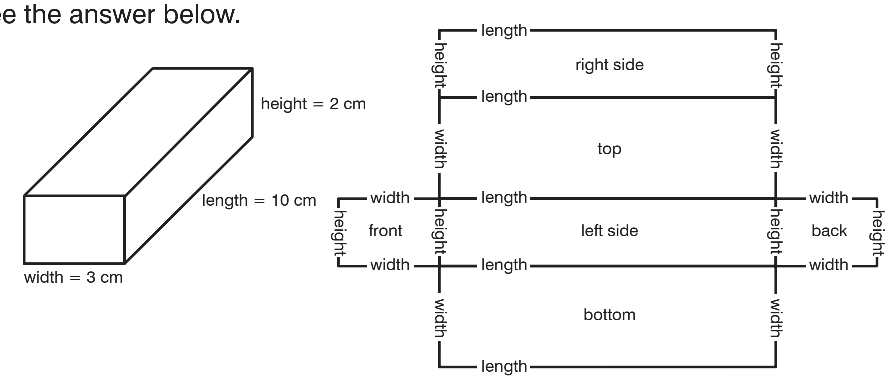
a) $10\text{ cm} \times 6\text{ cm} \times 2\text{ cm}$ b) $2\text{ m} \times 3.4\text{ m} \times 5\text{ m}$ c) $3\text{ km} \times 4.6\text{ km} \times 7.2\text{ km}$

ANSWERS: a) 120 cm^3 b) 34 m^3 c) 99.36 km^3

Remind students to include the correct units in their answers.

Volume of a rectangular prism and area of faces. Draw a rectangular prism on the board, mark the dimensions (say, 2 cm, 3 cm, and 10 cm) and have students find the volume of the prism. **ASK:** What is the length of this prism? (10 cm) The width? (3 cm) The height? (2 cm) Write that information on the board. Invite a volunteer to write the volume of the prism in terms of length, width, and height.

Ask students to sketch the net for the prism and label the top, bottom, side, front, and back faces on the net. Then ask them to mark each edge of the prism as length, width, or height. Label the first several edges together, and have students label the rest of the edges individually. See the answer below.



Have students find the area of each face on the net and then write each area in terms of length, width, and height (**EXAMPLE:** top face = length × width).

ASK: What does the expression "length × width" represent in the formula of the volume of the prism? What do you find when you multiply length by width? (the area of the top face—or the bottom face—of the prism) Rewrite the formula as "area of top face of prism × height."

Remind students that order does not matter in multiplication, so length × width × height can be rewritten as, say, length × height × width. What does the expression "length × height" represent in the formula? (the area of the left or right side) Ask students to rewrite the formula using the area of one of the side faces. Rewrite the formula for the volume as height × width × length. Have students identify the expression "height × width" as the area of the front face and rewrite the formula using the area of the front face.

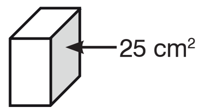
Have students find the volume of the prism with length 4 cm, width 6 cm, and height 7 cm in three ways: using the area of the top face, the area of the front face, and the area of one of the side faces. Did they get the same answer all three ways?

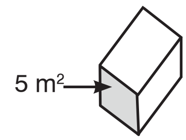
Finding dimensions of a prism given its volume. Draw a prism on the board, and give students the area of the bottom face and the volume. **EXAMPLE:** volume 32 cm^3, area of bottom face 16 cm^2. Mark the height as *h*, and ask students to write an equation for the volume of the prism.

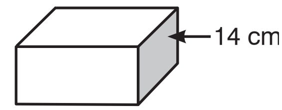
Then have them solve the equation. Have students use this method to find the height of several more prisms. Proceed to problems where the missing dimension is not height, but length or width.

EXTRA PRACTICE:

1. a) Volume = 75 cm^3 b) Volume = 12 m^3 c) Volume = 105 cm^3

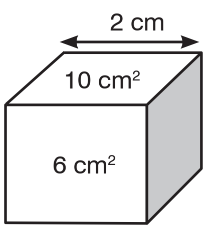






ANSWERS: a) 3 cm b) 2.4 m c) 7.5 cm

2. The area of the base of a right prism is 8 cm^2 and its volume is 32 cm^3. What is its height? (4 cm)



3. Find the volume of the prism in the margin.

   ANSWER: The height of the prism is 6 $cm^2 \div 2$ cm = 3 cm, so the volume is 3 cm $\times$ 10 cm^2 = 30 cm^3. Another answer: The length of the prism is 10 $cm^2 \div 2$ cm = 5 cm, so the volume is 6 $cm^2 \times 5$ cm = 30 cm^3.

Proceed to problems where the volume and one linear dimension are given, and students need to find the area of the face perpendicular to that direction. **EXAMPLE:** volume 35 cm^3, height 4 cm. What is the area of the bottom face? (35 $cm^3 \div 4$ cm = 8.75 cm^2)

Word problems practice:

Jon brought a cake to class to celebrate his birthday. The cake was a rectangular prism 28 cm by 30 cm by 7 cm.

a) What is the total volume of the cake? (5 880 cm^3)

b) There are 40 students in the class. How much cake will each person get?

   ________ $cm^3 \div 40$ = ________ (5 880 $cm^3 \div 40$ = 147 cm^3)

c) Use cm grid paper to draw the base of the cake. Cut the cake into 40 equal-sized pieces.

d) What are the dimensions of each piece? Use this to check your answer to part b). (**SAMPLE ANSWER:** 7 cm $\times$ 3 cm $\times$ 7 cm = 147 cm^3)

Extension

The volume of a rectangular prism is 24 cm^3 and its height is 2 cm. What can be the dimensions of the base of the prism? **SAMPLE ANSWER:** The base of the prism has area 24 $\div$ 2 = 12 cm^2, so the dimensions of the base could be 1 cm $\times$ 12 cm, 2 cm $\times$ 6 cm, 3 cm $\times$ 4 cm, 2.4 cm $\times$ 5 cm, and so on.

ME8-12 Volume of Polygonal Prisms

Pages 155–156

CURRICULUM EXPECTATIONS
Ontario: 6m40; 7m40; **8m1, 8m3, 8m5, 8m7, review**
WNCP: **8SS4, [C, R, V]**

VOCABULARY
volume
length
width
height
right prism
parallelogram
triangle
triangular prism
rectangular prism

Goals

Students will find the volume of right prisms.

PRIOR KNOWLEDGE REQUIRED

Can find volume of a rectangular prism
Can identify right prisms as prisms
Can identify the base of a prism
Can multiply or divide decimals
Is familiar with cubic units of measurement
Knows the formulas for the area of a triangle and a parallelogram and the connection between them

MATERIALS

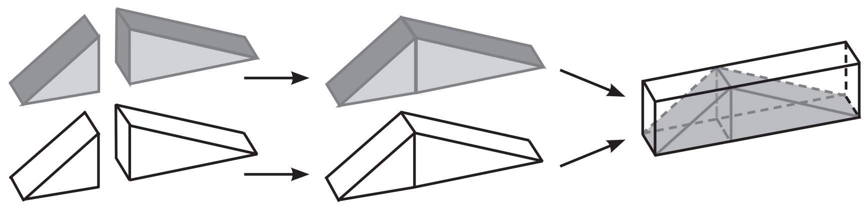
BLM Nets of 3-D Shapes (6) (p U-6)
two colours of paper to make prisms from the BLM (see below)
boxes of different shapes (not only rectangular)

Volume of rectangular prism = area of base × height. Draw a rectangular prism on the board, mark the dimensions (say, 2 cm, 3 cm, and 4 cm) and have students find the volume of the prism. Invite volunteers to write the volume of the prism in terms of the area of one of the faces and height, length, or width. Remind students that in the case of a rectangular prism, any pair of opposite faces can be bases and the dimension perpendicular to the bases is called *height.* So if we take, say, the bottom face to be the base, we can rewrite the formula for the volume of the prism as "height of prism × area of base of prism."

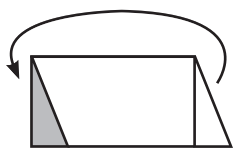
Volume of triangular prisms with a right triangle in the base. Ask students to think about how they could calculate the volume of such prisms. Ahead of time, photocopy **BLM Nets of 3-D Shapes (6)** onto two pages of different colours and make two copies of each prism, say, green and blue. The prisms on the BLM are both right rectangular, but they have different triangles as bases. Show students two identical prisms (one green, one blue) side by side, then show how you can put them together to make a rectangular prism. **ASK:** What fraction of the volume of the rectangular prism does each triangular prism make? (half)

Volume of triangular prisms with a scalene triangle in the base. Review finding the area of triangles by splitting them into two right triangles.

Join the congruent faces (numbered 1) of the two green prisms together so that they make a single triangular prism with a scalene obtuse base and show that prism to students. Repeat with the blue prisms. Place the green and blue prisms side by side to emphasize that they are identical. Do they have the same volume? (yes) Show how you can make a rectangular prism by separating the parts of the blue prism and attaching each smaller blue prism to the green one (attach the blue face 2 to the green face 2 and the blue face 3 to the green face 3 to make a single rectangular blue-and-green prism). What fraction of the rectangular prism is the green prism? (half) Summarize: The volume of a triangular prism with any triangle in the base is half the volume of the rectangular prism with the same height and a base that is twice as large as the base of the triangular prism.

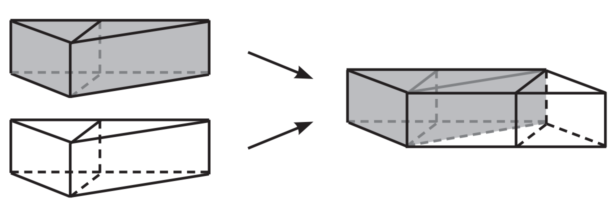


The triangular prisms of each colour can be joined into larger triangular prisms and then combined to form a rectangular prism.

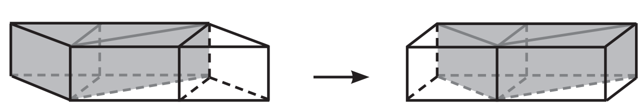


Volume of prisms with a parallelogram in the base. Draw a parallelogram on the board and review with students how they can convert a parallelogram to a rectangle with the same area by cutting off a triangle and shifting it to the other side. Remind students that the area of the parallelogram is base × height. Point out that the word "base" has a different meaning here—the base of a parallelogram is the length of the side to which we draw a perpendicular (but the base of a prism is a face, which can itself be a parallelogram).

Display the two green and two blue prisms again. Join the prisms of each colour into larger prisms with a scalene triangle in the base, as above, but this time place the prisms so that they stand on their bases. Show how you can combine the prisms to make a larger prism with a parallelogram in the base (join face 3 of the combined blue prism to face 3 of the combined green prism as shown below). Ask students to identify the base and the height of the parallelogram in the base of the prism. (Again, emphasize that the word base refers to two different things here—the length of the side of a parallelogram and the parallelogram itself, which is the base of the prism.)

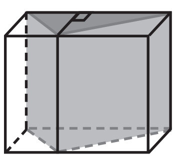


ASK: How could we convert this parallelogram-based prism into a rectangular prism? Take suggestions, then shift the smallest blue prism to the other side of the combined blue-and-green prism, so that face 2 is joined to face 2 to obtain a rectangular prism as before.



ASK: What is the length of the new prism? (the base of the parallelogram) What is its width? (the same as the height of the parallelogram in the base) What is its height? (the same as the height of the parallelogram-based prism) What is the volume of this prism? (base of parallelogram $\times$ height of parallelogram $\times$ height of the prism) What do the first two terms in the product make? (area of parallelogram) Ask students to rewrite the formula for the volume of the parallelogram-based prism using the area of the base of the prism. (area of base $\times$ height of prism)

Ask students to find the volume of a prism with height 7 cm and a parallelogram in the base that has base 5 cm and height 4 cm. Repeat with more prisms.



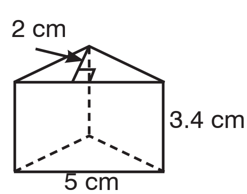
Volume of triangular prisms $=$ area of base $\times$ height of prism. Explain that now that you have a nice formula for the volume of a prism with a parallelogram in the base (area of base $\times$ height of prism), you would like to go back to the volume of a triangular prism, to see whether a similar formula would work there. Draw a triangular prism inside a rectangular prism, as shown at left, and ask students what the volume of the triangular prism should be. (half the volume of the rectangular prism) The volume of the rectangular prism is area of base $\times$ height of prism. Let's choose the top face of the rectangular prism to be the base, so that it contains the base of the triangular prism and both prisms have the same height. So the volume of the triangular prism is:

$$\frac{1}{2} \text{ volume of rectangular prism}$$
$$= \frac{1}{2} \text{ area of base of rectangular prism} \times \text{height of prism}$$
$$= \frac{1}{2} \text{area of rectangle} \times \text{height of prism}$$
$$= \text{ area of triangle} \times \text{height of prism}$$
$$= \text{ area of base of triangular prism} \times \text{height of prism}$$

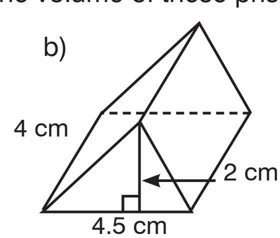
This means that the same formula (area of base $\times$ height of prism $=$ volume of prism) works for triangular prisms as well.

EXTRA PRACTICE: Find the volume of these prisms.

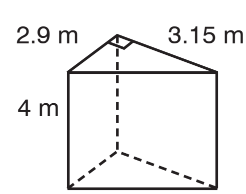
a)



b)



c)

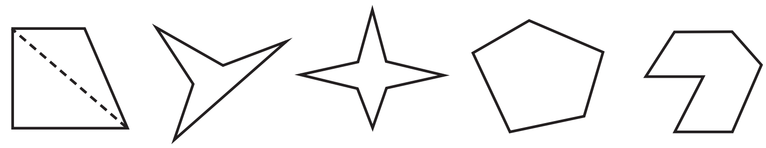


PROCESS ASSESSMENT

8m5, 8m7, [C, CN]
Workbook p 156 Question 2

ANSWERS: a) 17 cm^3 b) 18 cm^2 c) 18.27 m^3

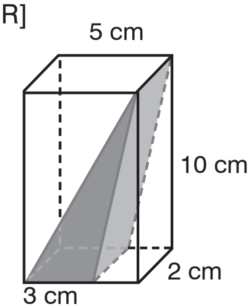
Volume of a prism with any polygon in the base. Combine any three of the green and blue triangular prisms (all standing on a base) so that they make a right prism with a complicated polygon in the base. Ask students how they could find the volume of this prism. Draw the shape of the base on the board and invite a volunteer to show how they would split the prism into triangular prisms by splitting the base into triangles. Point out that any polygon can be decomposed into triangles. Ask students to show how to do so for each of the following polygons.



Have students draw two different polygons (with at least 5 sides) and show how to cut them into triangles. Ask students to think about what measurements they will need to take to find the area of each polygon. Then have students take the measurements and find the area of the polygons they drew and the volume of prisms with those polygons as bases and height 20 cm.

PROCESS ASSESSMENT

8m3, [R]



EXTRA PRACTICE:
Valerie's teacher says that a triangular prism has volume that is half the volume of a rectangular prism of the same height. Valerie looks at this picture and thinks that the volume of the grey triangular prism is half the volume of the rectangular prism, so the volume of the triangular prism is 5 cm $\times$ 2 cm $\times$ 10 cm $\div$ 2 $=$ 50 cm^3. Is she correct? Explain.

ANSWERS: No, the triangular prism is not half the rectangular prism, it is smaller than that. In this triangular prism the front face is the base, so if we take the front face to be the base in the rectangular prism, we can see that the triangle is less than half the 5 cm $\times$ 10 cm rectangle. In fact, the volume of the triangular prism is (3 cm $\times$ 10 cm $\div$ 2) $\times$ 2 cm $=$ 30 cm^3.

PROCESS ASSESSMENT

8m1, [PS]

ACTIVITY

Give students a variety of right prisms (shapes made from **BLM Nets of 3-D Shapes** and boxes) and have them measure the prisms and find their volume.

Extension

A wealthy king had a treasure chest in the shape of a rectangular prism. He ordered his carpenters to create a larger chest for his treasure.

a) The first carpenter doubled the length of the box and left the width and the height the same. The second carpenter doubled the width of the box and left the length and the height the same. The third carpenter doubled the height of the box and left the length and the width the same. Who made the largest chest for the king's treasure? (nobody—they all have the same volume)

b) The fourth carpenter doubled the length, the width, and the height of the king's old treasure chest to create his new chest. How many times larger was the new chest than the old one? (8 times)

c) The fifth carpenter, being jealous of the money the fourth carpenter was paid, decided to make a chest that had the same volume as the chest of the fourth carpenter. He wanted his chest to have the same height as the king's old chest, but he decided that the length of the new chest would be two times more than the length of the old chest. How many times wider than the king's old chest did this carpenter make his chest? (4 times)

ME8-13 Volume of Cylinders

Pages 157–158

CURRICULUM EXPECTATIONS
Ontario: **8m2, 8m3, 8m5, 8m7, 8m37**
WNCP: **8SS4, [C, R, V]**

VOCABULARY
circle
radius
diameter
pi, π
approximately equal to ($\approx$)
cylinder
base
volume
height
right prism

Goals

Students will find the volume of cylinders.

PRIOR KNOWLEDGE REQUIRED

Can find the volume of a right prism using the area of the base
Can identify right prisms as prisms
Can identify the base of a prism
Can multiply or divide decimals
Is familiar with cubic units of measurement
Can find area of a circle
Is familiar with fractional notation for division
Is familiar with variables

MATERIALS

a cylinder
round pennies and 12-sided pennies
10 pennies per student

Review characteristics of circles. Review with students the formula for the area of a circle, the terms radius and diameter, and the meaning of π. Remind them that $\pi \approx 3.14$, and because this is an approximation and not an exact value, any calculation involving this value is only an approximation. Point out that because we use the value of pi rounded to two decimal places, it makes sense to round the answers to two decimal places as well.

Introduce cylinders. Remind students how regular polygons with many sides look almost like a circle. **ASK:** What will a right prism with a polygon with many sides in the base look like? Introduce the term *cylinder*, show students a cylinder, and explain that the circles are also called *bases*, just as the non-rectangular sides of prisms were called bases. To illustrate that a cylinder and a prism with a regular polygon with many sides in the base look very much alike, you can show two stacks of pennies, one made from round pennies, the other from 12-sided pennies.

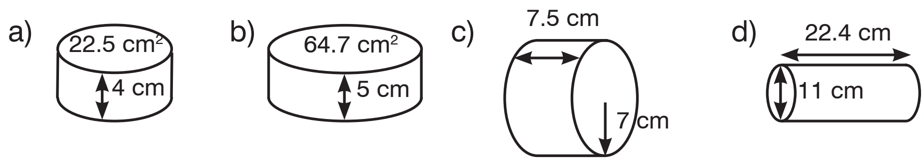
PROCESS ASSESSMENT

8m3, 8m7, [R, C]
Workbook Investigation H

Develop the formula for the volume of a cylinder. Review the formula for the volume of polygonal prisms: area of base $\times$ height of prism. Ask students to predict, based on what they know about the volume of right prisms, what the formula for the volume of cylinders should be. Draw students' attention to the stacks of pennies again. The stacks are so much alike that the volume should be almost the same. Have students do Parts A to D of the Investigation on Workbook page 157 individually. If necessary, remind students that the notation $\frac{a}{b}$ means $a \div b$, and that $a \times b$ can be written as ab. Continue through parts E to H of the Investigation as a class.

Practice finding the volume of cylinders. Use questions similar to Questions 2 and 3 on Workbook page 158.

EXTRA PRACTICE: Find the volume of each cylinder.



PROCESS ASSESSMENT

8m2, 8m5, [C, CN]
Workbook Question 4

ANSWERS: a) ≈ 90 cm^3 b) ≈ 323.5 cm^3 c) $\approx 1\ 153.95$ cm^3 d) $\approx 2\ 127.66$ cm^3

Word problems practice:

1. A glass of water is full to the brim.

   a) The glass of water measures 68 mm across and is 9 cm tall. How much space (in whole cm^3) does the glass of water take up?

   b) The sides of the glass are 3 mm thick, and its bottom is 9 mm thick. 1 cm^3 of water is 1 mL. How much water (in whole mL) does the glass hold?

   ANSWERS:
   a) Volume $= \pi \times 3.4^2 \times 9 \approx 327$ cm^3
   b) Inner volume $= \pi \times 3.1^2 \times 8.1 \approx 244$ cm^3, so the capacity of the glass is 244 mL.

2. A railway car is a cylinder 326 cm in diameter. It is 17.4 m long. What is its volume (in m^3, rounded to two decimal places)? (145.16 m^3)

Extension

Lipstick A is 13 mm wide and 25 mm long, and it costs \$5.95. Lipstick B is 15 mm wide and 17 mm long, and it costs \$5.87. Which one is larger? How much do they cost per mm^3? Which one is cheaper (per mm^3)? (Volume: A $\approx 3\ 316.63$ mm^3, B $\approx 3\ 002.63$ mm^3; cost: A ≈ 0.18 cents/mm^3, B ≈ 0.20 cents/mm^3. Lipstick A is larger and cheaper.)

ME8-14 Capacity

Page 159

CURRICULUM EXPECTATIONS
Ontario: 6m40; 7m40**; 8m2, 8m3, 8m5, 8m6, 8m7**
WNCP: **8SS4, [C, R, V]**

VOCABULARY
capacity
volume
length
width
height
right prism
cylinder
triangular prism
rectangular prism
decimetre (dm)

Goals

Students will solve problems related to capacity of right prisms and cylinders.

PRIOR KNOWLEDGE REQUIRED

Can find the volume of a rectangular prism
Can identify right prisms as prisms
Can identify the base of a prism
Can multiply or divide decimals
Is familiar with cubic units of measurement
Can find area of a circle
Can find the volume of a cylinder

MATERIALS

small graduated cylinders
centicubes
cans and boxes of different shapes

Capacity. Explain that the *capacity* of a container is how much it can hold. Write the term on the board.

Remind students that capacity is measured in litres (L) and millilitres (mL). **ASK:** Where have you seen the prefix "milli" before and what did it mean? (millimetre; one thousandth) How many millilitres are in 1 litre? (1 000) In 2 litres? (2 000) In 7 litres? (7 000) What do you do to change litres to millilitres? (multiply by 1 000)

Write on the board:

1 metre = 1 000 millimetres	1 m = 1 000 mm
1 litre = 1 000 millilitres	1 L = 1 000 mL

Ask students to think of three quantities that are measured in litres and three that are measured in millilitres. (**EXAMPLES:** mL—cup of juice, small carton of milk, dosage of liquid medicine, bottle of perfume; L—big carton of juice, gas tank of a car, bag of potting soil)

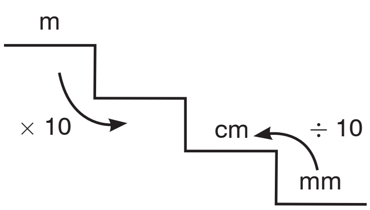
PROCESS EXPECTATION

Representing, Connecting

Connection between mL and cm³. Hold up a graduated cylinder with some water in it. Drop a centicube (= 1 cm^3) into the cylinder. **ASK:** your students if they can see how much liquid is displaced by the cube. (no, the amount is too small) What can you do to find how much water is displaced by one cube? (One answer: Drop in 10 cubes and divide the displacement by 10.) If possible, have all students drop centicubes into graduated cylinders and measure the displacement. (should be 10 mL) What is the capacity of 1 cm^3? (1 mL)

Show a small rectangular box and ask students how they could measure its capacity. We know the capacity of 1 cm^3. What is the capacity of 10 cm^3? Of 20 cm^3? Invite volunteers to measure the sides of the box and calculate its volume. What is the capacity of the box?

Draw a cube on the board and tell students that it has a capacity of 1 L. **ASK:** What are the dimensions of the cube? How many millilitres are in 1 L? How do you find the volume of the cube? (You multiply the side by itself 3 times.) Which number is multiplied by itself 3 times to get 1 000? So how long is the side of the cube? (10 cm) Mark the cube sides as 10 cm $\times$ 10 cm $\times$ 10 cm, and invite volunteers to write both the volume of the cube and its capacity (in mL and L) beside the cube.



Connection between L and dm^3. Remind students that 10 centimetres make 1 decimetre. You can draw the conversion "stairs" at left on the board and add decimetres to the empty step. Draw a cube, mark its sides as 1 dm $\times$ 1 dm $\times$ 1 dm, and ask students to find its volume in dm^3 and in cm^3. (1 dm^3 and 1 000 cm^3) What is the capacity of this cube? (1 000 mL = 1 L)

Finding the capacity of prisms. Draw a box on the board and write its dimensions: 30 cm $\times$ 40 cm $\times$ 50 cm. **ASK:** What is the capacity of the box? Let students find the capacity in millilitres first, then ask them to convert it to litres.

PROCESS EXPECTATION

Reflecting on other ways to solve the problem

Ask students if they can solve the problem another way. (**HINT:** a cube 1 dm $\times$ 1 dm $\times$ 1 dm has capacity 1 L) They can convert the dimensions to decimetres and get the result in litres: 3 dm $\times$ 4 dm $\times$ 5 dm = 60 dm^3, so the capacity is 60 L.) Did students get the same answer?

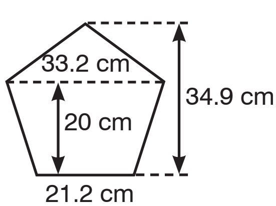
Give students more problems of this kind. Include triangular prisms, polygonal prisms, and cylinders. **EXAMPLES:**

1. A cylindrical water barrel is used to collect rain water. It is 70 cm wide. After a heavy rain, the height of the water in the barrel is 7 cm. How much water is in the barrel?

   ANSWER: volume $\approx 3.14 \times 35^2 \times 7 \approx 26.9$ cm^3 $\approx$ 26.9 L

2. A can of soup has diameter 6.5 cm and height 9.3 cm. The can is made of tin that is 1 mm thick. Find the capacity of the can, rounded to the nearest mL.

   ANSWER: Inner diameter = 6.3 cm (inner radius 3.15 cm), inner height = 9.1 cm, so capacity is 284 mL.



3. Find the volume and the capacity (in L, rounded to one decimal place) of a prism with height 12 cm and the base as shown.

   SOLUTION: Split the pentagon into a trapezoid (with bases 33.2 and 21.2 cm and height 20 cm) and a triangle (with base 33.2 cm and height 34.9 − 20 = 14.9 cm). The area of the trapezoid is 544 cm^2, and the area of the triangle is 247.34 cm^2. The area of the base of the prism is thus 791.34 cm^2, and the volume is 9 496.08 cm^3. The capacity is about 9.5 L.

EXTRA PRACTICE:

1. Find the volume and the capacity of the rectangular prisms with these dimensions:

   a) 1 m $\times$ 1 km $\times$ 1 m   b) 5 cm $\times$ 0.3 m $\times$ 2 m   c) 1 mm $\times$ 1 m $\times$ 1 km

   ANSWERS:

   a) Volume: 1 m $\times$ 1 000 m $\times$ 1 m $=$ 1 000 m^3
   $=$ 1 000 $\times$ (1 m $\times$ 1 m $\times$ 1 m)
   $=$ 1 000 $\times$ (10 dm $\times$ 10 dm $\times$ 10 dm)
   $=$ 1 000 000 dm^3, capacity 1 000 000 L

   b) Volume: 0.05 m $\times$ 0.3 m $\times$ 2 m $=$ 0.03 m^3 $=$ 30 000 cm^3, capacity 30 000 mL $=$ 30 L

   c) Volume: 0.001 m $\times$ 1 m $\times$ 1 000 m $=$ 1 m^3, capacity 1 000 L

2. Daniela wants to find the volume of an apple. She puts the apple into a glass jar with 600 mL of water. The jar is a cylinder 12 cm in diameter. The water reaches a height of 9.8 cm. What is the volume of the apple? (Volume of apple and water together $= \pi \times 6^2 \times 9.8$ cm^3 $\approx$ 1 107.8 cm^3, so the volume of the apple is $\approx$ 407.8 cm^3.)

Word problems practice:

1. A recipe for pumpkin pie filling calls for $1\frac{3}{4}$ cups of pumpkin puree, 1 cup of sugar, 2 beaten eggs, and $\frac{3}{4}$ cups of cream. A cup has capacity 240 mL, and a beaten egg is about 0.2 cups.

   a) Convert the measurements to decimals.

   b) What is the total capacity of the ingredients?

   c) Katie has a round pan 25 cm in diameter and 4 cm deep. She made the pie shell about 0.5 cm thick, and it reaches to the top of the pan. If she uses the recipe above for the filling, will she have too little filling for her pie shell, too much filling, or just enough?

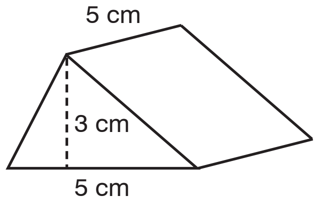
   d) About how far will the top of the filling be from the top of the shell (and the top of the pan)? Round your answer to the nearest millimetre (which is a tenth of a centimetre).

   ANSWERS:

   b) $1.75 + 1 + 0.4 + 0.75 = 3.9$ cups $=$ 936 mL

   c) The inner radius of the shell is $12.5 - 0.5 = 12$ cm and the height is 3.5 cm, so the capacity of the pie shell is about 1 583 mL. The recipe will not produce enough filling to fill the shell to the top.

   d) The area of the base of the inside of the pie shell is 472.16 cm^2, so the filling will fill about 936 cm^3 $\div$ 472.16 cm^2 $\approx$ 2.0 cm, so there will be about 4.0 cm $-$ 0.5 cm $-$ 2.0 cm $=$ 1.5 cm between the top of the pie shell and the top of the filling.



2. Ellie made a triangular prism from modelling clay, as shown. She rolled it into a cylinder with diameter 3 cm. What is the height of the cylinder she made?

   ANSWER: Volume of prism $= 5 \text{ cm} \times 3 \text{ cm} \times 5 \text{ cm} \div 2 = 37.5 \text{ cm}^3$.
   Height of cylinder $\approx 37.5 \text{ cm}^3 \div (3.14 \times 1.5^2) \text{ cm}^2 \approx 5.3 \text{ cm}$.

ACTIVITY

Give students cylindrical cans and boxes in the shape of different prisms (such as chocolate boxes) and have them find the volume and the capacity by taking the necessary measurements.

Extensions

1. A cylindrical can of soup measures 97 mm from top to bottom and 67 mm across.

   a) Find the volume of the can.

   b) The label says that the can contains 284 mL of soup. Examine a similar can and explain why the volume on the label can be different from the volume you found in a). (**ANSWER:** The can measurements include the rim of the can and its thickness. The inner radius and height, which determine capacity, will be smaller.)

2. When baking brownies, it is important that you keep the height of the brownie mix in the pan very close to what the recipe calls for. How should you change the recipe if it asks you to use a 10" square pan and you only have an 8" round pan? (An 8" round pan has a diameter of 8" and a radius of 4".)

   SOLUTION: A 10" square pan has area of base 100 in^2. An 8" round pan has area of base about 50.24 in^2, which is about half of the base of the square pan. To keep the height the same, we need to halve the volume, so we need to make only half of the recipe.

ME8-15 Changing Units of Area and Volume

Page 160

CURRICULUM EXPECTATIONS
Ontario: 6m39; 7m36; **8m2, 8m3, 8m5, 8m33**
WNCP: 7N2, **8SS4, [C, CN, R]**

VOCABULARY
metre, centimetre, millimetre, kilometre
metre/centimetre/millimetre/kilometre squared (m^2, cm^2, mm^2, km^2)
metre/centimetre/millimetre/kilometre cubed (m^3, cm^3, mm^3, km^3)

Goals

Students will convert measurements of area and volume.

PRIOR KNOWLEDGE REQUIRED

Is familiar with decimals
Can multiply or divide decimals by powers of 10
Is familiar with units of measurement for length, area, volume, and capacity
Can convert between linear measurements
Can find the area of polygons and circles
Can find the volume of prisms and cylinders

MATERIALS

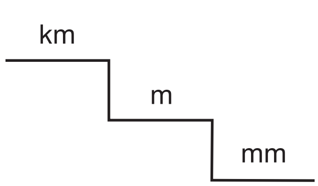
BLM Square Metre (p R-51)

Make sure students know how to multiply and divide decimals by 10, 100, and 1 000 by shifting the decimal point. You can use the questions below as a diagnostic test. If necessary, review NS8-46 to NS8-51.

Fill in the blanks:

a) $2.7 \times 100 =$ ______
b) $29 \div 100 =$ ________
c) $.45 \times 100 =$ ________
d) $3.6 \div 100 =$ ______
e) $2.32 \times 1\,000 =$ ________
f) $254 \div 1\,000 =$ ________
g) $.36 \times 1\,000 =$ ________
h) $5.07 \div 1\,000 =$ ________
i) $.043 \times 1\,000 =$ ________
j) $.79 \div 1\,000 =$ ________
k) $4.3 \times 10\,000 =$ ________
l) $37 \div 10\,000 =$ ________
m) $.18 \times 10\,000 =$ ________
n) $5.9 \div 10\,000 =$ ________
o) $6.253 \times 10\,000 =$ ________
p) $34.56 \div 10\,000 =$ ________
q) $41.31 \times 1\,000\,000 =$ ________________
r) $3\,278 \div 1\,000\,000 =$ ________________

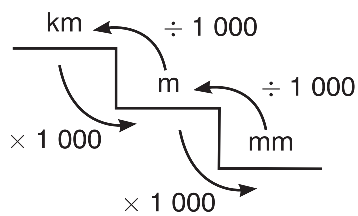
Review relationships. Review the relationships between units of length (metre, kilometre, millimetre), capacity (litre and millilitre), and mass (gram, kilogram, milligram). Discuss the meaning of the prefixes kilo and milli (both mean 1000, but kilo is used for larger units and milli is used for smaller units).



Do we multiply or divide? Draw the diagram in the margin on the board.

ASK: Which unit is the largest? Which unit is the smallest? Let's look at a measurement in metres: 2 000 m. How many kilometres is that? (2 km) To convert the measurement in metres to the new unit, kilometres, did we need more or fewer of the new units? (fewer) How many times fewer? (1 000 times fewer) To get the measurement in kilometres from a measurement in metres, should we multiply by 1 000 or divide by 1 000?

(divide) Why? (The new unit is larger than the old unit, so we need fewer new units.) Add an arrow from m to km on the diagram and label the division by 1 000.



Discuss the conversion from kilometres to metres, then from metres to millimetres and vice versa, adding more arrows to the diagram. The finished diagram should look like the diagram at left.

Choose pairs of units and ask students if they need to multiply or divide by 1 000 to convert a measurement from one to the other. **EXAMPLES:** from m to km, from km to mm, from mm to m.

Convert between units by multiplying or dividing by 1 000. Fill in the blanks in the following two questions together, then have students practise converting units individually using the same thinking.

Convert 275 mm to m:

The new units are ________ times ________ . (1 000 / bigger)
So I need ________ times ________ units. (1 000 / fewer)
I ________________ by ________ . (divide / 1 000)
275 mm = ________ m (0.275)

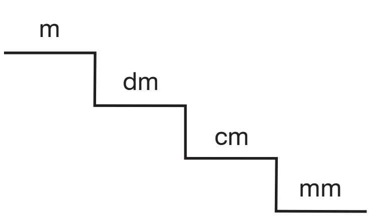
Convert 27.5 km to m:

The new units are ________ times ________ . (1 000 / smaller)
So I need ________ times ________ units. (1 000 / more)
I ________________ by ________. (multiply / 1 000)
27.5 km = ________ m (27 500 m)

EXAMPLES:
Change the units:

a) 245 m to km b) 2.67 m to mm c) 0.76 km to m
d) 345 mm to m e) 36.9 m to km f) 1 560 m to mm
g) 3.587 mm to m h) 8 765 km to mm i) 76.98 mm to m

ANSWERS:
a) 0.245 km b) 2 670 mm c) 760 m d) .345 m e) .036 9 km
f) 1 560 000 mm g) .003 587 m h) 8 765 000 000 mm i) 0.076 98 m



Review the relationships between m, cm, dm, and mm. Draw the diagram at left on the board. Have students copy the diagram and add arrows and multiplication or division prompts. They should explain, orally and/or in writing, why they multiply or divide in each case and by how much. Review answers on the board.

EXAMPLE: Convert 47 cm to metres. To get from centimetres to metres, I go up 2 steps. This means the new unit is 10 × 10 = 100 times larger, so I need 100 times fewer units. So I divide by 100. To divide by 100, I shift the decimal point 2 places to the left: 47 cm = 0.47 m.

Ask students to tell whether they need to multiply or divide, and by how much, to convert measurements from:

a) m to cm b) cm to mm c) cm to m d) mm to cm

Bonus e) mm to km f) km to cm

SAMPLE ANSWER: a) We need more centimetres than metres and there are 2 steps from one to the other in the diagram, so we have to multiply by 100 to convert a measurement in metres to a measurement in centimetres.

Now have students perform some conversions between these units.

EXTRA PRACTICE:
Change the units.

a) 240 m = _______ cm b) 2.61 mm = _______ cm
c) 0.78 cm = _______ m d) 38.5 dm = _______ m

Bonus
e) 36.9 km = _______ cm f) 234 568.9 dm = _______ km
g) 1 234 567 890 mm = _______ km

ANSWERS: a) 24 000 cm b) .261 cm c) 0.007 8 m d) 3.85 m
Bonus e) 3 690 000 cm f) 23.456 89 km g) 1 234.567 89 km

Review the names of the units of area: metres squared (m^2) and centimetres squared (cm^2).

1 m^2 = 10 000 cm^2. Draw a very large square on the board and mark its sides as 1 m. **ASK:** What is the area of this square? (1 m^2) Write the area below the square. Divide the square into a 10 × 10 grid, and **ASK:** How many squares are in the large square? (100) How do you know?

ASK: How many centimetres are in a metre? Change the markings on the sides to show 1 m = 100 cm. Ask whether the smaller squares are 1 cm × 1 cm. (no) What is the side length of each smaller square? (10 cm) What do you have to do to get squares 1 cm × 1 cm? (divide the smaller squares into another 10 × 10 grid) Show the division on one of the squares. (As an alternative, you can photocopy **BLM Square Metre** on a transparency and project it on the board; cover the labels to start and uncover them as you go.) How many small squares are in the medium square? (10 × 10 = 100) How do you know? What is the area of the medium square? (100 cm^2) How many small squares will fit into the largest square? (10 000) What is the area of the largest square in cm^2? (10 000 cm^2) How do you know? (There are 100 medium squares in the large square, and 100 small squares can fit in each medium square, so in total there will be 100 × 100 cm^2 = 10 000 cm^2). Write the equation for the area of the largest square on the board.

Find the area a different way and compare the two methods. ASK: How many small squares fit along the side of the large square? (100) Why? (because 1 m = 100 cm) How do you find the area of a square? What would that give for the large square? (100 cm × 100 cm = 10 000 cm^2) What is the area of the large square in metres? Ask students to write the equality for the area: 1 m^2 = 1 m × 1 m = 100 cm × 100 cm = 10 000 cm^2.

PROCESS EXPECTATION

Reflecting on other ways to solve a problem

Now compare these equalities: $100 \times 100\ cm^2 = 10\ 000\ cm^2$ and $100\ cm \times 100\ cm = 10\ 000\ cm^2$. How are they the same? How are they different? In the first equality, the first number does not have units—it is the number of times the medium square appears—and the second number is the area of the medium square. In the second equality, both numbers are length measurements, so they are measured in centimetres and produce centimetres squared when multiplied.

Converting between m^2 and cm^2. Draw a rectangle on the board and mark the sides as 120 cm and 200 cm. Ask students to find the area of the rectangle in cm^2. ($24\ 000\ cm^2$) Then ask them to convert the lengths to metres and to find the area in m^2. ($1.2 \times 2 = 2.4\ m^2$) Ask students to compare the answers. Which measurement has a larger numeral? Which measurement has a larger unit? Do you need more or fewer cm^2 than m^2? To get the measurement in cm^2, what do you have to do to the measurement in m^2? (multiply by 10 000) Why do you multiply? (because the new unit is smaller we need more units, so we have to multiply by 10 000) Repeat with a rectangle $60\ cm \times 70\ cm$, and again with a rectangle $25\ cm \times 40\ cm$.

Write several measurements on the board and ask students to convert the units between cm^2 and m^2.

PROCESS ASSESSMENT

8m2, [C]

Practise converting measurements of area. At first, ask students to explain why they need to multiply or to divide by 10 000, as in Question 2 on Workbook page 160, then have students just do the conversion.

EXTRA PRACTICE:
Change the units:

a) $23\ m^2 =$ ____________ cm^2	b) $2.61\ m^2 =$ ____________ cm^2
c) $7\ 865\ cm^2 =$ ____________ m^2	d) $38.5\ cm^2 =$ ____________ m^2
e) $0.076\ 5\ m^2 =$ ____________ cm^2	f) $0.54\ m^2 =$ ____________ cm^2
g) $137\ 845\ cm^2 =$ ____________ m^2	h) $0.5\ cm^2 =$ ____________ m^2
i) $0.002\ 3\ m^2 =$ ________ cm^2	j) $0.000\ 06\ m^2 =$ ________ cm^2
k) $456\ cm^2 =$ ____________ m^2	l) $4.72\ cm^2 =$ ____________ m^2

ANSWERS:

a) $230\ 000\ cm^2$	b) $26\ 100\ cm^2$	c) $0.786\ 5\ m^2$	d) $.003\ 85\ m^2$
e) $765\ cm^2$	f) $5\ 400\ cm^2$	g) $13.784\ 5\ m^2$	h) $0.000\ 05\ m^2$
i) $23\ cm^2$	j) $0.6\ cm^2$	k) $0.045\ 6\ m^2$	l) $0.000\ 472\ m^2$

Review the names of the units of volume: metres cubed (m^3), millimetres cubed (mm^3) and centimetres cubed (cm^3).

$1\ cm^3 = 1\ 000\ mm^3$. Draw a cube on the board and mark its sides as 1 cm. **ASK:** What is the volume of this cube? ($1\ cm^3$) Write the volume below the cube. Draw a copy of the cube and mark the sides as 10 mm. **ASK:** Are these cubes the same or different? Why? (same, 10 mm = 1 cm) Ask students to find the volume of the cube in millimetres cubed.
Draw a square and mark its sides 1 cm = 10 mm. Have students find the area in cm^2 and in mm^2. Then compare the units of area and volume: a square is 2-dimensional, so $1\ cm^2 = 10\ mm \times 10\ mm = 100\ mm^2$

(we multiply the linear factor 10 two times), and a cube is 3-dimensional, so 1 cm^3 = 10 mm × 10 mm × 10 mm = 1 000 mm^3 (we multiply the linear factor 10 three times).

Review that 1 m^3 = 1 000 000 cm^3. Draw a square and a cube again, and mark the sides of both as 1 m = 100 cm. Again, have students find the area and volume, in smaller units and in larger units, and again draw attention to the fact that the linear factor is multiplied by itself twice for area and three times for volume.

PROCESS EXPECTATION

Connecting

Connect to powers. ASK: How do mathematicians write an expression such as 100 × 100 in a short way? (100^2) How do they write 100 × 100 × 100?(100^3) Where do we see a similar notation? (in the units cm^2 and cm^3) Point out that to know what number to multiply or to divide, we need to look at the units. For example, to convert 345 m^2 to km^2, we can think this way:

The old units are m^2 and the new units are km^2. We know 1 km = 1 000 m, and we are dealing with square units, so the new units are 1 000^2 times larger. This means we need 1 000 000 fewer units, and so we divide by 1 000 000: 345 m^2 = 0.000 345 km^2.

Have students say whether they need to multiply or to divide, and by how much, to convert:

m^2 to mm^2 (× 1 000^2)	cm^2 to m^2 (÷ 100^2)
km^3 to m^3 (× 1 000^3)	m^3 to cm^3 (× 100^3)

Practise converting areas and volumes. Have students perform conversions such as:

a) 500 m^2 = ____________ cm^2	b) .9 m^3 = ____________ cm^3
c) 3 cm^2 = ____________ m^2	d) 1 950 cm^3 = ____________ m^3
e) 15.4 cm^2 = ____________ m^2	f) 0.05 m^3 = ____________ cm^3
g) 4 200 m^3 = ____________ mm^3	h) .7 mm^2 = ____________ cm^2
i) 2.3 km^3 = ____________ m^3	j) 145.4 mm^2 = ____________ m^2
k) 15.34 cm^3 = ____________ mm^3	l) 0.007 dm^3 = ____________ cm^3

ANSWERS:
a) 5 000 000 cm^2 b) 900 000 cm^3 c) 0.000 3 m^2 d) .001 95 m^3
e) .001 54 m^2 f) 50 000 cm^3 g) 4 200 000 000 000 mm^3 h) .007 cm^2
i) 2 300 000 000 m^3 j) 0.000 145 4 m^2 k) 15 340 mm^3 l) 7 cm^3

PROCESS ASSESSMENT

8m3, [ME]
Workbook Question 3

Measurements need to use the same units in calculation of volume. Remind students that when a prism or a cylinder has measurements given in different units, say metres and centimetres, you would need to convert some of the measurements before multiplying. Metres cannot be multiplied by centimetres! For example, if a prism has sides 2.2 m × 37 cm × 50 cm, we could convert all the measurements to centimetres and find the volume in cm^3: 220 cm × 37 cm × 50 cm = 407 000 cm^3. Alternatively, we could convert all the measurements to metres and find the volume in m^3: 2.2 m × .37 m × .5 m = 0.407 m^3. These answers agree, since there are 1 000 000 cm^3 in 1 m^3. Have students find the volume of prisms and cylinders with mixed measurements by converting to each of the units used, and check that the answers agree.

a) 105 cm $\times$ 0.6 m $\times$ 24 cm b) 2 m $\times$ 3.4 m $\times$ 58 cm
c) radius 3.1 m, height 46 cm

ANSWERS:
a) 151 200 cm^3 = 0.151 2 m^3 b) 3 944 000 cm^3 = 3.944 m^3
c) $\approx$13.880 684 m^3 = 13 880 684 cm^3

Point out to students that an easy way to make sure you do not mistakenly multiply metres by centimetres is to write all the measurements with the units at every stage of a calculation, as in the example above.

Extensions

1. Which has larger area? Which has larger perimeter?
   a) a rectangle 30 cm $\times$ 1.5 m or a rectangle 920 mm $\times$ 45 cm
   b) a square 1 m $\times$ 1 m or a rectangle 70 cm $\times$ 130 cm
   c) a square 1 m $\times$ 1 m or a rectangle 90 cm $\times$ 110 cm
   d) a square 1 m $\times$ 1 m or a rectangle 99 cm $\times$ 101 cm
   e) a square 1 m $\times$ 1 m or a circle of radius 63.7 cm?

   ANSWERS:
   Areas:

   a) 30 cm $\times$ 150 cm = 4 500 cm^2 and 92 cm $\times$ 45 cm = 4 140 cm^2, so the first rectangle is larger

   b) the rectangle has area .7 $\times$ 1.3 = .91 m^2, so the square is larger

   c) the rectangle has area .9 $\times$ 1.1 = .99 m^2, so the square is larger

   d) the rectangle has area .99 $\times$ 1.01 = .999 9 m^2, so the square is larger

   e) the circle has area $\approx$ $0.637^2 \times 3.14 \approx$ 1.274 1 m^2, so the circle is larger

PROCESS EXPECTATION
Connecting

   Perimeters: In a), the first rectangle has a larger perimeter. All three rectangles in b), c), and d) have the same perimeter as the 1 m $\times$ 1 m square. The circumference of the circle in d) is about the same as the perimeter of the square. Point out that the square has the largest area of all rectangles with the same perimeter, and the circle is the most economical of shapes—it will cover the largest area of all shapes with the same distance around.

2. Which insect travels faster: an insect moving 24 mm per second on an insect moving 24 m per hour?

   ANSWER: 24 m/h = 24 000 mm / 3 600 s = 24 000 $\div$ 3 600 mm/s = about 6.67 mm/s, so the first insect is faster. Here is another way to see this: The first insect is moving at 24 mm/s = 0.024 m/s. There are 3 600 seconds in an hour, so in an hour this insects moves 3 600 $\times$ 0.024 m, meaning its speed is 3 600 $\times$ 0.024 m/h = 86.4 m/h, which is more than 24 m/h. Again, the first insect is faster.

CONNECTION ↘

Science

3. Lisa has to convert units of density, from g/cm^3 to kg/m^3. Does she need to multiply or to divide, and by how much?

ANSWER: 1 g/cm^3 is a rate: 1 g : 1 cm^3 (or $\frac{1\,g}{1\,cm^3}$ in fractional notation).

Multiply both sides of the rate by 1 000 to change grams to kilograms:

1 g : 1 cm^3 = 1 000 g : 1 000 cm^3 = 1 kg : 1 000 cm^3

(in fractional notation: $\frac{1\,g}{1\,cm^3} = \frac{1\,000\,g}{1\,000\,cm^3} = \frac{1\,kg}{1\,000\,cm^3}$)

Multiply by 1 000 again to convert cm^3 to m^3:

1 kg : 1 000 cm^3 = 1 000 kg : 1 000 000 cm^3 = 1 000 kg : 1 m^3

(in fractional notation: $\frac{1\,kg}{1\,000\,cm^3} = \frac{1\,000\,kg}{1\,000\,000\,cm^3} = \frac{1\,000\,kg}{1\,m^3}$)

This means 1 g/cm^3 = 1 000 kg/m^3. Since we need 1 000 of the new units for 1 of the old units, the unit is smaller. Note that the new unit is smaller even though it uses kg and m^3, which are both larger than g and cm^3! This happens because we need more units from the old denominator to get the unit in the denominator of the new ratio, than we do units from the old numerator to get the unit in the numerator of the new ratio. The first time we multiplied by 1 000, we turned g to kg, but we had to multiply by 1 000 a second time to turn cm^3 to m^3.

4. Can you convert 500 cm^3 to dm^2? Why or why not? (No, because cm^3 are units of volume, and dm^2 are units of area.)

ME8-16 Surface Area of Prisms

Pages 161–163

CURRICULUM EXPECTATIONS
Ontario: 6m41; 7m41; **8m1, 8m2, 8m7, review**
WNCP: **8SS2, 8SS3, [C, R, PS]**

VOCABULARY
surface area
length
width
height
right prism
rectangular prism

Goals

Students will find the surface area of right prisms.

PRIOR KNOWLEDGE REQUIRED

Can find area of polygons
Can perform basic operations with decimals
Is familiar with square units of measurement
Can draw a net for a right prism
Can convert units of area
Can find the volume of right prisms

MATERIALS

empty cartons from medicine, soup, tea, etc

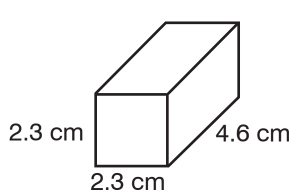
Introduce surface area. Review area for rectangles, parallelograms, and triangles. Tell students that the *surface area* of a prism is the sum of the areas of all faces of the shape. Ask students when they might need to know the surface area of a prism. (**EXAMPLE:** to calculate the amount of paper needed to wrap a present—see Extension 1)

Find the surface area of rectangular prisms by adding the areas of all faces. Present a rectangular prism. Invite volunteers to measure the sides of the prism. Ask students to draw the faces of the prism and to mark the dimensions of each face. Have students check that they drew all the faces. How many should there be? (6) Ask students to write a multiplication statement for the area of each face and to add the results for all the faces. Point out to students that because they are measuring area, the measurement units are cm^2 and not cm^3, even though this is a 3-D shape.

Identify identical faces and use multiplication to find the surface area. Draw a cube on the board and mark the faces (top, bottom, right, left, front, back). Ask students to name pairs of opposite faces. **ASK:** Which faces are the same as other faces? Which faces come in pairs? How can we use this to shorten the calculation of the surface area? Draw several rectangular prisms and mark dimensions on them. Have students find the areas of the top, front, and right side faces, and then double these areas to find the surface area of each prism. **EXAMPLES:**

3 cm × 4 cm × 5 cm   4.5 m × 2 m × 3 m   2.3 km × 1.2 km × 4 km

Students can also measure some empty prism-shaped cartons and find their surface area.



Bonus Find the surface area of the prism at left using as few calculations as you can.

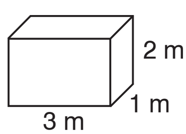
EXTRA PRACTICE:
The surface areas of the front, top, and right faces of a prism add to 450 cm². How can you find the total surface area of the prism?

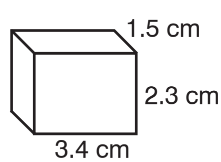
Sketching nets for prisms with given dimensions. Draw several right rectangular prisms on the board and write the dimensions beside them.

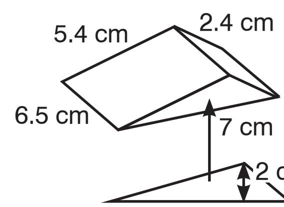
EXAMPLES: a) 3 cm $\times$ 3.4 cm $\times$ 6 cm b) 12.2 cm $\times$ 4 cm $\times$ 20.2 cm

Ask students to sketch the nets for the prisms. Then ask students to mark on the nets which face is which (top, bottom, right, etc.). One by one, go through the faces on the net and have students identify their dimensions. Do the first example as a class and have students do the rest on their own, but share answers on the board. Leave the pictures on the board for future use.

Finding volume and surface area of prisms using nets. Review with students how to find the volume of prisms. Then have students find the surface area and the volume of the prisms they sketched nets for above. Point out that to find the volume students only need the dimensions of the prisms, and these can be read from the nets. **ASK:** How can we use nets when finding surface area? (the area of the net is the same as the surface area of the shape) Then present several more sketches of prisms with dimensions and have students find the volume and the surface area of the prisms. **EXAMPLES:**

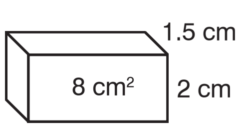






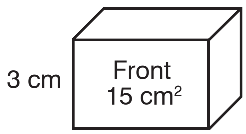
Finding missing dimensions. Have students solve several problems in which they have to find the missing dimension in a 2-D shape given the area and the dimension. **EXAMPLE:** A rectangle has area 72 cm². Its width is 4 cm. What is its length? ($72 \div 4 = 18$ cm) Next, move to problems involving 3-D shapes, as in Question 12 on Workbook p. 163. Then ask students to find the surface area of the prisms in Question 12. Students who have trouble solving Question 13 on Workbook page 163 can try to organize their search by using a table with headings, *a, b, c,* and $a \times c$. They can start by looking for pairs of numbers *a* and *c* that multiply to 18, and then find the number *b* that produces $b \times c = 6$.

EXTRA PRACTICE:



1. Find the missing length, the volume, and the surface area of the prism at left.

   ANSWER:
   Missing length = 4 cm
   Volume = 8 cm² $\times$ 1.5 cm = 12 cm³
   Surface area = $2 \times$ (2 cm $\times$ 4 cm + 1.5 cm $\times$ 4 cm + 1.5 cm $\times$ 2 cm)
   = $2 \times$ (8 cm² + 6 cm² + 3 cm²) = 34 cm²



2. The prism in the margin has volume 90 cm^3. Find the dimensions and the surface area of the prism.

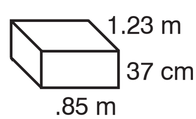
   ANSWER:
   length = 15 cm^2 ÷ 3 cm = 5 cm, width = 90 cm^3 ÷ 15 cm^2 = 6 cm,
   surface area =
   2 × (3 cm × 5 cm + 3 cm × 6 cm + 5 cm × 6 cm) = 126 cm^2

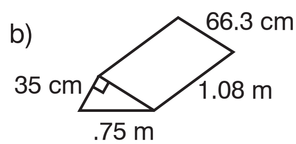
3. Sally's teacher tells her that she can find the surface area of a prism by adding the areas of three faces and then multiplying by 2. Which of the following will give Sally the right answer? Explain.

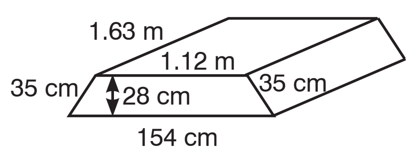
   i) (area of top + area of bottom + area of right side) × 2
   ii) (area of top + area of left side + area of back) × 2
   iii) (area of top + area of right side + area of front) × 2
   iv) (area of bottom + area of right side + area of left side) × 2
   v) (area of bottom + area of left side + area of front) × 2
   vi) (area of bottom + area of front + area of top) × 2

   ANSWER: ii), iii), and v) because we need to choose one from each pair of sides: top and bottom, left and right, front and back.

Prisms with mixed units. Review the need to keep track of the units used in calculations of volume and surface area. Review the formula for the volume of prisms: base of prism × height of prism. Have students practise finding the surface area and volume of prisms with measurements given in mixed units. **EXAMPLES:**

a) 

b) 

c) 

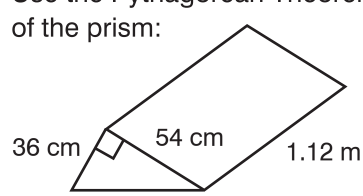
ANSWERS:
a) surface area = 5.610 2 m^2, volume = .386 835 m^3
b) surface area = 2.136 09 m^2, volume = 0.125 307 m^3
c) surface area = 6.412 6 m^2, volume = .630 832 m^3

PROCESS ASSESSMENT

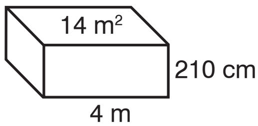
8m1, 8m2, [R, PS]
Workbook Question 14

Tell students that you heard someone say that in a) the surface area is larger than the volume. Is that correct? Have students explain the answer. (no, you cannot compare m^3 to m^2)

Bonus Use the Pythagorean Theorem to find the surface area of the prism:



ANSWER: Surface area = 19 392.8 cm^2 = 1.939 28 m^2

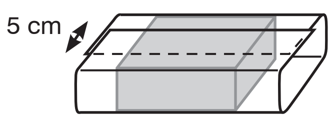


EXTRA PRACTICE:
Find the volume and the surface area of the prism in the margin.

ANSWER:
Missing length = 3.5 m
Volume = 14 m^2 × 2.1 m = 29.4 m^3
Surface area = 2 × (2.1 m × 4 m + 14 m^2 + 3.5 m × 2.1 m)
= 2 × (8.4 m^2 + 14 m^2 + 7.35 m^2) = 59.5 m^2

Extensions

1. Project: Which way of wrapping a present uses the smallest amount of paper?



   a) Find a box with all three dimensions different, such as a shoe box. Use old newspapers as wrapping paper.

   b) Wrap the box so that at least 5 cm of paper overlap at the first wrapping. Unwrap and check how much paper you used.

   c) Turn the box 90° and repeat b) with a new sheet of paper. Turn the box again, in a different direction, and repeat.

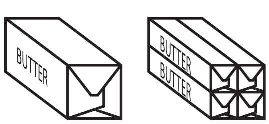
   d) Which way uses the least amount of paper?

   e) Find the area of the overlap during the first step of wrapping in each case. Does the smallest initial overlap area correspond to the smallest amount of paper used?

   f) Find the surface area of the box.

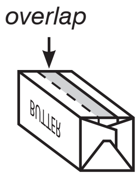
   g) For each way of wrapping, find how much paper is used to cover the unwrapped sides. Is there a correspondence between the largest amount of paper used in total and the greatest amount of paper used to cover the sides?

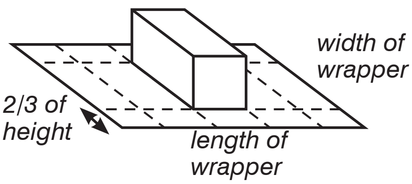
   Answers will vary for all a) to g), depending on the boxes students use!



2. a) A big block of butter has dimensions 15 cm × 7.5 cm × 7.5 cm. The same amount of butter can be bought in four smaller sticks of the same length, but half the width and height of the larger pack. What are the dimensions of the smaller sticks? (15 cm × 3.75 cm × 3.75 cm)

   Draw the nets for the large block of butter and the small stick. Find the surface area of both blocks. What is the additional surface area of the butter when it is sold in four sticks? (450 cm^2)





b) A wrapper for the block of butter is a rectangle that covers three of the larger sides of the block once, and has an overlap on the fourth larger side. The overlap is 1/3 of the width of the side. What is the length of the wrapper for the bigger block? for the smaller block? (bigger block: 4×7.5 cm $+ 2.5$ cm $= 32.5$ cm; smaller block: 4×3.75 cm $+ 1.25$ cm $= 16.25$ cm)

When the wrapper is folded to cover the two square faces of the block of butter, it covers only 2/3 of the height of the square, on both sides. What is the width of the wrapper for the bigger block? For the smaller block? (bigger block: 2×5 cm $+ 15$ cm $= 25$ cm, smaller block: 2×2.5 cm $+ 15$ cm $= 20$ cm)

How much more wrapping do four smaller sticks use? (1 300 cm^2 $-$ 812.5 cm^2 $=$ 487.5 cm^2)

c) Is the additional amount of wrapping the same as the additional surface area of butter? (no, the additional amount of wrapping is larger by 37.5 cm^2) Explain why there is a difference. (The additional amount of wrapping is larger because there is some overlap on each of the four sticks and it is more than the overlap on one large butter block.)

3. Research project:

a) One litre of paint covers 7 m^2. How much paint would someone need to paint a wall with dimensions 6 m by 3 m? What if the wall has a door that is 2 m high by 80 cm wide? What if the wall also has a window that is a 1 m by 1 m square?

b) A room has a closet that is 1 m deep, 2 m wide and 2.5 m high. A door that is 2 m high and 80 cm wide leads to it. I want to paint the sides, back, and top of the closet, but neither the floor nor the door. How much paint do I need?

c) Choose a room in your school or at home. Calculate the amount of paint needed to repaint the room. Consider all surfaces and fixtures, such as doors, windows, closets, electrical outlets, built-in shelves, or ledges. What will you paint and what does not need to be painted? Do you want to use more than one colour? Will you need more than one coat of paint? (You will if you are using a dark colour, or painting over a dark colour.)

4. A wealthy king has a treasure chest in the shape of a rectangular prism 30 cm wide, 40 cm long, and 25 cm high. He ordered his carpenters to design a chest that can hold twice as much treasure.

a) The first carpenter doubled the length of the box and left the width and the height the same. The second carpenter doubled the width of the box and left the length and the height the same. The third carpenter doubled the height of the box and left the length and the width the same. The chest with the smallest surface area uses the least amount of wood and so is the least expensive. Which carpenter made the least expensive chest?

ANSWER:
1st carpenter: $2 \times (30 \times 80 + 30 \times 25 + 25 \times 80) = 10\ 300\ \text{cm}^2$
2nd carpenter: $2 \times (60 \times 40 + 60 \times 25 + 25 \times 40) = 9\ 800\ \text{cm}^2$
3rd carpenter: $2 \times (30 \times 40 + 30 \times 50 + 50 \times 40) = 9\ 400\ \text{cm}^2$

The 3rd carpenter made the least expensive chest.

b) None of the three carpenters got the job of making the chest for the king's treasure. Instead, he ordered the chest from a fourth carpenter, who suggested a chest with a volume that was 101.4% of what the king wanted but that would cost less than the chests of the other carpenters. Her chest had a square at the base and was 40 cm high. What were the dimensions of this carpenter's chest? What was its surface area? Compare the surface area of this chest to the chest with the smallest area in part a).

ANSWER:
Volume: 101.4% of $(30 \times 25 \times 40 \times 2)\ \text{cm}^3 = 60\ 840\ \text{cm}^3$

The height is 40 cm, so the base should be $60\ 840 \div 40 = 1\ 521\ \text{cm}^2$, and $1\ 521 = 39^2$, so the box is 39 cm $\times$ 39 cm $\times$ 40 cm.

The surface area is $2 \times (39 \times 39 + 39 \times 40 + 39 \times 40) = 9\ 282\ \text{cm}^2$, 118 cm^2 smaller than the box made by the 3rd carpenter.

NOTE: The last box is the closest to a cube, which is the most economical of prisms. Of all prisms with the same volume, a cube will have the smallest surface area. Compare this to a square, which is the most economical of all rectangles. Of all rectangles with the same area, the square will have the smallest perimeter. See Extension 1 of ME8-15.

ME8-17 Surface Area of Cylinders

Pages 164–165

CURRICULUM EXPECTATIONS
Ontario: 6m41; **8m1, 8m2, 8m5, 8m6, 8m7, 8m38, 8m39**
WNCP: **8SS2, 8SS3, [C, CN, R, V, ME, PS]**

VOCABULARY
cylinder
circumference
radius
length
width
height
right prism
rectangular prism

Goals

Students will find the surface area of cylinders.

PRIOR KNOWLEDGE REQUIRED

Can find the area of a rectangle, a parallelogram, and a circle using the right formula
Can find the circumference of a circle
Can perform basic operations with decimals
Is familiar with square units of measurement

MATERIALS

empty toilet paper rolls
tailor's measuring tape or string and a ruler (to measure circumference)
cans and paper
BLM Nets of 3-D Shapes (4, 11) (pp U-4, U-11)

Develop the formula for the area of the side face of a cylinder. Give each student an empty toilet paper roll. Have students measure and record its height and diameter, and the circumference of the circle in the base. Then have students cut the roll vertically (as in Question 1 on Workbook page 164) and lay it flat. What shape do they get? (a rectangle) What is the width of this rectangle? What is its length? What measurements of the cylinder are these equal to? (length = circumference, width = height of cylinder)

Give students cans of different sizes and paper. Ask students to check whether a rectangle that has length equal to the circumference of a cylinder and width equal to the height of a cylinder will wrap around the can precisely. **ASK:** Did that work for all cans? So what formula could we use for finding the area of the side face of a cylinder? (circumference of base × height of cylinder)

PROCESS ASSESSMENT

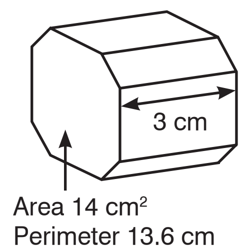
8m1, 8m2, [R, PS]
Workbook Question 3

Draw a cylinder on the board and mark its diameter as 6 cm and its height as 2 cm. **ASK:** What will the area of the side face be? Can you find the circumference of this cylinder? Which formula will you use? Have students find the area of the side face. (circumference × height = $(\pi \times 6 \text{ cm}) \times 2$ cm $\approx 37.68 \text{ cm}^2$) Repeat with a cylinder with height 3 cm and radius 5 cm. (area of side face = $(2 \times 5 \times \pi) \times 3 = 30\pi \approx 94.2 \text{ cm}^2$) Then ask students to predict a formula for the area of the side face of a cylinder with radius r and height h. ($2\pi rh$)

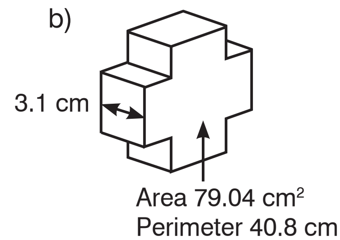
Remind students that since we use $\pi \approx 3.14$, which is only exact to two decimal places, our answers would not be correct for larger number of decimal places. For the moment we multiply by π in a problem, we should round the answers to at most two decimal places and use the approximately equal sign.

Analogue of the formula circumference of base $\times$ height for prisms. Hold up a prism with a many-sided polygon in the base (e.g., **BLM Nets of 3-D Shapes (11)**) so that the bases are horizontal. **ASK:** How is this prism the same as a cylinder and how is it different? Look at the formula for the area of the side face of a cylinder: circumference of base $\times$ height of cylinder. What would you replace "circumference of base" with to get a formula that will work for a prism? (perimeter of base) Have students use this formula to find the surface area of prisms given the height of the prism, and the area and circumference of the base. **EXAMPLES:**

a)



b)



ANSWERS:
a) $14 \times 2 + 13.6 \times 3 = 68.8$ cm^2 b) $79.04 \times 2 + 40.8 \times 3.1 = 284.56$ cm^2

PROCESS ASSESSMENT

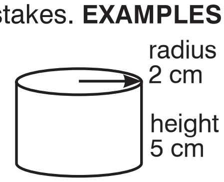
8m1, 8m2, [R, PS]
Workbook Question 8

PROCESS EXPECTATION

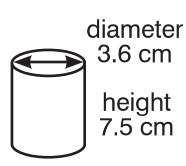
Mental Math and Estimation

Surface area of a cylinder. ASK: To find the surface area of the prism, what did you add to the surface area of the side faces? (the area of the bases) To find the surface area of a cylinder, what will you add to the area of the side face? (the area of the bases) What are the bases of a cylinder? (circles) What is the area of a circle? To summarize, the surface area of a cylinder will be the sum of three components: area of the bottom base, area of the top base, and area of the side face, which we find as area of a rectangle. Have students find the total surface area of several cylinders. Do the first one or two examples as a class, then have students work individually. Have students do at least half of the problems by hand. If they use a calculator for the rest of the problems, have them estimate the answer first, to avoid mistakes. **EXAMPLES:**

a)

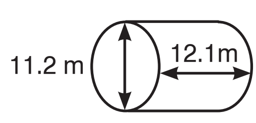


Area of rectangle $= 2\pi rh = 2 \times \pi \times 2 \times 5 \approx 62.8$ cm^2
Area of top base $= \pi \times 2^2 \approx 12.56$ cm^2
Area of bottom base $= \pi \times 2^2 \approx 12.56$ cm^2
Total surface area ≈ 87.92 cm^2

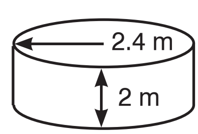


Area of rectangle $=$
Area of top base $=$
Area of bottom base $=$
Total surface area $\approx$

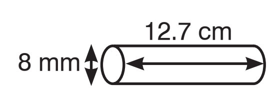
c)



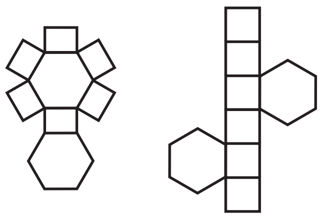
d)



e)



ANSWERS: b) 105.12 cm^2 c) 622.47 m^2 d) 66.316 8 m^2 e) 32.91 cm^2



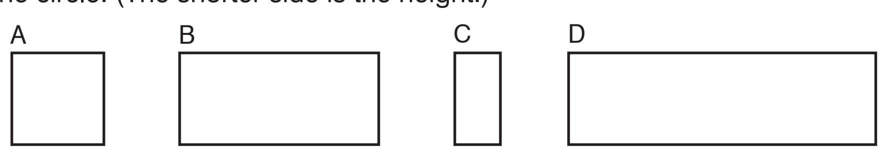
Nets of cylinders are similar to nets of prisms. Show students a hexagonal prism (see **BLM Nets of 3-D Shapes (4)**) and ask them to sketch a net for it. Encourage multiple solutions. Make sure that the two nets at left are shown. Then show a prism with a regular 12-sided polygon in the base (see **BLM Nets of 3-D Shapes (11)**) and ask students to sketch a net for it. If you need to construct an exact net for this prism, which net would be more convenient to draw? Why? (a net in which the side faces are joined side by side in a long rectangle, because you can draw all the rectangles together) Then ask students to think about what a net for a cylinder would look like. How would it be similar to the net of a prism with a 12-sided polygon in the base? What would you need to draw differently? (There is only one long rectangle for the side face, and circles instead of polygons.) What will the dimensions of the rectangle be? (circumference of base × height of prism)

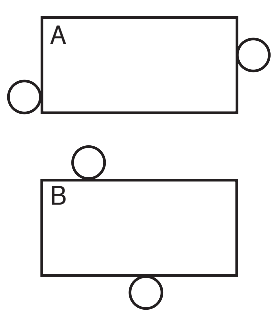
PROCESS EXPECTATION

Visualizing

Other nets of cylinders. Give students another empty roll of toilet paper, and have them cut it diagonally, as in Question 1 b) on Workbook page 164. to lay the cut tube flat. What shape did they get? (a parallelogram) **ASK:** How do we find the area of a parallelogram? (base × height) What is the height of this parallelogram? (the height of the cylinder) What is the base of this parallelogram? (the circumference of the base of the cylinder) Have students do the Activity below.

Matching nets to cylinders. Draw a cylinder on the board and mark its diameter as 10 cm and its height as 15 cm. Ask students what the dimensions of the rectangle that can be folded into the side face should be (about 31.4 cm × 15 cm) Draw several rectangles as shown below and ask which of them could be the rectangles that fold into the side face of this cylinder. (B and C) How do you know? What is wrong with the others? (the rectangle in question is about twice as long as it is wide, and only these two rectangles have these proportions) Ask students to tell which side would be the height and which side would be the circumference of the circle. (The shorter side is the height.)



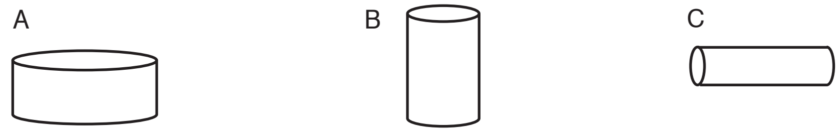


Draw the two nets at left. **ASK:** Which of these nets could be a net for a cylinder? What is wrong with the other picture? (Picture A will work as a net, but picture B will not because the circles are too small.) Approximately what should the ratio between the side that becomes the edge of the cylinder and the diameter of the circle be? Why? (The ratio between the side and the diameter is the ratio between the circumference of the circle and its diameter, which is π, so the first ratio should be about 3 : 1.)

PROCESS EXPECTATION

Connecting

Draw three cylinders below and have students decide which one the cylinder folded from net A will look like. (C)



Have students sketch the nets for the other two cylinders.

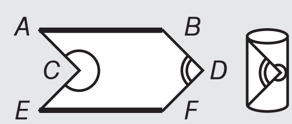
ACTIVITY

PROCESS EXPECTATION

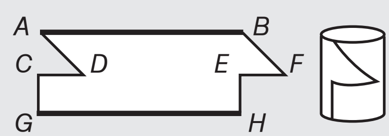
Connecting

Have students construct a variety of quadrilaterals (students should try at least one of each special quadrilateral and one general quadrilateral), then cut them out and try to roll them into a cylinder. Which ones will roll into the side face of a cylinder? What properties do such quadrilaterals have? Students can also experiment with other polygons.

ANSWER: Quadrilaterals that can roll into the side face of a cylinder have parallel sides of the same length and adjacent angles that add to 180°, so they have to be parallelograms. Other polygons need pairs of parallel sides of the same length, with pairs of angles at the sides that will become the edges of the side faces adding to 180°, and other angles adding in pairs to 360°. See an example of a hexagon and an octagon that will roll into a cylinder below. The thick lines mark the sides that will be glued to circles. **EXAMPLES:**

a) 

$\angle A + \angle B = 180°$
$\angle E + \angle F = 180°$
$\angle C + \angle D = 360°$

b) 

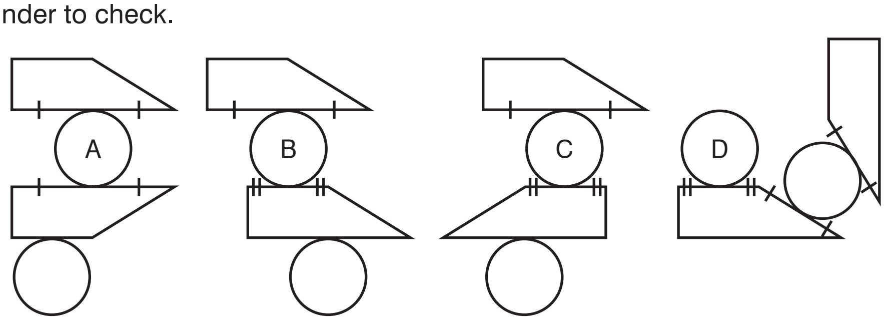
$\angle A + \angle B = 180°$
$\angle G + \angle H = 180°$
$\angle C + \angle E = 360°$
$\angle D + \angle F = 360°$

Extension

PROCESS EXPECTATION

Visualizing

Which of the pictures below works as a net for a cylinder? What is wrong with the others? Draw a similar picture, cut it out, and try to fold it into a cylinder to check.



EXPLANATION: B is the only net that works.

In A, both trapezoids would be glued to one of the circles using the longer base, and to the other with the shorter base. These sides do not add to the same length, and are either longer or shorter than the circumference of the circle.

In C, when you use the circle with the letter C as the bottom base and fold the trapezoids up to become the side faces, you see that each non-base side of one trapezoid will attach to the side of a different length in the other trapezoid, so the sides do not fit.

In D, the circles are attached to the adjacent sides of one of the trapezoids. If you try to wrap the trapezoid attached to both circles around them, the circles will meet. In a cylinder, the circular faces do not meet.

ME8-18 Surface Area and Volume

Pages 166–167

CURRICULUM EXPECTATIONS
Ontario: 6m42; 7m41, 7m42;
8m1, 8m7, 8m38, 8m39
WNCP: **8SS2, 8SS3, 8SS4**
[C, R, T, V, PS]

VOCABULARY
volume
length
width
height
right prism
radius
circumference
surface area
mat plan

PROCESS EXPECTATION

Organizing data

Goals

Students will find solve problems involving volume and surface area of prisms and cylinders.

PRIOR KNOWLEDGE REQUIRED

Can find the volume of a rectangular prism
Can find the volume of a cylinder
Can find the area and circumference of a circle
Can identify the base of a prism
Can multiply or divide decimals
Is familiar with cubic and square units of measurement
Can find the area of polygons
Can find the surface area of cylinders and prisms

Review some relevant prior knowledge. Review formulas for the areas of triangles, rectangles, parallelograms, and circles. Review finding missing dimensions of rectangles and triangles when given the area. Review finding the volume of right prisms (volume = area of the base × height) and the surface area of right prisms (add areas of all faces, nets can help keep track of faces). Remind students that writing the units at every step of the calculation helps both to avoid multiplying dimensions in different units and to ensure the right number of dimensions are multiplied (two for area, three for volume).

Finding dimensions of prisms with a given volume using an organized search. Tell students that you want to find all possible whole-number dimensions of a rectangle with area 60 cm^2. Have students solve the problem, then discuss possible solutions. Solutions include: produce a factor rainbow (each arc in the factor rainbow matches a rectangle); use a T-table to search for factors in an organized way; find the prime factorization of 60 and list all possible factors, then pair them; draw rectangles in an organized way. Point out that it is preferable to perform your search in an organized way because of the large number of possible solutions. All of the solutions above have some sort of organized structure. Then present a harder problem: Find all right rectangular prisms with volume 60 cm^3. **ASK:** How are these problems the same and how are they different? (We are again looking for numbers that multiply to 60, but there are 3 numbers involved in the second problem, and only 2 in the first.) Will a factor rainbow help here? (no) Why not? (we need triples, not pairs of numbers) Will drawing prisms be a convenient strategy? (no) Why not? (We sketch prisms, we do not draw them precisely the way we do rectangles, and it will take too much time to draw all possible prisms)

Suggest that students use a chart with three columns (tell them to leave space for a fourth column to be added later) and headings height, width, and length. Point out that when we have three values that can change,

PROCESS EXPECTATION

Splitting into simpler problems

it makes sense to solve a simpler problem first. In this case, we can set the first value, look at what the other two values can be in an organized way, then modify the first value and repeat. So we will start with height 1 cm for the prism, and see what length and width the prism can have. What do we know about the area of the base of a prism with height 1 cm and volume 60 cm^3? (area of the base = 60 cm^2) Have students list all possible lengths and widths for a prism with height 1 cm and volume 60 cm^3. How did they do that? Point out that this was a problem that they solved before—finding all rectangles that have area 60 cm^2.

height	width	length
1	1	60
1	2	30
1	3	20
1	4	15
1	5	12
1	6	10
2	2	15
2	3	10
2	5	6
3	4	5

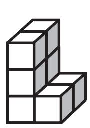
ASK: Do all prisms with volume 60 cm^3 have height 1? (no) Tell students that they will now look for all prisms of height 2 cm. What will the area of the base be? (30 cm^2) **ASK:** Does it make sense to include prisms with width 1 cm? Why not? (because any prism with width 1 cm was already listed as a prism with height 1 cm) Have students find all other prisms with height 2 cm. Next, use height 3 cm (only one prism is possible) and height 4 cm. Ask students to look at the first prism with height 4 cm that they've written (it is either $4 \times 1 \times 15$ or $4 \times 3 \times 5$). What do they notice? (it is already in the list, row 4 or row 10) Does it make sense to continue the search? Why not? (Since we are only looking for prisms that have height 4, and width at least 4 (all smaller widths are already accounted for), the prisms will have height $\times$ width at least 16, so the length is definitely smaller than 4. We have already listed all prisms with these dimensions.)

ASK: Which of these prisms will have the smallest surface area? Have students add a column for surface area and find the surface area of all these prisms. The prism with the smallest surface area is the one closest to a cube— the last one in the table.

PROCESS EXPECTATION

Mental math and estimation, Technology

Ask students to estimate the volume of a cube with sides 3.92 cm. Will it be a lot more, a lot less, or about the same as the volume of the prisms they found? Have them explain their answer. Then have students check the volume using a calculator. **ASK:** Will the surface area of the cube with sides 3.92 cm be more, less, or about the same as the surface area of the last prism you found? Again, have students explain their thinking and check by using a calculator.



3	1
3	1

Mat plans. Remind students that a *mat plan* shows what a shape made of connecting cubes or 1-cm cubes would look like from the top. The number of cubes stacked above each square in the mat plan is listed in the square. For example, show students an L-shape made of connecting cubes (or draw one on the board) and draw the mat plan for it (see sample in margin).

PROCESS EXPECTATION

Visualizing

ANSWER:

2		
2	2	2

Ask students to find the volume of the L-shape. **ASK:** How can you do that from the mat plan? (add the numbers on the mat plan). Then ask students whether this shape is a prism. (yes) What are its bases? (the L-shaped faces) Have students draw a mat plan of the shape as if it was standing on a base (see answer at left). What do they notice? (all numbers in the mat plan are the same) Does turning the shape on the side change its volume? (no) Remind students that the mat plan for any prism standing on its base will have the same numbers in all squares. Point out that this provides us with a self-checking mechanism for the mat plan after we turned the shape.

ASK: Which of the mat plans will be more useful for finding the surface area of the shape? (the second one) How will you find the surface area from it? (The shape has the same height everywhere, and the mat shows the base of the shape. Find the perimeter of the base and multiply by the height. Then add twice the area of the base.) Have students find the surface area of the shape. (perimeter of base $= 10$ cm, so area of side face is $2 \times 10 = 20$ cm^2, area of base is 4 cm^2, and total surface area $= 20 + 2 \times 4 = 28$ cm^2)

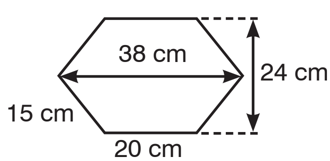
3	1	2
3	1	2

ANSWER:

2		
2		2
2	2	2

Have students find the volume and the surface area of the shape with mat plan in the margin. Students who have trouble visualizing the shape can use connecting cubes to construct it, identify it as a prism, turn it base down, and draw the second mat plan (see answer in margin). **ANSWER:** volume $= 12$ cm^3, surface area $= 40$ cm^2.

Word problems involving volume or surface area. Work through the following problems as a class.



1. Pam has a box with a hexagonal base, a lid, and rectangular sides. The box is 40 cm tall and its base (which is the same as the lid) is shown at left.

   a) Sketch the box.
   b) Sketch a net for the box and mark the dimensions on the faces.
   c) Find the surface area of the box.

   ANSWER: perimeter of the base $= 100$ cm, so side faces have total area 4 000 cm^2. Splitting the base into two trapezoids, with bases 20 cm and 38 cm and height 12 cm, the area of each base is $(38 + 20) \div 2 \times 12) = 696$ cm^2, so the surface area of the box is 5 392 cm^2.

   d) Pam needs 5 mL of paint for each 100 cm^2 of area. How much paint will she need to paint the outside of the box (including the lid)? ($53.92 \times 5 = 269.6$ mL of paint)

2. A rectangular box without a lid has volume 40 000 cm^3. It is 40 cm wide and 50 cm long.

   a) What is the height of the box?

   b) What are the length and the width of the lid (the missing face)?

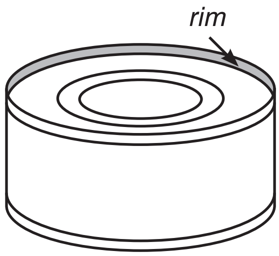
   c) What is the surface area of the box? Remember, there is no lid.

   d) The material for the box costs \$12.35 per m^2. How much will the material for the box cost?

   ANSWERS: a) 20 cm b) 40 cm $\times$ 50 cm

   c) 40 cm $\times$ 50 cm $+ 2 \times$ (40 cm $\times$ 20 cm $+$ 50 cm $\times$ 20 cm) $=$ 3 800 cm^2 $= 0.38$ m^2

   d) $0.38 \times \$12.35$ which rounds to \$4.69.



Finding the height of a cylinder from the volume and radius. Show students a can that has a rim on top. Explain that you know the capacity of this cylinder and thus you can find the volume. You can measure the diameter of the can, but you cannot measure the height because of the rim. Tell students that the diameter of the can is 5 cm and its capacity is 156 mL. What is the inside height of the can? Solve the problem as a class. (**PROMPTS:** What is the volume of the can? Which units, mm or cm, will be the most convenient for this problem? What is the radius of the base of the can? What is the area of the base of the can? The area of the base of the can is $\pi \times 2.5^2$ cm^2 $\approx$ 19.63 cm^2. The inside volume of the can is 156 cm^3, so the height is 156 $\div$ 19.63 $\approx$ 7.95 cm.)

Have students practise finding the heights of cylinders given their radius (or diameter) and volume (or capacity). **EXAMPLES:**

1. A cylinder has volume 125 cm^3. It has radius 42 mm. What is the height of the cylinder? ($\approx$ 2.26 cm)

2. A can has capacity 376 mL. It has inner diameter 6.8 cm. What is the inner height of the can (rounded to the nearest mm)? (10.4 cm)

Review finding the radius of a circle from the circumference or area. Review the formulas for the area and circumference of a circle, and how to work backwards when you know the circumference or area to find the radius. Solve several problems like the following:

1. A circle has area 10 m^2. What is its radius, rounded to the nearest centimetre? (**ANSWER:** 10 m^2 = $\pi \times r^2$, so $r^2 \approx 10 \div 3.14 \approx 3.19$ m^2, so $r \approx 1.79$ m)

2. A circle has area 15.6 cm^2. What is its diameter (rounded to the nearest millimetre)? (**ANSWER:** 15.6 cm^2 = $\pi \times r^2$, so $r^2 \approx 15.6 \div 3.14 \approx 4.97$ m^2, so $r \approx 2.28$ cm, and $d \approx 4.6$ cm)

3. A can has circumference 34 cm. What is its diameter in millimetres, rounded to one decimal place? (**ANSWER:** 34 cm = $\pi \times d$, so $d \approx 34 \div 3.14 \approx 10.83$ cm = 108.3 mm)

Finding radius or diameter from the volume and height of cylinders. Present the following problem:

> A round can has capacity 1.145 L. It is 15.5 cm tall. How many such cans will fit in a box 30 cm $\times$ 40 cm $\times$ 32 cm?

ASK: To know how many cans will fit into the box, what do we need to know? (the width of the can) The can is round. What shape it is? (a cylinder) What do we know about this cylinder? (capacity and height) What does "width" mean in terms of a cylinder? (diameter) Do we know a formula for a cylinder that involves capacity, diameter, and height? (no) Is there a formula that we know that could help us? What can we find easily from diameter? From capacity? Which formula that we know involves radius and height? (surface area and volume both use radius and height) Have students find the volume of the can. (1 145 cm^3) Again, suggest that students work backwards from the formula for the volume to find the radius of the can from the volume and the height. (1 145 cm^3 = $\pi \times r^2 \times$ 15.5 cm, so $r^2 \approx 1145 \div 15.5 \div 3.14 \approx$ 23.525 8 cm^2, $r \approx$ 4.85 cm, and $d \approx$ 9.7 cm) How many cans will fit at the bottom of the box? ($3 \times 4 = 12$ cans) How many cans will fit in the box in total? (24 cans)

SAY: Suppose we need a label that will cover the side face of the can and have an overlap of 1 cm. What shape should the label be? (rectangle) What should the dimensions of the label be? What should the height of the label be? (the same as the height of the can) What should the length of the label be? (the circumference of the base + 1 cm) Have students find the circumference of the base and the length of the label. ($C = \pi \times$ 9.7 cm $\approx$ 30.5 cm, so the length of the label should be about 31.5 cm.)

Have students practise finding the diameter or the radius of cylinders given the volume, capacity, or surface area. Include some problems with prisms.
EXAMPLES:

1. A round can is 10.2 cm tall. Its capacity is 398 mL. What is the radius of the can (rounded to the nearest millimetre)? What is the surface area of the can (in whole cm^2)? (radius $\approx$ 3.5 cm, surface area $\approx$ 302 cm^2)

2. A round can has circumference 257.4 mm. It is 3 cm tall. What is its capacity in whole mL? (**ANSWER:** $257.4 = 2\pi r$, so $r \approx 257.4 \div 6.28 \approx$ 40.99 mm, so volume $= \pi r^2 h \approx \pi \times 4.1^2 \times 3 \approx$ 158 cm^3, and capacity $\approx$ 158 mL)

3. a) A rectangular prism has width = length = 30 cm. Its surface area is 1 m^2. What is its height?

   b) A cylinder has width 30 cm and surface area 1 m^2. Without calculating, predict whether it is taller than the prism in a). Explain your prediction.

   c) Find the height of the cylinder in b) and check your prediction.

   d) Calculate the volumes of both the prism and the cylinder to see which is larger.

PROCESS ASSESSMENT

8m1, [PS, V]
Question 12

ANSWERS:

a) Let h be the height of the prism. Then its surface area is $2 \times (30 \times 30 + 30 \times h + 30 \times h)$ cm^2 = 1 800 cm^2 + $120 \times h$ cm^2 = 10 000 cm^2. $h \approx$ 68.3 cm.

b) The base of the cylinder is a circle with diameter 30 cm, which would fit inside the square that is the base of the prism in a), and the circumference of the circle is smaller than the perimeter of the square. The surface area of the prism and the cylinder are the same, so to compensate for the smaller bases and smaller perimeter, the height of the cylinder has to be larger than the height of the prism. The cylinder will be taller.

c) The bases of the cylinder each have area $\pi \times 15^2$ cm$^2 \approx 706.5$ cm^2, so the side face should have area about $10\,000 - 2 \times 706.5 = 8\,587$ cm^2. Height of cylinder $\approx 8\,587 \div (3.14 \times 30) \approx 91.2$ cm.

d) Volume of prism $\approx 30 \times 30 \times 68.3 = 61\,470$ cm^3.
Volume of cylinder $\approx 706.5 \times 91.2 = 64\,432.8$ cm^3.
The cylinder has larger volume.

Bonus A cylindrical thermos has circumference 26 cm and inner diameter 6.5 cm. Its inner height is 8 cm.

a) What is the capacity of the thermos?

b) What is the thickness of the thermos walls?

c) If the thickness of the bottom is twice the thickness of the walls, what is the outer height of the thermos without the lid?

d) What is the volume of the thermos when it is full (not including the lid)?

ANSWERS:
a) capacity $= \pi \times 3.25^2 \times 8 \approx 265.33$ mL

b) outer radius $= 26 \div (2\pi) \approx 4.14$ cm, so the thickness of the walls is about $4.14 - 3.25 = 0.89$ cm $= 8.9$ mm

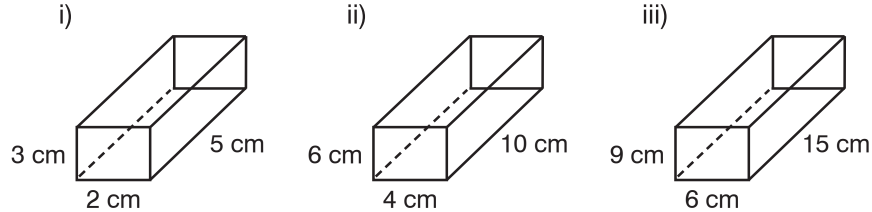
c) The bottom is 1.78 cm thick, and the outer height is 9.78 cm.

d) The volume is about $\pi \times 4.14^2 \times 9.78 \approx 526.38$ cm^3.

Extensions

1. a) Find the surface area and volume of each right rectangular prism

i) ii) iii)



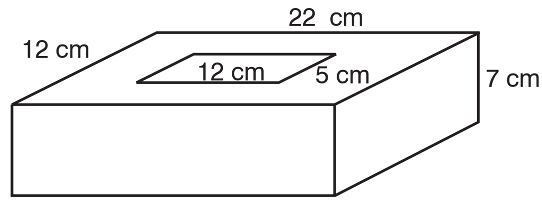
ANSWERS:

Surface area: 62 cm^2	Surface area: 248 cm^2	Surface area: 558 cm^2
Volume: 30 cm^3	Volume: 240 cm^3	Volume: 810 cm^3

b) How does the surface area of a right rectangular prism change when each side length is multiplied by the same amount? (it is multiplied by the square of the scale factor) How does the volume of a right rectangular prism change when each side length is multiplied by the same amount? (it is multiplied by the third power of the scale factor)

c) If each dimension is enlarged by 50% (so the new dimensions are 150% of the old dimensions), what percentage do the surface area and volume increase by? HINT: a 50% increase is the same as multiplying by what factor? (each dimension is multiplied by 1.5, so surface area is multiplied by $1.5^2 = 2.25$ and volume is multiplied by $1.5^3 = 3.375$)

2. Find the surface area and volume of the tissue box shown below. Remember to leave out the opening on the top.



ANSWERS:
Volume: 12 cm $\times$ 22 cm $\times$ 7 cm $=$ 1 848 cm^3

Surface area: 2 $\times$ (12 cm $\times$ 22 cm $+$ 12 cm $\times$ 7 cm $+$ 22 cm $\times$ 7 cm) $-$ 12 cm $\times$ 5 cm $=$ 1 004 cm^2 $-$ 60 cm^2 $=$ 944 cm^2

3. You are designing a cereal box for a cereal company. The box needs to have a volume of 2 000 cm^3. There are many possible boxes you could make with this volume.

a) Verify that the three sets of measurements (in cm) below produce boxes with volume 2 000 cm^3.

$1 \times 1 \times 2\,000$ $\quad$ $2 \times 25 \times 40$ $\quad$ $5 \times 25 \times 16$

b) Calculate the surface area of each box above. (8 002 cm^2, 2 260 cm^2, 1 210 cm^2)

c) If it costs 25¢ for each cm^2 for the material to make the box, which box would you recommend? ($5 \times 25 \times 16$)

d) Find three more boxes with the same volume and calculate the surface area of each. Now which box would you recommend? (Answers will vary.)

e) The cereal company wants the front of the box to be at least 20 cm wide and 20 cm high and the depth of the box to be at least 4 cm. Find two boxes satisfying these conditions that each have a volume of 2 000 cm^3. Which box would you recommend? (A: $20 \times 20 \times 5$ and B: $20 \times 25 \times 4$, Surface area of A $=$ 1 200 cm^2, Surface area of B $=$ 1 360 cm^2, so box A is better.)

NAME ______________________ DATE ______________________

Nets of 3-D Shapes — List of 3-D Shapes

Right prisms

1. Cube
2. Pentagonal prism
3. Rectangular prism
4. Hexagonal prism
5. Triangular prism with an obtuse scalene base
6. Triangular prisms with right scalene bases
7. Right prism with a trapezoid base
8. Right prism with a parallelogram base
9. Right prism with an irregular hexagonal base
10. Right octagonal prism
11. Right dodecagonal prism

Skew prisms

12. Skew prism with three different pairs of parallelogram faces
13. Skew rectangular prism
14. Skew square prism
15. Skew triangular prism

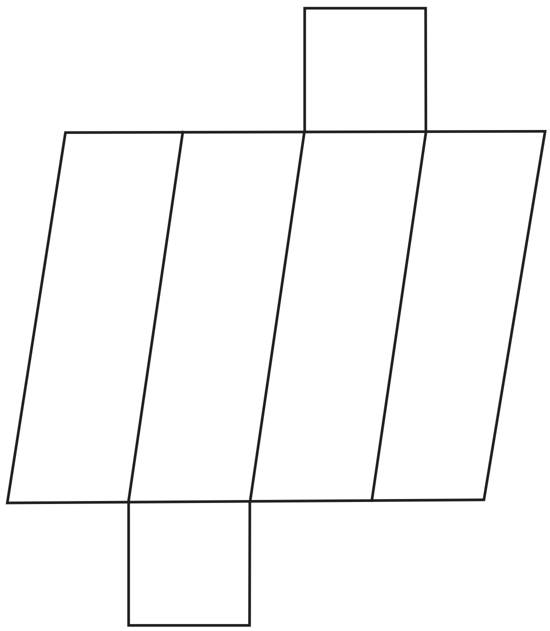
Other shapes

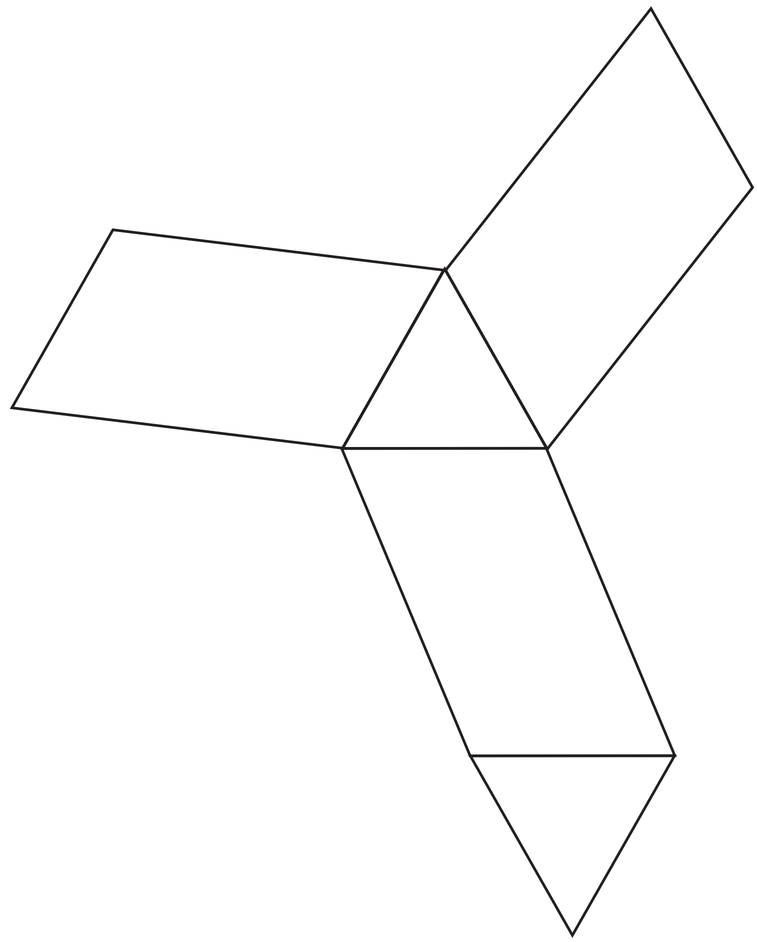
16. Skew rectangular pyramid
17. Right pentagonal pyramid
18. Right hexagonal pyramid
19. Tetrahedron
20. Square antiprism
21. Truncated square pyramid
22. Dodecahedron
23. Icosahedron
24. Octahedron

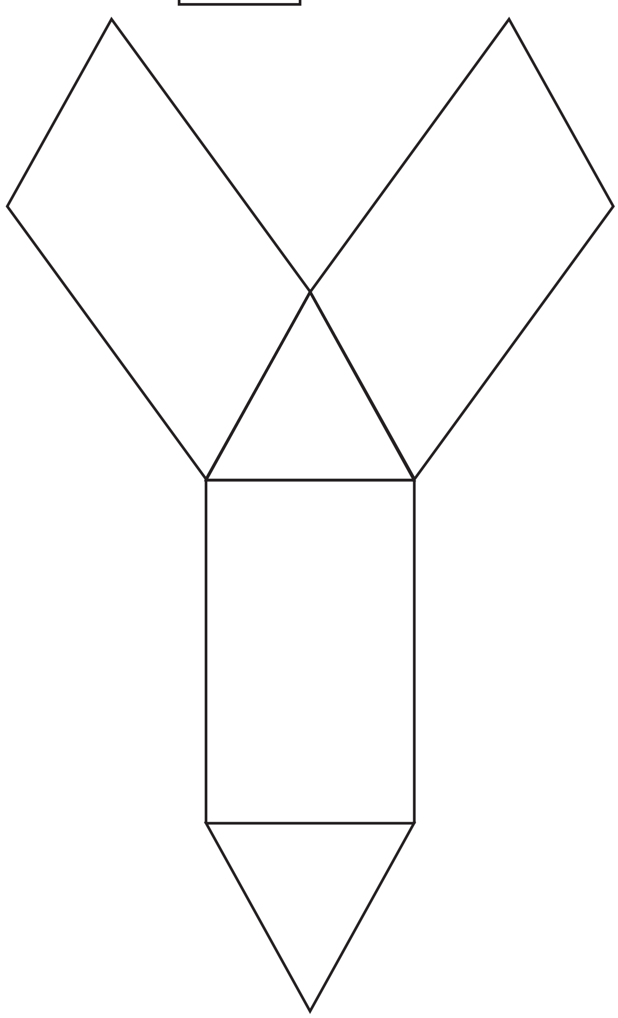
NAME ______________________ DATE ______________________

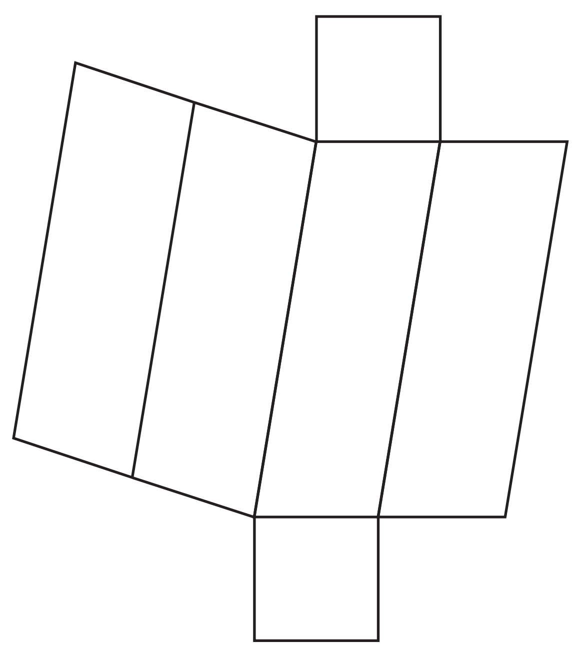
A Net or Not a Net?

Does this picture make a net of a 3-D shape? If yes, what shape?
Predict, then cut out the net and fold it to check your prediction.









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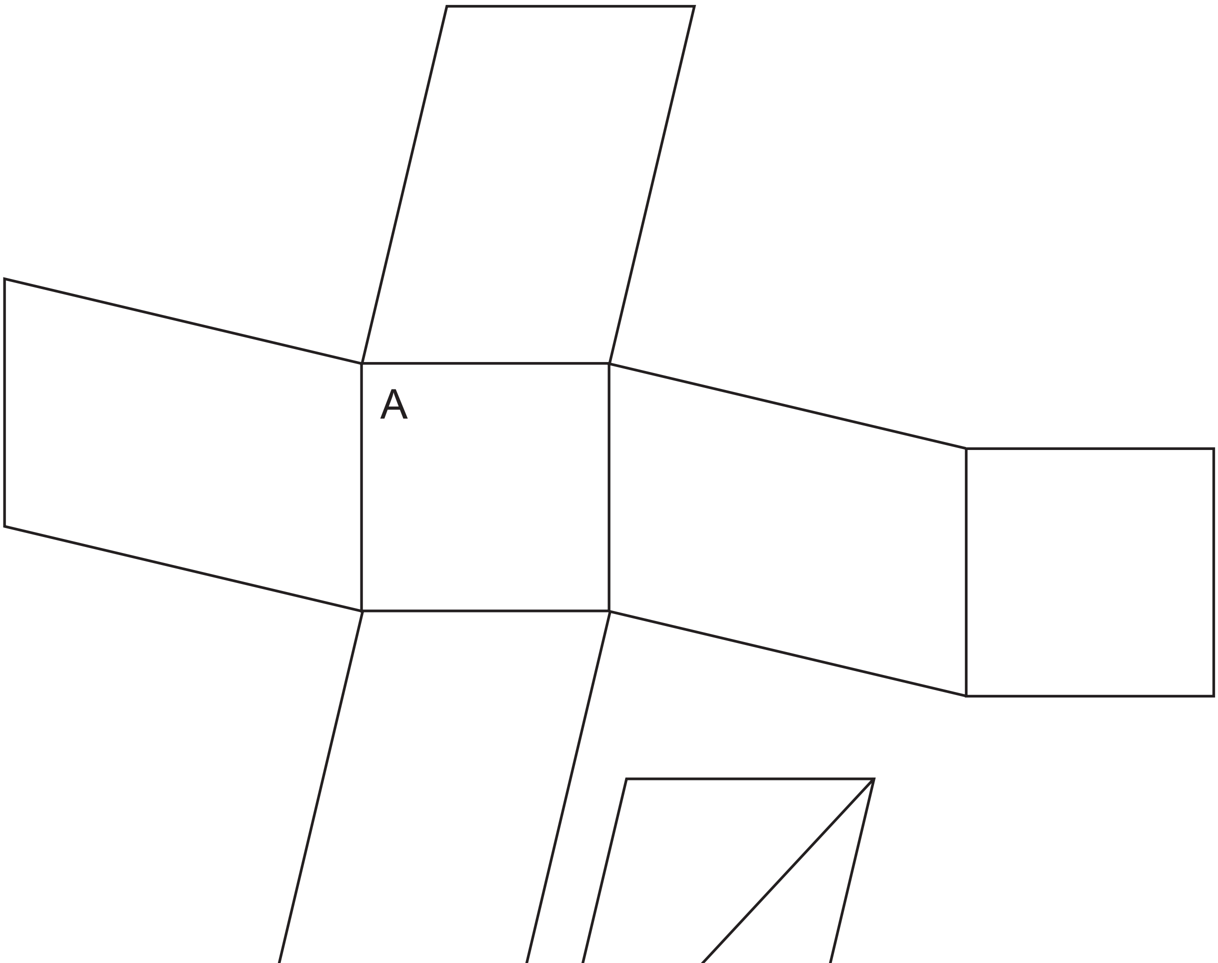
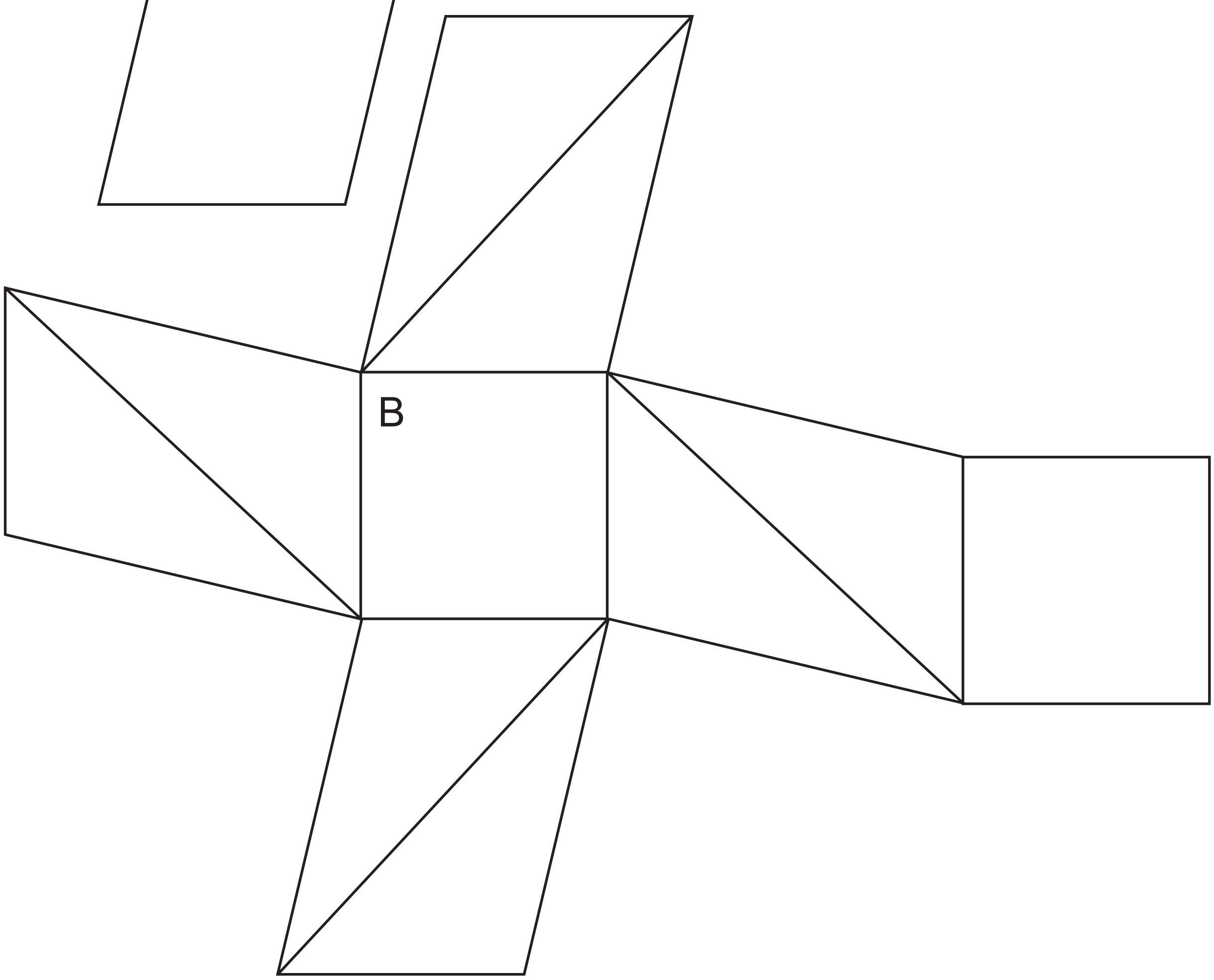
Is It a Net?





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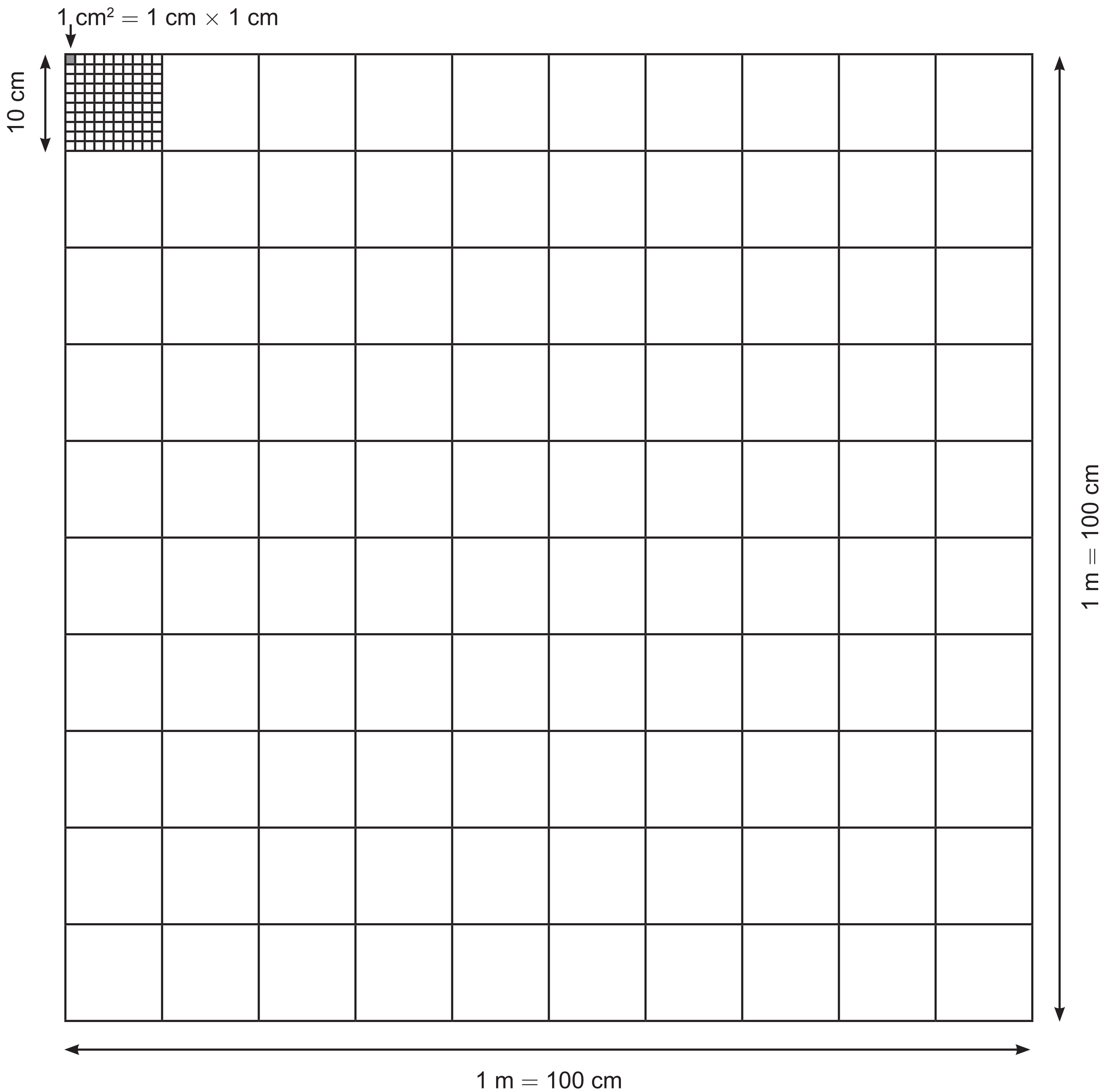
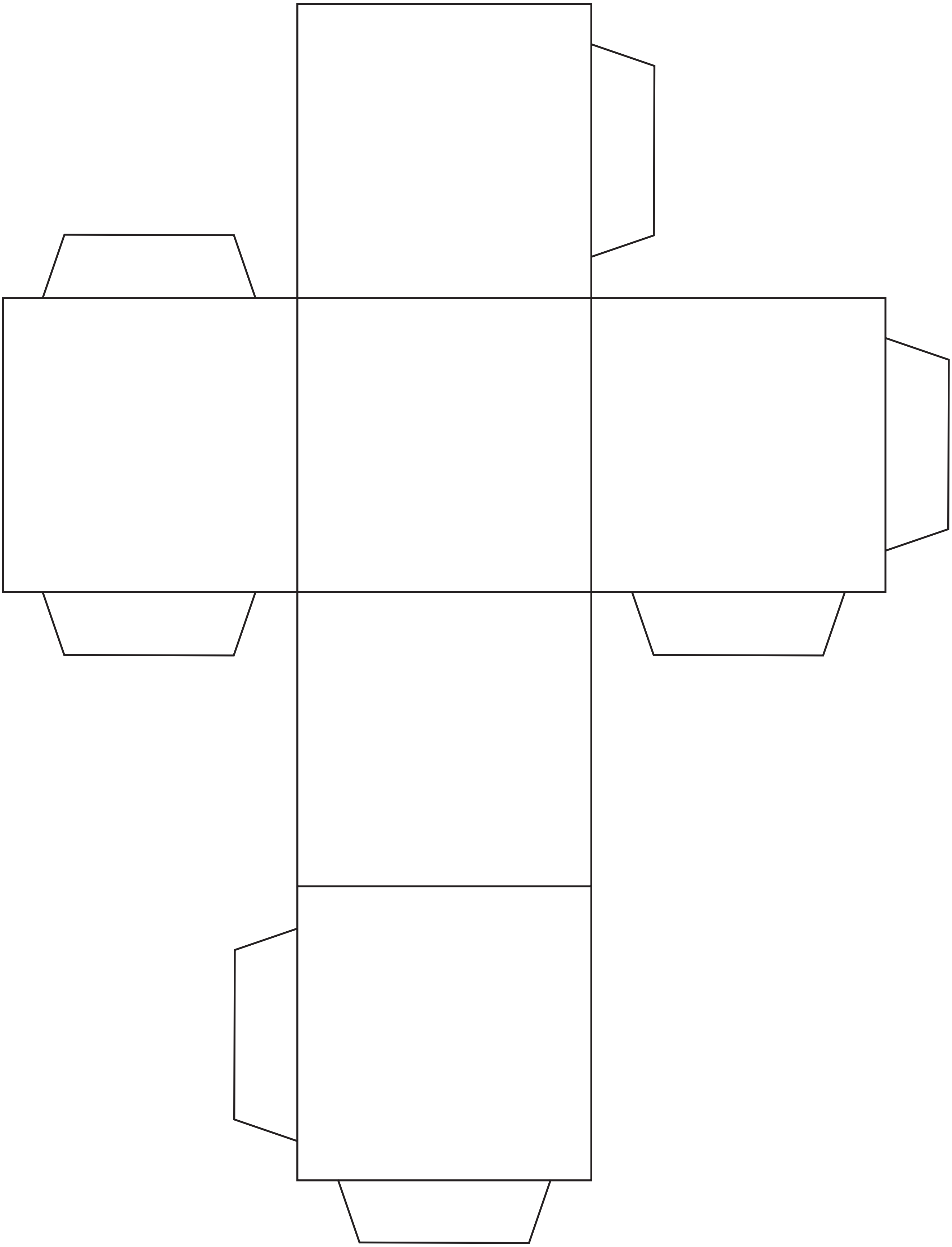
Square Metre



$1\ m^2$ $= 1\ m \times 1\ m = 100\ cm \times 100\ cm =$ **$10\,000\ cm^2$**

NAME ______________________________ DATE ______________________________

Nets of 3-D Shapes (1)



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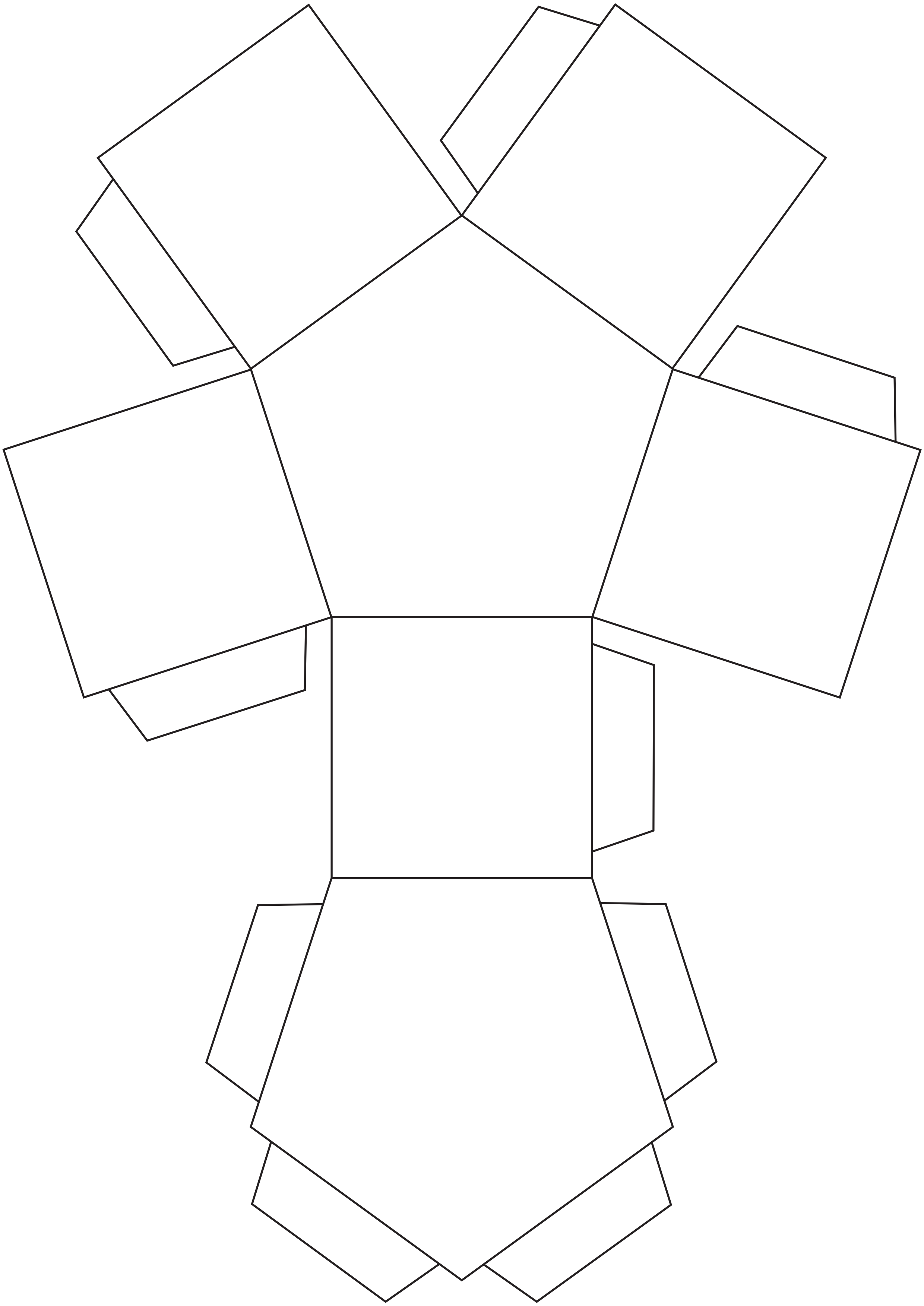
Nets of 3-D Shapes (2)



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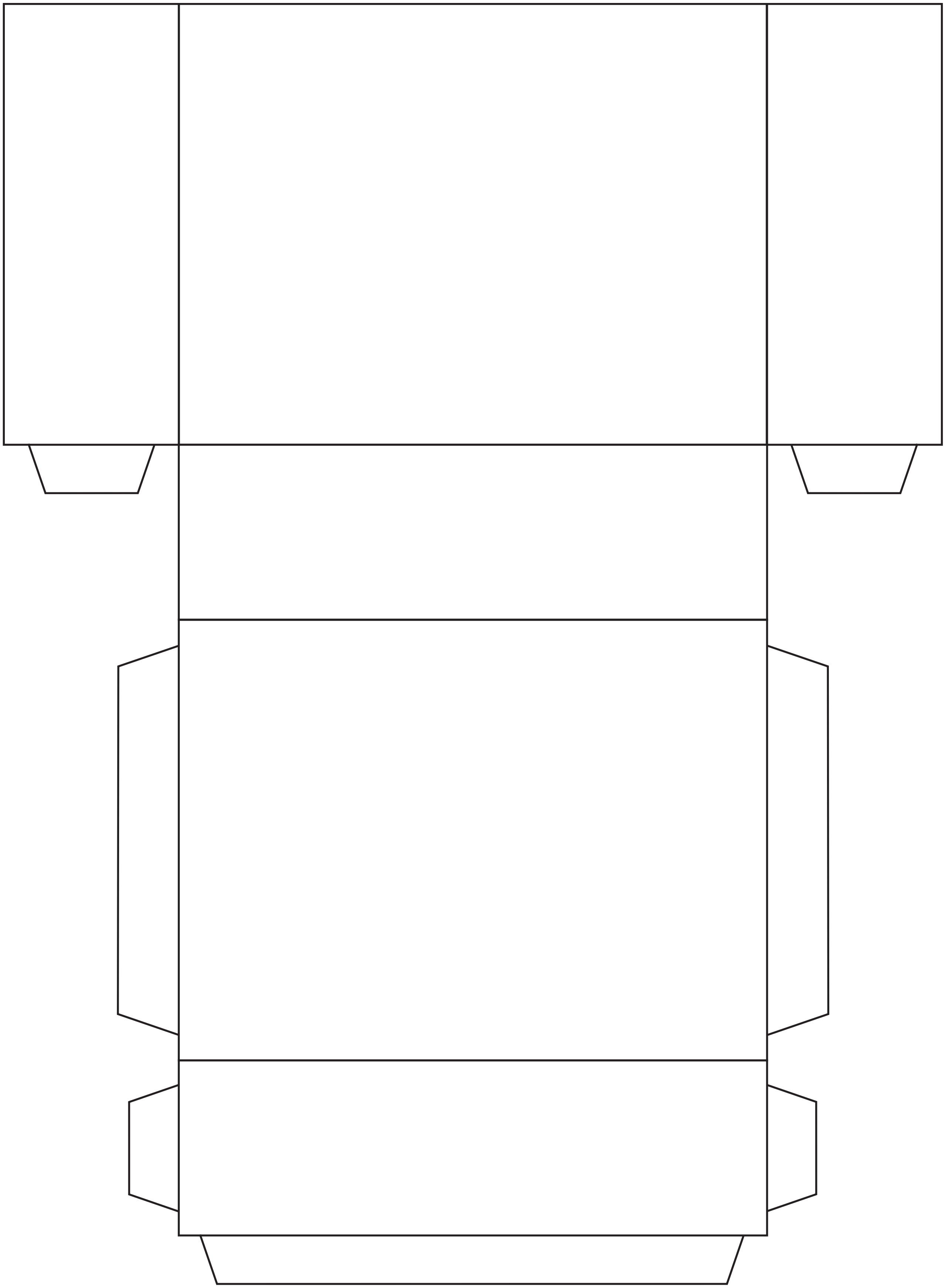
Nets of 3-D Shapes (3)



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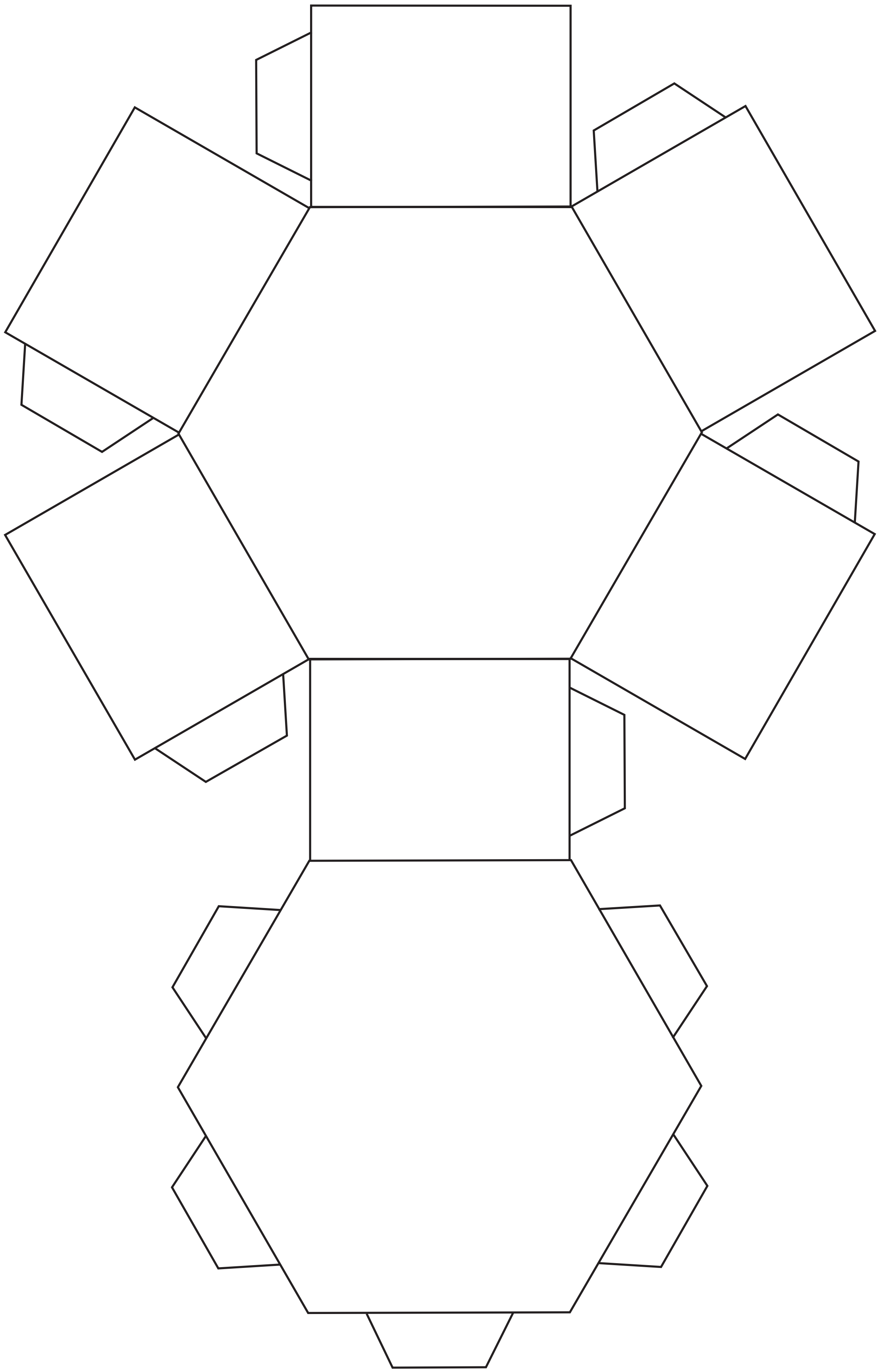
Nets of 3-D Shapes (4)



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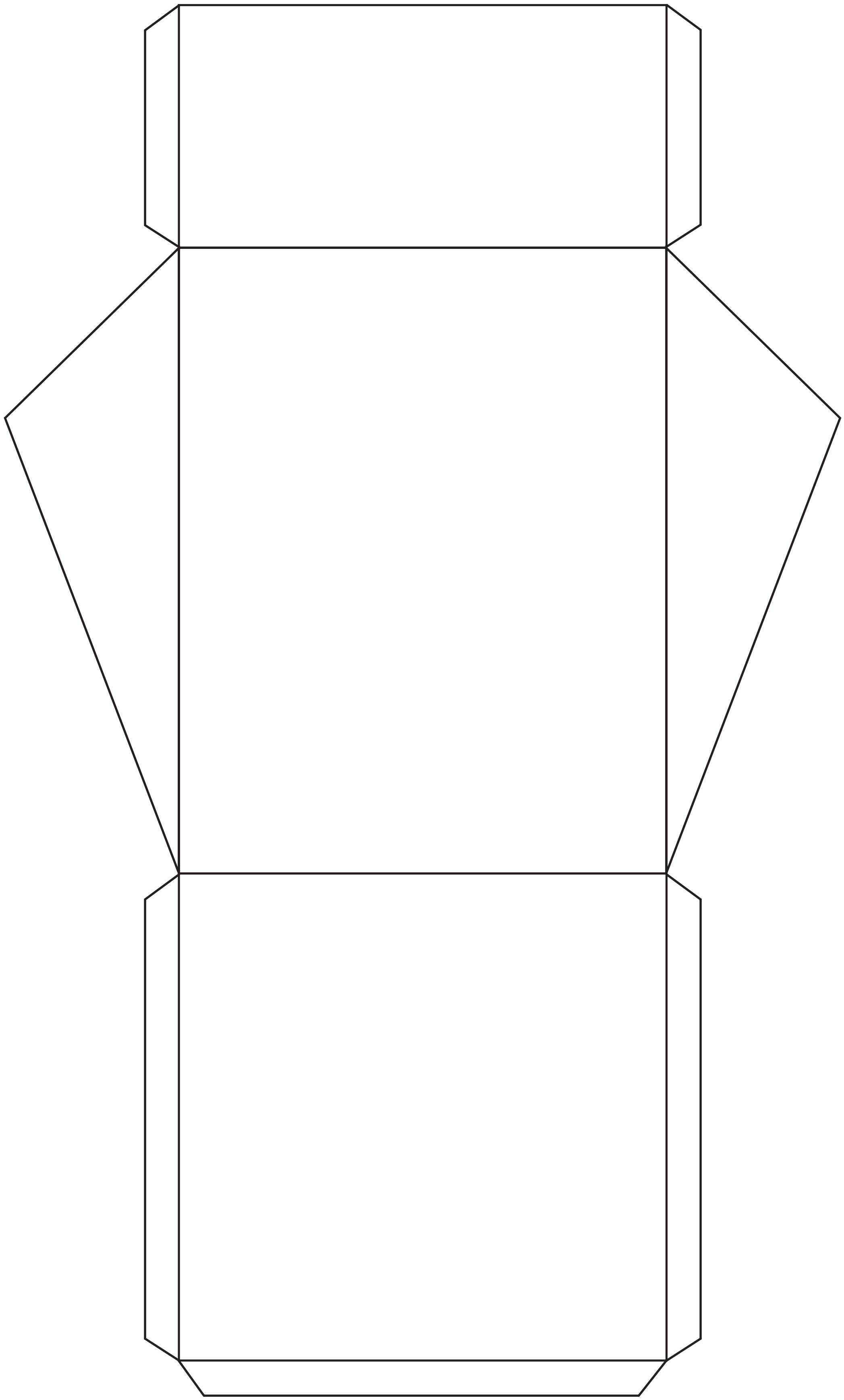
Nets of 3-D Shapes (5)



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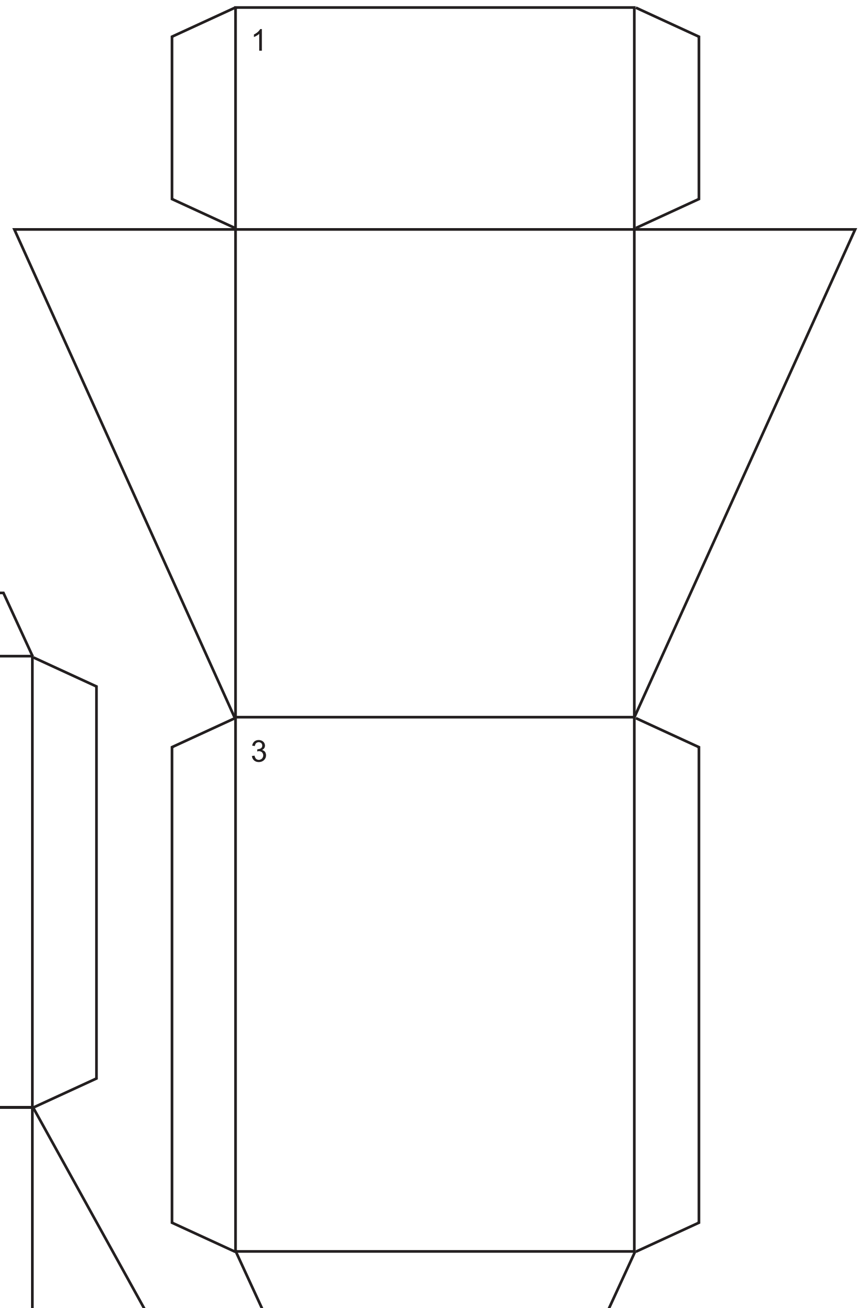
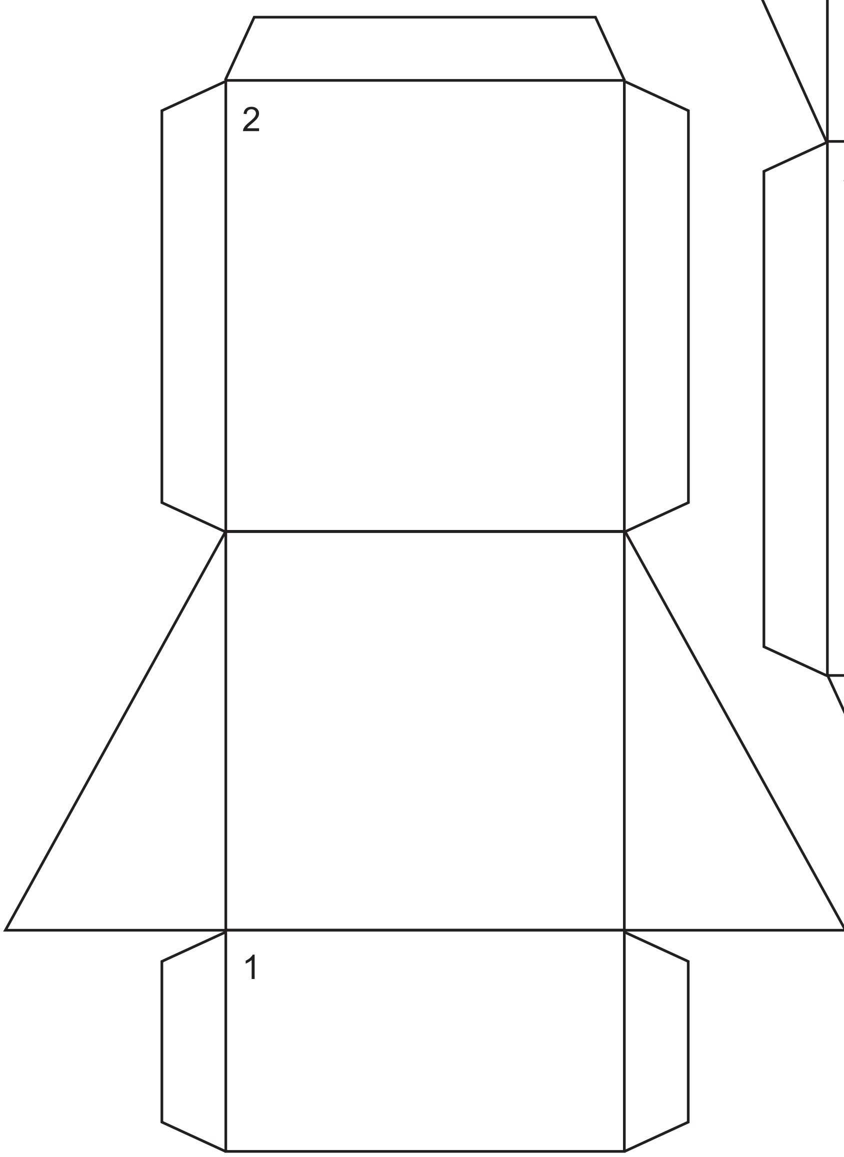
Nets of 3-D Shapes (6)





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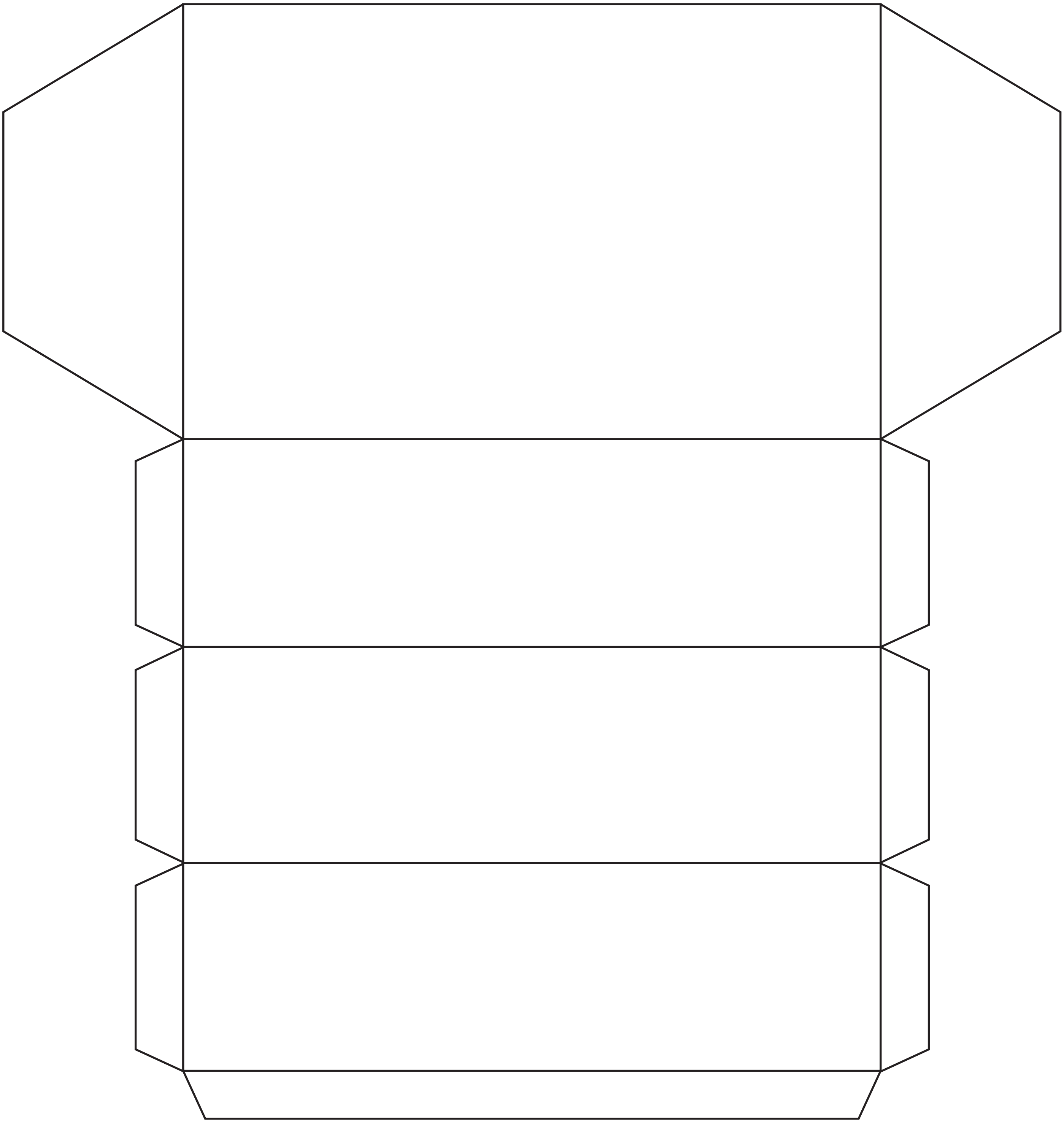
Nets of 3-D Shapes (7)



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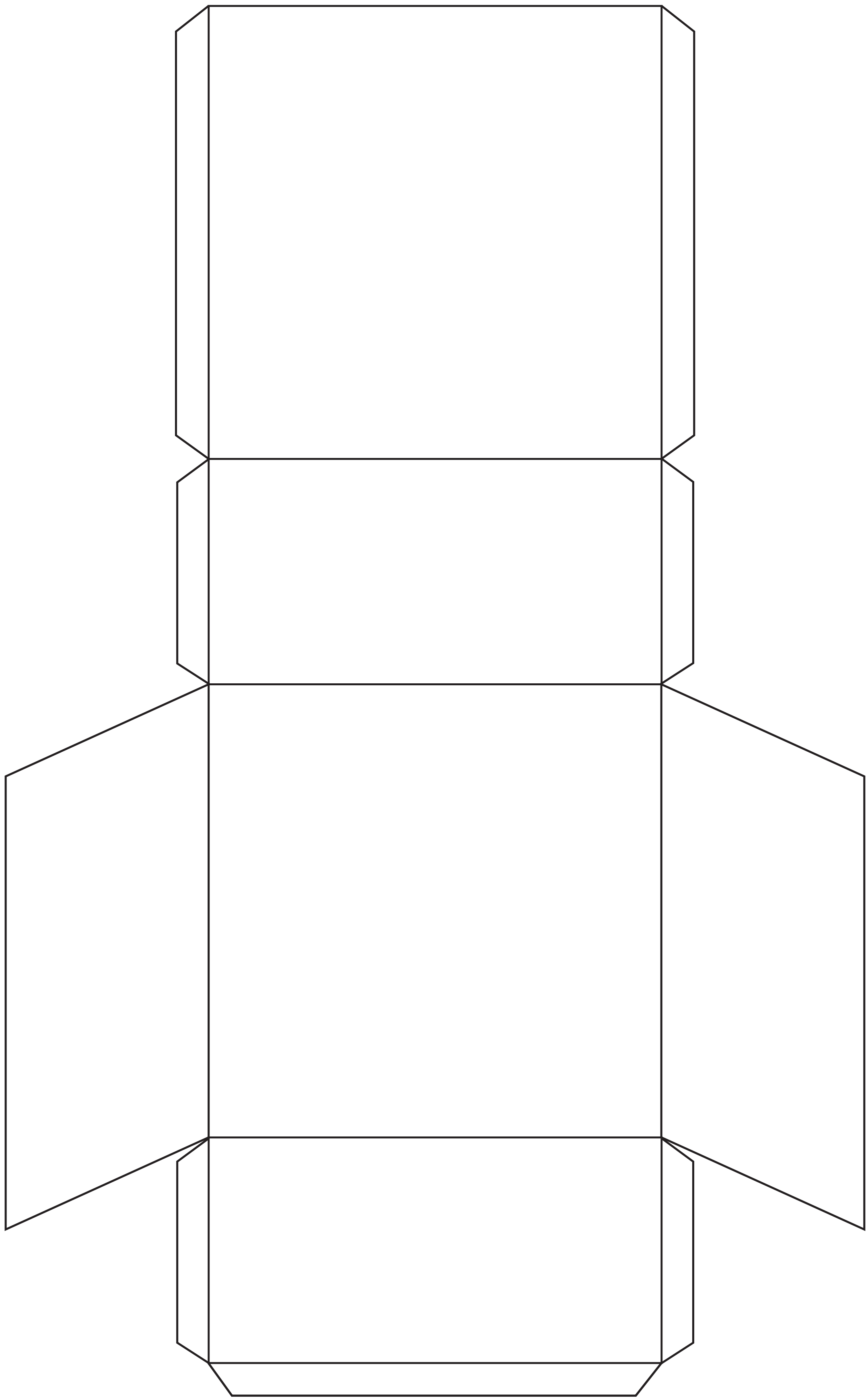
Nets of 3-D Shapes (8)



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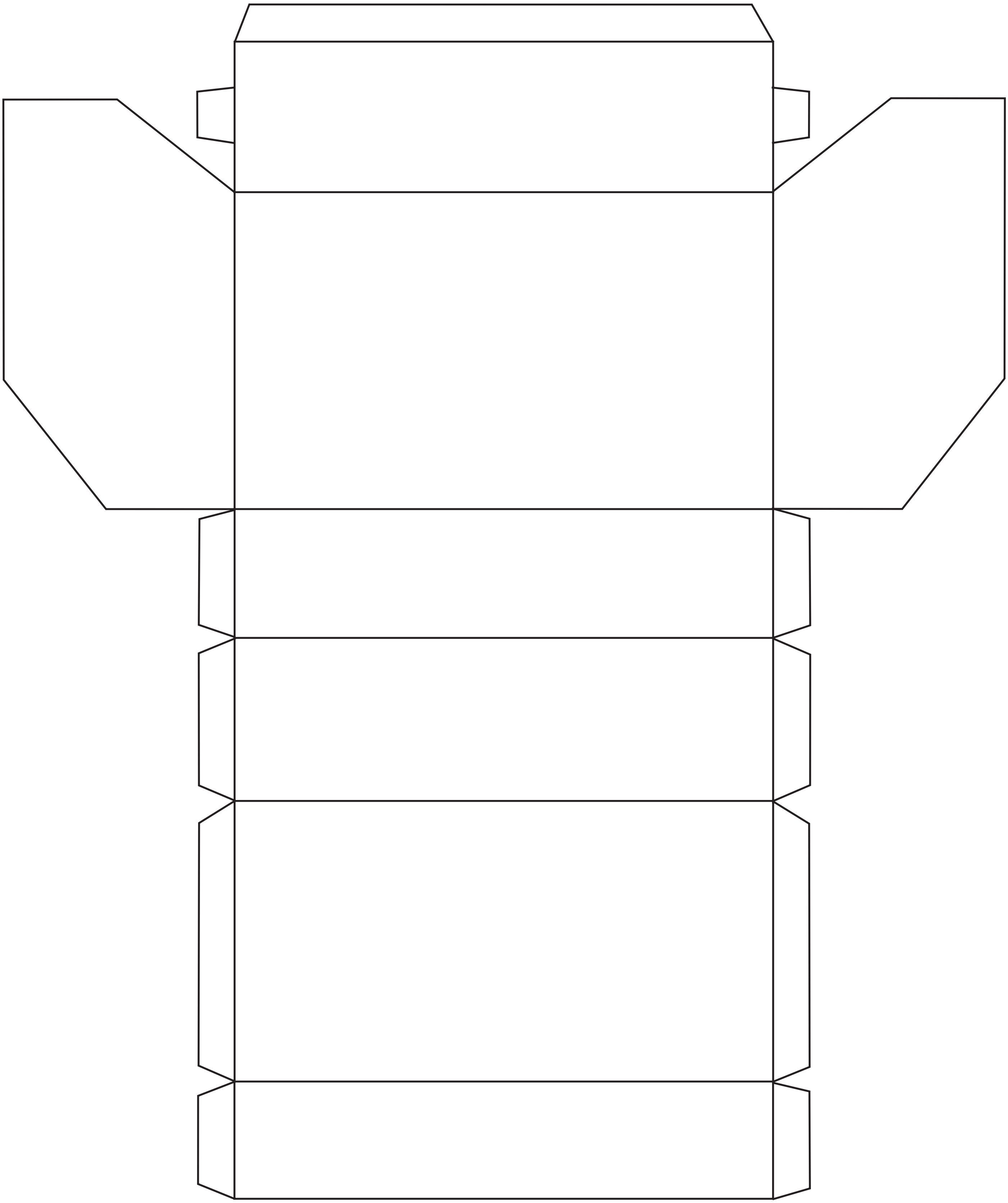
Nets of 3-D Shapes (9)



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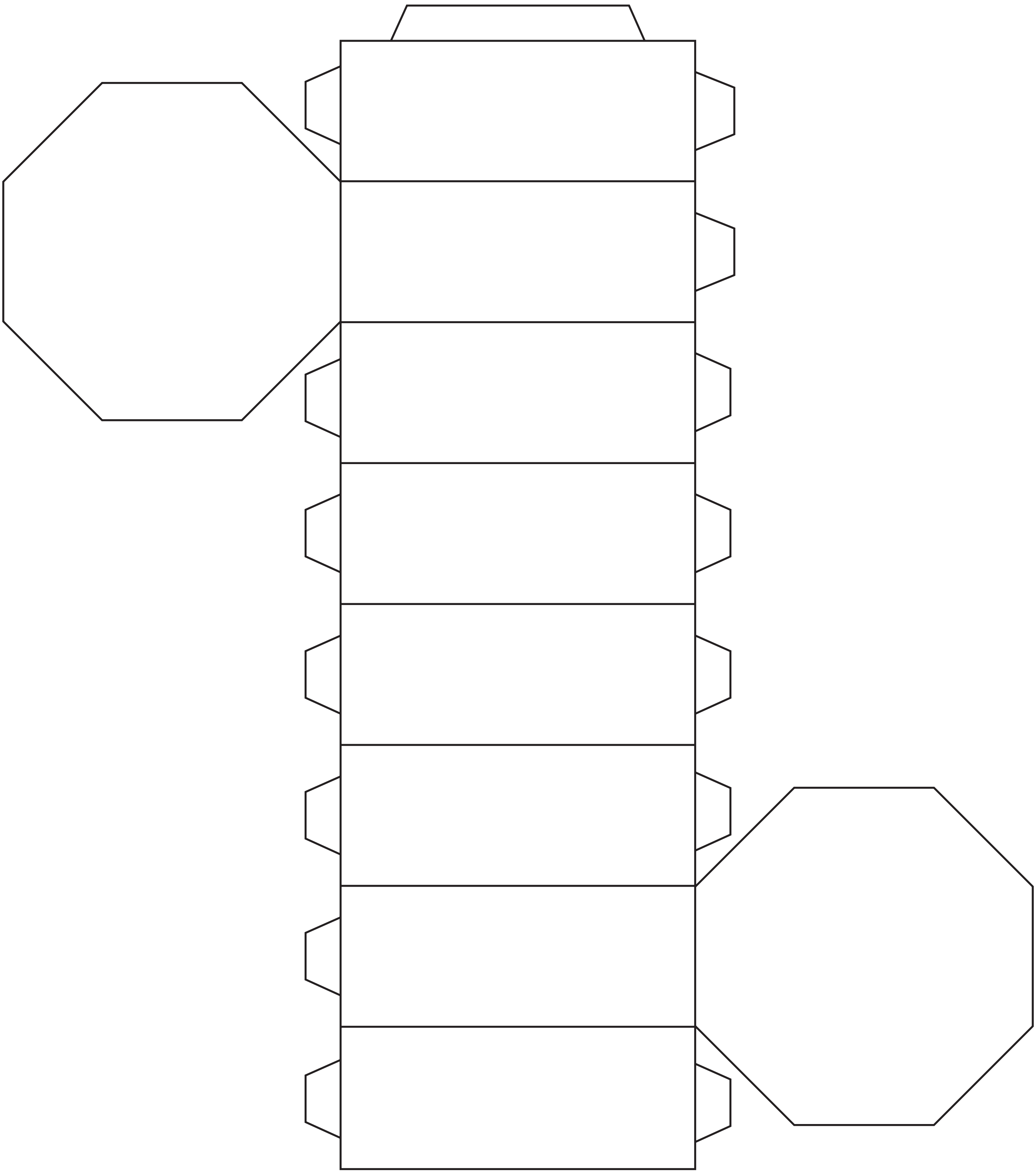
Nets of 3-D Shapes (10)



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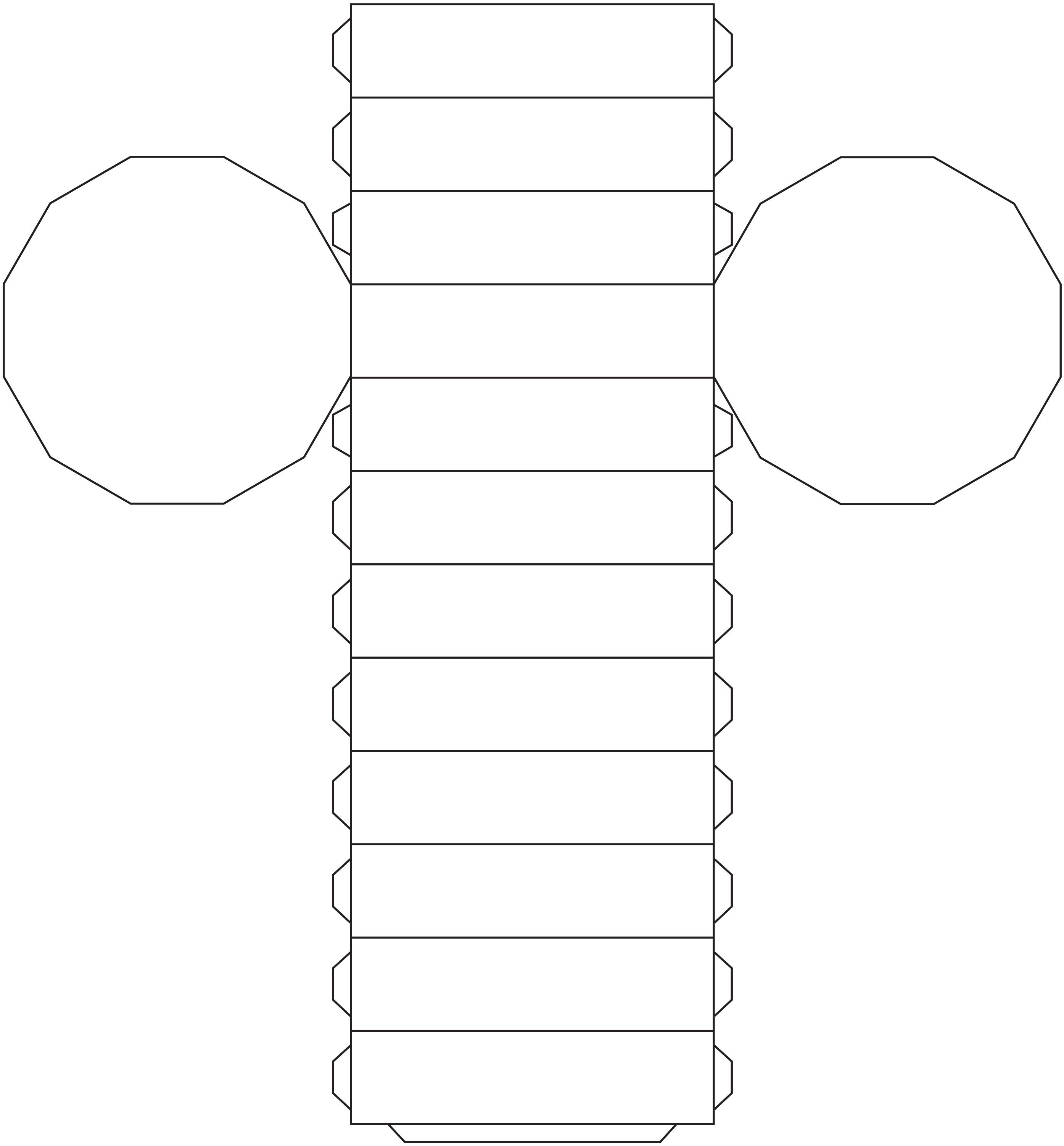
Nets of 3-D Shapes (11)



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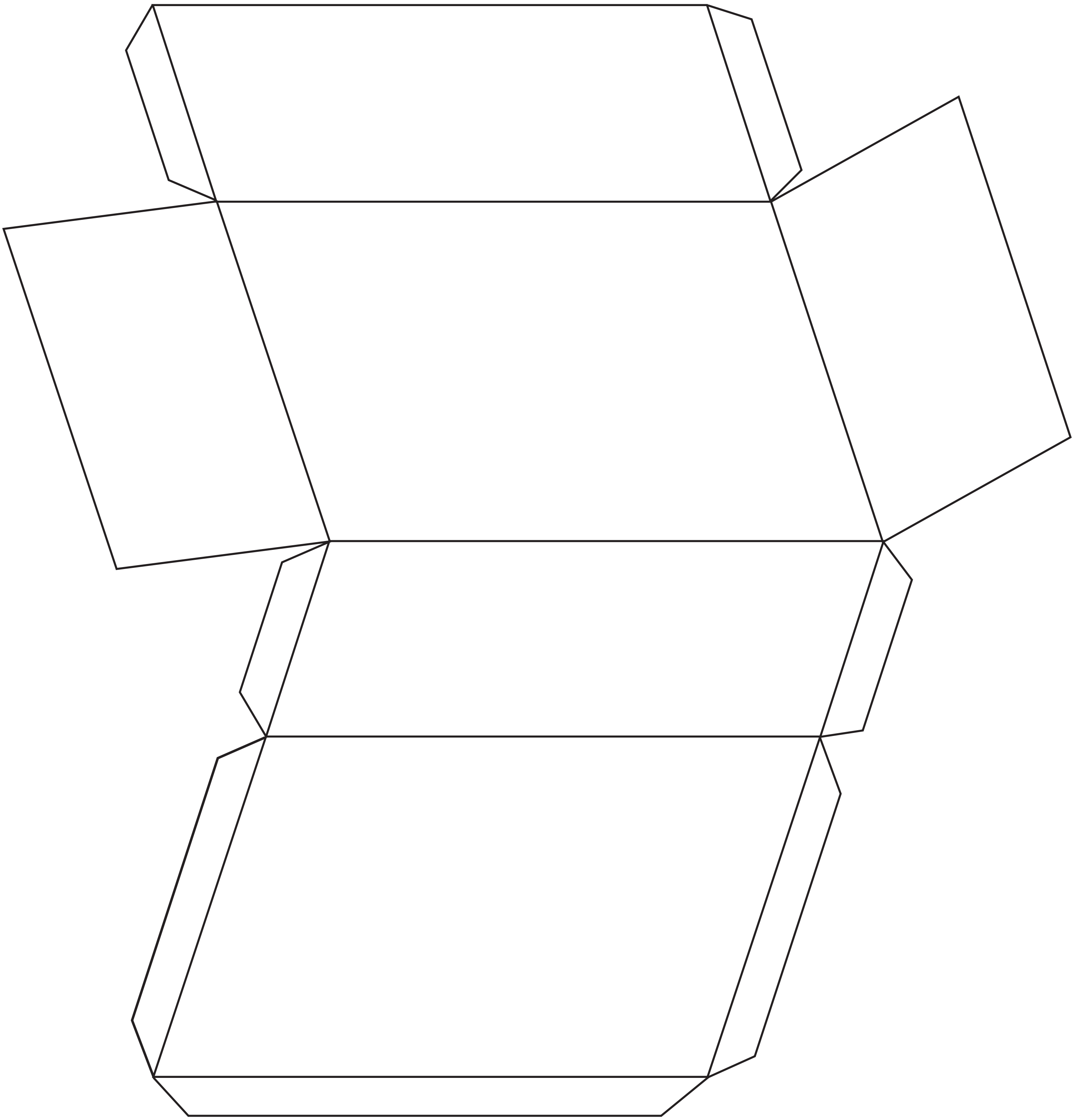
Nets of 3-D Shapes (12)



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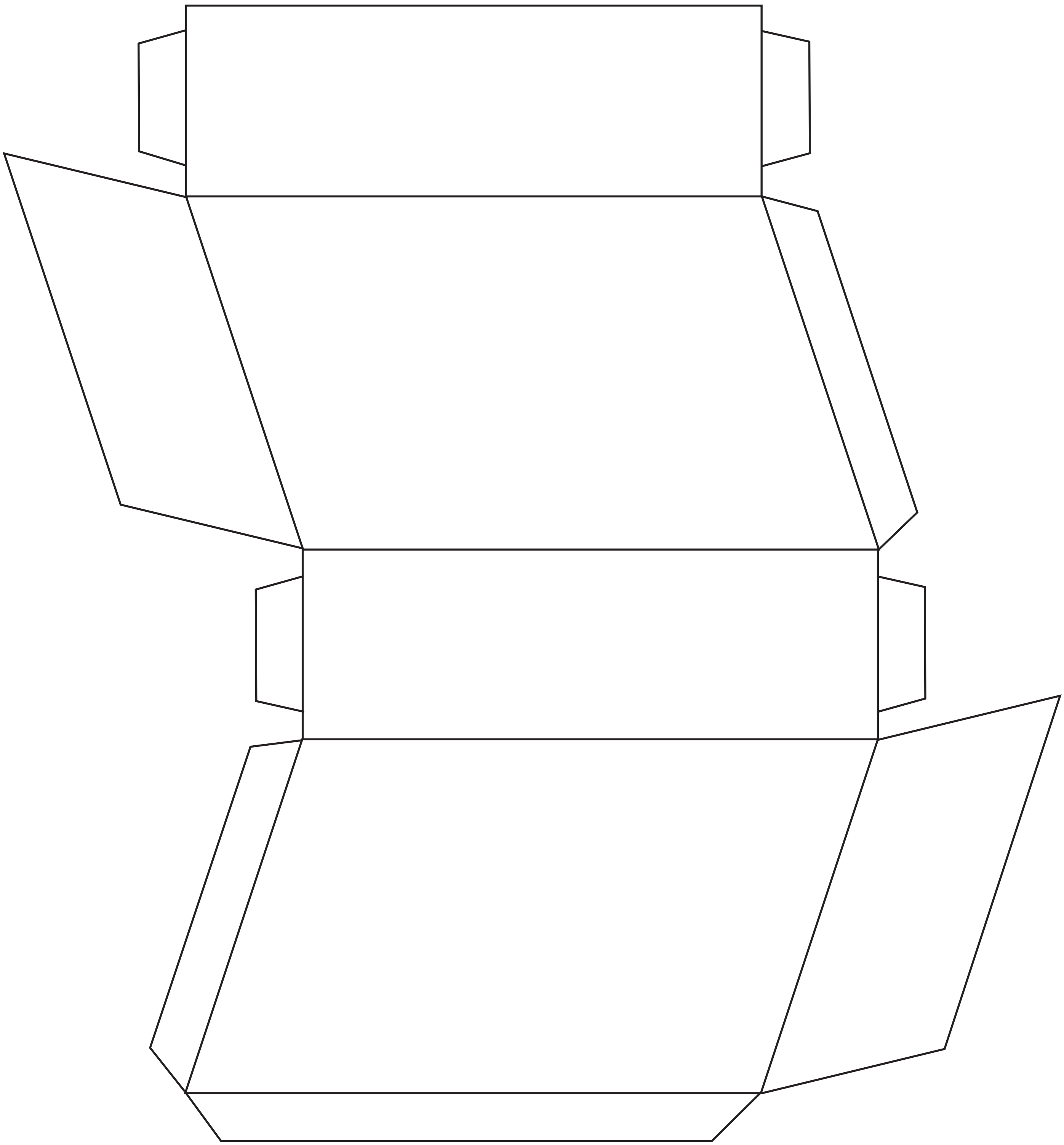
Nets of 3-D Shapes (13)



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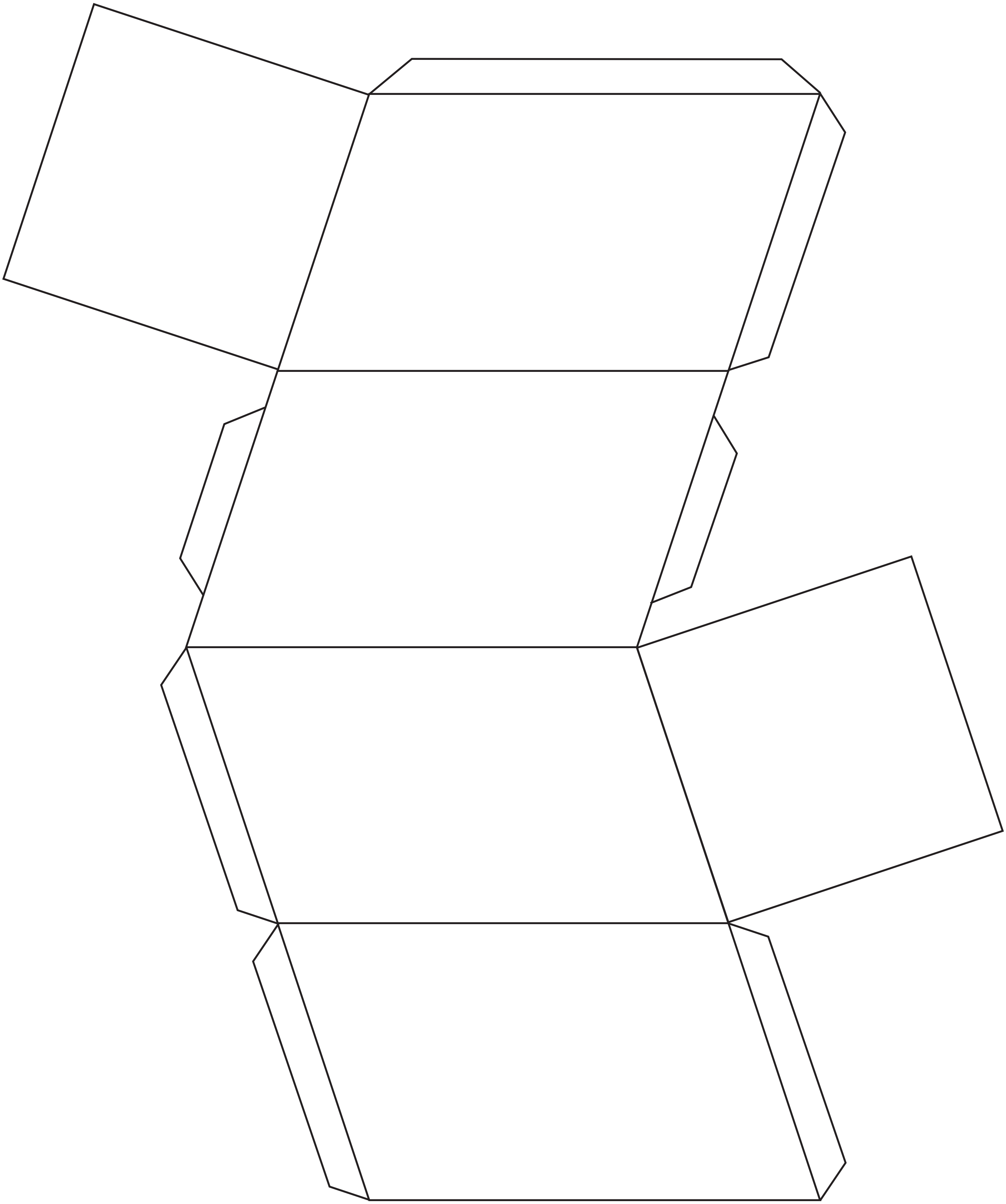
Nets of 3-D Shapes (14)



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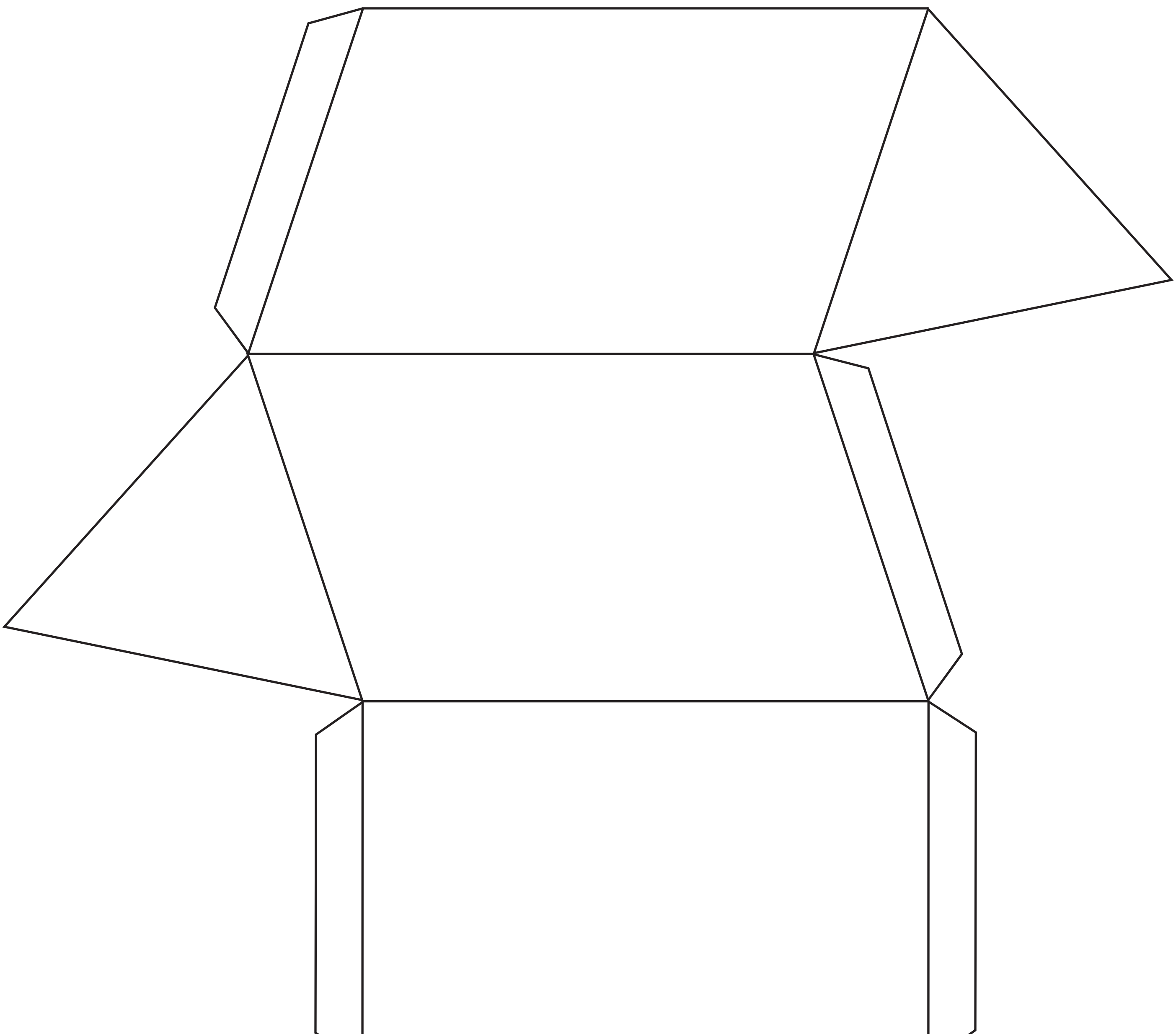

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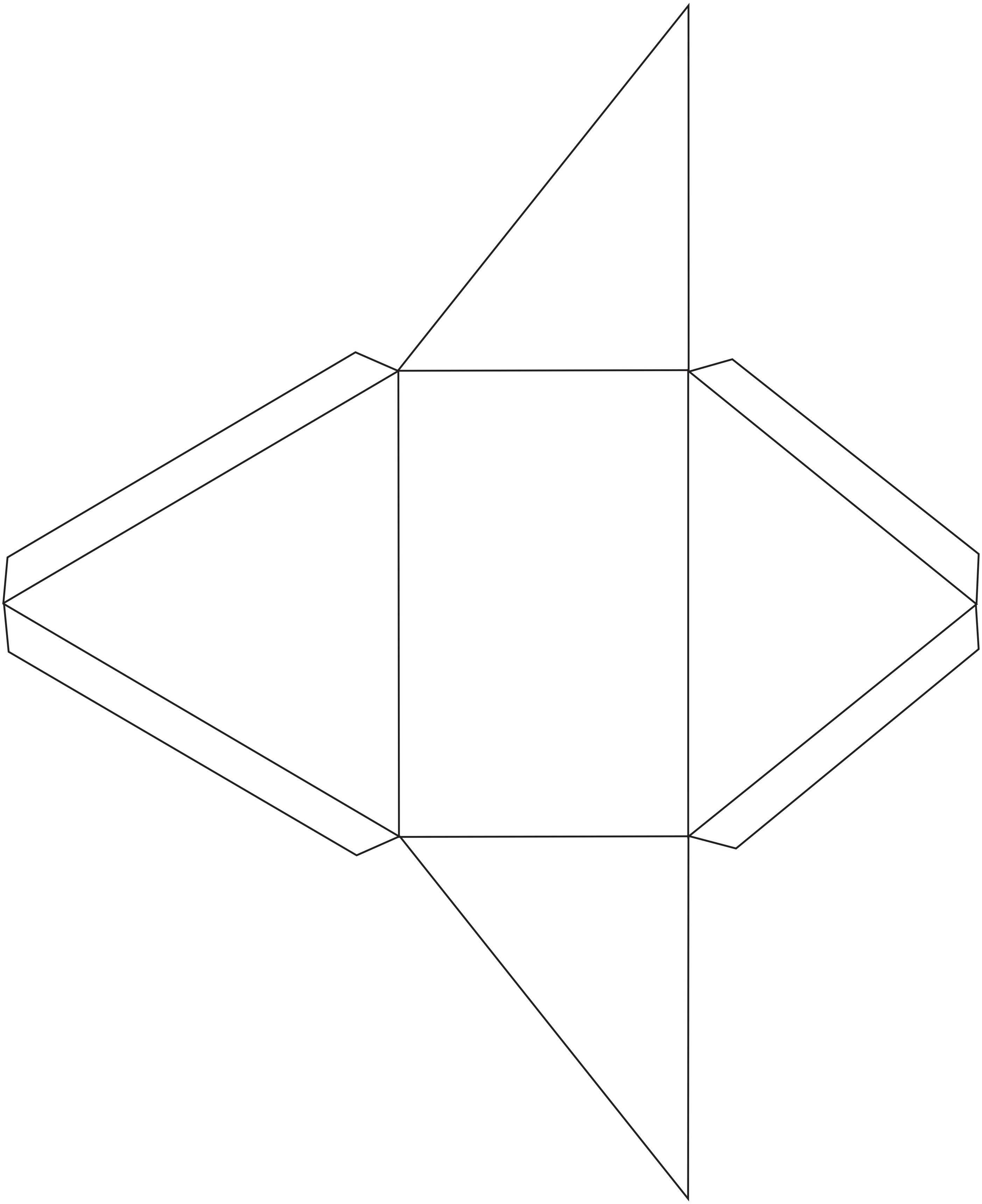
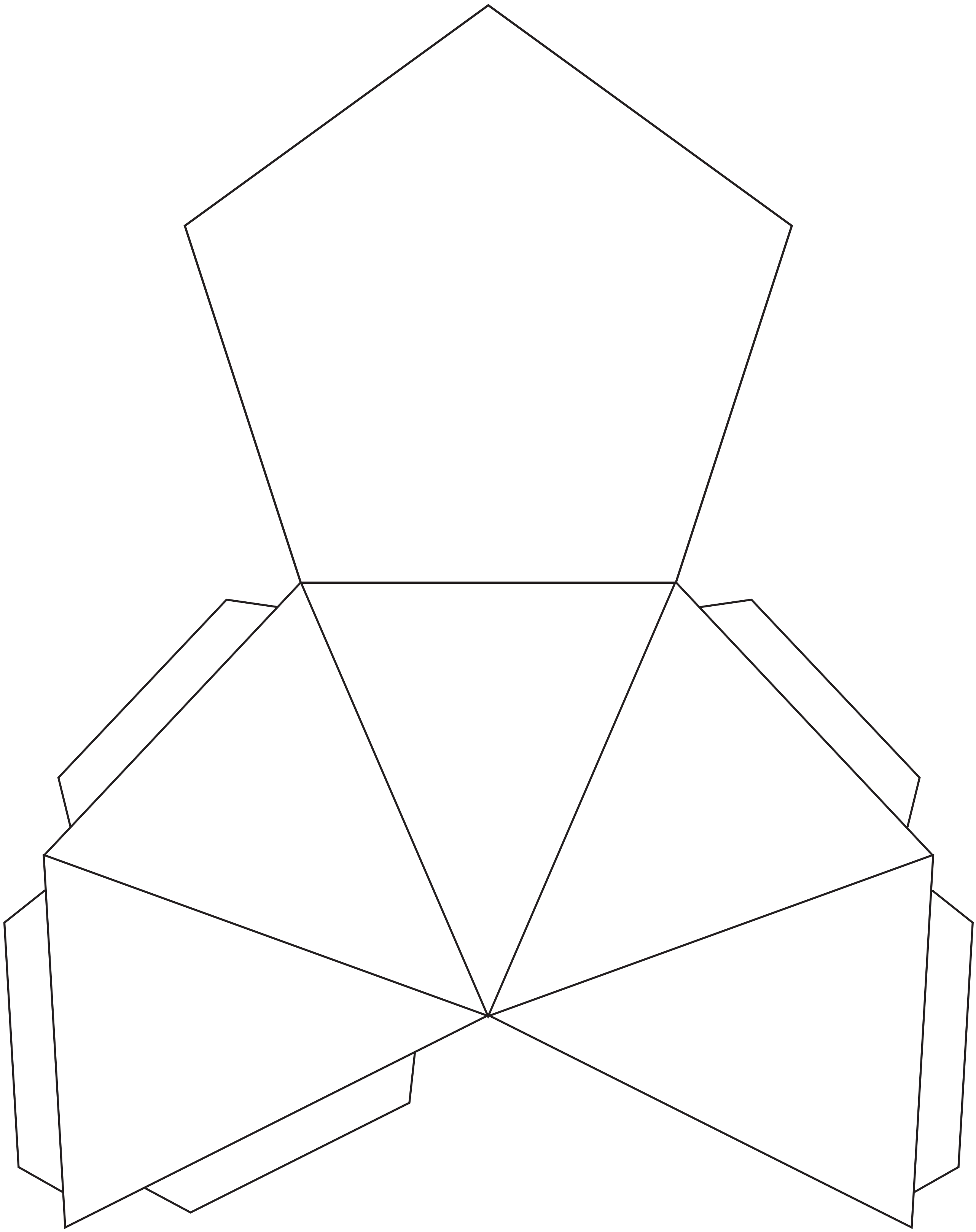
Nets of 3-D Shapes (16)



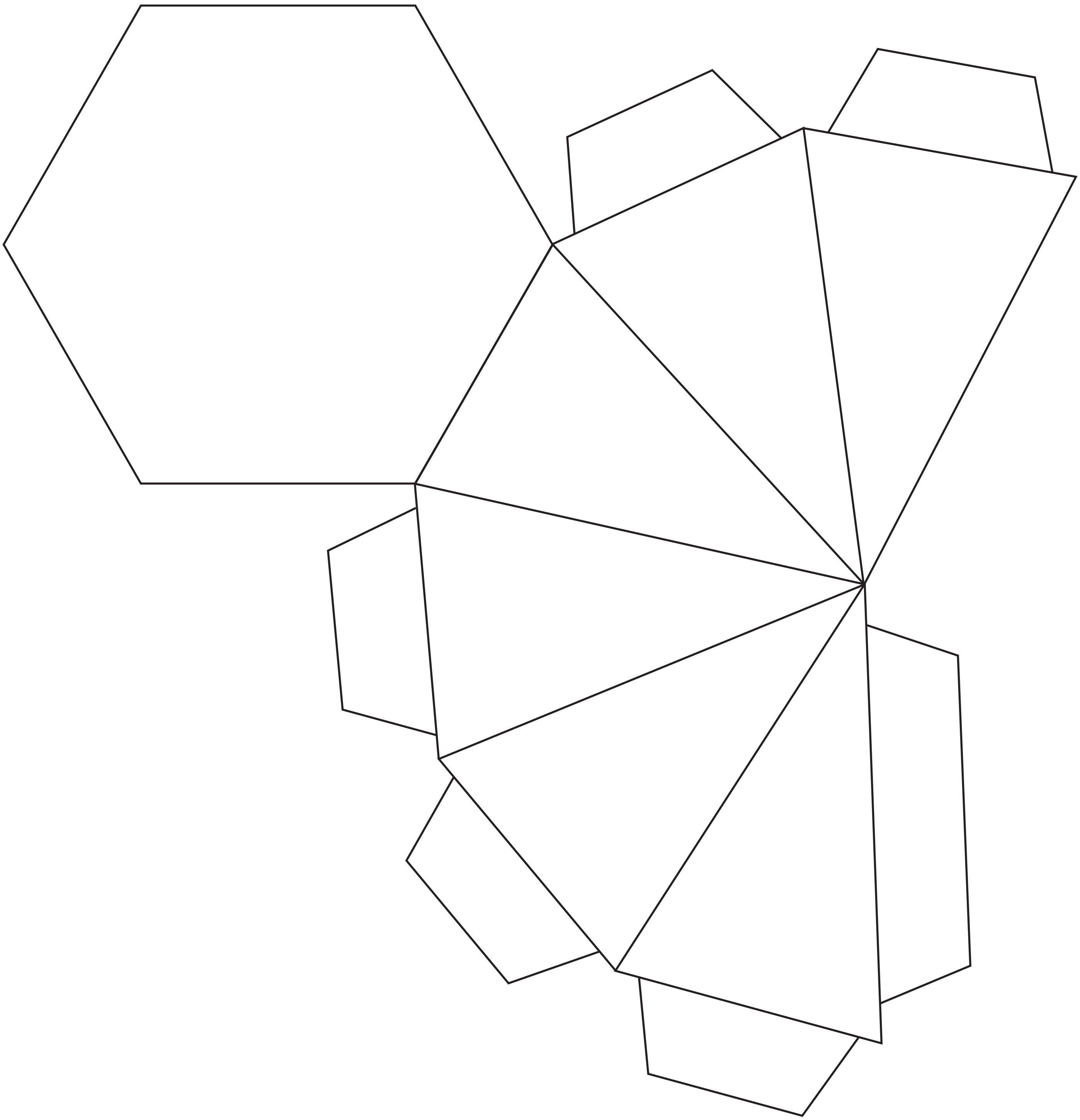
NAME ________________________ DATE ________________________

Nets of 3-D Shapes (17)



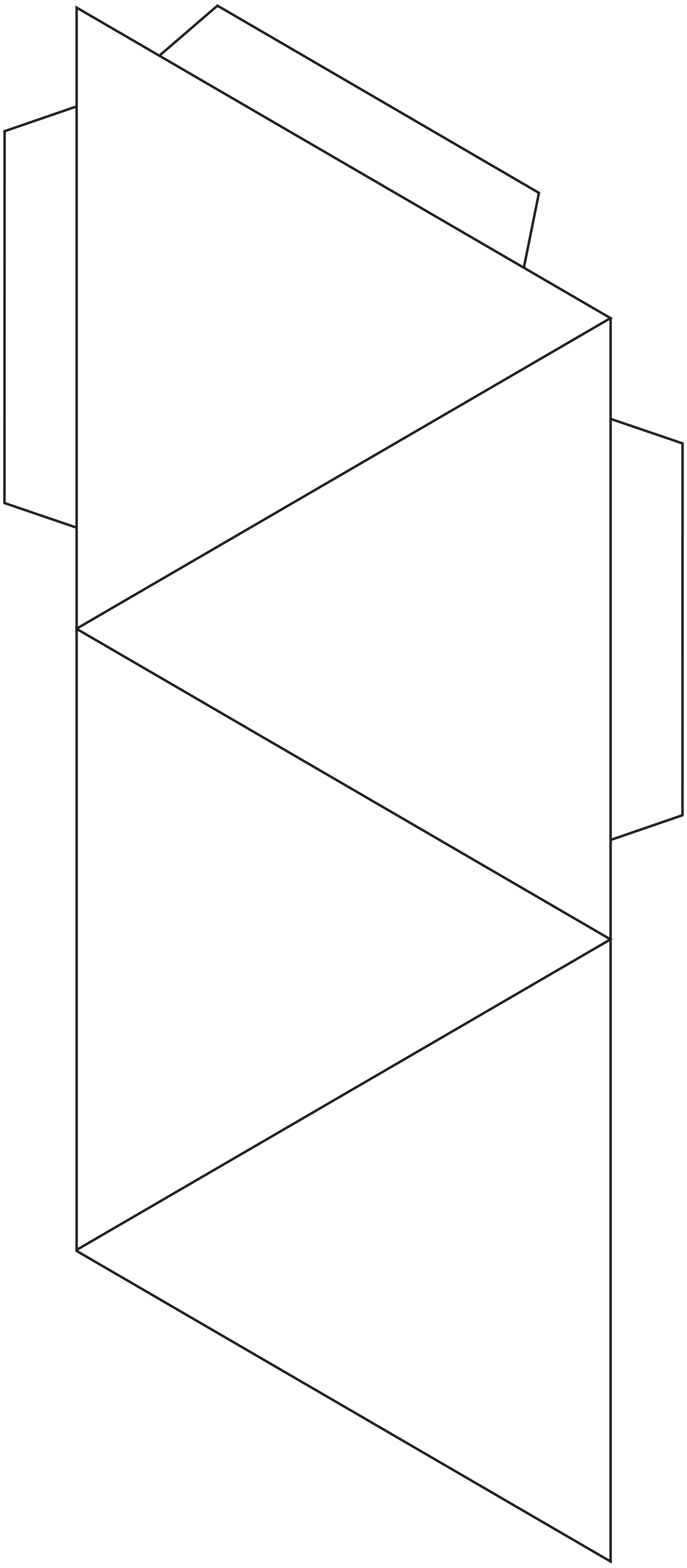
NAME ______________________ DATE ______________________

Nets of 3-D Shapes (18)



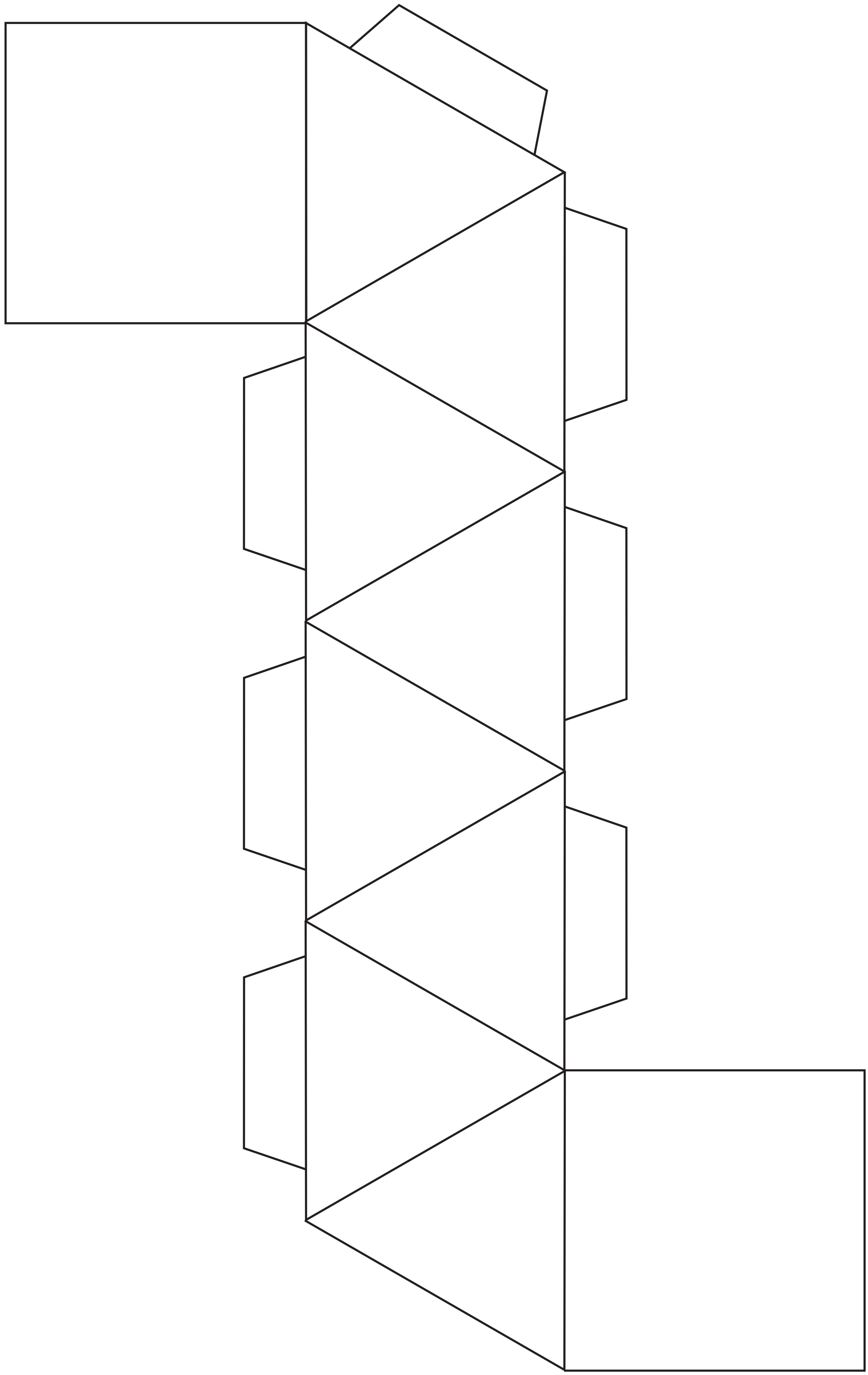
NAME ______________________ DATE ______________________

Nets of 3-D Shapes (19)



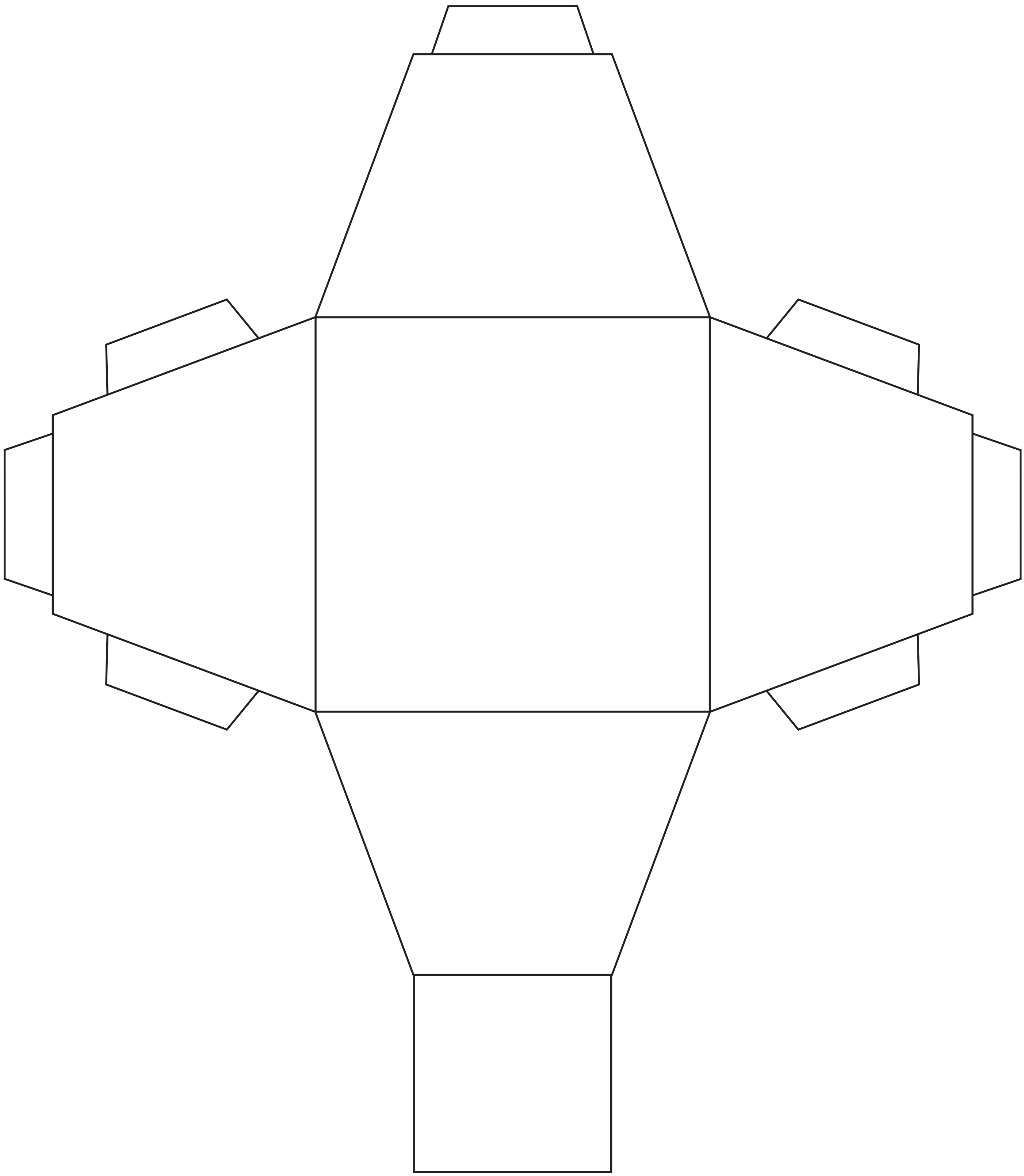
NAME ______________________ DATE ______________________

Nets of 3-D Shapes (20)



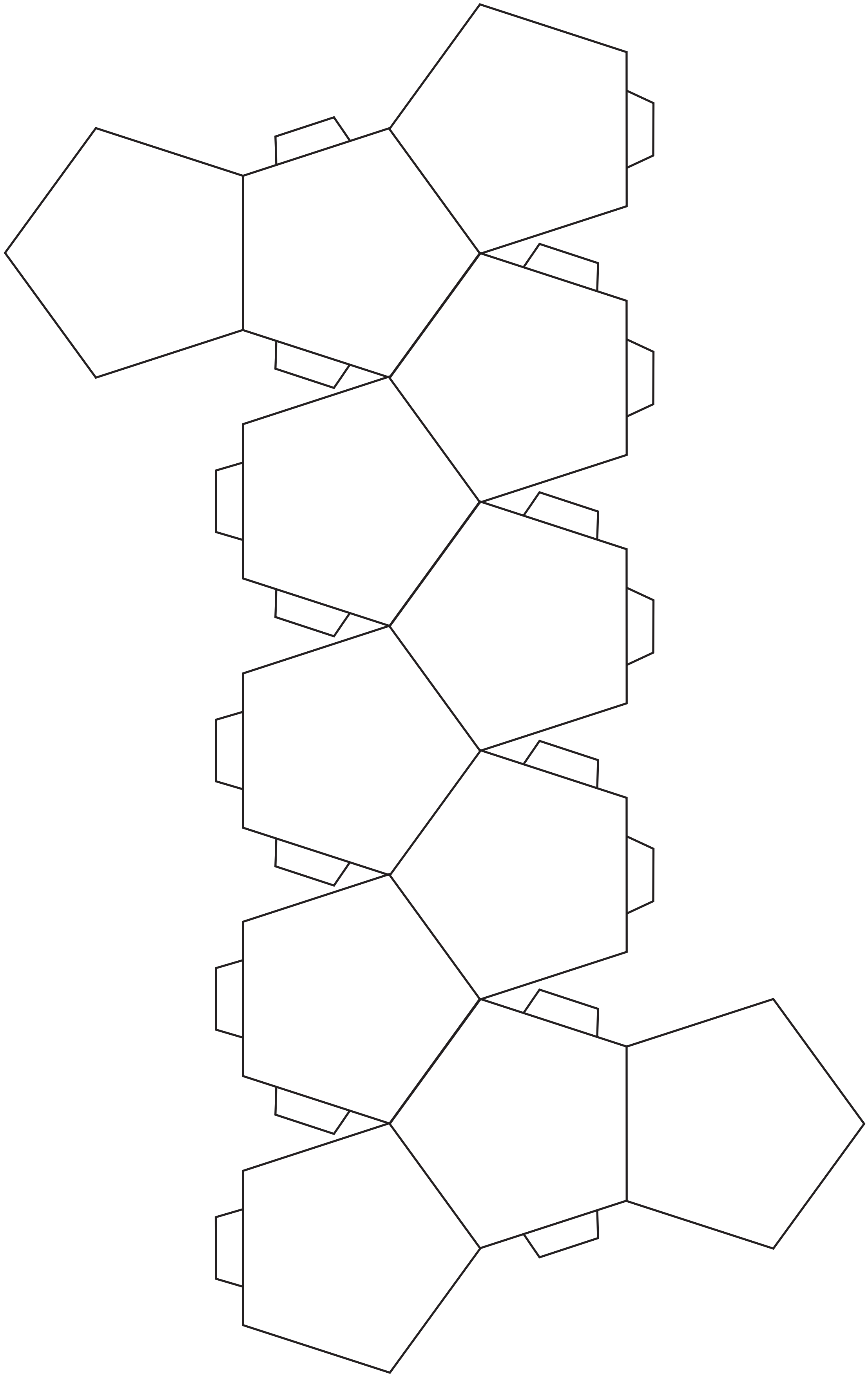
NAME ______________________ DATE ______________________

Nets of 3-D Shapes (21)



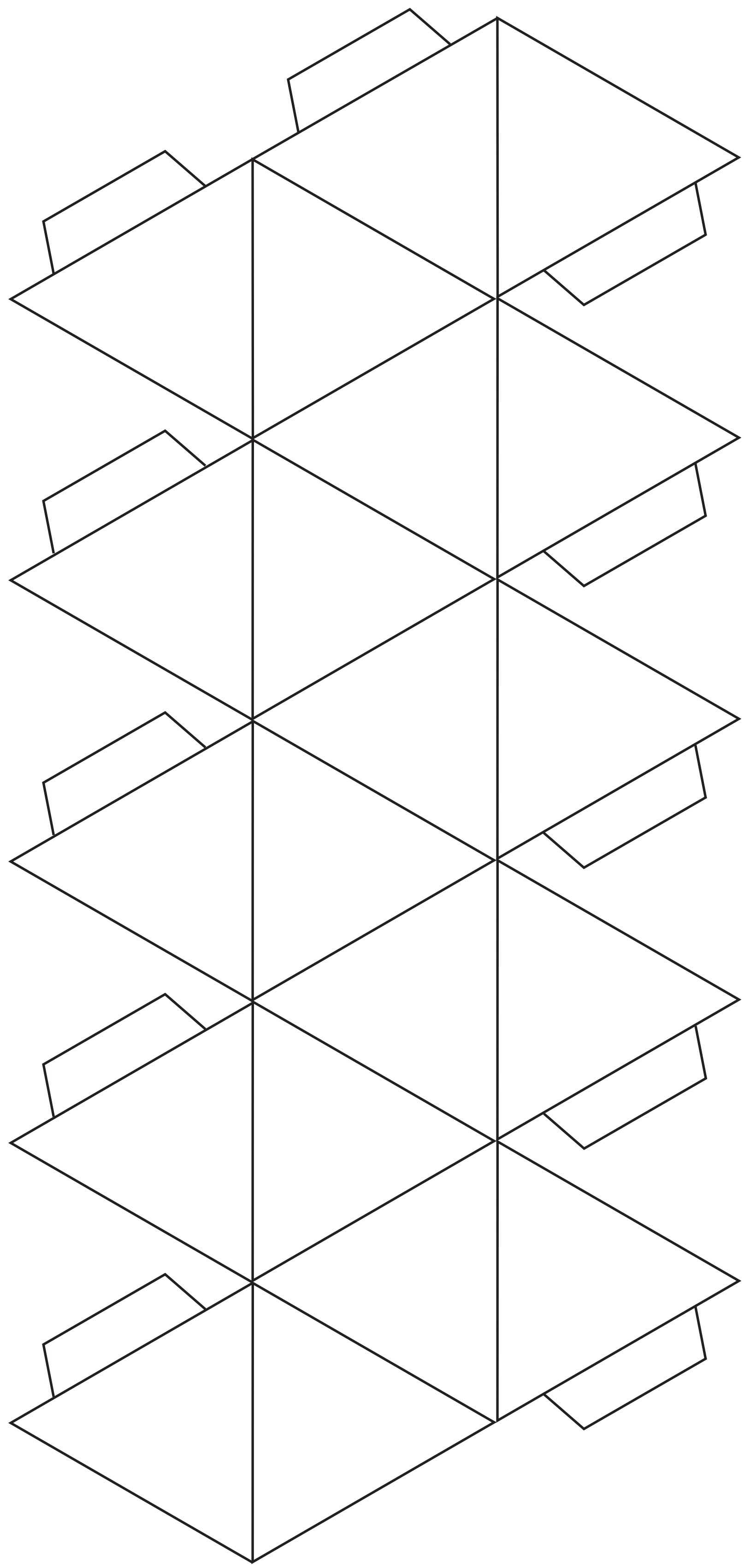
NAME ______________________ DATE ______________________

Nets of 3-D Shapes (22)



NAME ______________________ DATE ______________________

Nets of 3-D Shapes (23)



NAME ______________________ DATE ______________________

Nets of 3-D Shapes (24)