Number Sense – AP Book 8, Part 1: Unit 3

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1. Teacher to check rectangle drawings.
   a) 2 rectangles: 1×4, 2×2
   Factors: 1, 2, 4
   b) 1 rectangle: 1×5
   Factors: 1, 5
   c) 2 rectangles: 1×6, 2×3
   Factors: 1, 2, 3, 6
   d) 1 rectangle: 1×7
   Factors: 1, 7
   e) 2 rectangles: 1×8, 2×4
   Factors: 1, 2, 4, 8
   f) 2 rectangles: 1×9, 3×3
   Factors: 1, 3, 9
2. 4, 9
3. a) Teacher to check.
   b) 1, 4, 9, 16, 25
4. There is no whole number that, when multiplied by itself, equals 20.
5. No – it will have only two factors: 1 and itself. (1 is <u>not</u> prime.)
6. Teacher to check (square with sides = 6).
7. Teacher to check. (2 rectangles: 1×10, 2×5)
8. 1, 4, 9, 16, 25; 36, 49, 64, 81, 100
9. b) $3 \times 3 = 9$
   c) $8 \times 8 = 64$
   d) $0 \times 0 = 0$
   e) $7 \times 7 = 49$
10. b) 100 64 81 64 81 100
    c) 25 144 49 25 49 144
    d) 9 5 10 16 4 4 5 9 10 16
    e) 50 49 81 64 85 49 50 64 81 85

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1. b)

1st	2nd
1	12
2	6
3	4
4	-
5	-

Teacher to check charts for questions below – to save space, answers are given as ordered pairs.

c) (1, 15), (3, 5)
d) (1, 14), (2, 7)
e) (1, 25), (5, 5)
f) (1, 5)
g) (1, 26), (2, 13)
h) (1, 30), (2, 15), (3, 10), (5, 6)
i) (1, 42), (2, 21), (3, 14), (6, 7)
j) (1, 72), (2, 36), (3, 24), (4, 18), (6, 12), (8, 9)
k) (1, 63), (3, 21), (7, 9)
l) (1, 100), (2, 50), (4, 25), (5, 20), (10, 10)
m) (1, 64), (2, 32), (4, 16), (8, 8)
n) (1, 91), (7, 13)

2. **6:** 1 2 3 6

8: 1 2 3 4 8

12: 1 2 3 4 6 12

3. **3:** 1 3

4: 1 2 4

5: 1 5

6: 1 2 3 6

7: 1 7

8: 1 2 4 8

9: 1 3 9

10: 1 2 5 10

11: 1 11

12: 1 2 3 4 6 12

13: 1 13

14: 1 2 7 14

15: 1 3 5 15

16: 1 2 4 8 16

17: 1 17

18: 1 2 3 6 9 18

19: 1 19

20: 1 2 4 5 10 20

4. a) 1, 4, 9, and 16
   b) *Gaps:* +3, +5, +7, +9, +11
   Sequence: 1, 4, 9, 16, 25, 36
   perfect squares
   c) They all have a factor that pairs with itself.
   d) All numbers with an odd number of factors are perfect squares.
   Yes, this is also true.

INVESTIGATION

A. It has an odd number of factors. They are: 1, the number itself, and its square root (which pairs with itself).

B.

$25 = 5^2$	1, 5, 25
$36 = 6^2$	1, 2, 3, 4, 6, 9, 12, 18, 36
$49 = 7^2$	1, 7, 49
$64 = 8^2$	1, 2, 4, 8, 16, 32, 64
$81 = 9^2$	1, 3, 9, 27, 81
$100 = 10^2$	1, 2, 4, 5, 10, 20, 25, 50, 100

C. 2^2, 3^2, 5^2, and 7^2

D. 2, 3, 5, and 7

E. The squares of prime numbers have exactly 3 factors.

F. Only the squares of prime numbers have exactly 3 factors.

G. $11^2 = 121$, $13^2 = 169$, $17^2 = 289$

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1. a) 3×3
   b) 7×7
   c) 0×0
   d) 5×5
2. b) 4
   c) 3
   d) 6
   e) 1
   f) 10
   g) 9
   h) 8
3. b) 30
   c) 5
   d) 5
   e) 15
   f) 1
   g) 3
   h) 10

BONUS 17

4. a) $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{49}$, $\sqrt{64}$

   b) $\sqrt{4}$, 5, $\sqrt{8^2}$, 3^2, $\sqrt{100}$, 4^2

5. b) ≠
   $\sqrt{25}$; 3 + 4
   5; 7

   c) ≠
   $\sqrt{144}$; 13 – 5
   12; 8

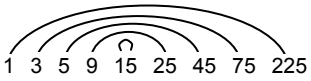
   d) =
   $\sqrt{25}$; 10 ÷ 2
   5; 5

6. a) 12

   b) 14

7. a) It's the factor that's paired with itself.

   b)



$\therefore \sqrt{225} = 15$

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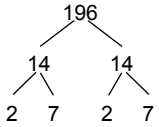
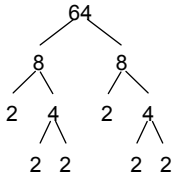
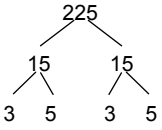
1. b)



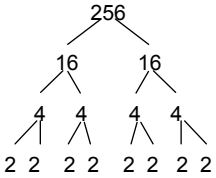
   c)



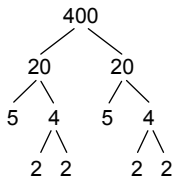
   d)



   e)



   f)



2. b) 2 × 7 × 2 × 7;
   2 × 7;
   2; 1

   c) 2 × 2 × 2 × 2 × 2 × 2;
   8 = 2 × 2 × 2;
   6; 3; 8

   d) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2;
   16 = 2 × 2 × 2 × 2;
   8; 4; 16

   e) 3 × 5 × 3 × 5;
   15 = 3 × 5;
   0; 0; 15

   f) 5 × 2 × 2 × 5 × 2 × 2;
   20 = 5 × 2 × 2;
   4; 2; 20

3. 6 times;

   The number of 2s that occur in a number's prime factorization (e.g. 3 times for 56) will double in the number's square (e.g. 6 times for 56^2).

4. No;

   The number of 2s in a perfect square's prime factorization will always be even (a multiple of 2) since it will be double the number of 2s in the prime factorization of the number's square root.

INVESTIGATION

A. 2 × 3 × 3 × 2 × 3 × 3;
   four times

B. 2 × 5 × 5 × 5 × 2 × 5 × 5 × 5;
   six times

C a) ii) 2 × 2 × 7 × 2 × 2 × 7 = 784;
   $\sqrt{784} = 28$

   iii) 48 = 2 × 2 × 2 × 2 × 3;
   2 × 2 × 2 × 2 × 3 × 2 × 2 × 2 × 2 × 2 × 3 = 2 304;
   $\sqrt{2\,304} = 48$

   iv) 35 = 5 × 7;
   5 × 7 × 5 × 5 × 7 = 1 225;
   $\sqrt{1\,225} = 35$

   v) 91 = 7 × 13;
   7 × 13 × 7 × 13 = 8 281;
   $\sqrt{8\,281} = 91$

   vi) 27 = 3 × 3 × 3;
   3 × 3 × 3 × 3 × 3 × 3 × 3 = 729;
   $\sqrt{729} = 27$

   vii) 63 = 3 × 3 × 7;
   3 × 3 × 7 × 3 × 3 × 7 = 3 969;
   $\sqrt{3\,969} = 63$

   b) The square root of a number squared is the original number.

D. a) ii) 2 × 3 × 3 = 18;
   $18^2 = 324$

   iii) 5 625 = 3 × 3 × 5 × 5 × 5 × 5;
   3 × 5 × 5 × 5 = 75;
   $75^2 = 5\,625$

   iv) 576 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3;
   2 × 2 × 2 × 3 = 24;
   $24^2 = 576$

   v) 1 936 = 2 × 2 × 2 × 2 × 11 × 11;
   2 × 2 × 11 = 44;
   $44^2 = 1\,936$

   vi) 11 025 = 3 × 3 × 5 × 5 × 7 × 7;
   3 × 5 × 7 = 105;
   $105^2 = 11\,025$

   vii) 27 225 = 3 × 3 × 5 × 5 × 11 × 11;
   3 × 5 × 11 = 165;
   $165^2 = 27\,225$

   b) When the prime factorization of a number is "halved", the result is the original number's square root.

E. If a number has an even occurance of all its prime factors, these factors can be "halved" and the result is a ***whole*** number. This is the square root of the original number, which means the original number was a perfect square.

5. a) $6\,300 = 2^2 \times 3^2 \times 5^2 \times 7$;
   No

   b) $6\,400 = 2^8 \times 5^2$;
   Yes

   c) $2\,268 = 2^2 \times 3^4 \times 7$;
   No

   d) $243 = 3^5$;
   No

   e) $729 = 3^6$;
   Yes

   f) $1\,296 = 2^4 \times 3^4$;
   Yes

6. a) *Gaps:*
   +16, +32, +64, +128, +256, +512

   Sequence:
   128, 256, 512, 1 024

   b) $2 = 2^1$ $64 = 2^6$
   $4 = 2^2$ $128 = 2^7$
   $8 = 2^3$ $256 = 2^8$
   $16 = 2^4$ $512 = 2^9$
   $32 = 2^5$ $1\,024 = 2^{10}$

   c) *Circle:*
   4, 16, 64, 256, 1 024

   d) Yes, every even-numbered term is a perfect square:
   100^{th} term = 2^{100}
   and $\sqrt{2^{100}} = 2^{50}$.

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1. a) 1.96

   b) 0.64

   c) 6.25

2. a)
   $4^2 = 16 < 19 < 25 = 5^2$, so:
   $4 = \sqrt{16} < \sqrt{19} < \sqrt{25} = 5$

   b) 20.25; less

In c) to i), answers may vary – teacher to check.

Sample:

c) $\sqrt{19} \approx 4.5$

d) $4.5 \times 4.5 = 20.25$

e) too high

f) $4.4 \times 4.4 = 19.36$

g) closer

h) $4.3 \times 4.3 = 18.49$ which is further away from 19 than 19.36 is (0.51 vs 0.36)

i) $\sqrt{19} \approx 4.4$

3. $\sqrt{19} \approx 4.36 \approx 4.4$

Teacher to check if this agrees with 2 i), above.

4. a) between 4^2 and 5^2

b) between 3^2 and 4^2

For c) to i), teacher to check grids.

c) between 6^2 and 7^2

d) between 4^2 and 5^2

e) between 5^2 and 6^2

f) between 7^2 and 8^2

g) between 8^2 and 9^2

h) between 8^2 and 9^2

i) between 6^2 and 7^2

5. b) 9, 16; 3^2, 4^2; 3, 4

c) 81, 100; 9^2, 10^2; 9, 10

d) 49, 64; 7^2, 8^2; 7, 8

e) 36, 49; 6^2, 7^2; 6, 7

f) 81, 100; 9^2, 10^2; 9, 10

g) 9, 16; 3^2, 4^2; 3, 4

h) 49, 64; 7^2, 8^2; 7, 8

i) 4, 9; 2^2, 3^2; 2, 3

j) 64, 81; 8^2, 9^2; 8, 9

6. Teacher to check.

7. a) 3.5

b) 4.7

c) 3.9

d) 5.5

8. Answers will vary – teacher to check.

9. a) 40

b) Yes, the window of his calculator would need to cut the answer off after a certain number of decimal places.

Plus, the square of the answer needs to be a whole number.

10. b) 25, 5^2; 5.20, 5

c) 25, 5^2; 4.58, 5

d) 49, 7^2; 6.63, 7

11. *n*

12. a) 5

b) 6

c) 4

d) 9

e) 8

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1. a) $\frac{3}{7}$; $\frac{3}{7}$; $3\frac{3}{7}$, 3, 7, 3.4

b) $\approx 3\frac{6}{7} \approx 3.9$

c) $\approx 3\frac{1}{7} \approx 3.1$

d) $\approx 3\frac{5}{7} \approx 3.7$

e) $\approx 3\frac{4}{7} \approx 3.6$

2. Teacher to check student's rough work.

a) $2\frac{1}{5}$

b) $2\frac{2}{5}$

c) $2\frac{3}{5}$

d) $2\frac{4}{5}$

3. Teacher to check if estimates were correct.

a) 2.2

b) 2.4

c) 2.6

d) 2.8

4. Teacher to check number lines.

a) $5.64 \approx 5.6$

b) $7.07 \approx 7.1$

c) $9.21 \approx 9.2$

5. a) $5.66 \approx 5.7$

b) $7.07 \approx 7.1$

c) $9.22 \approx 9.2$

6. Teacher to check.

7. Teacher to check.