Measurement – AP Book 8, Part 2: Unit 6

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1. a) Teacher to check shading of base.
   i) hexagonal
   ii) pentagonal
   iii) triangular
   b) rectangles
   c) i) 6
   ii) 5
   iii) 3
2. a) △ = 2
   ▭ = 3
   b) ▭ = 2
   ▭ = 4
   c) ▭ = 5
   ⬠ = 2
3. a) Right prisms: *A,* C, E
   Not prisms: B, D
   b) B isn't because it only has one base (it's a pyramid);
   D isn't because all its sides are pentagons.
4. Teacher to check.
5. Teacher to check.
6. Teacher to check.
7. C; A; B
   Teacher to check that prism dimensions are marked correctly.
8. a) Answers may vary – teacher to check.
   Sample answers:
   200 = 1 × 1 × 200
   200 = 2 × 10 × 10
   200 = 4 × 5 × 10
   b) Teacher to check.

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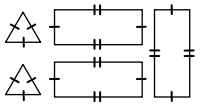
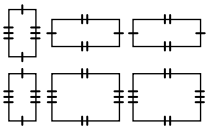
1. a) Rectangular (square) prism
   b)



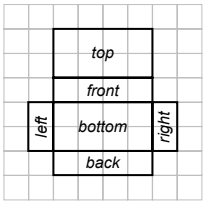
   triangular prism
   c)



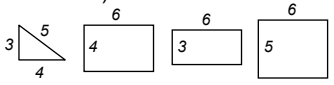
   rectangular prism
2. Teacher to check that bases are shaded correctly.

# base sides	3	4	5	6
# non-bases	3	4	5	6

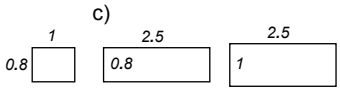
   They are equal.
3. b) Position of faces may vary – teacher to check.



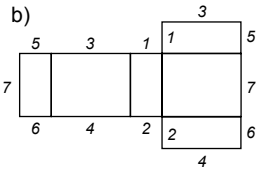
4. All measurements below are in metres.
   b)



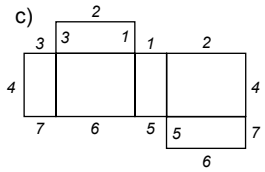
   number of copies:
   2 **1** **1** **1**
   c)



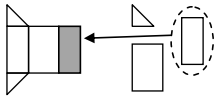
   number of copies:
   2 **2** **2**
5. a) *Circle:*
   the first net only
   b) *Circle:*
   the middle net only
   c) In a): the 2nd net has 6 side faces; the 3rd net has side faces that are attached to the incorrect edges of the base faces.
   For b): in the 1st net, the middle side face isn't wide enough – it should match the hypotenuse of the base triangle; in the 3rd net, the bases don't face the same direction.
6. b)



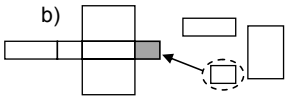
   c)



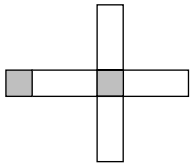
7. Position of added face may vary – teacher to check.
   a)



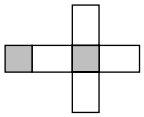
   b)



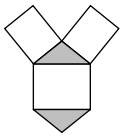
8. Nets may vary slightly – teacher to check.
   NOTE: Bases have been shaded here.
   a) rectangular prism:



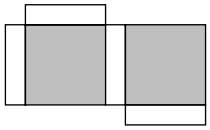
   b) rectangular prism:



   c) triangular prism:



   d) rectangular prism:



9. E, A, D, B, C

BONUS B, A

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1. a) 20 16
   b) 20 16
   c) 3 3
   d) 60 48
2. a) $\ell \times w$
   b) h
   c) $\ell \times w \times h$
   d) V = length × width × height
3. a) 2; 2; 2; 8 blocks
   b) 2; 3; 2; 12 m^3
   c) 2; 4; 2; 16 cm^3
   d) 2; 3; 5; 30 mm^3

INVESTIGATION

A. ii) 6 cm^2
   iii) 15 cm^2
   iv) 12 cm^2

B. ii) 6 cm^3
   iii) 15 cm^3
   iv) 12 cm^3

C. The numbers are the same but the units change from cm^2 to cm^3.

D. i) 2
   ii) 1
   iii) 2
   iv) 4

E. i) 2 cm
   ii) 1 cm
   iii) 2 cm
   iv) 4 cm

F. The numbers are the same but the units change from layers to cm.

G. i) 24 cm^3
   ii) 6 cm^3
   iii) 30 cm^3
   iv) 48 cm^3

H. height

4. 15; 4; 15 × 4 = 60
5. 12 × 2 = 24; 12 × 2 = 24
6. a) 15; 15;
   The numbers are the same, but the units aren't (cm^2 vs cm^3).

b) 8; 8;
The numbers are the same but the units aren't (layers vs cm).

c) Answers may vary – teacher to check.

Sample explanation:
Put simply, we know the number of blocks is the same so the volume must also be the same.

Put more formally, the "numbers" are the same in both equations – just the units change. The resulting *product* of the units, however (cm^3), is the same.

7. a) height
b) length
c) height
d) width
e) length
f) width

8. a) $10 \times 7 = 70$
b) width; $35 \times 2 = 70$
c) length; $14 \times 5 = 70$

Yes, you get the same answer.

9. a) 70 cm^3
b) 40 cm^3
c) 40 cm^3
d) 60 cm^3

10. a) – d) Teacher to check.
e) The prisms and their dimensions/volume don't change – they have simply been rotated in space.

11. a) i) 20; 3; 60
ii) 12; 5; 60
iii) 15; 4; 60
b) All 3 prisms have the same volume; Their dimensions are all the same – just their "direction" is different.

12. A and B have the same volumes;
All of their dimensions are the same – they've just been rotated. In C, there is a 3 cm edge where the others have a 2 cm edge.

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INVESTIGATION 1

A.

ii)	iii)
11	8
6	3
66	24
11	8
6	3

B. Yes

INVESTIGATION 2

A. a) one half
b) one half
c) one half

Sample explanation:
A diagonal cuts a rectangle into two equal (but inverted) triangles.

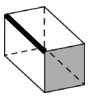
B. a) one half
b) one half
c) one half

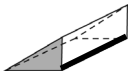
Sample explanation:
From A, we know that the base area of each triangular prism is half of its "cut" rectangular prism. Since their heights are the same, the volume is simply being halved.

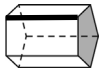
C. 2

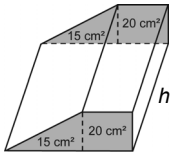
D. height;
triangle; height;
base; height

1. Any edge that is parallel to those marked below is also correct.

a) 

b) 

c) 

2. a) 

b) You can think of this prism as two separate prisms put together: a triangular prism (at left) and a rectangular prism.
Volume of TP
= area of base × h
= $15 \times h$
Volume of RP
= area of base × h
= $20 \times h$

c) Yes;
Can see this by the distributive property or by substitution:
$h = 1 \rightarrow V = 35$ cm^3
$h = 2 \rightarrow V = 70$ cm^3
$h = 3 \rightarrow V = 105$ cm^3

d) Volume of prism
$= (15 \times h) + (20 \times h)$
$= (15 + 20) \times h$
$= 35 \times h$
But 35 is the area of the prism's base, so: volume = area of base × height.

3. a) Base area: 15 cm^2
Volume: 75 cm^3
b) Base area: 36 cm^2
Volume: 108 cm^3

4. Estimates will vary – teacher to check.
a) Base area: 42.5 cm^2
Volume: 425 cm^3
b) Base area: 33.6 cm^2
Volume: 336 cm^3
c) Base area: 112.53 cm^2
Volume: 1125.3 cm^3

5. Answers will vary – teacher to check.

6. V = base area × h
$\therefore$ base area $= V \div h$
$= 600 \div 15$
$= 40$ cm^2

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1. a) 24 cm^3
b) 30 cm^3
c) 28 cm^3
d) 18 cm^3

INVESTIGATION

A. Volume = base × height;
A cylinder is a prism too, just with a round base.

B. He did this to account for the thickness of the can;
5 cm;
Again, to account for the thickness of the can.

C.

r^2	$h \times r^2$	$\frac{V}{hr^2}$
23.04	253.44	≈ 3.141
16.81	218.53	≈ 3.112
16	172.8	≈ 3.125
6.25	50	≈ 3.12

D. The last column: $\frac{V}{hr^2}$

E. hr^2, hr^2

F. πr^2

G. height

H. They both multiply the area of the base by the height. This makes sense since a cylinder is like a prism with a round base.

I. Answers will vary, depending on prediction.

2. a) 36 cm^3
b) 120 cm^3
c) 120 cm^3

3. a) base = 64π cm^2
≈ 201 cm^2
height = 10 cm
volume ≈ 2010 cm^3

b) base = 49π cm^2
≈ 154 cm^2
height = 20 cm
volume ≈ 3 080 cm^3

4. a) 30 × 20 = 600
b) 30 × 20 = 600
Yes, they are the same; Regardless of shape, both jars have same base area and height – and, as such, the same area.
∴ they will hold the same number of candies.

5. a) NOTE: Student measurements may vary a bit. Teacher to check.
Sample answers:
Diameter = 19 mm;
Radius = 9.5 mm;
Height of 1 coin = 1.5 mm;
Height of 10 coins = 14.5 mm.
The height found by stacking 10 coins and dividing will be more accurate;
It's hard to measure anything involving a part of a mm – it's too small to see clearly. The 10-coin method allows these "parts" to accumulate enough to be measurable.
$V = \pi r^2 h$
$= \pi(9.5)^2\left(\frac{14.5}{10}\right)$
$= \pi(9.5)^2(1.45)$
≈ 410.91 mm^3
≈ 0.411 cm^3
b) The volume of 10 pennies is about 4.11 cm^3 or 4.11 mL.
∴ the water level should rise to just above 34 mL.

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1. a) 1000 mL
b) 1000 cm^3
2. 1890 cm^3
3. 2L = 2000 cm^3
∴ the area of the base = 2000 ÷ 25 = 80 cm^2.
4. 250 mL = 250 cm^3
For the bottom section of the carton:
$V = l \times w \times h$
so: $250 = 49h$
$h \approx 5.1$ cm
∴ total height ≈ 9.6 cm
5. a) $V = \pi r^2 h$
$= \pi(3.4)^2(12.2)$
≈ 442.84 cm^3
b) 8 cans used in total so ≈ 3.54 L of juice
6. a) Total base area of:
<u>2 circular pans</u>
$= 2 \times \pi r^2$
$= 2 \times \pi(4.5)^2$
≈ 127.17 inch^2
<u>1 rectangular pan</u>
= 13 × 9
= 117 inch^2
For the same volume, the one with the lower base area needs more height.
∴ the cake mix will be higher in the rectangular pan.
b) rectangular pan
7. Volume of mixture = 11.25 cups = 2700 mL
Base area of the pan = 20 × 30 = 600 cm^2
∴ the mixture will be 4.5 cm high in the pan.
8. a) Tegan cannot carry the full aquarium: the water alone (i.e. not including the aquarium itself) weighs 162 kg.
b) She will need to make 27 trips.
c) Answers will vary – teacher to check.

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1. a) 10 mm
b) 100 mm^2
c) 1000 mm^3
d) 100, 100; 10 000
e) 100, 100, 100; 1 000 000
f) 1000, 1000, 1000; 1 000 000 000
2. b) 1000, smaller; 1000, more; multiply, 1000; 460 000 mm^3
c) 1000^2, bigger; 1 000 000, fewer; divide, 1 000 000; 0.015 km^2
d) 10^3, bigger; 1000, fewer; divide, 1000; 0.46 cm^3
e) 1000^2, bigger; 1 000 000, fewer; divide, 1 000 000; 0.000 004 m^2
f) 100^3, smaller; 1 000 000, more; multiply, 1 000 000; 400 000 cm^3
g) 100^2, smaller; 10 000, more; multiply, 10 000; 52 000 cm^2
h) 100^3, smaller; 1 000 000, more; multiply, 1 000 000; 10 000 cm^3
i) 1000^2, bigger; 1 000 000, fewer; divide, 1 000 000; 0.000 000 01 km^2
j) 100^3, bigger; 1 000 000, fewer; divide, 1 000 000; 0.0024 m^3
3. He forgot to convert to the same units first:
<u>Incorrect</u> ×
4 × 80 × 50 = 16 000 cm^3
<u>Correct</u> ✓
4 × 0.8 × 0.5 = 1.6 m^3
or 400 × 80 × 50
= 16 000 000 cm^3
4. a) 0.16 m = 16 cm
So: $16 = 2\pi r$
$r \approx 2.55$ cm
∴ Area = πr^2
≈ 20.4 cm^2
b) No;
There are lots of dimensions giving this perimeter, each with a different area.
c) Yes;
Here, the base must be 0.04 m × 0.04 m (4 cm × 4 cm) so the area is 0.16 m^2 or 16 cm^2.
5. 0.15 m^2
= (10 000 × 0.15) cm^2
= 1500 cm^2
It would cost about $150 (1500 × $0.10).
This is an approximation since there will be a little space between the tiles, some will break, etc.
6. a) $V = \pi r^2 h$
$= \pi(4)^2(1.2)$
≈ 60.29 m^3
Since 1 m^3 = 1000 L, this is 60 290 L.
It's more reasonable to use m^3, since the volume is so large.
b) ≈ $452.18

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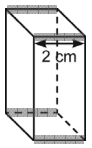
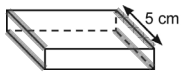
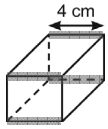
1. b) 

c) 

d) 

2. a) 1 cm
b) 3 cm
c) 5 cm

3. Teacher to check that students have shaded the face *opposite* to one that is marked.

4. a) back: 6 cm^2
   bottom: 12 cm^2
   left: 8 cm^2
   b) back: 15 cm^2
   bottom: 6 cm^2
   right: 10 cm^2
   c) back: 12 m^2
   bottom: 18 m^2
   left: 6 m^2

5. top + bottom
   = 15 cm^2 × 2 = 30 cm^2
   right + left
   = 10 cm^2 × 2 = 20 cm^2

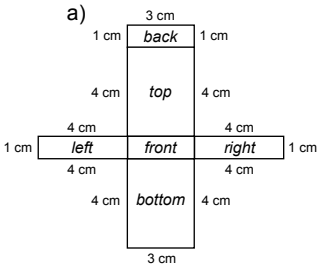
6. a) 22 cm^2
   b) 52 m^2
   c) 76 mm^2

7. Miki forgot to include the "invisible" sides in her calculation. She needs to multiply her answer by two: 80 cm^2.

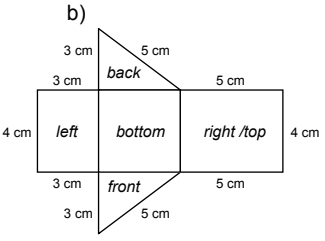
8. a) She is correct; the front/back face areas = the right/left face areas
   b) 2(8 × 8) + 4(5 × 8) = 288 cm^2

9. Exact assignment of face names may vary – teacher to check.

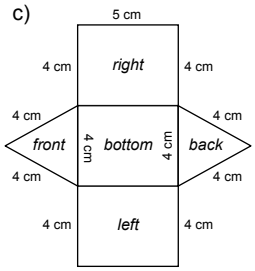
   a)



   b)



   c)



10. a) 38 cm^2
    b) 60 m^2
    c) ≈ 73.86 m^2
    NOTE:
    The triangle height
    = $\sqrt{12}$ ≈ 3.46 m.
    The surface area of a prism *is equal to* the area of its net;
    The net is like a "flattened" version of the prism: it is made from polygons that are exactly the same size/shape as the different faces of the prism.

11. a) 4 m
    b) 3 m
    c) 7 m

12. a) 5 m
    b) 5 m
    c) 4 m

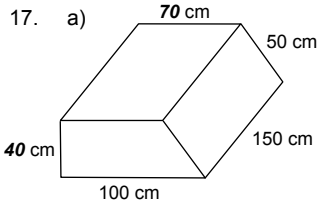
13. There is only one length combination that works:
    a = 6 m
    b = 3 m
    c = 2 m
    Although 4 is a factor of 12, no edge can be 4 m long. For this to be true, 4 would also need to be a factor of either 6 or 18. This only happens with 2, 3 and 6.

14. Surface area
    = $2\ell w + 2\ell h + 2wh$
    = $2(\ell w + \ell h + wh)$

15. In cm: 22 000 cm^2
    In m: 2.2 m^2

16. NOTE:
    Do not include the floor or ceiling, just the walls.
    Surface area, including door/windows = 66 m^2
    Door = 2 m^2
    Windows = 6 m^2
    ∴ area to paint = 58 m^2, which would cost $23.20.

17. a)



    b) Volume
    = 510 000 cm^3
    = 0.51 m^3
    Surface area
    = 45 800 cm^2
    = 4.58 m^2

18. a) SA = $(4n + 2)$ cm^2
    b) 82 cm^2

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1. a) rectangle
   b) parallelogram
   c) i) base = 15 cm
      height = 12 cm
      ii) base = 15 cm
      height = 12 cm
   d) Yes;
   The two tubes are identical (∴ their areas are the same) – they are just cut differently.
   e) $b \times h$ = 180 cm^2

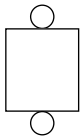
2. a) 21 cm^2
   b) ≈ 125.6 cm^2
   c) ≈ 621.72 cm^2

3. a) SA = $C \times h$
   b) SA = $\pi d \times h$
   c) SA = $2\pi r \times h$

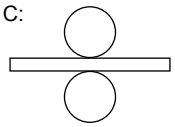
4. A, B, C, E;
   The four correct nets are made from rectangles and/or parallelograms.
   The two others aren't (both include a trapezoid).

5. a) 244.4 *rectangle*
   9π ≈ 28.26 *top circle*
   + ≈ 28.26 *bottom circle*
   ≈ 300.92 cm^2
   b) Net 2;
   The dimensions are correct in both cases but in Net 1, the circumference and height have been reversed.

6. a) B, D, E
   b) A:



   C:



   c) The surface area of a cylinder *is equal to* the area of its net;
   The circles in the net are the same size as the two bases of the cylinder, and the rectangle of the net – when "glued" – creates the curved face of the cylinder.

7. a) ≈ 157.0 *rectangle*
   25π ≈ 78.5 *top circle*
   + ≈ 78.5 *bottom circle*
   ≈ 314.0 cm^2
   b) ≈ 56.52 *rectangle*
   $\pi(4.5)^2$ ≈ 63.59 *top circle*
   + ≈ 63.59 *bottom circle*
   ≈ 183.70 cm^2
   c) ≈ 113.04 *rectangle*
   4π ≈ 12.56 *top circle*
   + ≈ 12.56 *bottom circle*
   ≈ 138.16 cm^2

8. Surface area of the can
   = $2\pi r^2 + 2\pi rh$
   = $2\pi r(r + h)$

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1. Answers will vary – teacher to check.

   Sample answers:

   a) 2 cm × 2 cm × 3 cm

   b) 1 cm × 2 cm × 4 cm

   c) 2 cm × 3 cm × 3 cm

2. Answers will vary – teacher to check.

   Sample answer:

   - 2 cm × 12 cm × 1 cm; surface area = 76 cm^2
   - 2 cm × 2 cm × 6 cm; surface area = 56 cm^2
   - 1 cm × 3 cm × 8 cm; surface area = 70 cm^2

   Of these, the 2nd uses the least amount of material.

   NOTE:
   The closer the prism is to a cube, the smaller its surface area will be.

3. a) 42 cm^2

   b) 18 cm^3

4. Volume = 240 m^3
   Surface area = 280 m^2

   Strategies will vary – teacher to check.

5. b) Explanations will vary – teacher to check.

   Sample explanation:
   Given a circle and square of the same area, the circle's circumference will be less than the square's perimeter. This means that the area of the cylinder's curved face will be smaller than the sum of the side face areas in the prism.
   ∴ the cylinder will require less material since its overall surface area is less.

   c) The containers both have the same volume since their heights and base areas are equal: 2000 cm^3

6. a) The volumes of the two containers are equal. From the first container, we know $V = 98\pi$ cm.
   ∴ h = 3.92 cm

   b) SA #1 ≈ 252.77 cm^2
   SA #2 ≈ 280.09 cm^2

   c) Cost #1
   ≈ $0.20 or 20¢
   Cost #2
   ≈ $0.22 or 22¢
   ∴ the first can is cheaper to make.

7. a) Area L ≈ 7.065 cm^2

   b) Area S ≈ 7.065 cm^2

   c) They hold the same volume: their height and base area are both equal.

8. She can multiply 20 cm^2 by 2 to get 40 cm^2 since the surface area of the front = back, top = bottom and right = left.

9. Capacity
   = 10 000 mL or 10 L
   Volume
   = 10 000 cm^3 or 0.01 m^3
   Surface area
   = 4 200 cm^2 or 0.42 m^2

10. a) <u>Cylinder A:</u>
    V ≈ 508.68 cm^3
    SA ≈ 621.72 cm^2

    <u>Cylinder B:</u>
    V ≈ 628 cm^3
    SA ≈ 408.2 cm^2

    b) B

    c) A

11. a) SA = 160 cm^2
    V = 100 cm^3

    b) A cube that is 5 cm × 5 cm × 5 cm:
    SA = 150 cm^2 (<160)
    V = 125 cm^3 (>100)

12. b) Volume of rect prism = 300 cm^3
    Volume of cylinder ≈ 282.6 cm^3

    c) Satya should pour the water from the full cylinder into the empty prism.
    *If the prism overflows, the cylinder is larger.
    *If there is still space in the prism, it is larger.

13. a) Teacher to check.

    b) <u>Tube A:</u>
    We first calculate the radius of the base ≈ 3.50 cm
    V ≈ 1077.02 cm^3
    SA ≈ 692.37 cm^2

    <u>Tube B:</u>
    We first calculate the radius of the base ≈ 4.46 cm
    V ≈ 1374.11 cm^3
    SA ≈ 741.11 cm^2

    c) B

    d) B;
    to solve, use SA ÷ V

14. $42\pi \approx 131.88$ cm^2, when radius = 3 cm and height = 4 cm