Introduction

1. Key Components of the JUMP Math Resources

Assessment & Practice Books Parts 1 and 2, Grades K–8

- Solid foundation for each of the strands in the curriculum at grade level
- Extensive review going back up to two grades
- All strands complete the curriculum at grade level

Teacher Resources, Grades K–8

Overview of JUMP Math
Mental Math Unit
Detailed Table of Contents (Parts 1 and 2)
Lesson plans provide clear explanations and explicit guidance on how to:

- introduce one concept at a time;
- explore concepts and make connections in a variety of ways;
- assess students quickly;
- extend learning with extra bonus questions and activities;
- develop problem solving skills; and
- support material for each strand.

Blackline Masters (extra practice, games, manipulatives)
Advanced Problem-Solving Lessons
Answer Keys (for Assessment & Practice Books and Unit Tests), Grades 3–8
Unit Tests, Grades 3–8
Curriculum Correlations (WNCP, ON)
SMART Board-compatible interactive whiteboard lessons, Grades 1–8 (available separately)

2. The Myth of Ability

There is a prevalent myth in our society that some people are born with mathematical talent—and others simply do not have the ability to succeed.

Recent discoveries in cognitive science are challenging this myth of ability. The brain is not hard-wired; it continues to change and develop throughout life. Steady, incremental learning can result in the emergence of new abilities. The brain, even when damaged, is able to rewire itself and learn new functions through rigorous instruction. As Philip E. Ross points out in his 2006 *Scientific American* article "The Expert Mind," this fact has profound implications for education:

> The preponderance of psychological evidence indicates that experts are made not born. What is more, the demonstrated ability to turn a child quickly into an expert—in chess, music, and a host of other subjects—sets a clear challenge before the schools. Can educators find ways to encourage the kind of effortful study that will improve their reading and math skills? Instead of perpetually pondering the question, "Why can't Johnny read?" perhaps educators should ask, "Why should there be anything in the world that he can't learn to do?"[1]

JUMP Math builds on the belief that every child can be successful at mathematics by

- promoting positive learning environments and building confidence through praise and encouragement;
- maintaining a balanced approach to mathematics by concurrently addressing conceptual and procedural learning, explicit and inquiry-based learning;

- achieving understanding and mastery by breaking mathematics down into sequential, scaffolded steps, while still allowing students to make discoveries;
- keeping all students engaged and attentive by "raising the bar" incrementally;
- guiding students strategically to explore and discover the beauty of mathematics as a symbolic language connected to the real world; and,
- using continuous assessment to ensure all students are engaged and none are left behind.

JUMP Math is an approach to teaching mathematics that has been developed by John Mighton and a team of mathematicians and educators who are dedicated to excellence in mathematics teaching and learning.

3. About JUMP Math

> Nine years ago I was looking for a way to give something back to my local community. It occurred to me that I should try to help kids who needed help with math. Mathematicians don't always make the best teachers because mathematics has become obvious to them; they can have trouble seeing why their students are having trouble. But because I had struggled with math myself, I wasn't inclined to blame my students if they couldn't move forward.
> — John Mighton[2]

John Mighton is a mathematician, bestselling author, award-winning playwright, and the founder of JUMP Math, a national charity dedicated to improving mathematical literacy.

JUMP Math grew out of John's work with a core group of volunteers in a "tutoring club" held in his apartment to meet the needs of the most challenged students from local schools. Over three years, John developed the early material—simple handouts for the tutors to use. This period was one of experimentation in developing the JUMP Math method through countless hours of one-on-one tutoring. Eventually, John began to work in local inner-city schools, placing tutors in classrooms. This led to the next period of innovation—working through the JUMP Math method in classrooms.

Teachers responded enthusiastically to their students' success and wanted to adapt the method for classroom use. John and a group of volunteers and teachers developed workbooks to meet teachers' needs for curriculum-based resources. These started out as a series of three remedial books with limited accompanying teacher materials, released in the fall of 2003. The effectiveness of these workbooks led quickly to the development of grade-specific, curriculum-based Assessment & Practice Books and teacher resources, first released in 2004.

John documented his experience in two national bestselling books, *The Myth of Ability* (2003) and *The End of Ignorance* (2007). As a playwright, he has won several national awards, including the Governor General's Literary Award for Drama, the Dora Award, the Chalmers Award, and the Siminovitch Prize. John was granted a prestigious Ashoka Fellowship as a social entrepreneur for his work in fostering mathematical literacy by building students' self-confidence and competence through JUMP Math. In 2010 John was appointed an Officer of the Order of Canada.

In only ten years, JUMP Math has grown from a gathering around John's kitchen table to a thriving organization reaching more than 50,000 students with high-quality learning resources and professional development for 3,000 teachers. JUMP Math is working in hundreds of schools across Canada and internationally, and has established a network of dedicated teachers who are mentoring and assisting teachers new to the program. As well, JUMP Math supports community organizations in reaching struggling students through homework clubs and after-school programs. Through the generous support of our sponsors, JUMP Math donates learning resources to classrooms and

homework clubs across Canada. JUMP Math has inspired thousands of community volunteers and teachers to reach out to struggling students by donating their time as tutors and mentors.

4. JUMP Math Works

JUMP Math is a learning organization committed to evaluation and evidence-based practice. JUMP Math is a leader in encouraging and supporting third-party research to study the efficacy of the JUMP Math approach to mathematics teaching and learning.

Hospital for Sick Children 2008–2010

Cognitive scientists from The Hospital for Sick Children in Toronto conducted a randomized-controlled study of the effectiveness of the JUMP Math program. Studies of such scientific rigour remain relatively rare in mathematics education research in North America. The results showed that, on well-established measures of math achievement, students who received JUMP instruction outperformed students who received the methods of instruction their teachers would normally use.

Ontario Institute for Studies in Education (OISE) 2007–2008

Researchers from OISE at the University of Toronto, led by Dr. Joan Moss, completed a one-year study on the efficacy of JUMP Math. Preliminary data indicate that:

- JUMP Math's Grade 5 resources for the curriculum in multiplication provide a greater variety of representations of concepts and more practice than provincially recommended programs; and
- JUMP Math significantly improves the conceptual understanding of math for struggling students.

Vancouver School Board 2006–2007

Over the school year, 68 JUMP Math teachers were surveyed. The teachers indicated that:

- the JUMP Math methodology enhances student retention and transfer, promotes independent thinking, and creates excitement and curiosity; and
- JUMP Math develops teacher confidence and self-efficacy.

Borough of Lambeth (London, UK) 2006–2009

During the summer of 2006, 24 public schools in Lambeth participated in a pilot study on JUMP Math. As a result of the pilot's impact on student behaviour, confidence, and achievement—as well as teachers' strong reaction to JUMP Math as an effective teaching tool—a total of 35 local schools adopted the program for the 2006–2007 school year. In this second implementation, 69% of the students who initially performed two years below age-related expectations moved up multiple levels after using JUMP Math, and either reached or surpassed their desired level by the end of the school year. In 2008–2009, it was shown that 57% of students who were initially at grade level progressed three years ahead of national expectations for their age.

JUMP Math Evaluation Pilot 2007–2008

After using JUMP Math as their exclusive math program during a 5-month pilot study, a class of Grade 3 students in British Columbia showed:

- an increase in math achievement equivalent to 9 months of instruction (80% more than the expected achievement in 5 months); and
- a statistically significant decrease in math anxiety and a statistically significant increase in positive attitude toward math.

In 2010, JUMP Math was recommended by the Canadian Language and Literacy Research Network and the Canadian Child Care Federation in a report entitled "Foundations for Numeracy: An Evidence-Based Toolkit for the Effective Mathematics Teacher."[3]

In 2009, JUMP Math was recognized by the UK Government in the document "What Works for Children with Mathematical Difficulties?"[4]

See the JUMP Math website, www.jumpmath.org, for more details and updates on research.

5. What Is the JUMP Math Approach?

JUMP Math is a balanced approach to teaching mathematics that supports differentiated instruction. JUMP Math covers the full curriculum for both Ontario and Western Canada through the Assessment & Practice Books, lesson plans, and a range of support materials.

The JUMP Math Assessment & Practice Books are not intended to be used without instruction. Teachers should use the Assessment & Practice Books and accompanying lesson plans in the Teacher Resources for dynamic lessons in which students are allowed to discover and explore ideas on their own. The careful scaffolding of the mathematics in the Assessment & Practice Books make them an excellent tool for teachers to use for guided practice and continuous assessment.

The JUMP Math approach to teaching mathematics emphasizes:

- Confidence-building
- Guided practice
- Guided discovery
- Continuous assessment
- Rigorously scaffolded instruction
- Mental math
- Deep conceptual understanding

Confidence-building

JUMP Math recognizes that math anxiety is a significant barrier to learning for many students. The JUMP approach has been shown to reduce math anxiety by building on success in small steps. Raising the bar incrementally—a key component of the JUMP Math materials—encourages engagement and confidence. The research in cognition that shows the brain can be altered by education also shows that the brain can't register the effects of education if it is not attentive. However, a child's brain can't be truly attentive unless the child is confident and excited and believes that there is a point in being engaged in the work. When students who are struggling become convinced that they cannot keep up with the rest of the class, their brains begin to work less efficiently, as they are never attentive enough to consolidate new skills or develop new neural pathways. That is why it is so important to give students the skills they need to take part in lessons and to give them opportunities to show off by answering questions in front of their classmates. To do this, try to constantly assess what struggling students know.

Guided practice

Research in psychology has shown that our brains are extremely fallible: our working memories are poor, we are easily overwhelmed by too much new information, and we require a good deal of practice to consolidate skills and concepts. Repetition and practice are essential. Even mathematicians need constant practice to consolidate and remember skills and concepts. New research in cognition shows how important it is to practise and build component skills before students can understand the big picture. The Assessment & Practice Books are designed to provide guided practice when used with the lesson plans. The multiple representations of

mathematics in JUMP Math combine with guided practice in small steps to promote mastery and understanding of key concepts.

Guided discovery

In "Students Need Challenge, Not Easy Success,"[5] Margaret Clifford makes the case that students benefit when they are given prompt, specific feedback on their work and when they are allowed to take "moderate risks." JUMP Math provides moderate risks by providing tasks that are within students' grasp. As students' confidence grows, their risk tolerance grows as well, and they are ready to take more and more steps by themselves.

The lesson plans in the JUMP Math Teacher Resources show you how to build lessons around the material in the Assessment & Practice Books by creating similar tasks and questions. As much as possible, when students are ready, allow them to think about and work on these questions and tasks independently rather than teaching them explicitly. When you feel your students have sufficient confidence and the necessary basic skills, let them explore more challenging or open-ended questions.

Students are more likely to become flexible and independent thinkers in math if you guide discovery through well-designed lessons. Hence, in creating discovery-based lessons it is important to balance independent work with practice and explicit hints and instruction. According to Philip E. Ross,[6] research in cognition shows that to become an expert in a game like chess it is not enough to play without guidance or instruction. The kind of training in which chess experts engage, which includes playing small sets of moves over and over, memorizing positions, and studying the techniques of master players, appears to play a greater role in the development of ability than the actual playing of the game.

Continuous assessment

The Assessment & Practice Books are designed to allow for continuous formative assessment; the books show teachers how to break material into steps and assess component skills and concepts in every area of the curriculum.

The point of constantly assigning tasks and quizzes is not to rank students or to encourage them to work harder by making them feel inadequate. Quizzes should instead be treated as opportunities for students to show off what they know, to become more engaged in their work by meeting incremental challenges, and to experience the collective excitement that can sweep through a class when students experience success together. Continuous assessment allows the teacher to differentiate instruction with small individual interventions.

Rigorously scaffolded instruction

Consistent with emerging brain research, JUMP Math provides materials and methods that minimize differences among students, allowing teachers to more effectively improve student performance in mathematics. In "Why Minimal Guidance During Instruction Does Not Work," Paul Kirschner, John Sweller, and Richard Clark argue that evidence from controlled studies almost uniformly supports direct, strong instructional guidance.[7] Even for students with considerable prior knowledge, strong guidance while learning helps take into account the limitations of a student's working memory: the mind can only retain so much of new information or so many component steps at one time.

Even in discovery-based lessons, in which there is little direct instruction, it is important to introduce new ideas through a series of well-designed tasks and explorations in which each new concept follows from the last; students are more likely to make discoveries if the progression of ideas makes sense to them and does not overwhelm them.

Mental math

Mental math is the foundation for all further study in mathematics. Students who cannot see number patterns often become frustrated and disillusioned with their work. Consistent practice in mental math allows students to become familiar with the way numbers interact, enabling them to make simple calculations quickly and effectively without always having to recall their number facts.

To solve problems, students must be able to see patterns in numbers and make estimates and predictions about numbers. It is a serious mistake to think that students who don't know number facts can get by in mathematics using a calculator or other aids. Students can certainly perform operations and produce numbers on a calculator, but unless they have number sense, they will not be able to tell if their answers are correct, nor be able to develop a talent for solving mathematical problems.

Deep conceptual understanding

JUMP Math scaffolds mathematical concepts rigorously and completely. JUMP Math materials were designed by a team of mathematicians and educators who have a deep understanding of, and a love for, mathematics. JUMP Math teaches symbolic and concrete understanding simultaneously, using a variety of approaches. JUMP Math materials offer multiple symbolic and concrete representations for all key mathematical concepts, and provide guided practice for mastery, allowing students to master and understand each representation completely before moving on.

JUMP Math shows teachers how to see the big ideas of mathematics in even the smallest steps, how to make sense of the individual steps in a mathematical procedure or problem, and how to relate them to the wider concept. JUMP Math teaches fundamental rules, algorithms, and procedures of mathematics for mastery, but students are enabled to discover those procedures themselves (as well as being encouraged to develop their own approaches) and are guided to understand the concepts underlying the procedures fully.

6. Building Confidence with the Introductory Unit

> In the twenty years that I have been teaching mathematics to children, I have never met an educator who would say that students who lack confidence in their intellectual or academic abilities are likely to do well in school. Our introductory unit has been carefully designed and tested with thousands of students to boost confidence. It has proven to be an extremely effective tool for convincing even the most challenged student that they can do well in mathematics.
> —John Mighton, in conversation

In recent years, research has shown that students are more likely to do well in subjects when they believe they are capable of doing well. It seems obvious, then, that any math program that aims to harness the potential of every student would start with an exercise that builds the confidence of every student. Getting Ready for JUMP Math: Introductory Unit Using Fractions, which can be downloaded from www.jumpmath.org, was designed for just this purpose. The Introductory Unit does not teach fractions in depth: you will find a more comprehensive approach to teaching fractions in the relevant JUMP Math Assessment & Practice Books and lesson plans. We recommend that teachers only use the unit for several weeks, preferably at the beginning of the school year.

The individual steps that teachers will follow in teaching the unit are extremely small, so even students who struggle most needn't be left behind. Throughout the unit, students are expected to:

- discover or extend patterns or rules on their own,
- see what changes and what stays the same in sequences of mathematical expressions, and
- apply what they have learned to new situations.

Students become very excited at making these discoveries and meeting these challenges as they learn the material. For many, it is the first time they have ever been motivated to pay attention to mathematical rules and patterns or to try to extend their knowledge in new cases.

The Introductory Unit Using Fractions, which consists of student worksheets and a short teacher's manual, has been specifically designed to build confidence by:

- Requiring that students possess only a few simple skills — These skills can be taught to even the most challenged students in a very short amount of time. To achieve a perfect score on the final test in the unit, students need only possess three skills: they must be able to skip count on their fingers, add one-digit numbers, and subtract one-digit numbers.
- Eliminating heavy use of language — Mathematics functions as its own symbolic language. Since the vast majority of children are able to perform the most basic operations (counting and grouping objects into sets) long before they become expert readers, mathematics is the lone subject in which the vast majority of children are naturally equipped to excel at an early age. By removing language as a barrier, students can realize their full potential in mathematics.
- Allowing you to continually provide feedback — In the Introductory Unit, the mathematics are broken down into small steps so that you can quickly identify difficulties and help as soon as they arise.
- Keeping students engaged through the excitement of small victories — Children respond more quickly to praise and success than to criticism and threats. If students are encouraged, they feel an incentive to learn. Students enjoy exercising their minds and showing off to a caring adult.

Since the Introductory Unit is about building confidence, work with your students to ensure that they are successful. Celebrate every correct answer. Take your time. Encourage your students. Point out that fractions are considered to be one of the most difficult topics in mathematics. Have fun!

7. Using JUMP Math in the Classroom

JUMP Math supports a balanced approach to teaching mathematics. In the Teacher Resources you will find lesson plans which include everything from group work to explorations. Below are some recommendations for using JUMP Math.

Teach at regular intervals.

Build a lesson around the material on a particular Assessment & Practice Book page by creating similar questions or exercises. Discuss one or two skills or concepts at a time with the whole class, allowing students to develop ideas by themselves, but giving hints and guidance where necessary. Ask questions in several different ways and allow students time to think before you solicit an answer, so that every student can put their hand up and so that students can discover the ideas for themselves. After presenting a particular concept, do not go on until all of the students are assessed and show a readiness to move ahead.

Give mini-quizzes.

Each time you cover a concept or skill, assign a mini-quiz consisting of several questions or a straightforward task to see exactly what students have understood or misunderstood. Write questions or instructions on the board and let students work independently in a separate notebook (or with concrete materials when indicated by the lesson plans). Depending upon the topic you

are working on, assign questions from the Assessment & Practice Book only after going through several cycles of explanations (or explorations) followed by mini-quizzes. Check the work of students who might need extra help first, then take up the answers to the quiz at the board with the entire class. If any of your students finish a quiz early, assign extra questions.

Assess continuously.

The secret to bringing an entire class along at the same pace is to use "continuous assessment." When students are not able to keep up in a lesson it is usually because they are lacking one or two basic skills that are needed for that lesson, or because they are being held back by a simple misconception that is not difficult to correct. Make an effort to spot mistakes or misunderstandings right away. If you wait too long to correct an error, mistakes pile upon mistakes so that it becomes impossible to know exactly where a student is going wrong. To spot mistakes, it helps to break material into small steps or separate concepts, so the Assessment & Practice Books are an ideal tool for assessing mistakes and misunderstandings.

Prepare bonus questions.

Be ready to write bonus questions on the board from time to time during the lesson for students who finish their quizzes or tasks early. Bonus questions and extensions are included in most of the lesson plans. While students who finish quickly are occupied with these questions, circulate around the class doing spot checks on the work of students who are struggling. The bonus questions you create should generally be simple extensions of the material (see How to Create Bonus Questions below).

If a student doesn't understand your explanation, assume there is something lacking in your explanation, not in your student.

Rephrase or reword explanations if a student doesn't understand. Sometimes lessons go too fast for a student or steps are inadvertently skipped. Taking time to reflect on what worked and didn't work in a lesson can help you reach even the most challenged students. When students are struggling always ask, "How could I have improved the lesson?"

In mathematics, it is always possible to make a step easier.

The exercises in the JUMP Math Assessment & Practice Books break concepts and skills into small steps and in a coherent order that students will find easy to master. The lesson plans in the Teacher Resources provide many examples of extra questions that can be used to fill in a missing step in the development of an idea if a problem occurs.

Introduce one piece of information at a time.

Teachers often inadvertently introduce too many new pieces of information at the same time. In trying to comprehend the final item, students can lose all memory and understanding of the material that came before, even though they may have appeared to understand this material completely as it was being explained.

According to Herb Simon, who won the Nobel Prize for his work on the brain, research in cognition shows that, "… a learner who is having difficulty with components can easily be overwhelmed by the processing demands of a complex task. Further, to the extent that many components are well mastered, the student wastes much less time repeating these mastered operations to get an opportunity to practice the few components that need additional effort."[8]

John Mighton once observed a student teacher who was trying to teach a boy in a Grade 7 remedial class how to draw mixed fractions. The boy was getting very frustrated as the intern kept

asking him to carry out several steps at the same time. Here is how John separated the steps to facilitate understanding and success:

> I asked the boy to simply draw a picture showing the number of whole pies in the fraction 2 1/2. He drew and shaded two whole pies cut into halves. I then asked him to draw the number of whole pies in 3 1/2, 4 1/2 and 5 1/2 pies. He was very excited when he completed the work I had assigned him, and I could see that he was making more of an effort to concentrate. I asked him to draw the whole number of pies in 2 1/4, 2 3/4, 3 1/4, 4 1/4 pies, then in 2 1/3, 2 2/3, 3 1/3 pies, and so on. (I started with quarters rather than thirds because they are easier to draw.) When the boy could draw the whole number of pies in any mixed fraction, I showed him how to draw the fractional part. Within a few minutes he was able to draw any mixed fraction. If I hadn't broken the skill into two steps (i.e., drawing the number of whole pies then drawing the fractional part) and allowed him to practise each step separately, he might never have learned the concept.

Before you assign Assessment & Practice Book pages, verify that all students have the skills they need to complete the work.

Before assigning a question from the Assessment & Practice Books, it is important to verify that all of your students are prepared to answer the question without your help (or with minimal help).

Never allow students to work ahead in the Assessment & Practice Books on material you haven't covered with the class.

Students who finish a page in the Assessment & Practice Book early should be assigned bonus questions similar to the questions in the Assessment & Practice Book or extension questions from the lesson plan. Write the bonus questions on the board or have extra pages prepared and ask students to answer the questions in their notebooks or on the pages themselves. While students are working independently on the bonus questions, you can spend extra time with anyone who needs help.

Raise the bar incrementally.

When a student has mastered a skill or concept, raise the bar slightly by challenging them to answer a question that is only incrementally more difficult or complex than the questions previously assigned.

Praise students' efforts.

We've found the JUMP program works best when teachers give their students a great deal of encouragement. Because the lessons are laid out in steps that any student can master, you'll find that you won't be giving false encouragement. One of the reasons that students love the program so much is that it's a thrill to be doing well at math!

Teach the number facts.

It is a serious mistake to think that students who don't know their number facts can always get by in mathematics using a calculator or other aids. Trying to do mathematics without knowing basic number facts is like trying to play the piano without knowing where the notes are. (See the Mental Math section of this guide for strategies to help students learn their number facts.)

Create excitement about math.

Engaging the entire class in lessons is not simply a matter of fairness; it is also a matter of efficiency. While the idea may seem counterintuitive, teachers will enable students who learn more quickly to go further if they take care of the students who struggle. Teachers can create a real sense of excitement about math in the classroom simply by convincing struggling students that they can do

well in the subject. The class will cover far more material in the year and students who excel will no longer have to hide their love of math for fear of appearing strange or different.

8. How to Create Bonus Questions

Students love to show off to their teachers by solving difficult-looking puzzles and surmounting challenges, and they also love to succeed in front of their peers. You can make math lessons more exciting (and also make time to check the work of students who need extra time) if you know how to create engaging bonus questions. Bonus questions generally shouldn't be based on new concepts and they don't have to be extremely difficult to capture the attention of students. Here are some strategies you can use to create questions that will look hard enough to interest students who work quickly, but that all of your students can aspire to answer.

1. Make the numbers in a problem larger or introduce several new terms or elements without introducing any new skills or concepts.

This is the simplest way to create bonus questions. Students of all ages love showing off with larger numbers or with more-challenging looking rules and procedures. If your students know how to add a pair of three digit numbers without regrouping, let them impress you by adding a pair of five or six digit numbers. If they can add two fractions let them add three or four. You will be amazed at how excited students become when they can apply their skills with larger numbers or more difficult-looking calculations. You can use this strategy in almost any lesson.

Some things to bear in mind: First, bonus questions shouldn't look tedious. You don't want to give students an endless series of calculations that appear to have no purpose. It helps if you are excited when you assign bonus questions and if you assign only a few questions at a time. Students should feel they are involved in a quest, faced with a series of increasingly more difficult challenges that they believe they can meet.

Students will not necessarily gain a deeper conceptual understanding of a particular mathematical idea when they work on bonus questions that involve larger numbers or that have more terms or elements. But they will still make important conceptual gains. In addition to generalizing from smaller to larger numbers, they will, for instance, develop the ability to hold more material in their working memory, to follow a series of steps in a procedure, to stay on task, and to see patterns and apply rules in increasingly complex situations. They will also consolidate their understanding and commit the material memory. Their behaviour, confidence, and level of engagement will also likely improve.

2. Make a mistake and ask your students to correct it.

Students love correcting a teacher's mistakes—and you can find a way to make mistakes in any lesson! For instance, if you are teaching T-tables, you might draw the following T-table on the board:

INPUT	OUTPUT
1	7
2	10
3	14
4	16

Tell your students you created the table by adding the same number repeatedly to the initial number, but you think you made a mistake. Ask them to find the mistake and explain where you went wrong.

3. Leave out several terms or elements in a sequence and ask your students to say what is missing.

For instance, you might ask students to say what numbers are missing in the following T-tables, assuming the tables were made by adding the same number repeatedly. **NOTE:** The problem on the right is harder than the one on the left. Students might solve the problem on the right by guessing and checking or using a number line.

INPUT	OUTPUT
1	8
2	
3	14
4	17

INPUT	OUTPUT
1	7
2	
3	15

When you create bonus questions, use number facts that your students are likely to know and give clear and concise instructions. Rather than giving lengthy explanations to struggling students, try to assign tasks that students can immediately see how to do. For instance, if the numbers in the charts above are too difficult for your students, use different numbers:

INPUT	OUTPUT
1	2
2	4
3	
4	8
5	10

INPUT	OUTPUT
1	5
2	
3	
4	20
5	25

Most students know how to count by twos or fives, so every student should see which numbers are missing from these charts. Once struggling students have succeeded with easier tasks, they will be more willing to take risks and to guess and check to solve more difficult problems.

4. Vary the task or problem slightly.

You could use more challenging patterns in the output column, such as decreasing patterns (as shown on the left) or patterns with a gap that changes (as shown on the right).

INPUT	OUTPUT
1	23
2	20
3	17
4	14

INPUT	OUTPUT
1	5
2	7
3	10
4	14

You could also add an extra column to the table, as you might do if you were following a recipe with three quantities that vary with each other. You could ask students to say how much of one ingredient they would need if they had a certain amount of another ingredient.

Number of pies	Number of cups of flour	Number of cups of cherries
1	2	3
2	4	6
3	6	9
4	8	12

5. Look for applications of the concepts.

You might tell students that the numbers in the output column represent a student's savings and ask them to predict how long it will take to save a certain amount of money. Or you could give them a T-table and ask them to draw a picture or describe a pattern that might be represented by the table.

6. Look for patterns in any work you assign, and ask students to describe the patterns.

For example, if you ask students to find and state the rule for the T-table below, you can ask students who finish early to describe the pattern in the ones and the tens digits of the numbers in the right-hand column.

INPUT	OUTPUT
1	7
2	12
3	17
4	22
5	27

(Answer: the ones digit repeats every second term (7, 2, 7, 2, 7,…) and the tens digit increases by 1 every second term (0, 1, 1, 2, 2,…). This happens because the numbers increase by 5 each time, so after two steps they have increased by 10.)

This kind of exercise can keep students who finish their work early occupied while you do a spot check on other students' work.

7. Use extension questions from the lesson plans.

As your students become more confident you will want to create questions that challenge them more and that extend the ideas in the lesson. Our lesson plans contain many suggestions for creating extension questions for your students.

9. Features of the JUMP Math Materials

Diversity of representation

What makes sense to one student does not always make sense to another. Students learn in different ways. Multiple representations help to reach a broader number of students. Students who see multiple representations of a concept also develop a deeper understanding of that concept and are better able to explain it. They can then begin to make connections between the various representations. The more ways you as a teacher know how to teach a concept, the better you will understand it yourself and the more able you will be to teach it in ways that meet the varying needs of your students.

Differentiated instruction

Because the math is broken into steps in the Assessment & Practice Books, they allow you to teach to the whole class while still addressing the needs of individual students (not leaving anyone behind and challenging those who are ready to move ahead). Part of the philosophy of JUMP Math is to teach to the collective—to ensure that the class meets success as a whole, thereby increasing excitement and momentum. In the Assessment & Practice Books, concepts and skills are introduced one step at a time, with lots of opportunities for practice. Struggling students can complete all of the

questions in the Assessment & Practice Book while students who excel can skip some questions and do extra work provided in the lesson plans.

Text and layout

In the Assessment & Practice Books, we tried to reduce the number of words per page and to use clear, simple language. This ensures that all students have equal access to the materials, regardless of their reading level. (However, in the lesson plans you will find many exercises that combine mathematics with language learning and communication.) Layout is simple and uncluttered to avoid distraction, which is particularly helpful for children with learning disabilities. Visual elements such as boxes, figures, background shading, and bold text emphasize changes in content to help students learn new steps or new ideas.

Review in the Assessment & Practice Books

At the beginning of the school year, teachers often find that many of their students have forgotten material taught in the previous year. That's why the Assessment & Practice Books for each grade level (after Grade 1) provide review that goes back one or two years in the curriculum. Research in education has shown that if teachers assess their students and start at the right level, they cover more curriculum in the year.

10. Hints for Helping Students Who Have Fallen Behind

Teach the number facts.

Struggling students often have a weak grasp of number facts. They have trouble solving problems or doing calculations because their working memories are overwhelmed trying to recall number facts. Research shows that automatic recall of number facts helps enormously in the learning of math.

Review older material.

Even mathematicians constantly forget new material, including material they once understood completely. Giving reviews doesn't have to create a lot of extra work for a teacher. John Mighton recommends that, once a month, you copy a selection of questions from the Assessment & Practice Books for units already covered onto a single sheet and make copies for the class. Students rarely complain about doing questions they already did a month or more ago (and quite often they won't even remember that they did those particular questions).

Make mathematical terms part of your spelling lessons.

In some areas of math (e.g., geometry) the greatest difficulty that students face is in learning the terminology. If you include mathematical terms in your spelling lessons, students will find it easier to remember the terms and to communicate about their work.

Set aside five minutes every few days to give extra review to struggling students.

Spending five minutes with a small group of students who need extra help can make the teaching component of lessons more productive. Ensuring that struggling students have the prerequisite skills and knowledge to participate fully in the lesson will enhance their learning and contribute to a positive learning environment for all your students.

Allow wait time.

Wait time is the time between asking the question and soliciting a response. Wait time gives students a chance to think about their answer and leads to longer and clearer explanations. It is

particularly helpful for more timid students, students who are slower to process information, and students who are learning English as a second language. Studies about the benefits of increasing wait time to three seconds or longer confirm that there are increases in student participation, better quality of responses, better overall classroom performance, more questions asked by students, and more frequent and unsolicited contributions.

11. Other Features of the JUMP Math Program

1. SMART Board-Compatible Interactive Whiteboard Lessons

The JUMP Math interactive whiteboard lessons will help you deliver our lessons more easily and effectively. Each interactive lesson is an abbreviated version of the corresponding lesson from the Teacher Resources. The interactive lessons allow you to project and manipulate key text, diagrams, exercises, activities, and bonus and extension questions. You can also call up and clone manipulatives and diagrams (such as base ten blocks, money, number lines, spinning dice, charts, graphs, and Venn diagrams) so students can come to the board and work out or demonstrate their answers and ideas using these tools. The interactive lessons also allow you to save examples of both your work and students' work.

If you do not have a SMART Board but do have access to a projector, the whiteboard lessons can still help you organize lessons and save time by not having to write text and create graphs and diagrams.

The interactive lessons are available at JUMP Math's online store:
http://jumpmath.utpshop.com/jumpmath/.

2. Professional Development

Teachers

Professional development sessions offered across Canada give teachers opportunities to:

- learn about JUMP Math's philosophy and guiding principles;
- consider their own comfort levels with mathematics and math instruction;
- watch a video of a lesson and discuss specific instructional techniques;
- learn how to use all of the JUMP Math resources; and
- take home practical ideas on how to support all students' learning in the classroom.

Community Volunteers

A new JUMP Math Essentials tutor program (Grades 3–8) is now available. There are three resource packages: Grades 3 & 4, Grades 5 & 6, and Grades 7 & 8. Each package includes a detailed tutor's guide and student worksheets. Detailed information is available at www.jumpmath.org.
Professional development sessions for tutors include instruction on how to:

- plan for 25 weeks of lessons;
- assess student requirements;
- engage students and get them excited about math; and
- use the support materials.

3. National Book Fund

JUMP Math is committed to supporting vulnerable communities in schools. We provide free resources and supports through the National Book Fund.

References

[1] Phillip E. Ross, "The Expert Mind," *Scientific American*, July 2006: 44.

[2] John Mighton, *The End of Ignorance* (Toronto: Alfred A. Knopf/Random House, 2007), 6.

[3] Canadian Child Care Federation and Canadian Language and Literacy Research Network (CLLRNet), "Foundations for Numeracy: An Evidence-Based Toolkit for the Effective Mathematics Teacher" (London, ON: CLLRNet, 2010), 44.

[4] Ann Dowker, "What Works for Children with Mathematical Difficulties? The Effectiveness of Intervention Schemes," Ref: 00086-2009BKT-EN (London, England: UK Government Department for Children, Schools and Families, 2009), 35–36.

[5] Margaret M. Clifford, "Students Need Challenge, Not Easy Success," *Educational Leadership* 48, no. 1 (1990): 22–26.

[6] Phillip E. Ross, "The Expert Mind," *Scientific American*, July 2006.

[7] Paul A. Kirschner, John Sweller, and Richard E. Clark, "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential and Inquiry-Based Teaching," *Educational Psychologist* 41, no. 2:75–86.

[8] John R. Anderson, Lynne M. Reder, and Herbert A. Simon, "Applications and Misapplications of Cognitive Psychology to Mathematics Education," *Texas Educational Review* (Summer 2008): 208.

Features of the Teacher Resources for Grade 8

Features of Units

Multiple units for the same strand. In each book, certain strands are covered by more than one unit, and units are not organized by strand.

This breaks up long strands, especially in Grade 7, and improves sequencing. The material in Grades 7 and 8 level is deeply interconnected; revisiting strands helps make the connections clearer and reduces the need to review material learned earlier in the year. It also enables you to report on several strands each term.

Strategies for immediate assessment. There are two strategies we use in some lesson plans.

Signalling. When a problem has a simple answer, such as one word, a short phrase, or a sign (e.g., $+$ or $-$), students can be asked to signal the answer. For example, if the answers to a question are "yes" or "no," students can signal thumbs up for yes and thumbs down for no. Or, if a student has given an answer, the others can signal agreement with a thumbs up or disagreement with a thumbs down. Students can also signal answers by making shapes or signs. For example, if the potential answers to a question are "$+$" or "$-$," students can make the sign they choose with their fingers or hands. Signalling is also useful when you have multiple choice questions: number the answers and have students hold up the number of fingers corresponding to the answer they think is correct.

You need to insist that students signal their answers at the same time, to minimize copying other answers. One way to do so is to give students adequate thinking time and then have students all show their answer on the count of three.

Scribe, stand, share. This is a way to check students' answers when there are many possible short answers. Ask students to write ("scribe") their answers in a list. Once all students have at least one answer listed, have all students stand up ("stand"). Ask a student who is standing to give one answer from his or her list ("share"). Correct the answer if necessary. The other students look to see if they have the answer on their lists. If they do, they put a checkmark beside it to show that it has been said. If they don't, they add it to their list with a checkmark. When students have no new answers on their list (all their answers have already been said), they sit down. Continue asking students who are standing to share one answer until everyone is sitting down. This process ensures that all students get all the right answers; all students are actively thinking of and recording answers; and many students have the chance to show off their answers or at least the fact that they had an answer.

Games. Like the other activities in the lesson plans, the games provided require individual work (working alone) or co-operation (working with a partner) instead of competition (working against an opponent). Points are sometimes awarded in a game, but only toward a team score. It is important not to ask individual teams what their point total is, since this question would promote competition among teams. There are several benefits to individual and cooperative activities.

Benefits of Individual Activities

1. Maximizing engagement. When students are working individually, there is no off-time when they are waiting for a partner to do a task.
2. Building confidence for group work. Working individually prepares students to work in groups. Only once students can successfully complete individual tasks will they have the confidence to share their work in group settings.

3. Facilitating assessment. It is easier to assess each student when they are doing individual work because you know exactly who you are assessing.

Benefits of Cooperative Activities

1. Building teamwork skills. Cooperating with others is an essential skill outside the classroom and throughout life.
2. Accelerating learning. In the setting of a cooperative game, players align their goals in order to win. When students are allowed to use their energy to pull each other along in the same direction, rather than work against each other, students are more likely to progress faster.
3. Promoting empathy. Empathy (understanding the feelings of others and compassionately acting on that understanding) has been shown effective in reducing bullying. When working towards common goals, students naturally mirror each other's emotions, an important aspect of empathy.
4. Avoiding hierarchy. When students succeed or fail together, it is less likely that some students will be considered superior to others. Avoiding hierarchy promotes the collective excitement that sweeps through a class when everyone experiences success together.

Problems and Puzzles. At the end of certain units in the Assessment & Practice Books, we provide a Problems and Puzzles page. This page usually does not require a lesson, but is intended as a review and to ensure that students can combine the concepts learned and can use them in new ways.

Using the Assessment & Practice Books as a diagnostic tool. Some lessons are listed as review only. In these lessons, you could have students do the Assessment & Practice Book pages as a diagnostic assessment and then teach the material as necessary. Be sure, however, that students work at the same pace. There is no point in having some students finish the exercises in five minutes while others need the whole class period. Use the bonus questions in the lesson plans to keep students who finish early engaged. Break the student pages up into chunks and do not allow any student to work ahead until all students have finished each chunk.

Advanced problem-solving lessons. Eleven problem-solving lessons and an introduction are provided at the end of several units in the Grade 8 Teacher Resources. You can teach the problem-solving lessons at any point in the grade after the unit in which the lessons can be found. Problem-solving lessons focus on one or more problem-solving strategies rather than on meeting curriculum requirements. These lessons apply concepts learned throughout the year, but they are not necessary to meet the curriculum. While regular lessons expose students to these problem-solving strategies, the problem-solving lessons provide a way to isolate and focus on the strategies. A detailed description of how to use the problem-solving lessons is included in the introduction to the lessons.

If your students do not require much review, we recommend that you cover more than one review lesson from the Teacher Resources per day. This will allow you to spend more time on the problem-solving lessons.

Features of Lesson Plans

Goals. The specific goals for student learning are stated at the beginning of each lesson plan.

Prior knowledge required. Any prior knowledge that students need in order to understand the concepts taught in a lesson is reviewed at the beginning of the lesson. The prior knowledge required is also listed at the beginning of each lesson plan. If you decide to teach the lessons in a different order than that presented in the Assessment & Practice Books, it is essential that you pay close attention to this list, to ensure that you have covered all the necessary background material.

Materials. The materials required for each lesson are listed at the beginning of the lesson plan. This list includes any materials you will need for Activities, but not Extensions; any materials needed for an Extension are listed within the Extension.

Vocabulary. We list vocabulary words at the beginning of each lesson plan. Make sure students are familiar with the vocabulary words. Make some of the words, such as geometrical terms, part of your spelling lessons.

Curriculum expectations. The curriculum expectations (Ontario and Western and Northern Canadian Protocol, or WNCP) that the lesson covers are stated at the beginning of each lesson plan. Only process or specific expectations covered by the core of the lesson (which includes Activities but not Extensions) are listed. We list the relevant expectations for up to 2 years prior and 1 year ahead. The expectations for the current year are in bold print, for quick identification.

Process expectations. The curricula for both Ontario and the WNCP include both specific expectations and process expectations. Many problem-solving strategies correspond to process expectations, and we use flags in the margin to highlight specific problem-solving strategies where they occur in a lesson. Sometimes a note points to a teacher-led introduction to the specific strategy; other times it points to problems students are expected to solve using the strategy. When more than one problem-solving strategy is used, we sometimes mention them all and sometimes mention just the most important one. Underneath the process expectation flag in the margin, we describe the particular strategy used (e.g., looking for a pattern, modelling, or organizing data). Sample flag:

PROCESS EXPECTATION

Organizing Data

All the descriptions under the process expectation flag correspond to specific process expectations either in Ontario, in the WNCP, or both; these are listed under Curriculum Expectations at the beginning of each lesson. “Problem solving” is the description used when students are asked to determine a way to get from what is known to what is being sought independently, without being given the steps; this corresponds to the WNCP process expectation [PS].

Assessment opportunities for process expectations, using specific exercises in the lesson or in the Assessment & Practice Book, are also highlighted in each lesson. Sample flag:

PROCESS ASSESSMENT

7m5, 7m7, [CN, C]

The expectations to assess are listed underneath the process assessment flag, using the notation for Ontario (e.g., 7m3) or the WNCP (e.g., [CN]), where applicable.

Connections. We use notes in the margin to highlight connections to other areas of mathematics, other subject areas, or the real world. These are different from the process expectation Connecting, which refers to using connections between different problems to help solve a given problem. Sample flag:

CONNECTION 

Real world

Descriptive subheadings. Subheadings in bold print summarize the main parts of the lesson and the order in which teachers should introduce concepts and work through different types of problems.

Support materials

Blackline masters (BLMs). Blackline masters provide:

- materials needed for games (e.g., game boards and playing cards);
- replacements for resources that you may not have in your classroom (e.g. hundreds charts, 1 cm grid paper, nets of 3-D shapes);
- extra pages for remedial practice, grade-level practice, or advanced practice; and
- summaries of concepts learned in a particular unit (e.g., step-by-step instructions for various geometric constructions, or area formulas for parallelograms and triangles). The summary BLMs available for a unit are listed in the Introduction to that unit.

Sample Lesson Plan Pages

Strand
NS = Number Sense
PA = Patterns and Algebra
ME = Measurement
G = Geometry
PDM = Probability and Data Management

Pages in the Assessment & Practice Book covered in this lesson

Grade

Lesson Number

Lesson Title

Curriculum correlations (from 2 grades below, if applicable), stated explicitly; grade level expectation in bold.

Explain the meaning of these terms and write them on the board as they appear in the lesson. Include them in your spelling tests from time to time.

Opportunities to teach problem-solving strategies are highlighted.

Sample questions (in text or margin) let students practise each concept separately.

The purpose of the lesson, stated explicitly.

Test and activate prior knowledge before using it.

Sample answers are provided.

The main idea or concept behind each part of the lesson is in bold at the beginning of a paragraph.

NS7-91

Subtraction Using a Thermometer

Page 140-141

CURRICULUM EXPECTATIONS
Ontario: **7m1, 7m2, 7m3, 7m4, 7m5, 7m6, 7m7, 7m26**
WNCP: **7N6, [C, CN, R, T, PS, V, ME]**

VOCABULARY
opposite
integer

PROCESS EXPECTATION
Modelling

EXAMPLES:
3 − 4 = ___ 2 − 6 = ___
3 − 5 = ___ 4 − 7 = ___

Goals

Students will subtract positive numbers from positive or negative numbers by using the model of temperature dropping on a thermometer. Students will then use patterns to predict how to subtract negative numbers from positive or negative numbers.

PRIOR KNOWLEDGE REQUIRED

Can add integers
Knows that integers that add to 0 have the same number but a different sign
Is familiar with negative temperatures

Remind students that in the last lesson, they subtracted integers by using a number line. In this lesson, they will subtract integers by using a thermometer model. Then they will compare the two methods to see if produce the same answer.

A thermometer as a vertical number line. Review adding and subtracting on a number line. Then turn the number line from a horizontal to a vertical position. Add by moving up and subtract by moving down. **ASK:** Where have you seen a vertical number line before? (examples: a thermometer, the vertical axis on a graph or the scale on a measuring cup) If no one suggests a thermometer, remind them. Then tell students that the temperature was 5°C and increased 2°C. **ASK:** What is the temperature now? (7°C) What operation did you use? (addition) Write on the board: 5 + 2 = 7. Then tell students that the temperature was 5°C and dropped 2°C. **ASK:** What is the temperature now? (3°C) What operation did you use? (subtraction) Write on the board: 5 − 2 = 3. **ASK:** If 5 − 2 shows the temperature dropping from 5°C by 2°C, what would 2 − 5 show? (the temperature dropping from 2°C by 5°C) Explain that although we can't take away 5 objects when we only have 2 objects, the temperature can drop by 5°C when it is only 2°C.

Draw a thermometer on the board. Show starting at 5°C and dropping 2°C. Ask a volunteer to show starting at 2°C and dropping 5°C. **ASK:** What is the temperature now? (−3°C) Write: 2 − 5 = −3.

Have students practise subtracting larger positive integers from smaller positive integers using the thermometer model. Students could copy a vertical thermometer from 5°C to −5°C into their notebooks to help them.

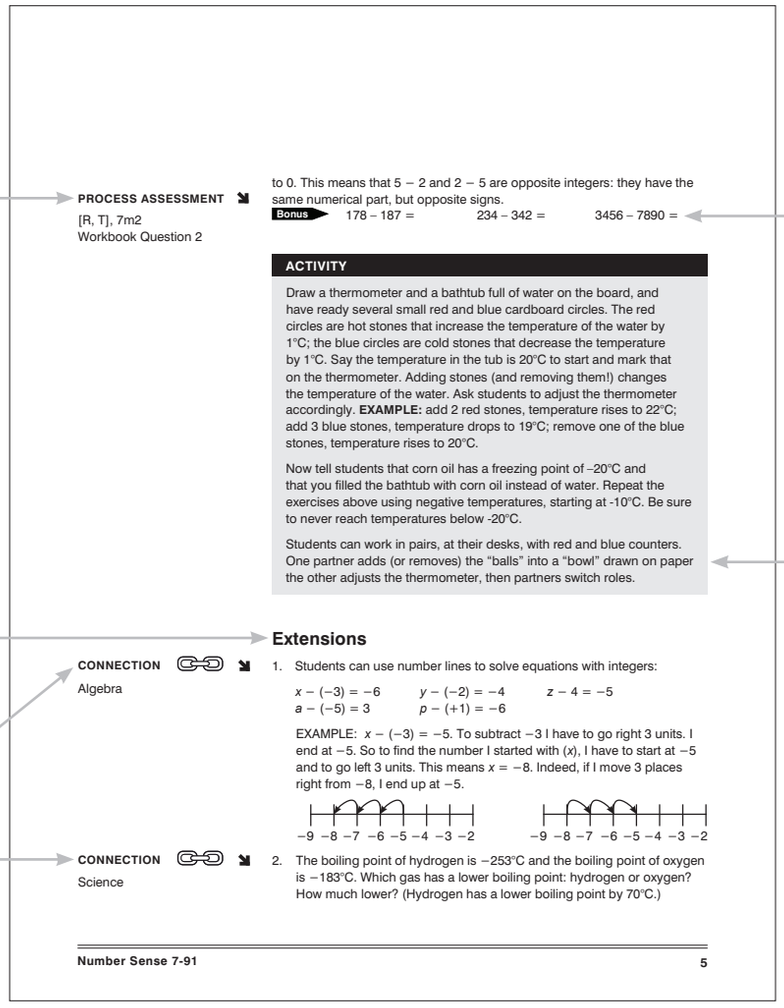
The relationship between *a* − *b* and *b* − *a*. Have students discover this relationship by completing Workbook Questions 1 and 2 (students will subtract only positive numbers from positive numbers). Then explain the relationship as follows: 5 − 2 + 2 − 5 = 5 − 5 = 0, so 5 − 2 and 2 − 5 add

Number Sense 7-91 **1**

Opportunities to evaluate process (through questions in the Assessment & Practice Book or in-class activities) are highlighted.

Optional extensions provide extra challenges for students who need them.

Connections to other strands (e.g., Algebra), other subjects (e.g., science), or to the real world are highlighted.



to 0. This means that 5 − 2 and 2 − 5 are opposite integers: they have the same numerical part, but opposite signs.

PROCESS ASSESSMENT
[R, T], 7m2
Workbook Question 2

Bonus 178 – 187 = 234 – 342 = 3456 – 7890 =

ACTIVITY

Draw a thermometer and a bathtub full of water on the board, and have ready several small red and blue cardboard circles. The red circles are hot stones that increase the temperature of the water by 1°C; the blue circles are cold stones that decrease the temperature by 1°C. Say the temperature in the tub is 20°C to start and mark that on the thermometer. Adding stones (and removing them!) changes the temperature of the water. Ask students to adjust the thermometer accordingly. **EXAMPLE:** add 2 red stones, temperature rises to 22°C; add 3 blue stones, temperature drops to 19°C; remove one of the blue stones, temperature rises to 20°C.

Now tell students that corn oil has a freezing point of –20°C and that you filled the bathtub with corn oil instead of water. Repeat the exercises above using negative temperatures, starting at -10°C. Be sure to never reach temperatures below -20°C.

Students can work in pairs, at their desks, with red and blue counters. One partner adds (or removes) the "balls" into a "bowl" drawn on paper the other adjusts the thermometer, then partners switch roles.

Extensions

CONNECTION Algebra

1. Students can use number lines to solve equations with integers:

   $x - (-3) = -6$ $y - (-2) = -4$ $z - 4 = -5$
   $a - (-5) = 3$ $p - (+1) = -6$

   EXAMPLE: $x - (-3) = -5$. To subtract −3 I have to go right 3 units. I end at −5. So to find the number I started with (x), I have to start at −5 and to go left 3 units. This means $x = -8$. Indeed, if I move 3 places right from −8, I end up at −5.

   −9 −8 −7 −6 −5 −4 −3 −2 −9 −8 −7 −6 −5 −4 −3 −2

CONNECTION Science

2. The boiling point of hydrogen is −253°C and the boiling point of oxygen is −183°C. Which gas has a lower boiling point: hydrogen or oxygen? How much lower? (Hydrogen has a lower boiling point by 70°C.)

Number Sense 7-91 5

Bonus questions that students can solve independently allow you to spend more time with struggling students.

Activities are in a grey box at the end of each lesson. Hands-on or whole-class activities help students consolidate their knowledge.

Sample Assessment & Practice Book Pages

Strand
PA = Patterns and Algebra
NS = Number Sense
ME = Measurement
G = Geometry
PDM = Probability and Data Management

Grade

Lesson number

Lesson title

G7-17 Constructing Triangles

A **straightedge** is any tool used to draw lines and line segments. It is like a ruler without marks. When we say "use a straightedge," you can use a ruler but you cannot use the marks on it.

1. Draw a line from the point to the intersection of the arcs, as shown in part a).

a) b) c)

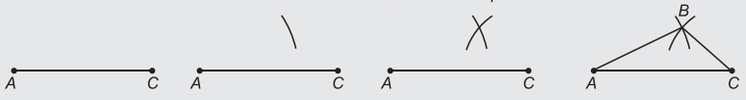
How to construct a triangle with given side lengths *AB*, *BC*, and *AC*, using a compass and a straightedge:

Step 1: Construct a line segment *AC*.

Step 2: Set your compass to the width of *AB*. Construct an arc centred at *A*.

Step 3: Set your compass to the width of *BC*. Construct an arc centred at *C* with radius *BC*, so that it intersects the first arc at one point.

Step 4: Label the intersection point of the arcs *B*. Use a straightedge to join *B* to *A* and *C*.



2. Construct triangles with the following side lengths using a compass and a straightedge.

Use a ruler to set your compass width to the given lengths:

a) 3 cm, 4 cm, 5 cm b) 8 cm, 8 cm, 12 cm c) 5 cm, 5 cm, 8 cm

Now set your compass width to the length of these lines:

d) e)

REMINDER ▶ We identify triangles according to the number of equal sides they have and the size of their angles.

number of equal sides:	0 – scalene △	2 – isosceles △	3 – equilateral △
size of angles:	3 acute – acute △	1 right – right △	1 obtuse – obtuse △

3. Are the angles of the triangles you constructed in Question 2 acute, right, or obtuse? Use a protractor or a set square to check. Then name the types of triangles you constructed.

a) *scalene right* b) _____ c) _____

d) _____ e) _____

Which types of triangles could you have identified from the side lengths, without drawing them?

Question numbers are in bold so that they are easy to find.

Teaching boxes contain definitions, explanations, examples, and step-by-step instructions.

Complete or partial answers (including intermediate steps where applicable) appear in italics.

Reminder boxes contain summaries of information taught previously, sometimes with a page reference.

Strand, grade, lesson number.

Background shading indicates questions that students must answer in a notebook.

The main question being investigated.

Parts of the investigation are labelled with letters instead of numbers.

Conceptual questions require students to reflect on their findings.

Bonus questions can appear at any point in a lesson. Students work on them independently, without additional teaching.

Investigations are visually separated from the main body of the lesson.

5. Draw a coordinate grid on grid paper.

a) Draw a scalene triangle on your grid.
b) Reflect the triangle through the *x*-axis, then reflect the image through the *y*-axis.
c) Reflect the original triangle through the *y*-axis, then reflect the image through the *x*-axis.
d) What do you notice about the images in parts b) and c)?

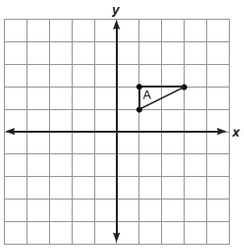
In Question 5, performing the transformations in reverse order did not change the result. Does this happen with any two transformations?

INVESTIGATION 1 ▶ Consider two transformations:

T: Translation 4 units left and 1 unit down.

R: Rotation 90° clockwise around the origin.

How does performing first *T* then *R* compare to performing first *R* then *T*?



A. Start with triangle A. Do *T* first, then perform *R*. Label the resulting triangle B.

B. Start with triangle A. Do *R* first, then perform *T*. Label the resulting triangle C.

C. Do triangles B and C coincide? What transformation would take B to C?

D. Can you obtain B from A by a single rotation, reflection or translation? Explain.

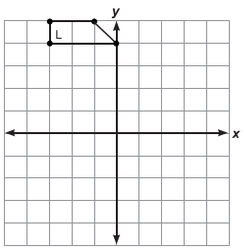
E. Start with triangle A. Do *R*. Which translation do you need to get triangle B?

INVESTIGATION 2 ▶ Consider two transformations:

F: Reflection through the *x*-axis.

R: Rotation 90° counter-clockwise around the origin.

How does performing first *F* then *R* compare to performing first *R* then *F*?



A. Start with trapezoid L. Do *F* first, then perform *R*. Label the resulting trapezoid M.

B. Start with trapezoid L. Do *R* first, then perform *F*. Label the resulting trapezoid N.

C. Do trapezoids M and N coincide? _____

D. Start with trapezoid L. Do *R*. Which reflection do you need to perform to obtain M? How do you know?

BONUS ▶ You can obtain M from L by a single reflection. Find the mirror line.

Mental Math
Addition and Subtraction

If any of your students don't know their addition and subtraction facts, teach them to add and subtract using their fingers by the methods taught below. You should also reinforce basic facts using drills, games and flash cards. There are mental math strategies that make addition and subtraction easier: some effective strategies are taught in the next section. (Until your students know all their facts, allow them to add and subtract on their fingers when necessary.)

To **ADD** 4 + 8, Grace says the greater number (8) with her fist closed. She counts up from 8, raising one finger at a time. She stops when she has raised the number of fingers equal to the lesser number (4):

She said "12" when she raised her 4th finger, so: **4 + 8 = 12**

1. Add:

a) 5 + 2 = _____ b) 3 + 2 = _____ c) 6 + 2 = _____ d) 9 + 2 = _____

e) 2 + 4 = _____ f) 2 + 7 = _____ g) 5 + 3 = _____ h) 6 + 3 = _____

i) 11 + 4 = _____ j) 3 + 9 = _____ k) 7 + 3 = _____ l) 14 + 4 = _____

m) 21 + 5 = _____ n) 32 + 3 = _____ o) 4 + 56 = _____ p) 39 + 4 = _____

To **SUBTRACT** 9 – 5, Grace says the lesser number (5) with her fist closed. She counts up from 5 raising one finger at a time. She stops when she says the greater number (9):

She has raised 4 fingers when she stopped, so: **9 – 5 = 4**

2. Subtract:

a) 7 – 5 = _____ b) 8 – 6 = _____ c) 5 – 3 = _____ d) 5 – 2 = _____

e) 9 – 6 = _____ f) 10 – 5 = _____ g) 11 – 7 = _____ h) 17 – 14 = _____

i) 33 – 31 = _____ j) 27 – 24 = _____ k) 43 – 39 = _____ l) 62 – 58 = _____

To prepare for the next section, teach your students to add 1 to any number mentally (by counting forward by 1 in their head) and to subtract 1 from any number (by counting backward by 1).

Students who don't know how to add, subtract or estimate readily are at a great disadvantage in mathematics. Students who have trouble memorizing addition and subtraction facts can still learn to mentally add and subtract numbers in a short time if they are given daily practice in a few basic skills.

Skill 1: Adding 2 to an Even Number

This skill has been broken down into a number of sub-skills. After teaching each sub-skill, you should give your students a short diagnostic quiz to verify that they have learned the skill. I have included sample quizzes for Skills 1 to 4.

i) Naming the next one-digit even number:

Numbers that have ones digit 0, 2, 4, 6 or 8 are called the even numbers. Using drills or games, teach your students to say the sequence of one-digit even numbers without hesitation. Ask students to imagine the sequence going on in a circle so that the next number after 8 is 0 (0, 2, 4, 6, 8, 0, 2, 4, 6, 8…) Then play the following game: name a number in the sequence and ask your students to give the next number. Don't move on until all of your students have mastered the game.

ii) Naming the next greatest two-digit even number:

CASE 1: Numbers that end in 0, 2, 4 or 6
Write an even two-digit number that ends in 0, 2, 4 or 6 on the board. Ask your students to name the next greatest even number. Students should recognize that if a number ends in 0, then the next even number ends in 2; if it ends in 4 then the next even number ends in 6, etc. For instance, the number 54 has ones digit 4: so the next greatest even number will have ones digit 6.

QUIZ

Name the next greatest even number:

a) 52 : _____ b) 64 : _____ c) 36 : _____ d) 22 : _____ e) 80 : _____

CASE 2: Numbers that end in 8
Write the number 58 on the board. Ask students to name the next greatest even number. Remind your students that even numbers must end in 0, 2, 4, 6, or 8. But 50, 52, 54 and 56 are all less than 58 so the next greatest even number is 60. Your students should see that an even number ending in 8 is always followed by an even number ending in 0 (with a tens digit that is one higher).

QUIZ

Name the next greatest even number:

a) 58 : _____ b) 68 : _____ c) 38 : _____ d) 48 : _____ e) 78 : _____

iii) Adding 2 to an even number:

Point out to your students that adding 2 to any even number is equivalent to finding the next even number: **EXAMPLE:** 46 + 2 = 48, 48 + 2 = 50, etc. Knowing this, your students can easily add 2 to any even number.

QUIZ

Add:

a) 26 + 2 = _____ b) 82 + 2 = _____ c) 40 + 2 = _____ d) 58 + 2 = _____ e) 34 + 2 = _____

Skill 2: Subtracting 2 from an Even Number

i) Finding the preceding one-digit even number:

Name a one-digit even number and ask your students to give the preceding number in the sequence. For instance, the number that comes before 4 is 2 and the number that comes before 0 is 8. (**REMEMBER:** the sequence is circular.)

ii) Finding the preceding two-digit number:

CASE 1: Numbers that end in 2, 4, 6 or 8
Write a two-digit number that ends in 2, 4, 6 or 8 on the board. Ask students to name the preceding even number. Students should recognize that if a number ends in 2, then the preceding even number ends in 0; if it ends in 4 then the preceding even number ends in 2, etc. For instance, the number 78 has ones digit 8 so the preceding even number has ones digit 6.

QUIZ

Name the preceding even number:

a) 48 : _____ b) 26 : _____ c) 34 : _____ d) 62 : _____ e) 78 : _____

CASE 2: Numbers that end in 0
Write the number 80 on the board and ask your students to name the preceding even number. Students should recognize that if an even number ends in 0 then the preceding even number ends in 8 (but the ones digit is one less). So the even number that comes before 80 is 78.

QUIZ

Name the preceding even number:

a) 40 : _____ b) 60 : _____ c) 80 : _____ d) 50 : _____ e) 30 : _____

ii) Subtracting 2 from an even number:

Point out to your students that subtracting 2 from an even number is equivalent to finding the preceding even number: **EXAMPLE:** $48 - 2 = 46$, $46 - 2 = 44$, etc.

QUIZ

Subtract:

a) $58 - 2 =$ _____ b) $24 - 2 =$ _____ c) $36 - 2 =$ _____ d) $42 - 2 =$ _____ e) $60 - 2 =$ _____

Skill 3: Adding 2 to an Odd Number

i) Naming the next one-digit odd number:

Numbers that have ones digit 1, 3, 5, 7, and 9 are called the odd numbers. Using drills or games, teach your students to say the sequence of one-digit odd numbers without hesitation. Ask students to imagine the sequence going on in a circle so that the next number after 9 is 1 (1, 3, 5, 7, 9, 1, 3, 5, 7, 9…). Then play the following game: name a number in the sequence and ask you students to give the next number. Don't move on until all of your students have mastered the game.

ii) Naming the next greatest two-digit odd number:

CASE 1: Numbers that end in 1, 3, 5 or 7
Write an odd two-digit number that ends in 1, 3, 5, or 7 on the board. Ask you students to name the next greatest odd number. Students should recognize that if a number ends in 1, then the next even number ends in 3; if it ends in 3 then the next even number ends in 5, etc. For instance, the number 35 has ones digit 5: so the next greatest even number will have ones digit 7.

QUIZ

Name the next greatest odd number:

a) 51 : _____ b) 65 : _____ c) 37 : _____ d) 23 : _____ e) 87 : _____

CASE 2: Numbers that end in 9
Write the number 59 on the board. Ask students to name the next greatest number. Remind your students that odd numbers must end in 1, 3, 5, 7, or 9. But 51, 53, 55, and 57 are all less than 59. The next greatest odd number is 61. Your students should see that an odd number ending in 9 is always followed by an odd number ending in 1 (with a tens digit that is one higher).

QUIZ

Name the next greatest odd number:

a) 59 : _____ b) 69 : _____ c) 39 : _____ d) 49 : _____ e) 79 : _____

iii) Adding 2 to an odd number:

Point out to your students that adding 2 to any odd number is equivalent to finding the next odd number: **EXAMPLE:** 47 + 2 = 49, 49 + 2 = 51, etc. Knowing this, your students can easily add 2 to any odd number.

QUIZ

Add:

a) 27 + 2 = _____ b) 83 + 2 = _____ c) 41 + 2 = _____ d) 59 + 2 = _____ e) 35 + 2 = _____

Skill 4: Subtracting 2 from an Odd Number

i) Finding the preceding one-digit odd number:

Name a one-digit even number and ask your students to give the preceding number in the sequence. For instance, the number that comes before 3 is 1 and the number that comes before 1 is 9. (**REMEMBER:** the sequence is circular.)

ii) Finding the preceding odd two-digit number:

CASE 1: Numbers that end in 3, 5, 7 or 9
Write a two-digit number that ends in 3, 5, 7 or 9 on the board. Ask students to name the preceding even number. Students should recognize that if a number ends in 3, then the preceding odd number ends in 1; if it ends in 5 then the preceding odd number ends in 3, etc. For instance, the number 79 has ones digit 9, so the preceding even number has ones digit 7.

QUIZ

Name the preceding odd number:

a) 49 : _____ b) 27 : _____ c) 35 : _____ d) 63 : _____ e) 79 : _____

CASE 2: Numbers that end in 1
Write the number 81 on the board and ask your students to name the preceding odd number. Students should recognize that if an odd number ends in 1 then the preceding odd number ends in 9 (but the tens digit is one less). So the odd number that comes before 81 is 79.

QUIZ

Name the preceding odd number:

a) 41 : _____ b) 61 : _____ c) 81 : _____ d) 51 : _____ e) 31 : _____

iii) Subtracting 2 from an odd number:

Point out to your students that subtracting 2 from an odd number is equivalent to finding the preceding even number: **EXAMPLE:** $49 - 2 = 47$, $47 - 2 = 45$, etc.

QUIZ

Subtract:

a) $59 - 2 =$ _____ b) $25 - 2 =$ _____ c) $37 - 2 =$ _____ d) $43 - 2 =$ _____ e) $61 - 2 =$ _____

Skills 5 and 6

Once your students can add and subtract the numbers 1 and 2, then they can easily add and subtract the number 3: Add 3 to a number by first adding 2, then 1 (**EXAMPLE:** $35 + 3 = 35 + 2 + 1$). Subtract 3 from a number by subtracting 2, then subtracting 1 (**EXAMPLE:** $35 - 3 = 35 - 2 - 1$).

NOTE: All of the addition and subtraction tricks you teach your students should be reinforced with drills, flashcards and tests. Eventually students should memorize their addition and subtraction facts and shouldn't have to rely on the mental math tricks. One of the greatest gifts you can give your students is to teach them their number facts.

Skills 7 and 8

Add 4 to a number by adding 2 twice (**EXAMPLE:** $51 + 4 = 51 + 2 + 2$). Subtract 4 from a number by subtracting 2 twice (**EXAMPLE:** $51 - 4 = 51 - 2 - 2$).

Skills 9 and 10

Add 5 to a number by adding 4 then 1. Subtract 5 by subtracting 4 then 1.

Skill 11

Students can add pairs of identical numbers by doubling (**EXAMPLE:** $6 + 6 = 2 \times 6$). Students should either memorize the 2 times table or they should double numbers by counting on their fingers by 2s.

Add a pair of numbers that differ by 1 by rewriting the larger number as 1 plus the smaller number (then use doubling to find the sum): **EXAMPLE:** $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$; $7 + 8 = 7 + 7 + 1 = 14 + 1 = 15$, etc.

Skills 12, 13 and 14

Add a one-digit number to 10 by simply replacing the zero in 10 by the one-digit number: **EXAMPLE:** $10 + 7 = 17$.

Add 10 to any two-digit number by simply increasing the tens digit of the two-digit number by 1: **EXAMPLE:** $53 + 10 = 63$.

Add a pair of two-digit numbers (with no carrying) by adding the ones digits of the numbers and then the tens digits: **EXAMPLE:** $23 + 64 = 87$.

Skills 15 and 16

To add 9 to a one-digit number, subtract 1 from the number and then add 10: **EXAMPLE:** $9 + 6 = 10 + 5 = 15$; $9 + 7 = 10 + 6 = 16$, etc. (Essentially, the student simply has to subtract 1 from the number and then stick a 1 in front of the result.)

To add 8 to a one-digit number, subtract 2 from the number and add 10: **EXAMPLE:** $8 + 6 = 10 + 4 = 14$; $8 + 7 = 10 + 5 = 15$, etc.

Skills 17 and 18

To subtract a pair of multiples of ten, simply subtract the tens digits and add a zero for the ones digit: **EXAMPLE:** $70 - 50 = 20$.

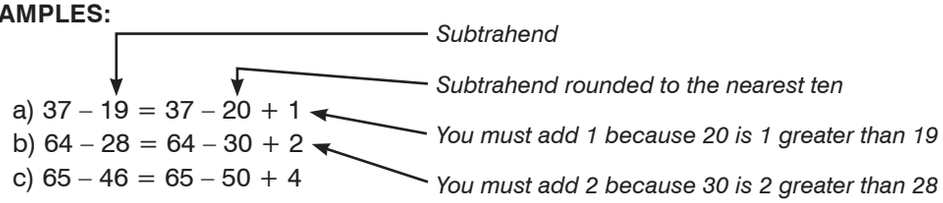
To subtract a pair of two-digit numbers (without carrying or regrouping), subtract the ones digit from the ones digit and the tens digit from the tens digit: **EXAMPLE:** $57 - 34 = 23$.

Mental Math
Further Strategies

Further Mental Math Strategies

1. Your students should be able to explain how to use the strategies of "rounding the subtrahend (the number you are subtracting) up to the nearest multiple of ten."

EXAMPLES:



PRACTICE QUESTIONS:

a) 27 – 17 = 27 – _____ + _____

b) 52 – 36 = 52 – _____ + _____

c) 76 – 49 = 76 – _____ + _____

d) 84 – 57 = 84 – _____ + _____

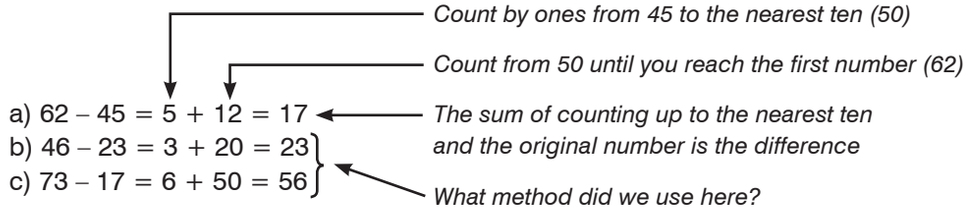
e) 61 – 29 = 61 – _____ + _____

f) 42 – 18 = 42 – _____ + _____

NOTE: This strategy works well with numbers that end in 6, 7, 8 or 9.

2. Your students should be able to explain how to subtract by thinking of adding.

EXAMPLES:



PRACTICE QUESTIONS:

a) 88 – 36 = _____ + _____ = _____

b) 58 – 21 = _____ + _____ = _____

c) 43 – 17 = _____ + _____ = _____

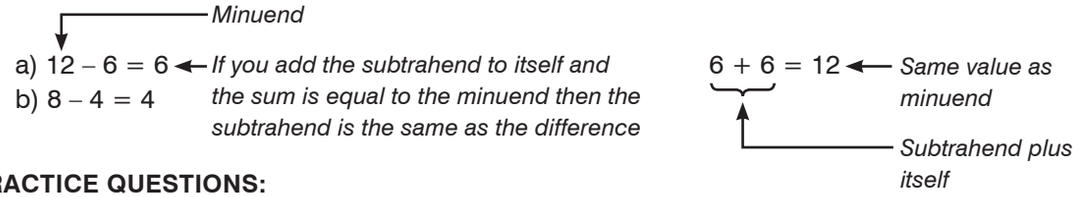
d) 74 – 28 = _____ + _____ = _____

e) 93 – 64 = _____ + _____ = _____

f) 82 – 71 = _____ + _____ = _____

3. Your students should be able to explain how to "use doubles."

EXAMPLES:



PRACTICE QUESTIONS:

a) 6 – 3 = _____

b) 10 – 5 = _____

c) 14 – 7 = _____

d) 18 – 9 = _____

e) 16 – 8 = _____

f) 20 – 10 = _____

Mental Math
Exercises

NOTE TO TEACHER: Teaching the material on these worksheets may take several lessons. Students will need more practice than is provided on these pages. These pages are intended as a test to be given when you are certain your students have learned the materials fully.

Skills 1, 2, 3 and 4

1. Name the even number that comes after the number. Answer in the blank provided:

   a) 32 _____ b) 46 _____ c) 14 _____ d) 92 _____ e) 56 _____

   f) 30 _____ g) 84 _____ h) 60 _____ i) 72 _____ j) 24 _____

2. Name the even number that comes after the number:

   a) 28 _____ b) 18 _____ c) 78 _____ d) 38 _____ e) 68 _____

3. Add. **REMEMBER:** adding 2 to an even number is the same as finding the next even number:

   a) 42 + 2 = _____ b) 76 + 2 = _____ c) 28 + 2 = _____ d) 16 + 2 = _____

   e) 68 + 2 = _____ f) 12 + 2 = _____ g) 36 + 2 = _____ h) 90 + 2 = _____

   i) 70 + 2 = _____ j) 24 + 2 = _____ k) 66 + 2 = _____ l) 52 + 2 = _____

4. Name the even number that comes before the number:

   a) 38 _____ b) 42 _____ c) 56 _____ d) 72 _____ e) 98 _____

   f) 48 _____ g) 16 _____ h) 22 _____ i) 66 _____ j) 14 _____

5. Name the even number that comes before the number:

   a) 30 _____ b) 70 _____ c) 60 _____ d) 10 _____ e) 80 _____

6. Subtract. **REMEMBER:** subtracting 2 from an even number is the same as finding the preceding even number:

   a) 46 – 2 = _____ b) 86 – 2 = _____ c) 90 – 2 = _____ d) 14 – 2 = _____

   e) 54 – 2 = _____ f) 72 – 2 = _____ g) 12 – 2 = _____ h) 56 – 2 = _____

   i) 32 – 2 = _____ j) 40 – 2 = _____ k) 60 – 2 = _____ l) 26 – 2 = _____

7. Name the odd number that comes after the number:

   a) 37 _____ b) 51 _____ c) 63 _____ d) 75 _____ e) 17 _____

   f) 61 _____ g) 43 _____ h) 81 _____ i) 23 _____ j) 95 _____

8. Name the odd number that comes after the number:

   a) 69 _____ b) 29 _____ c) 9 _____ d) 79 _____ e) 59 _____

9. Add. **REMEMBER:** Adding 2 to an odd number is the same as finding the next odd number:

a) 25 + 2 = _____ b) 31 + 2 = _____ c) 47 + 2 = _____ d) 33 + 2 = _____

e) 39 + 2 = _____ f) 91 + 2 = _____ g) 5 + 2 = _____ h) 89 + 2 = _____

i) 11 + 2 = _____ j) 65 + 2 = _____ k) 29 + 2 = _____ l) 17 + 2 = _____

10. Name the odd number that comes before the number:

a) 39 _____ b) 43 _____ c) 57 _____ d) 17 _____ e) 99 _____

f) 13 _____ g) 85 _____ h) 79 _____ i) 65 _____ j) 77 _____

11. Name the odd number that comes before the number:

a) 21 _____ b) 41 _____ c) 11 _____ d) 91 _____ e) 51 _____

12. Subtract. **REMEMBER:** Subtracting 2 from an odd number is the same as finding the preceding odd number.

a) 47 – 2 = _____ b) 85 – 2 = _____ c) 91 – 2 = _____ d) 15 – 2 = _____

e) 51 – 2 = _____ f) 73 – 2 = _____ g) 11 – 2 = _____ h) 59 – 2 = _____

i) 31 – 2 = _____ j) 43 – 2 = _____ k) 7 – 2 = _____ l) 25 – 2 = _____

Skills 5 and 6

13. Add 3 to the number by adding 2, then adding 1 (**EXAMPLE:** 35 + 3 = 35 + 2 + 1):

a) 23 + 3 = _____ b) 36 + 3 = _____ c) 29 + 3 = _____ d) 16 + 3 = _____

e) 67 + 3 = _____ f) 12 + 3 = _____ g) 35 + 3 = _____ h) 90 + 3 = _____

i) 78 + 3 = _____ j) 24 + 3 = _____ k) 6 + 3 = _____ l) 59 + 3 = _____

14. Subtract 3 from the number by subtracting 2, then subtracting 1 (**EXAMPLE:** 35 – 3 = 35 – 2 – 1):

a) 46 – 3 = _____ b) 87 – 3 = _____ c) 99 – 3 = _____ d) 14 – 3 = _____

e) 8 – 3 = _____ f) 72 – 3 = _____ g) 12 – 3 = _____ h) 57 – 3 = _____

i) 32 – 3 = _____ j) 40 – 3 = _____ k) 60 – 3 = _____ l) 28 – 3 = _____

15. Fred has 49 stamps. He gives 2 stamps away. How many stamps does he have left?

16. There are 25 minnows in a tank. Alice adds 3 more to the tank. How many minnows are now in the tank?

Skills 7 and 8

17. Add 4 to the number by adding 2 twice (**EXAMPLE:** 51 + 4 = 51 + 2 + 2):

a) 42 + 4 = _____ b) 76 + 4 = _____ c) 27 + 4 = _____ d) 17 + 4 = _____

e) 68 + 4 = _____ f) 11 + 4 = _____ g) 35 + 4 = _____ h) 8 + 4 = _____

i) 72 + 4 = _____ j) 23 + 4 = _____ k) 60 + 4 = _____ l) 59 + 4 = _____

18. Subtract 4 from the number by subtracting 2 twice (**EXAMPLE:** 26 – 4 = 26 – 2 – 2):

a) 46 – 4 = _____ b) 86 – 4 = _____ c) 91 – 4 = _____ d) 15 – 4 = _____

e) 53 – 4 = _____ f) 9 – 4 = _____ g) 13 – 4 = _____ h) 57 – 4 = _____

i) 40 – 4 = _____ j) 88 – 4 = _____ k) 69 – 4 = _____ l) 31 – 4 = _____

Skills 9 and 10

19. Add 5 to the number by adding 4, then adding 1 (or add 2 twice, then add 1):

a) 84 + 5 = _____ b) 27 + 5 = _____ c) 31 + 5 = _____ d) 44 + 5 = _____

e) 63 + 5 = _____ f) 92 + 5 = _____ g) 14 + 5 = _____ h) 16 + 5 = _____

i) 9 + 5 = _____ j) 81 + 5 = _____ k) 51 + 5 = _____ l) 28 + 5 = _____

20. Subtract 5 from the number by subtracting 4, then subtracting 1 (or subtract 2 twice, then subtract 1):

a) 48 – 5 = _____ b) 86 – 5 = _____ c) 55 – 5 = _____ d) 69 – 5 = _____

e) 30 – 5 = _____ f) 13 – 5 = _____ g) 92 – 5 = _____ h) 77 – 5 = _____

i) 45 – 5 = _____ j) 24 – 5 = _____ k) 91 – 5 = _____ l) 8 – 5 = _____

Skill 11

21. Add:

a) 6 + 6 = _____ b) 7 + 7 = _____ c) 8 + 8 = _____

d) 5 + 5 = _____ e) 4 + 4 = _____ f) 9 + 9 = _____

22. Add by thinking of the larger number as a sum of two smaller numbers:

a) 6 + 7 = 6 + 6 + 1 b) 7 + 8 = _____ c) 6 + 8 = _____

d) 4 + 5 = _____ e) 5 + 7 = _____ f) 8 + 9 = _____

Mental Math
Exercises

Skills 12, 13 and 14

23. a) 10 + 3 = _____ b) 10 + 7 = _____ c) 5 + 10 = _____ d) 10 + 1 = _____

    e) 9 + 10 = _____ f) 10 + 4 = _____ g) 10 + 8 = _____ h) 10 + 2 = _____

24. a) 10 + 20 = _____ b) 40 + 10 = _____ c) 10 + 80 = _____ d) 10 + 50 = _____

    e) 30 + 10 = _____ f) 10 + 60 = _____ g) 10 + 10 = _____ h) 70 + 10 = _____

25. a) 10 + 25 = _____ b) 10 + 67 = _____ c) 10 + 31 = _____ d) 10 + 82 = _____

    e) 10 + 43 = _____ f) 10 + 51 = _____ g) 10 + 68 = _____ h) 10 + 21 = _____

    i) 10 + 11 = _____ j) 10 + 19 = _____ k) 10 + 44 = _____ l) 10 + 88 = _____

26. a) 20 + 30 = _____ b) 40 + 20 = _____ c) 30 + 30 = _____ d) 50 + 30 = _____

    e) 20 + 50 = _____ f) 40 + 40 = _____ g) 50 + 40 = _____ h) 40 + 30 = _____

    i) 60 + 30 = _____ j) 20 + 60 = _____ k) 20 + 70 = _____ l) 60 + 40 = _____

27. a) 20 + 23 = _____ b) 32 + 24 = _____ c) 51 + 12 = _____ d) 12 + 67 = _____

    e) 83 + 14 = _____ f) 65 + 24 = _____ g) 41 + 43 = _____ h) 70 + 27 = _____

    i) 31 + 61 = _____ j) 54 + 33 = _____ k) 28 + 31 = _____ l) 42 + 55 = _____

Skills 15 and 16

28. a) 9 + 3 = _____ b) 9 + 7 = _____ c) 6 + 9 = _____ d) 4 + 9 = _____

    e) 9 + 9 = _____ f) 5 + 9 = _____ g) 9 + 2 = _____ h) 9 + 8 = _____

29. a) 8 + 2 = _____ b) 8 + 6 = _____ c) 8 + 7 = _____ d) 4 + 8 = _____

    e) 5 + 8 = _____ f) 8 + 3 = _____ g) 9 + 8 = _____ h) 8 + 8 = _____

Skills 17 and 18

30. a) 40 – 10 = _____ b) 50 – 10 = _____ c) 70 – 10 = _____ d) 20 – 10 = _____

    e) 40 – 20 = _____ f) 60 – 30 = _____ g) 40 – 30 = _____ h) 60 – 50 = _____

31. a) 57 – 34 = _____ b) 43 – 12 = _____ c) 62 – 21 = _____ d) 59 – 36 = _____

    e) 87 – 63 = _____ f) 95 – 62 = _____ g) 35 – 10 = _____ h) 17 – 8 = _____

Mental Math
Advanced

Multiples of Ten

STUDENT: In the exercises below, you will learn several ways to use multiples of ten in mental addition or subtraction.

EXAMPLE 1

542 + 214 = 542 + 200 + 10 + 4 = 742 + 10 + 4 = 752 + 4 = 756

827 – 314 = 827 – 300 – 10 – 4 = 527 – 10 – 4 = 517 – 4 = 513

Sometimes you will need to carry:

545 + 172 = 545 + 100 + 70 + 2 = 645 + 70 + 2 = 715 + 2 = 717

1. Warm up:

   a) 536 + 100 = _____ b) 816 + 10 = _____ c) 124 + 5 = _____ d) 540 + 200 = _____

   e) 234 + 30 = _____ f) 345 + 300 = _____ g) 236 – 30 = _____ h) 442 – 20 = _____

   i) 970 – 70 = _____ j) 542 – 400 = _____ k) 160 + 50 = _____ l) 756 + 40 = _____

2. Write the second number in expanded form and add or subtract one digit at a time. The first one is done for you:

   a) 564 + 215 = *564 + 200 + 10 + 5* = *779*

   b) 445 + 343 = _____ = _____

   c) 234 + 214 = _____ = _____

3. Add or subtract mentally (one digit at a time):

   a) 547 + 312 = _____ b) 578 – 314 = _____ c) 845 – 454 = _____

EXAMPLE 2

If one of the numbers you are adding or subtracting is close to a number with a multiple of ten, add the multiple of ten and then add or subtract an adjustment factor:

645 + 99 = 645 + 100 – 1 = 745 – 1 = 744

856 + 42 = 856 + 40 + 2 = 896 + 2 = 898

EXAMPLE 3

Sometimes in subtraction, it helps to think of a multiple of ten as a sum of 1 and a number consisting entirely of 9s (**EXAMPLE:** 100 = 1 + 99; 1000 = 1 + 999). You never have to borrow or exchange when you are subtracting from a number consisting entirely of 9s.

100 – 43 = 1 + 99 – 43 = 1 + 56 = 57 ← *Do the subtraction, using 99 instead of 100, and then add 1 to your answer.*

1000 – 543 = 1 + 999 – 543 = 1 + 456 = 457

4. Use the tricks you've just learned:

   a) 845 + 91 = _____ b) 456 + 298 = _____ c) 100 – 84 = _____ d) 1000 – 846 = _____

Mental Math
Game: Modified Go Fish

Purpose

If students know the pairs of one-digit numbers that add up to particular target numbers, they will be able to mentally break sums into easier sums.

EXAMPLE: As it is easy to add any one-digit number to 10, you can add a sum more readily if you can decompose numbers in the sum into pairs that add to ten.

$$7 + 5 = \underbrace{7 + 3}_{\text{These numbers add to 10.}} + 2 = 10 + 2 = 12$$

To help students remember pairs of numbers that add up to a given target number, here is a variation of "Go Fish" that we have found very effective.

The Game

Pick any target number and remove all the cards with value greater than or equal to the target number out of the deck. In what follows, the target number is 10, so you would take all the tens and face cards out of the deck (Aces count as one).

The dealer gives each player 6 cards. If a player has any pairs of cards that add to 10 they are allowed to place these pairs on the table before play begins.

Player 1 selects one of the cards in his or her hand and asks the Player 2 for a card that adds to 10 with the chosen card. For instance, if Player 1's card is a 3, they may ask the Player 2 for a 7.

If Player 2 has the requested card, the first player takes it and lays it down along with the card from their hand. The first player may then ask for another card. If the Player 2 doesn't have the requested card they say: "Go fish," and the Player 1 must pick up a card from the top of the deck. (If this card adds to 10 with a card in the player's hand they may lay down the pair right away). It is then Player 2's turn to ask for a card.

Play ends when one player lays down all of their cards. Players receive 4 points for laying down all of their cards first and 1 point for each pair they have laid down.

NOTE: With weaker students, start with pairs of numbers that add to 5. Take all cards with value greater than 4 out of the deck. Each player should be dealt only 4 cards to start with.

For students who have had a great deal of trouble sorting their cards and finding pairs that add to a target number, the following exercise helps:

Give your student only three cards; two of which add to the target number. Ask the student to find the pair that add to the target number. After the student has mastered this step with 3 cards repeat the exercise with 4 cards, then 5 cards, and so on.

NOTE: You can also give your student a list of pairs that add to the target number. As the student gets used to the game, gradually remove pairs from the list so that the student learns the pairs by memory.

Mental Math
Checklist #1

Student Name	Can Add 1 to Any Number	Can Subtract 1 from Any Number	Can Add 2 to Any Number	Can Subtract 2 from Any Number	Knows All Pairs that Add to 5	Can Double 1-Digit Numbers

Mental Math
Checklist #2

Student Name	Can Add Near Doubles. EXAMPLE: 6 + 7 = 6 + 6 + 1	Can Add a 1-Digit Number to Any Multiple of 10. EXAMPLE: 30 + 6 = 36	Can Add Any 1-Digit Number to a Number Ending in 9. EXAMPLE: 29 + 7 = 30 + 6 = 36	Can Add 1-Digit Numbers by "Breaking" them Apart into Pairs that Add to 10. EXAMPLE: 7 + 5 = 7 + 3 + 2 = 10 + 2	Can Subtract Any Multiple of 10 from 100. EXAMPLE: 100 – 40 = 60

Mental Math
Checklist #3

Student Name	Can Mentally Make Change from a Dollar. SEE: Unit on Money.	Can Mentally Add Any Pair of 1-Digit Numbers.	Can Mentally Subtract Any Pair of 1-Digit Numbers.	Student Can Multiply and Count by:							
				2	3	4	5	6	7	8	9

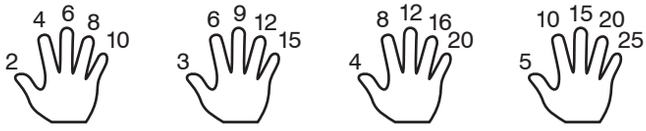
How to Learn Your Times Tables in a Week

Trying to do math without knowing your times tables is like trying to play the piano without knowing the location of the notes on the keyboard. Your students will have difficulty seeing patterns in sequences and charts, solving proportions, finding equivalent fractions, decimals and percents, solving problems etc. if they don't know their tables.

Using the method below, you can teach your students their tables in a week or so. (If you set aside five or ten minutes a day to work with students who need extra help, the pay-off will be enormous.) There is really no reason for your students not to know their tables!

Day 1: Counting by 2s, 3s, 4s and 5s

If you have completed the Fractions Unit you should already know how to count and multiply by 2s, 3s, 4s and 5s. If you don't know how to count by these numbers you should memorize the hands below:



If you know how to count by 2s, 3s, 4s and 5s, then you can multiply by any combination of these numbers. For instance, to find the product 3 × 2, count by 2s until you have raised 3 fingers.

3 × 2 = 6

Day 2: The Nine Times Table

The numbers you say when you count by 9s are called the **MULTIPLES** of 9 (zero is also a multiple of 9). The first ten multiples of 9 (after zero) are: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90. What happens when you add the digits of any of these multiples of 9 (**EXAMPLE:** 1 + 8 or 6 + 3)? The sum is always 9!

Here is another useful fact about the nine times table: Multiply 9 by any number between 1 and 10 and look at the tens digit of the product. The tens digit is always one less than the number you multiplied by:

$9 \times 4 = 36$ — *3 is one less than 4*

$9 \times 8 = 72$ — *7 is one less than 8*

$9 \times 2 = 18$ — *1 is one less than 2*

You can find the product of 9 and any number by using the two facts given above. For instance, to find 9 × 7, follow these steps:

STEP 1: $9 \times 7 =$ ___ ___

Subtract 1 from the number you are multiplying by 9: **7 – 1 = 6**

$9 \times 7 =$ 6 ___

Now you know the tens digit of the product.

Note

1. Make sure your students know how to subtract (by counting on their fingers if necessary) before you teach them the trick for the nine times table.

2. Give a test on **STEP 1** (above) before you move on.

STEP 2: $9 \times 7 = \underline{6}\ \underline{\ \ }$ $9 \times 7 = \underline{6}\ \underline{3}$

These two digits add to 9. *So the missing digit is 9 – 6 = **3***
(You can do the subtraction on your fingers if necessary).

Practise these two steps for all of the products of 9: 9×2, 9×3, 9×4, etc.

Day 3: The Eight Times Table

There are two patterns in the digits of the 8 times table. Knowing these patterns will help you remember how to count by 8s.

STEP 1: You can find the ones digit of the first five multiples of 8, by starting at 8 and counting backwards by 2s.

8
6
4
2
0

STEP 2: You can find the tens digit of the first five multiples of 8, by starting at 0 and count up by 1s.

08
16
24
32
40

(Of course you don't need to write the 0 in front of the 8 for the product 1×8.)

STEP 3: You can find the ones digit of the next five multiples of 8 by repeating step 1:

8
6
4
2
0

STEP 4: You can find the remaining tens digits by starting at 4 and count up by 1s.

48
56
64
72
80

Practise writing the multiples of 8 (up to 80) until you have memorized the complete list. Knowing the patterns in the digits of the multiples of 8 will help you memorize the list very quickly. Then you will know how to multiply by 8:

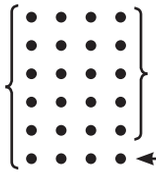
$8 \times 6 = 48$

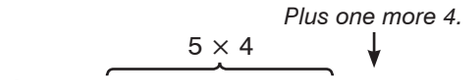
Count by eight until you have 6 fingers up: 8, 16, 24, 32, 40, 48.

Day 4: The Six Times Table

If you have learned the eight and nine times tables, then you already know 6×9 and 6×8.

And if you know how to multiply by 5 up to 5×5, then you also know how to multiply by 6 up to 6×5! That's because you can always calculate 6 times a number by calculating 5 times the number and then adding the number itself to the result. The pictures below show why this works for 6×4:





6 $\times$ 4 = 5 $\times$ 4 + 4 = 20 + 4 = 24

Similarly:

6 $\times$ 2 = **5** $\times$ 2 + 2; **6** $\times$ 3 = **5** $\times$ 3 + 3; **6** $\times$ 5 = **5** $\times$ 5 + 5.

Knowing this, you only need to memorize 2 facts:

ONE: $6 \times 6 = 36$ **TWO:** $6 \times 7 = 42$

Or, if you know 6×5, you can find 6×6 by calculating $6 \times 5 + 6$.

Day 5: The Seven Times Table

If you have learned the six, eight and nine times tables, then you already know:

6×7, 8×7 and 9×7.

And since you also already know $1 \times 7 = 7$, you only need to memorize 5 facts:

1. $2 \times 7 = 14$ **2**. $3 \times 7 = 21$ **3**. $4 \times 7 = 28$ **4**. $5 \times 7 = 35$ **5**. $7 \times 7 = 49$

If you are able to memorize your own phone number, then you can easily memorize these 5 facts!

NOTE: You can use doubling to help you learn the facts above. 4 is double 2, so 4×7 (= 28) is double 2×7 (= 14). 6 is double 3, so 6×7 (= 42) is double 3×7 (= 21).

Try this test every day until you have learned your times tables:

1. $3 \times 5 =$ _____	2. $8 \times 4 =$ _____	3. $9 \times 3 =$ _____	4. $4 \times 5 =$ _____
5. $2 \times 3 =$ _____	6. $4 \times 2 =$ _____	7. $8 \times 1 =$ _____	8. $6 \times 6 =$ _____
9. $9 \times 7 =$ _____	10. $7 \times 7 =$ _____	11. $5 \times 8 =$ _____	12. $2 \times 6 =$ _____
13. $6 \times 4 =$ _____	14. $7 \times 3 =$ _____	15. $4 \times 9 =$ _____	16. $2 \times 9 =$ _____
17. $9 \times 9 =$ _____	18. $3 \times 4 =$ _____	19. $6 \times 8 =$ _____	20. $7 \times 5 =$ _____
21. $9 \times 5 =$ _____	22. $5 \times 6 =$ _____	23. $6 \times 3 =$ _____	24. $7 \times 1 =$ _____
25. $8 \times 3 =$ _____	26. $9 \times 6 =$ _____	27. $4 \times 7 =$ _____	28. $3 \times 3 =$ _____
29. $8 \times 7 =$ _____	30. $1 \times 5 =$ _____	31. $7 \times 6 =$ _____	32. $2 \times 8 =$ _____

Contents of AP Books 8.1 and 8.2

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Unit 1: Number Sense

Unit 2: Patterns and Algebra

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Unit 4: Probability and Data Management

Unit 5: Geometry

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PART 2

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