Remote Estimation of Two-State Markov Sources: a State Estimation Entropy Perspective

BIRS Workshop

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DLR - Institute of Communications and Navigation

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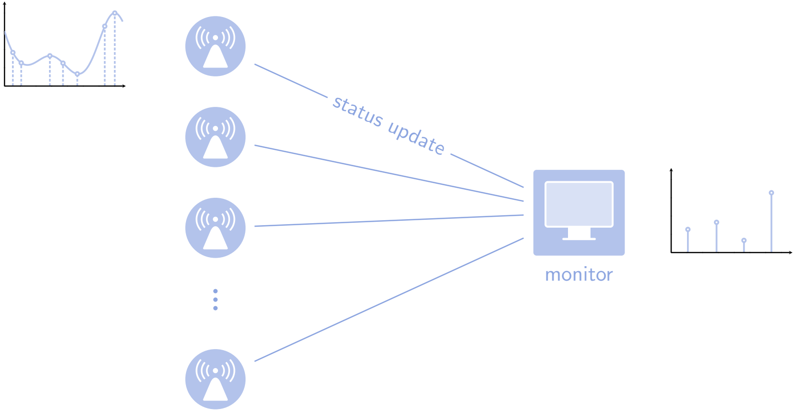
Monitoring in IoT Systems

How to maintain an accurate knowledge from remote sensors



Monitoring in IoT Systems

The challenge of maintaining an accurate knowledge from remote sensors

- monitor state of remotely-deployed sensor nodes
  - smart agriculture
  - environmental monitoring
  - asset tracking, ...
- possibly massive number of battery-powered, low-complexity devices
  - transmit only terminals
  - constraints on protocols complexity
- sporadic traffic, following non-regular patterns
  - high cost for coordination and resource assignment (grant-based)

Monitoring in IoT Systems

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- random access procedures commonly used for medium sharing
  - advanced schemes (modern random access)[1]
  - ALOHA-like access employed in most practical systems, e.g. LoRaWAN, Sigfox

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- non-trivial challenges hinder performance
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- goal is to maintain accurate knowledge of the monitored sources at the receiver
  - first step is definition of metrics that can capture this capability
  - pioneering role played by age of information (AoI)[2]

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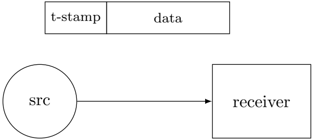
Age of Information

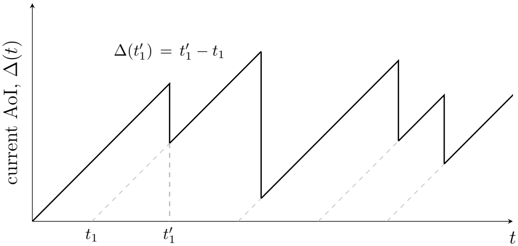
A measure of freshness of available information



current age of information

$$\delta(t) = t - \sigma(t)$$

$\sigma(t)$: time stamp of last rcvd update



New metrics to capture data significance

Complementing age of information

- AoI focuses on timeliness of delivered information
  - in some applications, knowledge at the receiver may be critical (e.g., actuation)
- other metrics proposed to try to capture this aspect
  - age of incorrect information[3]

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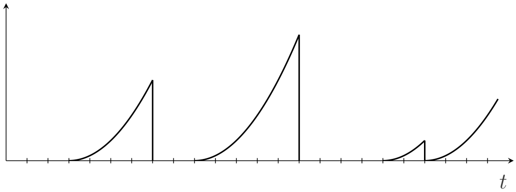

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  - information theory inspired metrics

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- mutual information[5]
  - $I(X_t; W^t) = H(X_t) - H(X_t \,|\, W^t)$
  - if close to 0, received samples W^t carry little information and deemed obsolete

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- entropy on tracked process conditioned on received samples[6,7]
  - $H(X_t \,|\, W^t)$
  - measure of uncertainty at the receiver on the current status of the source

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Remote monitoring in random access channels

Current status and open questions

DLR

- performance well-known in terms of traditional metrics (e.g., throughput, delay)
- recent results characterized behavior in terms of AoI
  - ALOHA-based systems[8,9,10,11]
  - modern random access schemes[12,13]
- first studies for age of incorrect information[14,15]
- behavior in terms of other metrics largely unexplored

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8. S. Kaul, R. Yates, "Status updates over unreliable multiaccess channels," in Proc. IEEE ISIT, 2017
9. R. Yates, S. Kaul, "Age of information in uncoordinated unslotted updating," in Proc. IEEE ISIT, 2020
10. X. Chen, K. Gatsis, H. Hassani, S. Bidokhti, "Age of information in random access channels," in IEEE Trans. Inf. Theory, 2022
11. O. Yavascan and E. Uysal, "Analysis of slotted ALOHA with an age threshold," IEEE J. Sel. Areas Commun., 2021.
12. A. Munari, "Modern random access: an age of information perspective on irregular repetition slotted ALOHA," IEEE Trans. Commun., 2021.
13. A. Munari, F. Lazaro, G. Durisi, G. Liva, "The dynamic behavior of frameless ALOHA: Drift analysis, throughput and age of information," IEEE Trans. Commun., 2023.
14. A. Nayak, A. Kalor, F. Chiariotti, P. Popovski, "A decentralized policy for minimization of age of incorrect information in slotted ALOHA systems," in Proc. IEEE ICC., 2023
15. A. Munari, "Monitoring IoT sources over random access channels: age of incorrect information and missed detection probability," in Proc. IEEE ICC, 2024.

Ideas and Contribution

State Estimation Entropy in Random Access Channels

- consider set of two-state Markov sources
- study behavior in terms of age of information and state estimation entropy

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  - random sampling and transmission strategy
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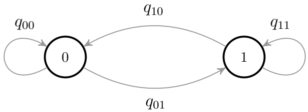
- analytical results via hidden Markov models and density evolution
- insights on impact of role played by these metrics in terms of protocol operations

Outline

System Model

Source model and channel access

- M independent sources, modeled as two-state Markov processes of alphabet $\{0, 1\}$



$$\pi_0 = \frac{q_{10}}{q_{10} + q_{01}}, \quad \pi_1 = \frac{q_{01}}{q_{10} + q_{01}}$$

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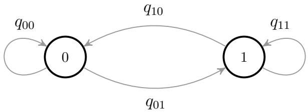

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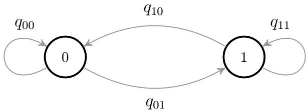

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- common channel to a receiver (slotted time)
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- slotted ALOHA access, collision channel model
  - destructive collisions, singleton slots always decoded
  - no feedback nor retransmissions

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$$Y_0 \ Y_1 \ Y_2 \ \dots \quad Y_n \in \{0,1,\texttt{I},\texttt{C},\ominus,\oplus\}$$

  - I: idle slot, C: slot with collision
  - $\ominus$, $\oplus$: observation of state (0 or 1) from another source

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  - $\mathtt{I}$: idle slot, $\mathtt{C}$: slot with collision
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- denote sequence of observations up to time n as $Y^n = [Y_0, Y_1, \ldots, Y_n]$

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  - observation $Y_n \in \{\mathtt{I}, \mathtt{C}, \ominus, \oplus\}$ carries information on the source of interest
    - $X_{n-1} = 0$, state known at receiver
    - $Y_n = \ominus \rightarrow X_n = 0$: a state change would have induced a collision

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State Estimation Entropy

- for a given sequence of observations $Y^n = y^n$, uncertainty at the receiver on current state of the source measured by the entropy

$$\mathsf{h}(y^n) = \mathsf{H}\left(X_n \,|\, Y^n = y^n\right)$$

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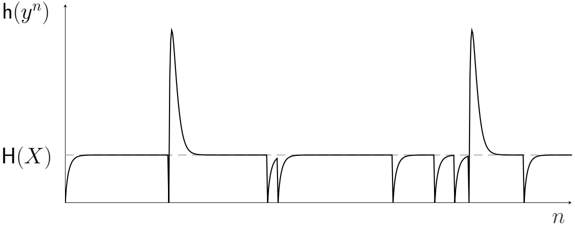

single node, random transmission policy; $q_{10} = 0.2$, $q_{01} = 0.01$, i.e., $\pi_0 \simeq 0.95$, $\pi_1 \simeq 0.05$
$\mathsf{H}(X) = -\pi_0 \log_2 \pi_0 - \pi_1 \log_2 \pi_1$

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- in particular, we are interested in the limiting behavior as n grows large, giving the average state estimation entropy

$$H_\infty = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \mathsf{H}(X_n \,|\, Y^n)$$

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$$P(x_n, y^n) = \sum_{x_{n-1}} P(x_n \mid x_{n-1}) \, P(y_n \mid x_n) \, P(x_{n-1}, y^{n-1})$$

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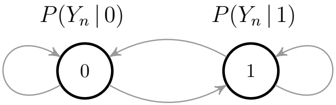

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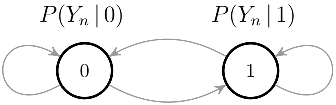

- observation probabilities can easily be computed
  - $\mathsf{P}[Y_n = \mathtt{I} \,|\, X_n = x_n] = (1-\alpha)^M$
  - $\mathsf{P}[Y_n = \ominus \,|\, X_n = x_n] = (M-1)\pi_0\, \alpha\, (1-\alpha)^{M-1}$
  - $\mathsf{P}[Y_n = 0 \,|\, X_n = 0] = \alpha(1-\alpha)^{M-1}$, $\mathsf{P}[Y_n = 0 \,|\, X_n = 1] = 0$

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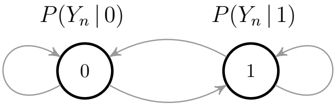

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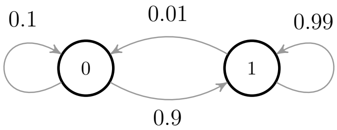

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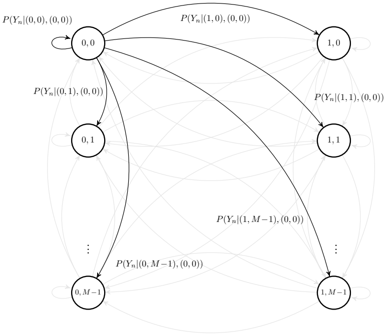

- S_n: number of sources (other than reference one) in state 0 at time n

- channel output depends on state through conditional probability function $P(Y_n \,|\, \sigma_{n-1}, \sigma_n)$

A Posteriori Probability (APP) Logarithmic Ratio

A sufficient statistics for $X_n \,|\, Y^n$

- consider a posteriori probability (APP) logarithmic ratio

$$\lambda_n := \ln \frac{\mathsf{P}[X_n = 0 \,|\, Y^n = y^n]}{\mathsf{P}[X_n = 1 \,|\, Y^n = y^n]}$$

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  - $X_n \to \Lambda_n \to Y^n$, i.e. $P(x_n \,|\, \lambda_n, y^n) = P(x_n \,|\, \lambda_n)$

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$$\ln \frac{\mathsf{P}[X_n = 0 \,|\, Y^n = y^n]}{\mathsf{P}[X_n = 1 \,|\, Y^n = y^n]} = \ln \frac{\mathsf{P}[X_n = 0 \,|\, \Lambda_n = \lambda_n]}{\mathsf{P}[X_n = 1 \,|\, \Lambda_n = \lambda_n]}$$

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- recalling that $\mathsf{P}[X_n = 0 \,|\, \Lambda_n = \lambda_n] + \mathsf{P}[X_n = 1 \,|\, \Lambda_n = \lambda_n] = 1$, we get

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- λ_n suffices to compute uncertainty at receiver at time n
  - recursive calculation of λ_n as function of λ_{n-1} and observation y_n

Outline

Recursive calculation of λ_n

Random transmission policy

- from forward recursion on HMM we have

$$P(x_n, y^n) = \sum_{x_{n-1} \in \{0,1\}} P(y_n \mid x_n) P(x_n \mid x_{n-1}) P(x_{n-1}, y^{n-1})$$

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- leaning on this, we obtain

$$\lambda_n = \ln \frac{\mathsf{P}[X_n = 0, Y^n = y^n]}{\mathsf{P}[X_n = 1, Y^n = y^n]} = \ln \frac{P(y_n \,|\, 0)}{P(y_n \,|\, 1)} + \ln \frac{q_{00} + q_{10} e^{-\lambda_{n-1}}}{q_{01} + q_{11} e^{-\lambda_{n-1}}} := f(y_n, \lambda_{n-1})$$

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- all quantities are known
  - q_{00}, q_{01}, q_{10}, q_{11} are source transition probabilities
  - observation probabilities $P(y_n \,|\, x_n)$ derived earlier

Recursive calculation of λ_n

Reactive transmission policy

- consider now reactive transmission strategy

Recursive calculation of λ_n

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- following the same approach, we obtain

$$P(\sigma_n, y^n) = \sum_{\sigma_{n-1}} P(y_n \,|\, \sigma_{n-1}, \sigma_n) P(\sigma_n \,|\, \sigma_{n-1}) P(\sigma_{n-1}, y^{n-1})$$

$$\lambda_n = \ln \frac{\sum\limits_{\sigma_n \in \{0\} \times \mathcal{S}} P(\sigma_n, y^n)}{\sum\limits_{\sigma_n \in \{1\} \times \mathcal{S}} P(\sigma_n, y^n)}$$

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- $P(y_n \,|\, \sigma_{n-1}, \sigma_n)$, $P(\sigma_{n-1}, \sigma_n)$ obtained from statistics of X_n and access strategy

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- $P(y_n \mid \sigma_{n-1}, \sigma_n)$, $P(\sigma_{n-1}, \sigma_n)$ obtained from statistics of X_n and access strategy
- however, complexity of calculation grows with M^2: impractical for larger networks

Recursive calculation of λ_n, reactive case

Myopic surrogate model

- to simplify calculation of λ_n, introduce myopic surrogate model

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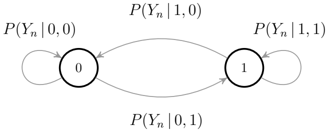
- under this hypothesis, observation no longer depends on S_n



Recursive calculation of λ_n, reactive case

Myopic surrogate model

- for the myopic HMM, the recursion simplifies to

$$P(x_n, y^n) = \sum_{x_{n-1} \in \{0,1\}} P(y_n \,|\, x_{n-1}, x_n) P(x_n \,|\, x_{n-1}) P(x_{n-1}, y^{n-1})$$

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$$:= g(y_n, \lambda_{n-1})$$

- complexity independent of the number of sources
- HMM gives in general an approximation, yet exact for symmetric sources $(q_{10} = q_{01})$

Outline

State Estimation Entropy Derivation

Statistics of the APP logarithmic ratio

- to compute state estimation entropy, we are interested in

$$\mathsf{E}[H_n] = \mathsf{H}(X_n \,|\, Y^n) = \mathsf{H}(X_n \,|\, \Lambda_n) = \mathsf{E}[\mathsf{h}(\Lambda_n)]$$

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- quantized density evolution, i.e. quantize values taken by λ_n

$$\mathsf{H}(X_n \,|\, Y^n) = \sum_{\lambda_n} P(\lambda_n)\, \mathsf{h}(\lambda_n)$$

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Quantized Density Evolution

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- distribution of Λ_n computed recursively, given distribution of Λ_{n-1} and conditional distribution of $Y_n \,|\, X_n$

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$$P(\lambda_n, x_n) = \sum_{x_{n-1}\in\{0,1\}} \sum_{\substack{y_n,\lambda_{n-1}: \\ f(y_n,\lambda_{n-1})=\lambda_n}} P(\lambda_{n-1}, y_n, x_n, x_{n-1})$$

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- similar approach for reactive case, relying on myopic surrogate model

Outline

Average AoI Performance

Random and reactive strategies

- for slotted ALOHA access, assuming i.i.d. behavior across slots, average AoI inversely proportional to throughput T

$$\Delta = \frac{1}{2} + \frac{M}{\mathsf{T}}$$

- minimal AoI attained for transmission probability $1/M$

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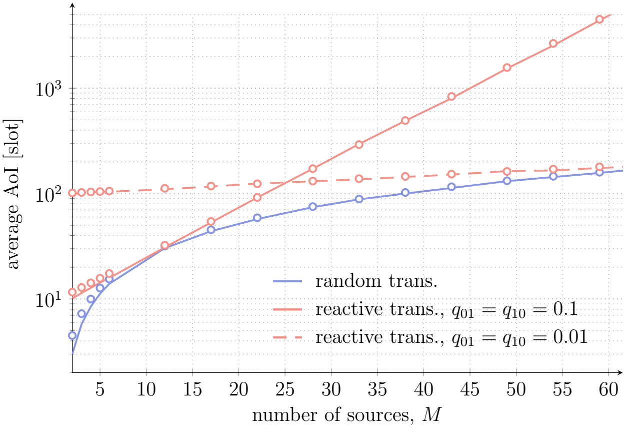
- reactive transmission strategy: transmission probability driven by source statistics

$$\tilde{\alpha} = \pi_0 q_{01} + \pi_1 q_{10}$$

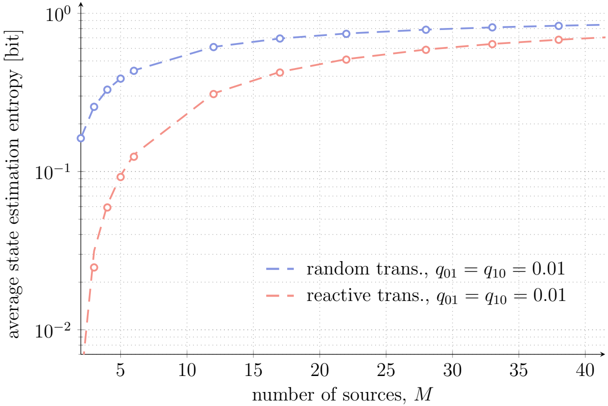
Average AoI Performance

Symmetric sources



Average State Estimation Entropy

Symmetric sources, random and reactive transmission policies



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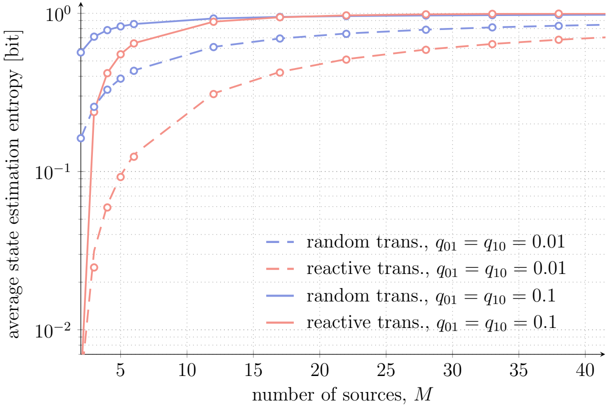


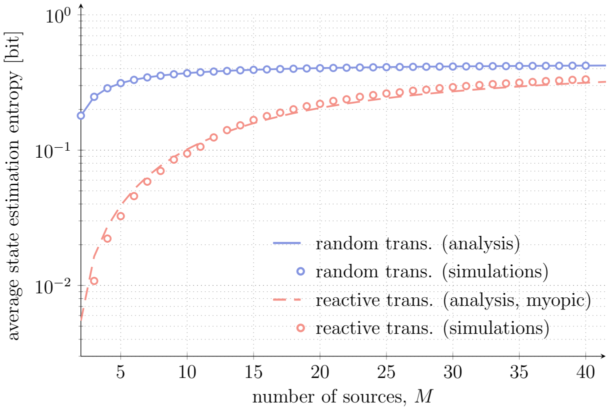
Average State Estimation Entropy

Symmetric sources, random and reactive transmission policies

- SEE increases in larger networks
  - more congestion, higher loss probability
  - H_∞ tends to source entropy as M grows
- lower SEE when sources transition less often ($q_{10} = q_{01} = 0.01$)
  - fewer updates needed to track evolution
  - lower channel congestion (reactive strategy)
- reactive strategy offers better performance
  - only relevant updates are sent
  - for $M = 2$, perfect knowledge at the receiver is achieved

Average State Estimation Entropy

Asymmetric sources, $q_{10} = 0.1$, $q_{01} = 0.01$



Concluding Remarks

- remote source monitoring in IoT systems
  - two-state Markov sources
  - slotted ALOHA-based channel access, random and reactive strategies
- analytical characterization of state estimation entropy
  - capture uncertainty at receiver
- different protocol operation insights when considering AoI and SEE
  - when freshness is of relevance, transmission strategy maximizing throughput is optimal
  - when uncertainty on source state is important, reactive strategy is convenient

Concluding Remarks

- metric choice based on targeted application
  - freshness relevant e.g., in tracking, monitoring (AoI valuable proxy for complex systems)
  - uncertainty on source state relevant for, e.g., actuation
- results also for error probability, false alarm and missed detection probability[16]
- open research topic
  - identify more advanced, optimal policies
  - tracking of more practical, possibly correlated, source processes

16. A. Munari, G. Cocco, G. Liva, "Remote Monitoring of Markov Sources over Random Access Channels: False Alarm and Detection Probability," in Proc. IEEE Asilomar 2023.

Additional Insights

State Estimation at the Receiver

Estimation Error Probability

- receiver aims at estimating the state of the reference source
  - denote by $\widehat{X}_n$ the estimate at time n of the state X_n

- evaluate performance in terms of state estimation error probability

$$P_e^{(n)} = \mathsf{P}\left[\widehat{X}_n \neq X_n\right]$$

- in particular, interested in average error probability

$$P_e = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} P_e^{(n)}$$

State Estimation at the Receiver

MAP and Decode&Hold

- P_e minimized via maximum a posteriori (MAP) estimator
  - APP logarithmic ratio combined with threshold test
  - $\hat{x}_n = 0$ if $\lambda_n > 0$, $\hat{x}_n = 1$ if $\lambda_n < 0$

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- MAP estimator entails computational complexity that may be critical, e.g. battery-powered or computationally-limited receivers
  - introduce simpler decode and hold (D&H) estimator

Decode and Hold Estimator

Definition and Trade-Offs

- decode and hold: update estimate only upon receiving update from source, otherwise keep last received value

$$\hat{X}_n = \begin{cases} Y_n & \text{if } Y_n \in \{0,1\} \\ \hat{X}_{n-1} & \text{if } Y_n \in \{\mathtt{I}, \mathtt{C}, \ominus, \oplus\}\,. \end{cases}$$

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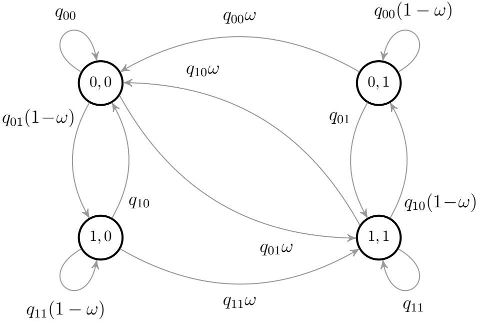
- simpler implementation, yet suboptimal performance
  - consider two-source network, reactive strategy
  - D&H in error as soon as a collision occurs
  - MAP estimator experiences no error: perfect knowledge out of collisions and idle slots

Decode and Hold Estimator

Markov Model, random transmission strategy

- error probability obtained by tracking Markov process $(X_n, \widehat{X}_n)$
- for random strategy, defining $\omega = \alpha(1-\alpha)^{M-1}$



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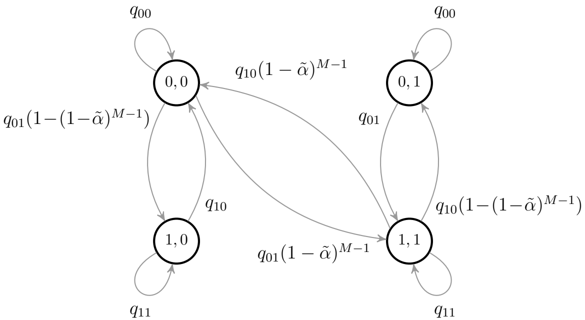
- computing the stationary distribution, we get

$$P_e = \frac{2q_{01}q_{10}\,(1-\omega)}{(q_{01}+q_{10})\,[\omega + (1-\omega)(q_{01}+q_{10})]}$$

Decode and Hold Estimator

Markov Model, reactive transmission strategy

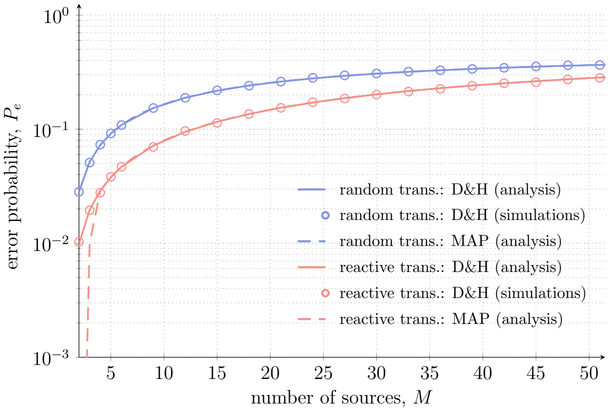
- similar approach holds for reactive strategy (myopic approximation)
  - average activation probability $\tilde{\alpha} = \pi_0\, q_{01} + \pi_1\, q_{10}$



$$P_e \approx \frac{1-(1-\tilde{\alpha})^{M-1}}{2-(1-\tilde{\alpha})^{M-1}}$$

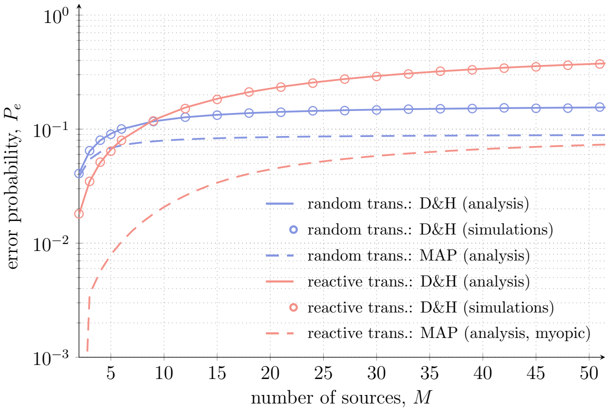
State Estimation Error Probability

MAP and D&H, symmetric sources ($q_{10} = q_{01} = 0.01$)



State Estimation Error Probability

MAP and D&H, asymmetric sources ($q_{10} = 0.01$, $q_{01} = 0.1$)



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  - difference only for low M
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MAP and D&H

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  - when more sources present, collisions become less informative
  - reactive strategy leads to better estimate

- for asymmetric sources, D&H performs significantly worse
  - D&H may need long time to recover from an error
  - reactive strategy performs worse than random one
    - random transmissions can lead to quicker recovery from erroneous estimate