Spectral Efficiency of Low Earth Orbit Satellite Constellations

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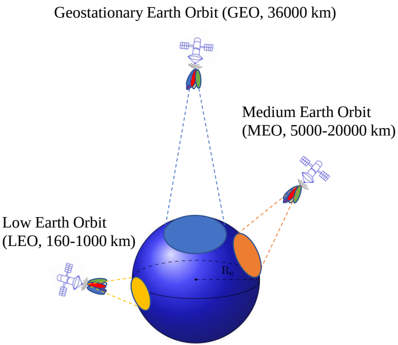
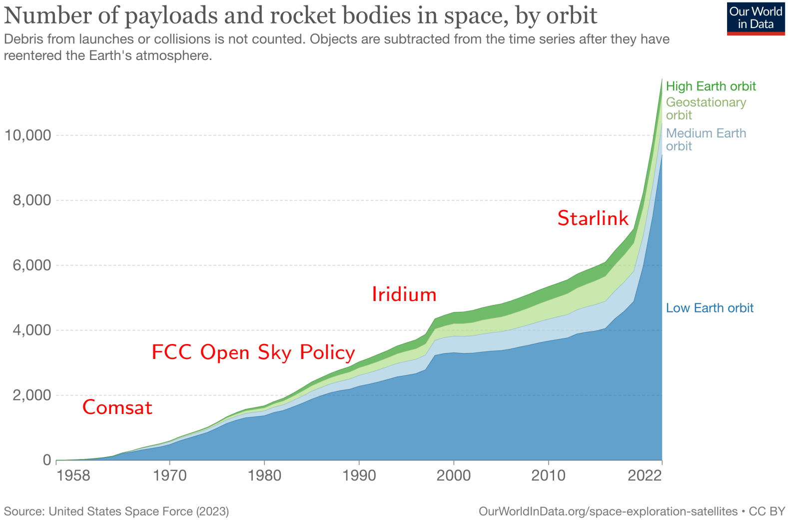
Satellites



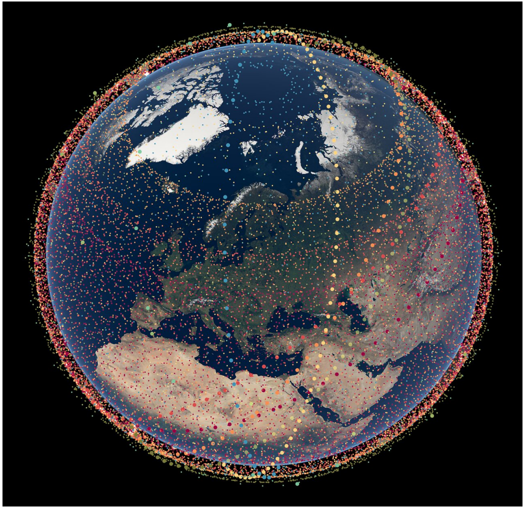
LEO and MEO can offer lower latency and higher throughput than GEO satellites.


Number of payloads and rocket bodies in space, by orbit
Debris from launches or collisions is not counted. Objects are subtracted from the time series after they have reentered the Earth's atmosphere.
Our World in Data
High Earth orbit
Geostationary orbit
Medium Earth orbit
Low Earth orbit
Starlink
Iridium
FCC Open Sky Policy
Comsat
0
2,000
4,000
6,000
8,000
10,000
1958
1970
1980
1990
2000
2010
2022
Source: United States Space Force (2023)
OurWorldInData.org/space-exploration-satellites • CC BY

Mega-constellations

- Starlink focuses on consumer broadband services. About 5000 operational satellites in orbit to date.
- Kuiper's focuses are similar. The initial design has 3,200+ satellites.
- Others include OneWeb, Telesat Lightspeed, etc.
- Required by the International Telecommunication Union (ITU) radio regulations and the FCC to coordinate to prevent harmful mutual interference. Mostly in the 10-12 GHz band.
- Fundamental questions:
  - What is the spectral efficiency? capacity?
  - Will interference be an issue?
  - How to best allocate satellite spectrum?

Starlink 2nd Generation plan



ESO/Jos Francisco (josefrancisco.org)

Propagation model

- TX and RX typically have a large number of antennas (up to 1000s).
- Line of sight.
- Need beamforming to close links.
- Each TX-RX channel matrix has rank-1:

$$H \propto \begin{bmatrix} e^{j\theta} \\ e^{j2\theta} \\ \vdots \\ \vdots \\ e^{jt\theta} \end{bmatrix} \begin{bmatrix} e^{j\phi} & e^{j2\phi} & \dots & e^{jr\phi} \end{bmatrix} = \mathbf{e}(\theta)\, \mathbf{e}^T(\phi)$$

Massive vector interference channel

n satellites, n ground terminals

$$
\begin{aligned}
Y_1 &= H_{1,1}X_1 + H_{1,2}X_2 + \cdots + H_{1,n}X_n \\
Y_2 &= H_{2,1}X_1 + H_{2,2}X_2 + \cdots + H_{2,n}X_n \\
&\vdots \\
Y_n &= H_{n,1}X_1 + H_{n,2}X_2 + \cdots + H_{n,n}X_n
\end{aligned}
$$

where

$$H_{k,l} \propto \left(h^2 + \text{ground\_distance}_{k,l}\right)^{-\frac{\alpha}{2}} \mathbf{e}(\theta)\, \mathbf{e}^T(\phi).$$

Potential variables to optimize:

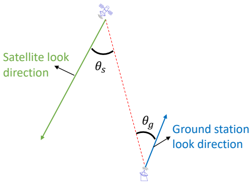
- n
- positions (which determines distances)
- look directions
- signaling (TDM/FDM/etc.)

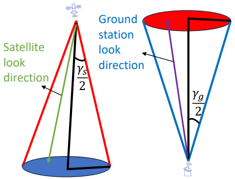
Assumptions

- B: available frequency resources.
- Satellite i's power spectral density constraints:

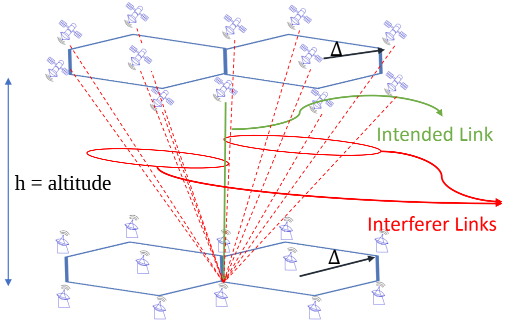
$$\int_B P_i(f)\,df \le P_{\max} \text{ and } P_i(f) \le \mathrm{PSD}_{\max}(f),\ \forall f \in B.$$

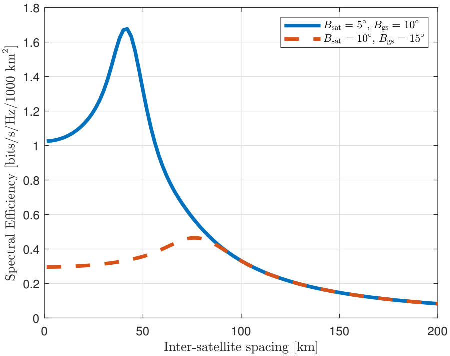
- All transmitters use Gaussian codebooks and spot beamforming;
- All receivers treat all interference as independent additive white Gaussian noise.
- $w_s(\cdot)$: satellite antenna/beam pattern; $w_g(\cdot)$: ground station antenna/beam pattern.
- The link gain is proportional to $d^{-\alpha} w_s(\theta_s) w_g(\theta_g)$.
- Look directions are upper bounded by γ_s, γ_g.





An example with a regular configuration





Altitude $h = 550$ km, the serving link SNR is 8 dB.

Problem and simplification

- If one is not limited by the number of satellites and ground stations, what is the highest spectral efficiency?
- If R bits/s/Hz/km^2 is achievable, there exists a fixed configuration that achieves R. Without loss of generality, assume a snapshot with no motions.
- It suffices to consider a single narrow subband.
- We consider a one-to-one satellite and ground station association. (Is this optimal?)

Single narrow subband

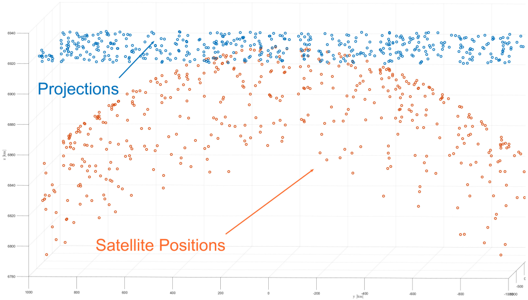
- Link i:

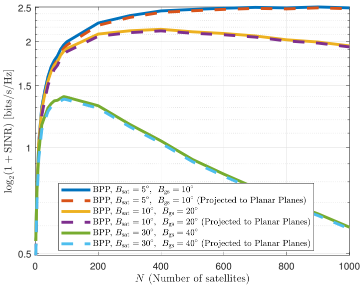
$$\mathrm{SINR}_i = \frac{p_i d_{i,i}^{-\alpha} w_{i,i}}{\sum_{j \neq i} p_j d_{j,i}^{-\alpha} w_{j,i} + \sigma^2}.$$

- The spectral efficiency:

$$R = \frac{1}{4\pi r_e^2} \sum_{i=1}^{n} \log_2(1 + \mathrm{SINR}_i).$$

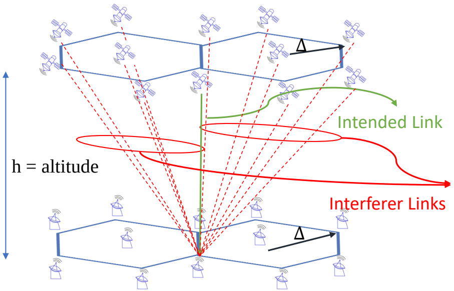
“Flat-Earth” approximation





- Suppose the field-of-view of the each ground station is flat.
- Suppose all ground stations are on one plane.
- Association minimizes the total distance.
- Associated satellite and ground station look toward each other.

Regular configuration

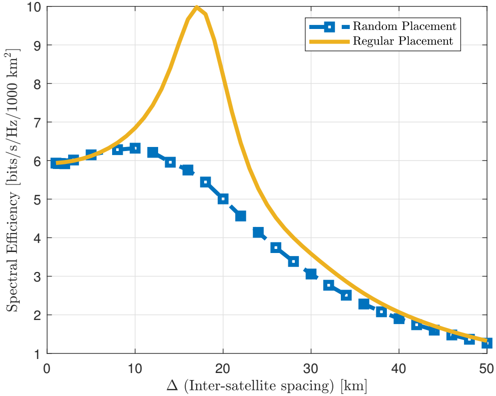


Assumption: The serving satellite is directly above.

Proposition

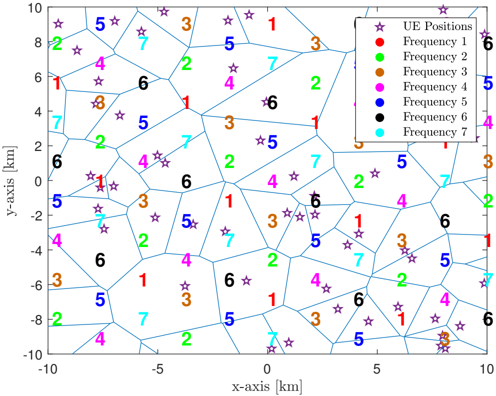
For the regular configuration, to maximize the spectral efficiency, p_i must be equal to psd_{max} for any i.

Random vs regular configuration



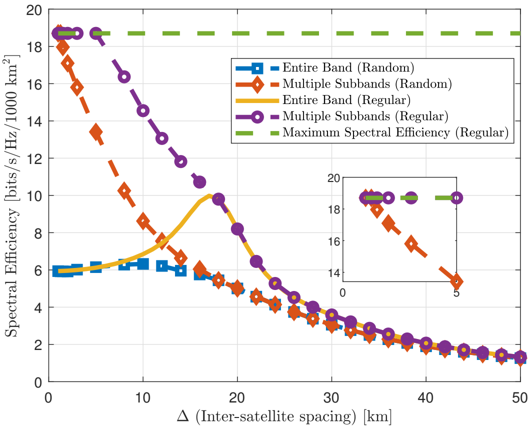
- Single-channel.
- Serving-link SNR is 8 dB, satellite beamwidth $= 2^\circ$, ground station beamwidth $= 5^\circ$.

Subband allocation



Consider hexagonal frequency re-use pattern.
Given a constellation, assign each satellite frequency bands included in its Voronoi region.

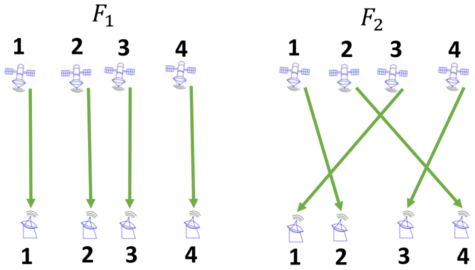
Performance with subband allocation



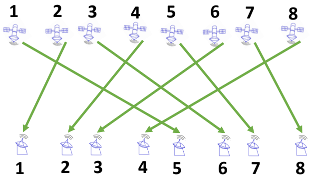
- $\mathrm{psd}_{\max} h^{-\alpha}/(|B|\sigma^2) = 18$ dB,
- $P_{\max} = \mathrm{psd}_{\max}/10$,
- 300 subbands,
- Satellite beam-width $= 2°$,
- Ground-station beam-width $= 5°$.

Look direction optimization

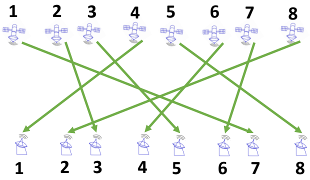
- Arranging the associations (look directions) slightly may reduce interference significantly.
- A small change of look direction does not significantly change the direct link gain.



Shuffling



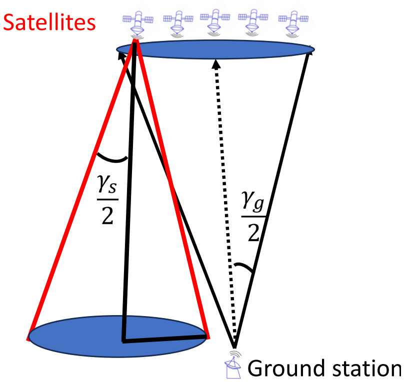
One round of shuffling;



Two rounds of shuffling.

Adding a shuffling round increases the minimum distance from the serving satellites to neighboring ground stations.

Upper bound for the regular configuration

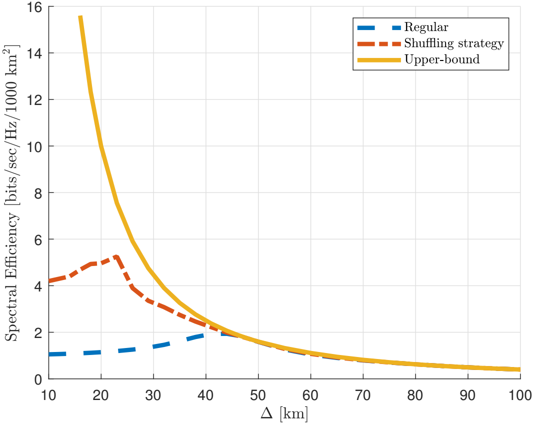


If X satellites are in the beam-region of a ground terminal, the spectral efficiency is upper bounded by

$$\frac{2}{\Delta^2\sqrt{3}}\log_2\left(1+\frac{\mathrm{psd}_{\max}h^{-\alpha}}{X\,I_{\min}+\sigma^2}\right),$$

where

$$I_{\min}\triangleq \mathrm{psd}_{\max}\left(h^2+h^2\tan^2(\gamma_g/2)\right)^{-\frac{\alpha}{2}}w_s(\gamma_s)w_g(\gamma_g).$$



- Single-channel.
- Serving link gain is 10 dB.
- The satellite and the ground station beamwidths are chosen as $(5^\circ, 10^\circ)$.
- We set beam-region widths as $\gamma_s = \gamma_g = 40^\circ$.
- For any given Δ, optimal shuffling parameters are numerically found.

Conclusion

- The current density of satellites is well below the interference-limited regime.
- Subband allocation and judicious association can lead to a substantial improvement in spectral efficiency.
- Open question: Is regular configuration near optimal?
- Open question: What is the capacity? the optimal satellite density?
- Most results presented here have been reported in Ozturk, Guo, Berry & Honig, "Spectral Efficiency of Low Earth Orbit Satellite Constellations." Under review. Can share upon email request.
- Other related work from my group and collaborators:

  - Hazlett, Guo & Honig, "From 'openskies' to traffic jams in 12 GHz: A short history of satellite radio spectrum," Journal of Law & Innovation, vol. 6, no. 1, pp. 6694, 2023.

  - Berry, Bustamante, Guo, Hazlett, Honig, Lohmeyer, Murtazashvili, Palo & Weiss, "Spectrum rights in outer space: Interference management for low Earth orbit (LEO) broadband constellations," Journal of Information Policy, under revision. Available at SSRN 4178793, 2024.

  - Ozturk, Berry, Guo, Honig & Lind, "Reducing Satellite Interference to Radio Telescopes Using Beacons." IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN), 2024. Also available at arXiv:2312.12692.

  - Mollakhani & Guo, "Fault-Tolerant Spectrum Usage Consensus for Low-Earth-Orbit Satellite Constellations." Under review. An earlier versionis available as arXiv preprint arXiv:2312.05213.